Polarons in cold atomic gases: a brief introduction

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Quantum Few- to Many-body Physics in Ultracold Atoms
WIPM

Outline

- What is polaron?
    - Condensed-matter background
    - Polarons in cold atoms: general picture
    - Early experiments
- How to characterize polarons in cold atomic gases
    - Chevy's ansatz
    - T-matrix formalism
- An example: polaron in alkaline-earth(-like) atoms
- Outlook

What is polaron?

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- Quasiparticle excitation describing a single impurity moving in its environment (in a general sense)

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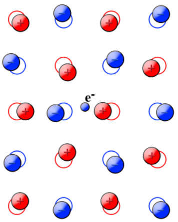
- Quasiparticle excitation describing a single impurity moving in its environment (in a general sense)
- Landau (localized electrons) and Pekar (phonon clouds) (1933-1951)
- The Fröhlich polaron (1950s)



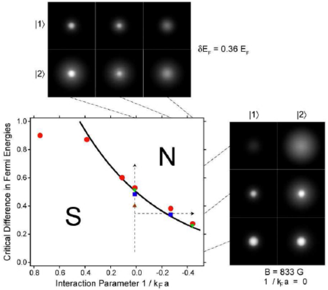
- Electron-phonon interaction in solids
- Phonons not affected by a single electron
- Brillouin-Wigner perturbation theory

$$E_p = E_0 + \frac{p^2}{2m^*} + O(p^4)$$

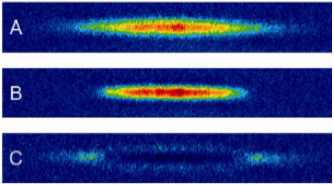
- Large (Fröhlich) vs. small polarons (Landau)

In cold atoms

- Pairing in polarized Fermi gases



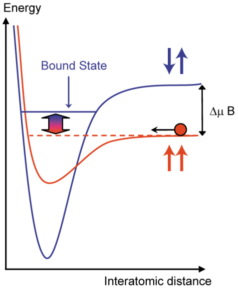
M. W. Zwierlein et al. Science 311, 492 (2006)

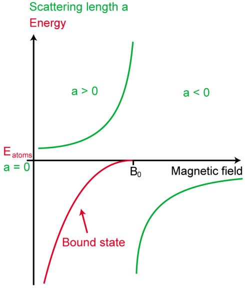


G. B. Partridge et al. Science 311, 503 (2006)

- Large-polarization limit: polaron physics
- Interaction tunable through Feshbach resonance

Feshbach resonance (s-wave interaction)

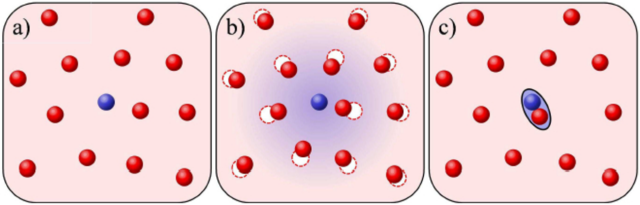




- Multi-channel resonant scattering
- The scattering length diverges as the threshold of an 'open' channel coincides with a bound state energy of a 'closed' channel
- Strong- ($a \to 0^+$) and weak-interaction ($a \to 0^-$) limits

Fate of impurity: limiting cases and those inbetween

- Weak-interaction limit: free impurity
- Strong-interaction limit: impurity-atom dimer

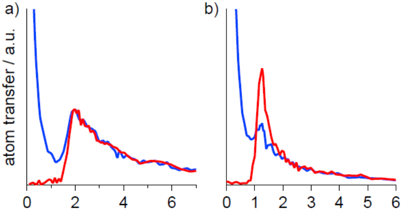


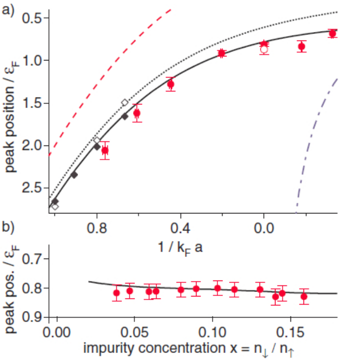
From Phys. Rev. Lett. 102, 230402 (2009)

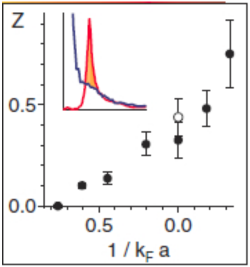
- Inbetween: polaronic branch and dimeronic branch
- Transition occurs between polaronic quasiparticles and many-body bound states
- Predicted by diagrammatic QMC calculations

N. Prokof'ev and B. Svistunov, Phys. Rev. B 77, 020408(R) (2008)

Early experiments on polaron-molecule transition I



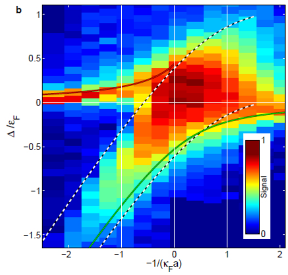




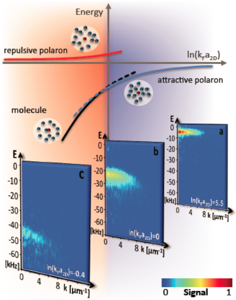
- rf-spectroscopy measurement
- Weakly interacting quasiparticle
- Polaron-molecule transition
- Consistent with QMC

A. Schirotzek, C.-H. Wu, A. Sommer, M. W. Zwierlein, Phys. Rev. Lett. 102, 230402 (2009)

Early experiments on polaron-molecule transition II



C. Kohstall, *et al.*, Nature 485, 615 (2012)

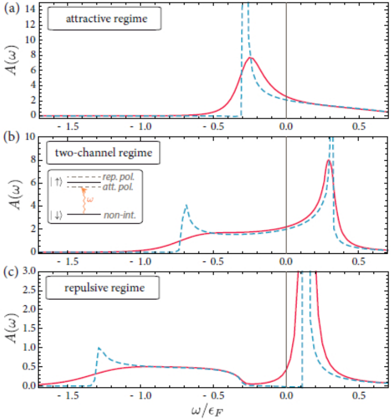


M. Koschorreck, *et al.*, Nature 485, 619 (2012)

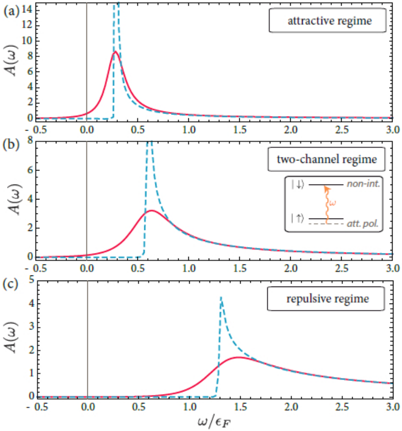
- Attractive and repulsive polarons in 3D and 2D
- Repulsive polaron: a metastable quasiparticle
- Molecule-hole continuum
- Consistent with calculations based on perturbation theory

Detection schemes

- Reverse rf spectroscopy



- Standard rf spectroscopy



R. Schimdt, M. Knap, D. A. Ivanov, J.-S. You, M. Cetina, E. Demler, Rep. Prog. Phys. 81, 024401 (2018)

How to characterize polarons theoretically?

- Variational approach: Chevy's ansatz
  F. Chevy, Phys. Rev. A 74, 063628 (2006)
- T-matrix approach
  R. Combescot, A. Recati, C. Lobo, F. Chevy, Phys. Rev. Lett. 98, 180402 (2007)
  M. Punk, W. Zwerger Phys. Rev. Lett. 99, 170404 (2007)
- $1/N$ expansion
  M. Veillette *e al.*, Phys. Rev. A 78, 033614 (2008)
- Fixed-node QMC
  C. Lobo, A. Recati, S. Giorgini, S. Stringari, Phys. Rev. Lett. 97, 200403 (2006)
- Diagrammatic QMC
  N. V. Prokof'ev, B. V. Svistunov, Phys. Rev. B 77, 020408(R) (2008)
- Other
  R. Schimdt, *et al.*, Rep. Prog. Phys. 81, 024401 (2018)

Chevy's ansatz (polarons in a polarized two-component Fermi gas)

- Polaron ansatz wave function

$$|\Psi\rangle_{\mathbf{Q}} = \left(\psi_{\mathbf{Q}} b^{\dagger}_{\mathbf{Q}} + \sum_{q<k_F, k>k_F} \psi_{\mathbf{kq}} b^{\dagger}_{\mathbf{Q}-\mathbf{k}+\mathbf{q}} a^{\dagger}_{\mathbf{k}} a_{\mathbf{q}}\right) |\mathrm{FS}\rangle_N$$

- Hamiltonian

$$H = \sum_{\mathbf{k}} \epsilon_k b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} + \sum_{\mathbf{k}} \epsilon_k a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + U \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^{\dagger}_{\mathbf{k}} b^{\dagger}_{\mathbf{k}'} b_{\mathbf{k}'+\mathbf{q}} a_{\mathbf{k}'-\mathbf{q}}$$

- Minimize $\langle\Psi|H|\Psi\rangle_{\mathbf{Q}}$ or $H|\Psi\rangle_{\mathbf{Q}} = E_P|\Psi\rangle_{\mathbf{Q}}$ (truncation)
- More particle-hole excitations possible
- Renormalization of bare interaction U with $U_0 = 4\pi\hbar^2 a_s/m$

$$\frac{1}{U} = \frac{1}{U_0} - \sum_{\mathbf{k}} \frac{1}{2\epsilon_k}$$

Equations for the coefficients (assuming $Q = 0$)

$$U\psi_0(\sum_{q<k_F}) + U\sum_{q<k_F, k>k_F}\psi_{\mathbf{kq}} = E_P\psi_0$$

$$(\epsilon_k - \epsilon_q + \epsilon_{\mathbf{q}-\mathbf{k}})\psi_{\mathbf{kq}} + U\psi_0 + U\sum_{k>k_F}\psi_{\mathbf{kq}} = E_P\psi_{\mathbf{kq}}$$

Therefore we have

$$\left(\frac{1}{U} - \sum_{\mathbf{k}}\frac{1}{A_{\mathbf{kq}}}\right)A_{\mathbf{q}} - \frac{1}{E_P}\sum_{\mathbf{q}}A_{\mathbf{q}} = 0, \text{ with } \begin{cases} A_{\mathbf{kq}} = E_P - (\epsilon_k - \epsilon_q + \epsilon_{\mathbf{q}-\mathbf{k}}) \\ A_{\mathbf{q}} = U\psi_0 + U\sum_{\mathbf{k}}\psi_{\mathbf{kq}} \end{cases}$$

Closed equation

$$E_P = \sum_{q<k_F}\frac{1}{\frac{1}{U} - \sum_{k>k_F}\frac{1}{E_P - \epsilon_k + \epsilon_q - \epsilon_{\mathbf{q}-\mathbf{k}}}}$$

Molecular wave function

$$|M\rangle_{\mathbf{Q}} = \sum_{k>k_F} \phi_{\mathbf{k}} b^{\dagger}_{\mathbf{Q}-\mathbf{k}} a^{\dagger}_{\mathbf{k}} |\mathrm{FS}\rangle_{N-1} + \sum_{k,k'>k_F} \phi_{\mathbf{k}\mathbf{k}'\mathbf{q}} b^{\dagger}_{\mathbf{Q}-\mathbf{k}-\mathbf{k}'+\mathbf{q}} a^{\dagger}_{\mathbf{k}} a^{\dagger}_{\mathbf{k}'} a_{\mathbf{q}} |\mathrm{FS}\rangle_{N-1}$$

Equations for the coefficients (assuming $Q = 0$)

$$A_{\mathbf{k}}\phi_{\mathbf{k}} = U\phi_{\mathbf{k}}(\sum_{q<k_F}) + U\sum_{k'>k_F}\phi_{\mathbf{k}'} + U\sum_{k'>k_F,q<k_F}\phi_{\mathbf{k}\mathbf{k}'\mathbf{q}} - U\sum_{k'>k_F,q<k_F}\phi_{\mathbf{k}'\mathbf{k}\mathbf{q}}$$

$$A_{\mathbf{k}\mathbf{k}'\mathbf{q}}\phi_{\mathbf{k}\mathbf{k}'\mathbf{q}} = U\sum_{k''>k_F}\phi_{\mathbf{k}''\mathbf{k}'\mathbf{q}} + U\sum_{k''>k_F}\phi_{\mathbf{k}\mathbf{k}''\mathbf{q}} + U\phi_{\mathbf{k}}$$

With

$$A_{\mathbf{k}} = E_M - 2\epsilon_k$$
$$A_{\mathbf{k}\mathbf{k}'\mathbf{q}} = E_M - \epsilon_{\mathbf{q}-\mathbf{k}-\mathbf{k}'} - \epsilon_k - \epsilon_{k'}$$

Note: i) $\phi_{\mathbf{k}\mathbf{k}'\mathbf{q}} = -\phi_{\mathbf{k}'\mathbf{k}\mathbf{q}}$; ii) E_M relative to the energy of $|\mathrm{FS}\rangle_{N-1}$.

Defining

$$G_{\mathbf{kq}} = \phi_{\mathbf{k}} + 2\sum_{\mathbf{k}'} \phi_{\mathbf{k}\mathbf{k}'\mathbf{q}},$$

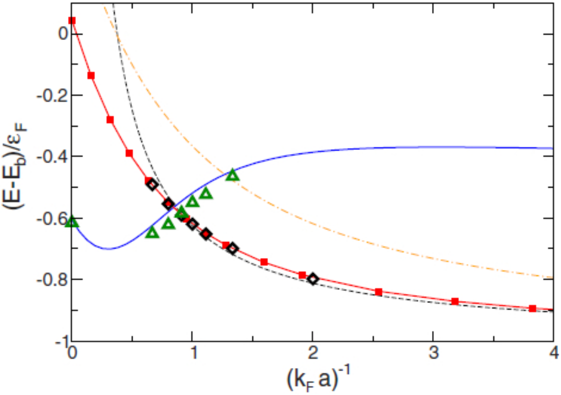
we have

$$\left(\frac{1}{U} - \sum_{k'>k_F} \frac{1}{A_{\mathbf{k}\mathbf{k}'\mathbf{q}}}\right) G_{\mathbf{kq}} + \sum_{k''>k_F} \frac{G_{\mathbf{k}''\mathbf{q}}}{A_{\mathbf{k}\mathbf{k}''\mathbf{q}}}$$
$$- \sum_{q'<k_F} \frac{G_{\mathbf{k}\mathbf{q}'}}{A_{\mathbf{k}}} - \sum_{k''>k_F} \left(\frac{1}{U} - \sum_{k'''>k_F} \frac{1}{A_{\mathbf{k}'''}}\right) \frac{\sum_{q'<k_F} G_{\mathbf{k}''\mathbf{q}'}}{A_{\mathbf{k}} A_{\mathbf{k}''}} = 0.$$

Simpler case: the bare molecular state

$$\frac{1}{U} = \sum_{k>k_F} \frac{1}{A_{\mathbf{k}}}.$$

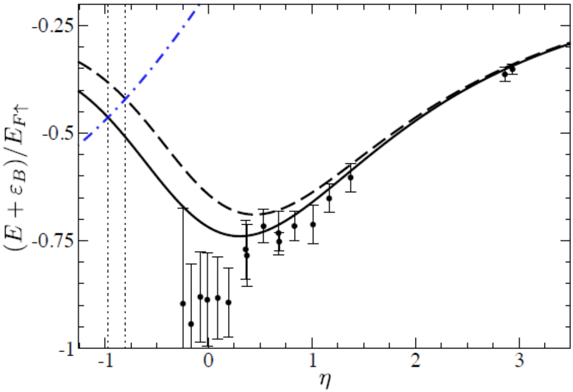
Comparison of results (3D)



- Compare $E_M - E_F$ with E_P ($E_b = -\hbar^2/ma_s^2$ subtracted)
- Crossing at $(k_F a_s)_c^{-1} = 0.84$ vs 0.90 (diagrammatic QMC)

M. Punk, P. T. Dumitrescu, W. Zwerger, Phys. Rev. A 80, 053605 (2009)

Comparison of results (2D)



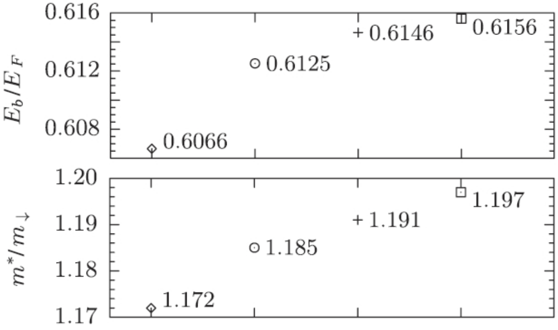
- Polaron-molecule transition exists in 2D
- Polaron with single (two) particle-hole pair(s)

M. M. Parish, Phys. Rev. A 83, 051603(R) (2011)

M. M. Parish, J. Levinsen, Phys. Rev. A 87, 033616 (2013)

M. Koschorreck *et al.*, Nature 485, 619 (2012)

Convergence of results on the single particle-hole level

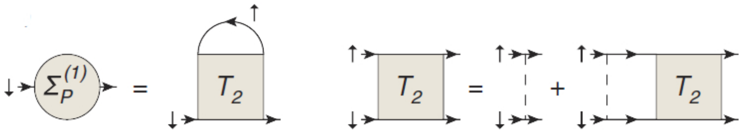


- Converge quickly at the lowest orders of particle-hole excitations
- Destructive interference between higher-order terms

R. Combescot, S. Giraud, Phys. Rev. Lett. 101, 050404 (2008)

T-matrix approach (polaron single-hole level)

$$G_{\downarrow}^{-1}(\mathbf{p},\omega) = \omega + i0^{+} - \epsilon_{p\downarrow} - \Sigma(\mathbf{p},\omega)$$
$$\Sigma(\mathbf{p},\omega) = \Sigma^{(1)}(\mathbf{p},\omega) + \Sigma^{(2)}(\mathbf{p},\omega) + ...$$
$$E_P = \mathrm{Re}\left[\Sigma(\mathbf{p}=0, E_P)\right]$$

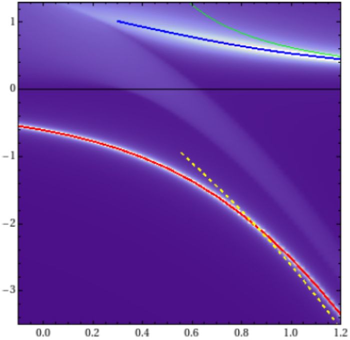


- Equivalent to the variational approach
- Better extendability, e.g., capable of analyzing losses

R. Combescot, A. Recati, C. Lobo, F. Chevy, Phys. Rev. Lett. 98, 180402 (2007)
G. M. Bruun and P. Massignan, Phys. Rev. Lett. 105, 020403 (2010)

General features in the spectral funcion

$$A_\downarrow(\mathbf{p}, E) = -2\mathrm{Im}\frac{1}{E + i0^+ - \epsilon_{\mathbf{p}\downarrow} - \Sigma(\mathbf{p}, E)}$$



- Attractive and repulsive polaron
- Broadening and decay channels
- Molecule-hole continuum

P. Massignan, G. Bruun, Eur. Phys. J. D 65, 83 (2011)

Polaron decay rate, impurity residue and effective mass

- Pole expansion (for well-defined quasiparticles)

$$G_{\downarrow}(\mathbf{p},\omega) \approx \frac{Z_P}{\omega - E_P - \frac{p^2}{2m^*} + i\frac{\Gamma_P}{2}}$$

- Impurity residue

$$Z_P = \left\{1 - \mathrm{Re}\left[\partial_\omega \Sigma(0, E_P)\right]\right\}^{-1}$$

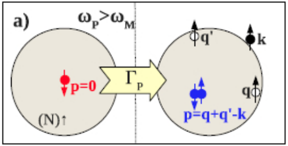
- Effective mass

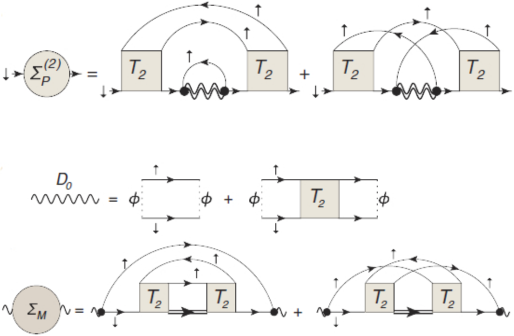
$$\frac{m}{m^*} = Z_P \left\{1 + \mathrm{Re}\left[\partial_{\epsilon_p} \Sigma(0, E_P)\right]\right\}$$

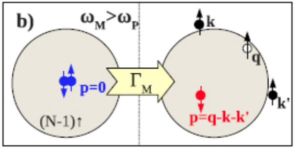
- Polaron decay rate

$$\Gamma_P = -2 Z_P \mathrm{Im}\left[\Sigma(0, E_P)\right]$$

Decay of polarons and molecules







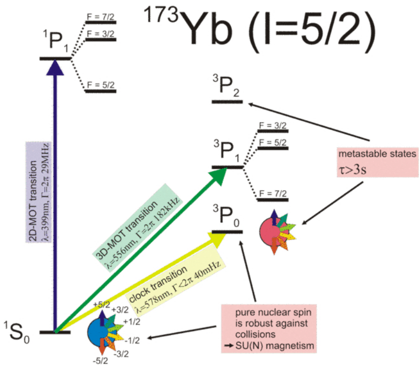
- Additional p-h pair due to energy-momentum conservation
- Other decay channels possible:
  repulsive to attractive polaron, repulsive polaron to bare impurity
- Diagrams for molecular state

G. M. Bruun and P. Massignan, Phys. Rev. Lett. 105, 020403 (2010)

Alkaline-earth(-like) atoms

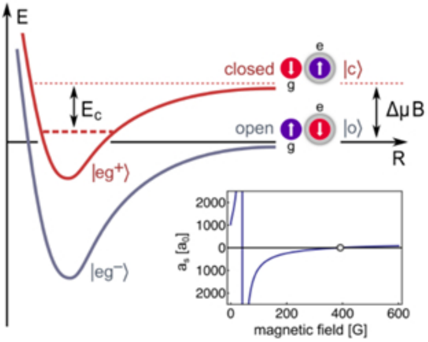
- Sr, Yb, Ca, etc.
- Two valence electrons: a rich level structure
- Even vs. odd isotopes

An example:



- Clock-state manifolds
- Versatile control

Orbital Feshbach resonance



Two-body interaction channels at short range

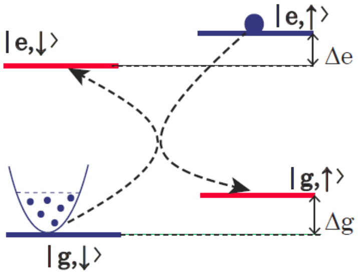
$$|\pm\rangle = \frac{1}{2}(|ge\rangle \pm |eg\rangle) \otimes (|\downarrow\uparrow\rangle \mp |\uparrow\downarrow\rangle)$$

R. Zhang, Y. Cheng, H. Zhai, P. Zhang, Phys. Rev. Lett. **115**, 135301 (2015)

G. Pagano *et al.*, Phys. Rev. Lett. **115**, 265301 (2015)

M. Höfer *et al.*, Phys. Rev. Lett. **115**, 265302 (2015)

Polaron problem in alkaline-earth(-like) atoms

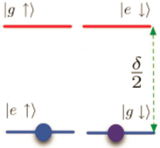


- Inter-orbital spin-exchange interaction
- Excitation of scattering states in the closed channel
- Effect of external magnetic field

J.-G. Chen, T.-S. Deng, WY, W. Zhang, Phys. Rev. A 94, 053627 (2016)

T.-S. Deng, Z.-C. Lu, Y.-R. Shi, J.-G. Chen, W. Zhang, WY, Phys. Rev. A 97, 013635 (2018)

Hamiltonian



$$H_0 = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} (a^{\dagger}_{g,\downarrow\mathbf{k}} a_{g,\downarrow\mathbf{k}} + a^{\dagger}_{e,\uparrow,\mathbf{k}} a_{e,\uparrow,\mathbf{k}})$$

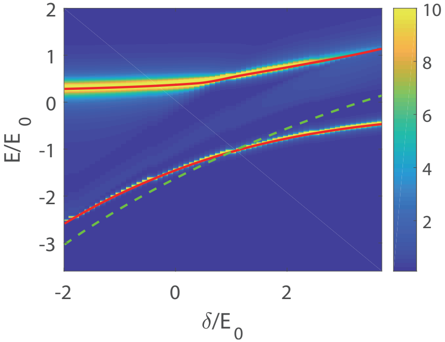
$$+ \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}} + \frac{\delta}{2})(a^{\dagger}_{e,\downarrow\mathbf{k}} a_{e,\downarrow\mathbf{k}} + a^{\dagger}_{g,\uparrow\mathbf{k}} a_{g,\uparrow\mathbf{k}}),$$

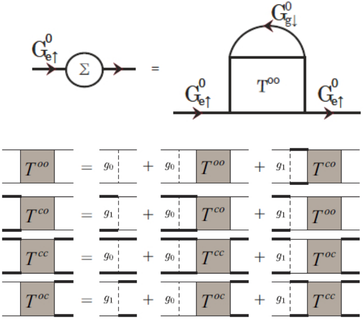
$$H_{\text{int}} = \frac{g_+}{2} \sum_{\mathbf{q}} A^{\dagger}_{+}(\mathbf{q}) A_{+}(\mathbf{q}) + \frac{g_-}{2} \sum_{\mathbf{q}} A^{\dagger}_{-}(\mathbf{q}) A_{-}(\mathbf{q}),$$

with

$$A_{\pm}(\mathbf{q}) = \sum_{\mathbf{k}} (a_{e,\downarrow,\mathbf{k}} a_{g,\uparrow,\mathbf{q}-\mathbf{k}} \mp a_{e,\uparrow,\mathbf{k}} a_{g,\downarrow,\mathbf{q}-\mathbf{k}})$$

The overall picture in terms of spectral function





$$A(0,E) = -2\mathrm{Im}G_{e\uparrow}(0,E)$$

$$g_0 = \frac{g_+ + g_-}{2}, \quad g_1 = \frac{g_- - g_+}{2}$$

J. Xu, R. Zhang, Y. Cheng, P. Zhang, R. Qi, H. Zhai Phys. Rev. A 94, 033609 (2016)

T.-S. Deng, Z.-C. Lu, Y.-R. Shi, J.-G. Chen, W. Zhang, WY, Phys. Rev. A 97, 013635 (2018)

Ground-state of the impurity system: attractive polaron vs molecule

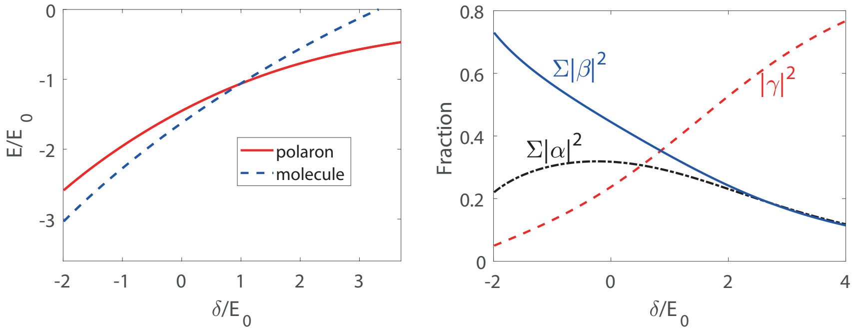
$$|P\rangle_{\mathbf{Q}} = \gamma a^{\dagger}_{e,\uparrow\mathbf{Q}}|g_{\downarrow}\rangle_N + \sum_{\substack{|\mathbf{k}|>k_F \\ |\mathbf{q}|<k_F}} \alpha_{\mathbf{kq}} a^{\dagger}_{e,\uparrow\mathbf{Q}+\mathbf{q}-\mathbf{k}} a^{\dagger}_{g,\downarrow\mathbf{k}} a_{g,\downarrow\mathbf{q}}|g_{\downarrow}\rangle_N$$
$$+ \sum_{\substack{\mathbf{k} \\ |\mathbf{q}|<k_F}} \beta_{\mathbf{kq}} a^{\dagger}_{e,\downarrow\mathbf{Q}+\mathbf{q}-\mathbf{k}} a^{\dagger}_{g,\uparrow\mathbf{k}} a_{g,\downarrow\mathbf{q}}|g_{\downarrow}\rangle_N,$$
$$|M\rangle_{\mathbf{Q}} = \sum_{|\mathbf{k}|>k_F} \alpha_{\mathbf{k}} a^{\dagger}_{e,\uparrow\mathbf{Q}-\mathbf{k}} a^{\dagger}_{g,\downarrow\mathbf{k}}|g\downarrow\rangle_{N-1} + \sum_{\mathbf{k}} \beta_{\mathbf{k}} a^{\dagger}_{e,\downarrow\mathbf{Q}-\mathbf{k}} a^{\dagger}_{g,\uparrow\mathbf{k}}|g\downarrow\rangle_{N-1}.$$

- Schrödinger's equation

$$(H_0 + H_{\text{int}})|\alpha\rangle = (E_\alpha + E_{\text{FS}})|\alpha\rangle, \quad \alpha = P, M$$

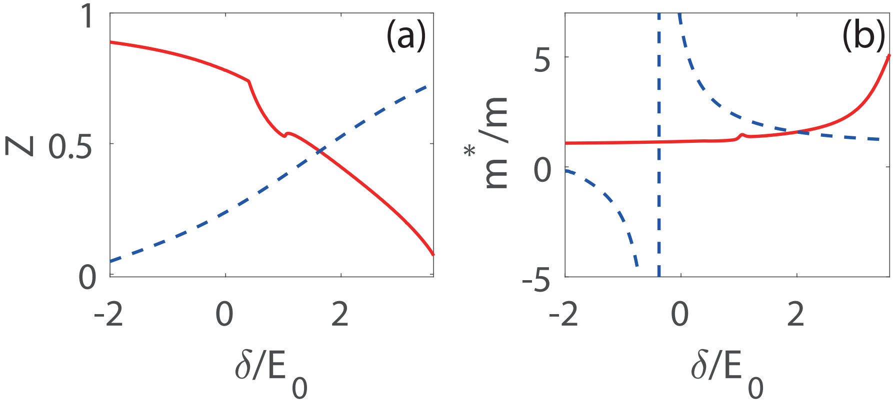
with E_{FS} the energy of the Fermi sea with N atoms.

Polaron energy and polaron-molecule transition



- Polaron-molecule transition in the attractive branch
- Variation of weights in the polaron wave function

Repulsive polaron: residue and effective mass



$$Z_{\pm} = \frac{1}{1 - \mathrm{Re}\left[\frac{\partial \Sigma(0,\omega)}{\partial \omega}\right]}\Bigg|_{\omega=E_{\pm}}, \quad \frac{m^*_{\pm}}{m} = \frac{1}{Z_{\pm}} \frac{1}{1 + \mathrm{Re}\left[\frac{\partial \Sigma(\mathbf{Q},\omega)}{\partial \mathbf{Q}^2}\right]}\Bigg|_{\mathbf{Q}=0,\omega=E_{\pm}}$$

Understanding the kinks

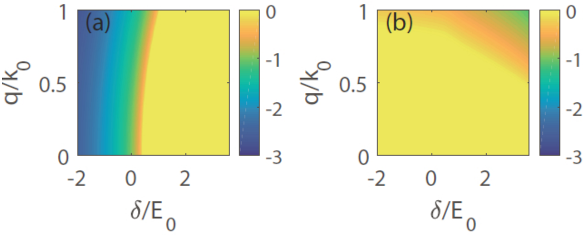
$$T^{oo}(\mathbf{q},\omega)=\frac{\frac{1}{2}(g_++g_-)-g_+g_-\chi^c}{1-\frac{1}{2}(g_++g_-)(\chi^o+\chi^c)+g_+g_-\chi^o\chi^c}$$

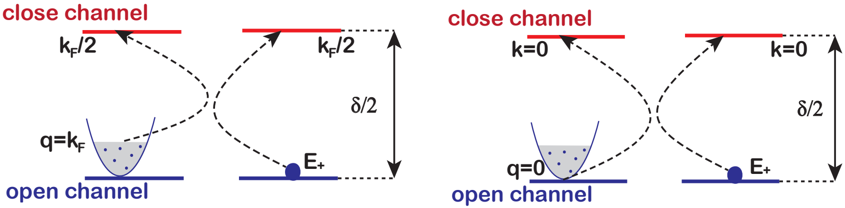
$$\chi^c(\mathbf{q},\omega)=\sum_{\mathbf{k}}\frac{1}{\omega+i0^+-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{q}-\mathbf{k}}-\delta}$$

$$\chi^o(\mathbf{q},\omega)=\sum_{|\mathbf{k}|>k_F}\frac{1}{\omega+i0^+-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{q}-\mathbf{k}}}$$

- Location of kinks: $\delta=E_+$, $\delta=E_++\frac{E_F}{2}$
- Contribution of the imaginary part of $\chi^c(\mathbf{q},\omega)$ to $\Sigma(\mathbf{q},E)$

Understanding pair propagators

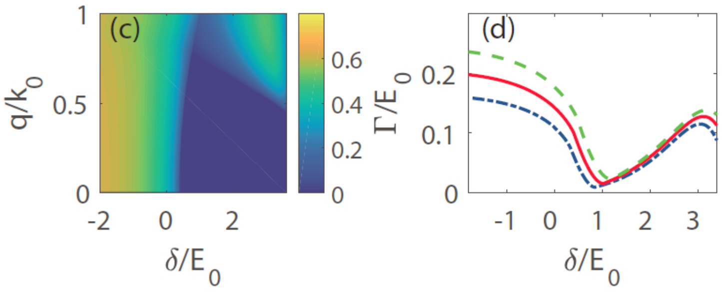




$$E_F + E_+ = \delta + \frac{E_F}{4} + \frac{E_F}{4}$$

$$E_+ = \delta$$

Impact on the decay of repulsive polaron



$$\Sigma(0,E) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\omega}{2\pi} G^0_{g\downarrow}(\mathbf{q},\omega) T^{oo}(\mathbf{q}, E+\omega)$$

$$\Gamma = -2Z_+[\mathrm{Im}\Sigma(0,E_+)]$$

- Decay into the bare impurity state in the attractive branch

F. Scazza *et al.*, Phys. Rev. Lett. 118, 083602 (2017)

Phase separation and itinerant ferromagnetism

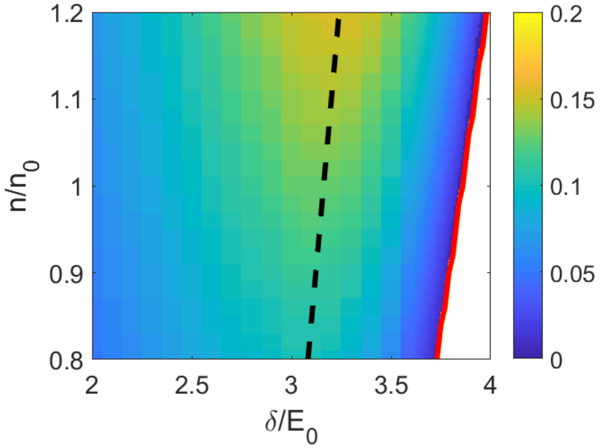
- Stoner's itinerant ferromagnetism:
  polarization + repulsive interaction $\rightarrow$ ferromagnetism
- Experimental evidence? (in alkali atoms)
  G.-B. Jo *et al.*, Science 325, 1521 (2009)
  C. Sanner *et al.*, Phys. Rev. Lett. 107, 175302 (2011)
  G. Valtolina *et al.*, Nat. Phys. 13, 704 (2017)
- In alkaline-earth-(like) aotms?

Criterion?

- Phase separation vs. polaron state: energetic considerations
- Long-lived polaron excitations: $\Gamma \ll E_F$

P. Massignan, Z. Yu, G. M. Bruun, Phys. Rev. Lett. 110, 230401 (2013)

Phase diagram



- OFR non-universal
- A favorable parameter window for probing ferromagnetism

Outlook

Variants of the polaron problem

- Polaron in a Bose-Einstein condensate

  Pitaevskii, Blume, Timmermans, Jaksch, Devereese, Li, Demler, Cui, Zhai...

  Exp: Jin, Arlt...

- Polaron in a Fermi superfluid

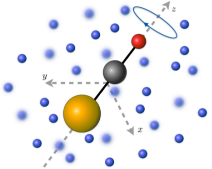
  Nishida, Cui, Yi

- Polaron in spin-orbit coupled Fermi gas

  Cui, Zhang, Yi

- Rotating impurities: angulon

  Lemeshko,...



M. Lemeshko, PRL (2017)

A probe to many-body system, a bridge between few and many...