Lecture Two: Review of Relevant Experiments

So far experiments on p-wave gases are mostly carried out on the Fermi alkali gas ^{6}Li and ^{40}K, and I will be focusing on this two elements. In Bose alkali gas mixture ^{85}Rb and ^{87}Rb, p-wave scattering resonance is also seen[1], but I will not discuss that.

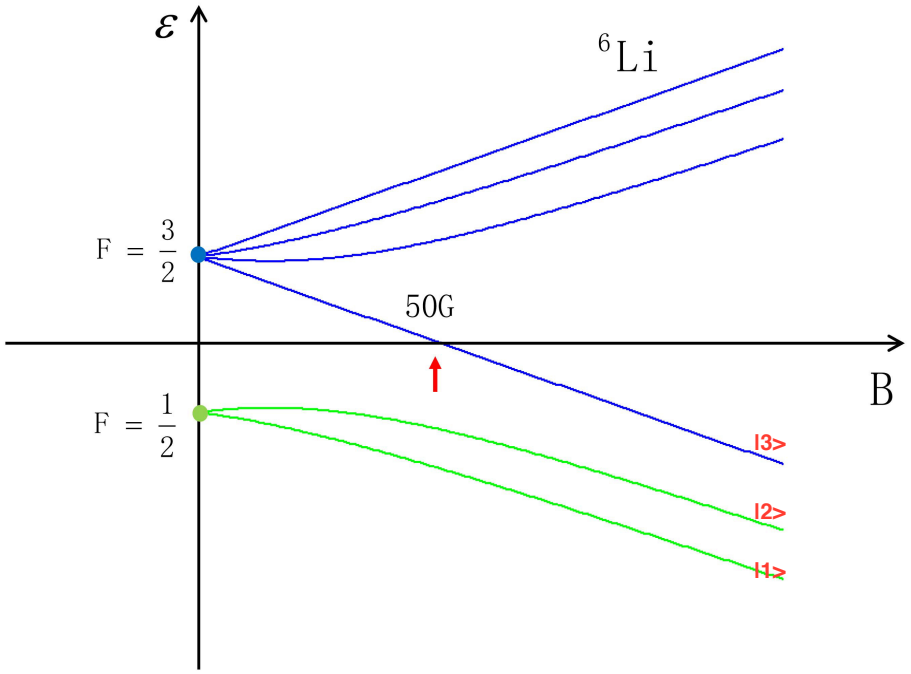

Figure 1: Energy diagram of ^{6}Li in an external magnetic field.

[1]PRA **94**, 062702 (2016)

1. Scattering properties of ^{6}Li.

scattering channel	theory	ENS	MIT	Comments
$\|1\rangle$ - $\|1\rangle$	159.15	160.2	159.14(0.4)	loss due to three-body
$\|1\rangle$ - $\|2\rangle$	185.15	186.2	185.09(0.2)	two-body dipole
$\|2\rangle$ - $\|2\rangle$	214.90	215.2	214.94(0.4)	two-body

Since p-wave scattering occurs through three angular momentum channels, $m = \pm 1$ (xy-channel) and $m = 0$ (z-channel), there are splittings due to magnetic dipole-dipole interaction. In the case of ^{6}Li, this effect is small. For example for the $|1\rangle$ - $|1\rangle$ scattering, the splitting is about 10mG with $m = 0$ higher. This is typically not resolved in experiments (either due to magnetic field fluctuations or temperature broadening).

2. Feshbach Resonance

Consider two ^{6}Li atoms in collision, each is described by a set of internal degrees of freedom, its valence electron spin $s_{1,2}$, nuclear spin $i_{1,2}$ and the relative orbital angular momentum ℓ. To discuss the Feshbach resonance, one necessarily is interested in the short-range potential between the two atoms, and we form $S = s_1 + s_2$ and $I = i_1 + i_2$. Now because of the total interchange of atoms obeys Fermi statistics, it turned out that

$$S + I + \ell = \text{even}. \tag{1}$$

In fact, this expression is independent of the statistics (Bose/Fermi) of the (composite) particle in question. For ^{6}Li, we have $s_{1,2} = 1/2$ and $i_{1,2} = 1$, so one finds $S = 0, 1$ and $I = 0, 1, 2$. Now the relevant closed channel molecule (singlet channel, $\nu = 38$) which gives rise to the p-wave Feshbach resonance observed in ^{6}Li is in the singlet channel with $S = 0$. One requires $I + \ell = \text{even}$.

S	I	ℓ	Comments
0	0,2	0	two s-wave scattering in ^{6}Li (543, 822)
0	1	1	p-wave scattering

Note that in this case (as well in ^{40}K), the p-wave resonance is due to coupling of p-wave scattering fermions to a p-wave closed channel molecule, and the inter-channel coupling is of s-wave symmetry.

As mentioned before, small dipolar interaction in ^{6}Li splits the resonance with $m = \pm 1$ from that of $m = 0$ about 10 mG.

A related question is the molecular fraction in the closed channel (molecule in the singlet channel, $\nu = 38$). This has been calculated and investigated experimentally by measuring the magnetic moment of the Feshbach molecules (linear superposition of closed channel molecules and open channel scattering fermions).

$$\psi_{\text{Feshbach}} = \sqrt{Z}\psi_{\text{closed channel molecule}} + \sqrt{1-Z}\psi_{\text{open channel fermions}} \tag{2}$$

This turns out to be $Z = 0.81$ close to coupled channel calculation which gives $Z = 0.82$. What p-wave Feshbach is special is that Z essentially stays the same as one tunes the magnetic field. In any simple model that one writes down usually $Z = 1$ at resonance. (problem here: experimental resolution and other closed channel states, needs full coupled channel calculation). [Recall s-wave case...]

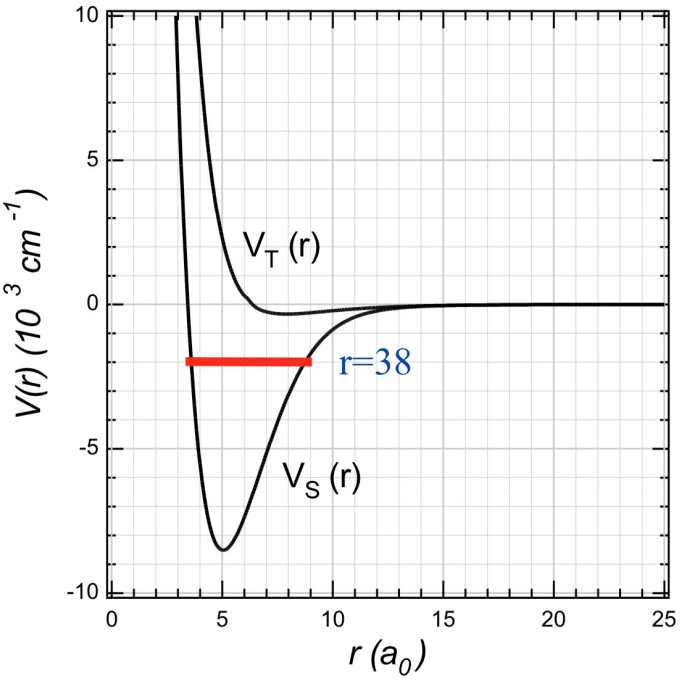

Figure 2: Energy diagram of ^{6}Li in an external magnetic field.

3. Molecular Energy measurement[2].

One unique feature of the p-wave scattering is the existence of centrifugal barrier which allows the existence of quasi-bound state. To measure the molecular binding energy, one can make sure of magneto-association spectroscopy, in which a modulated magnetic field with an amplitude δB (180mG of order of the width) and a fixed frequency ($\nu_{\rm mod} = 650$KHz). Then one changes the static magnetic field, and see the loss features of the cloud. The assumptions are when on resonance, there will be significant loss due to formation of p-wave

[2]PRA **77**, 053616 (2008)

molecules (quasi-molecules) and these, via atom-molecule scattering, leads to loss. In this way, one can map out the field dependences of molecular binding energy by changing $\nu_{\rm mod}$.

Note:

1. line shape similar for bound and quasi-bound state $|2\rangle - |2\rangle$ (much higher barrier for ^{6}Li) [cf. ^{40}K].

2. No temperature dependence of the loss feature (collision energy 100nK and temperature around 400nK).

3. No Rabi oscillation between atom pairs and molecules (quasi-bound state) is seen.

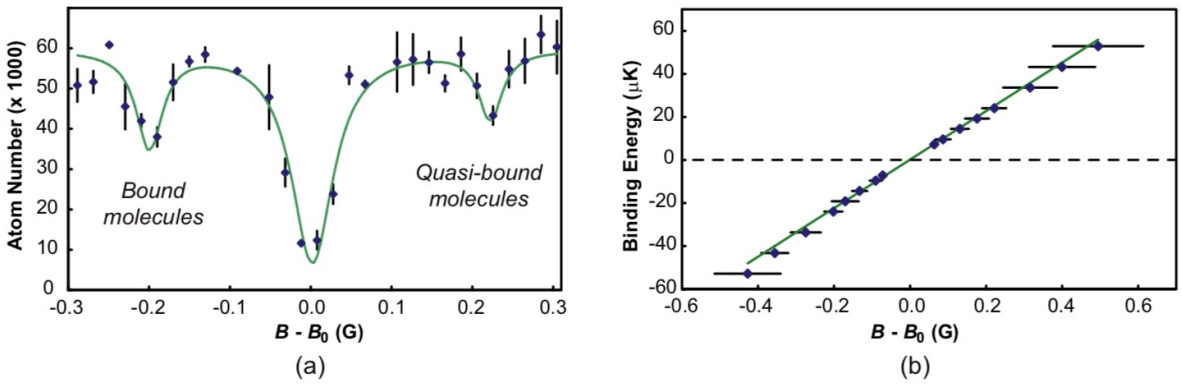

Figure 3: Binding energy of Li-6 in the $|1\rangle - |1\rangle$ states.Linear dependence on magnetic field.

4. Molecular conversion rate and loss[3].
In the same Swinburne experiment when the conversion to molecules are small, theoretical calculation (on the bound state side) as well experiments (by monitoring the remaining number of atoms) are done to see the field dependence of conversion rate. It is found that
1. The rate is similar for the bound state and quasi-bound state, which is a bit surprising;
2. Fermi-Golden-rule calculation match the bound side very well;
3. No special feature of molecules at resonance.

Initial experiments on ^{6}Li have mainly focused with measuring the loss of atoms close to p-wave resonance. In this regard, one should distinguish two different cases:
(1) The p-wave resonance for the lowest hyperfine-Zeeman states ($|1\rangle$-$|1\rangle$) where no dipole loss is allowed.
(2) The loss feature of the other combinations where dipole loss dominate ($|1\rangle$-$|2\rangle$, $|2\rangle$-$|2\rangle$).
Several known results are:
(1) Three-body loss for $|1\rangle$-$|1\rangle$ loss follow the $T^{-\lambda}$ with $\lambda > 2$ (2 for Fermi-Golden-rule, Wigner threshold law). This is generally observed in experiments away from resonance, but close to resonance, unitary limited loss is seen[4].

Incidentally, the loss measurement generally shows asymmetry in

[3]PRA **77**, 053616 (2008)
[4]arXiv:1803.07530v1

loss about the resonance.

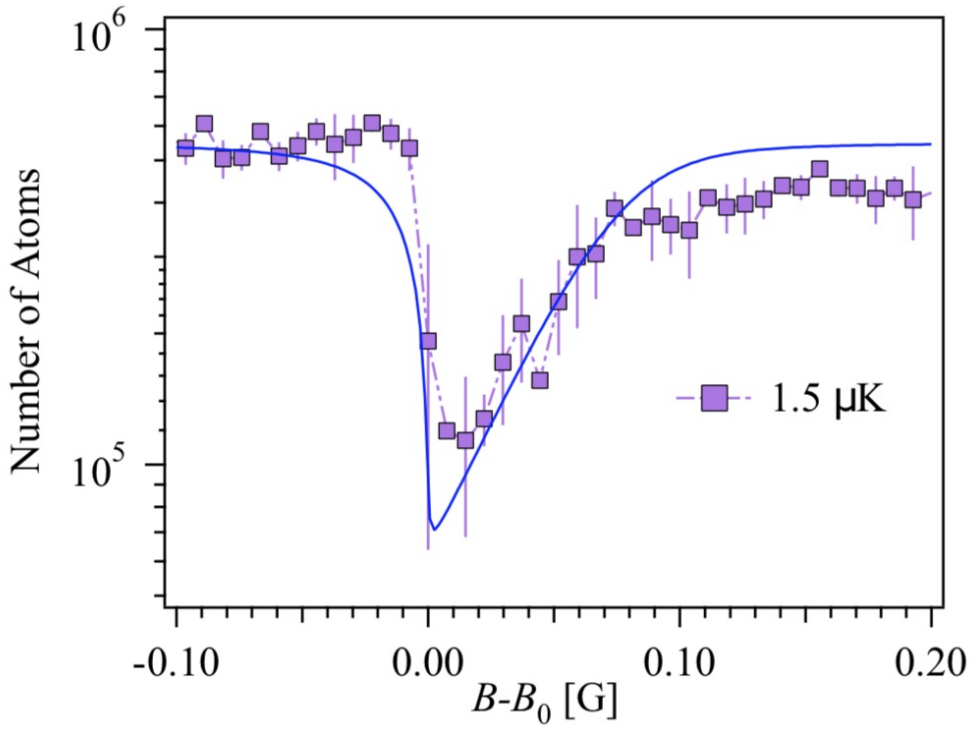

Figure 4: Loss feature of ^{6}Li in $|1\rangle - |1\rangle$ state. Note: shape edge at resonance and asymmetry profile; Cf. magneto-association spectrascopy.

The loss is described by the equation

$$\frac{\dot{N}}{N} = -L_3 \left\langle n^2 \right\rangle \tag{3}$$

L_3 at unitarity is expected to be (independent of interaction)

$$L_3 \propto \frac{1}{(k_B T)^2}, \tag{4}$$

In general, the three-body loss rate can be computed by including an imaginary part in the low-energy effective range expansion

$$k^3 \cot \delta_k = -\frac{1}{v} - \frac{i}{v_i} - \frac{1}{2R}k^2 \tag{5}$$

where v_i describes the loss of atoms due to three-body recombination [S-matrix no longer unitary]. L_3 is then the thermal average of the (energy-dependent) rate K_3

$$L_3 \propto \frac{1}{(k_B T)^{3/2}} \int_0^\infty K_3 \sqrt{E} \exp(-E/k_B T) dE \tag{6}$$

where K_3 is given by (Γ is resonance width)

$$K_3 \propto \frac{1}{E^2} \frac{1}{(E - E_b)^2 + \Gamma^2/4} \tag{7}$$

where $E_b = \frac{R}{mv}$, or in terms of $k_0 = \sqrt{R/v}$, $E_b = k_0^2/m$. k_0 would be a typical scale of the problem. On the bound state part, this would give the inverse of size of the molecule.

As for the interaction dependences of the three-body loss coefficient, it is known that for v small, one has

$$L_3 \propto V^{8/3} \tag{8}$$

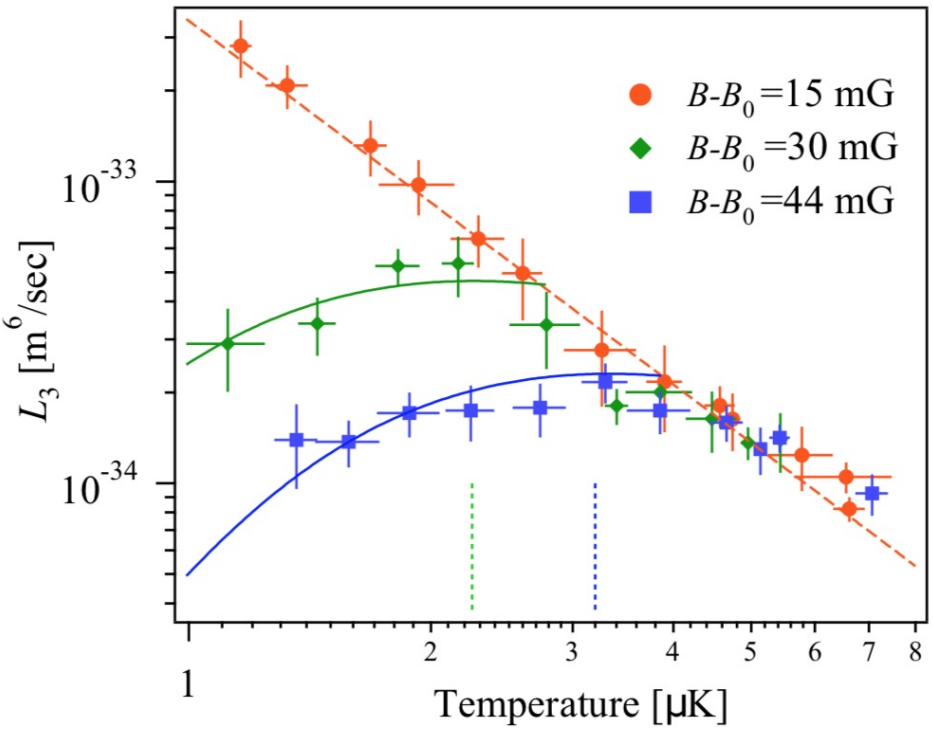

Figure 5: Temperature dependence of L_3.

5. Two-body dipolar loss.

There is a detailed theoretical analysis by Kurlov and Shlyapnikov in 1D, 2D and 3D based on the basis of Eq.(5), adapted to 2D and also 1D. The general conclusion is that in low dimensions, two-body loss rates are suppressed compare with 3D. Theoretical analysis of the loss data is very similar to that of L_3, except that one should include in the average the appropriate scattering cross sections.

Experiment performed by the Mukaiyama group which generally

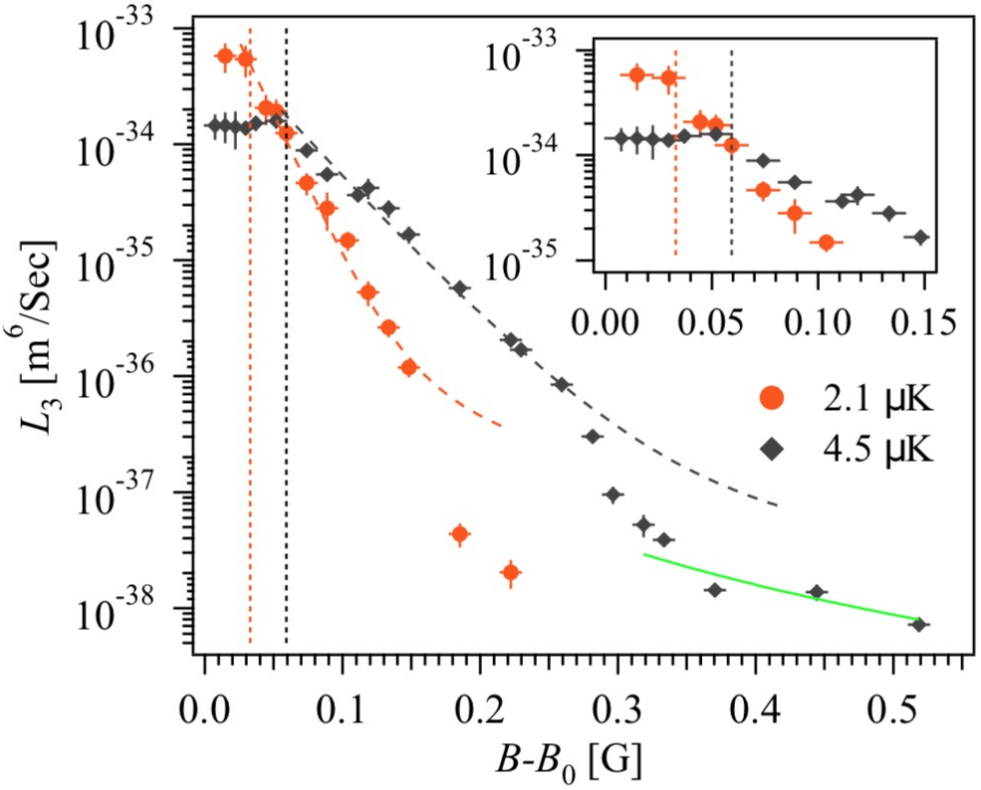

Figure 6: Interaction dependence of L_3.

agrees very well with the theoretical expectation, including the suppression of loss in 2D as compared with 3D. For example, the different temperature dependences of K_2 in the resonance and off-resonance regime is given below.

To Summarize the ^{6}Li p-wave case, the few-body aspects of the problem are fairly understood (theoretically three-body recombination still requires some work). The life time of the sample is limited to

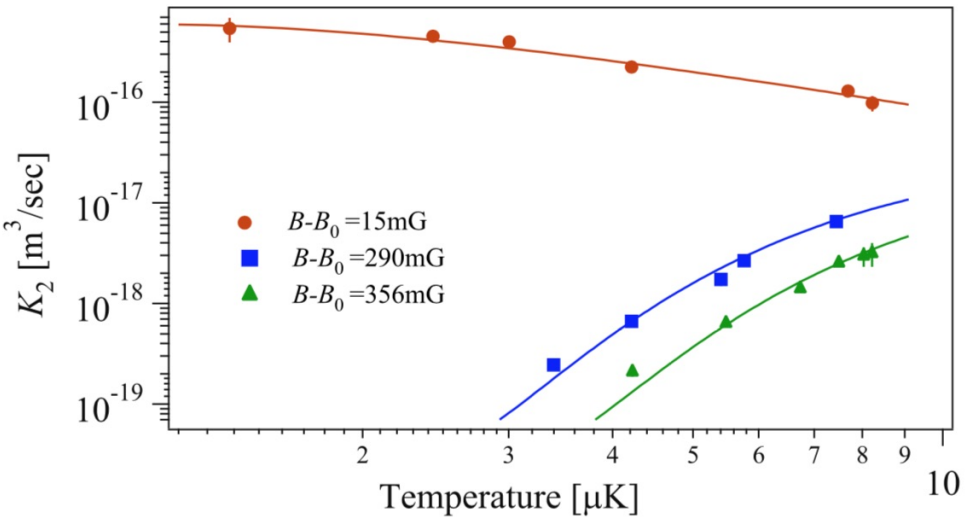

Figure 7: Temperature dependence of K_2 for ^{6}Li.

several hundred ms close to resonance, and it is still quite challenging to achieve interesting many-body states; even the normal state properties is barely investigated let alone the superfluid state.

One experiment by Mukaiyama group that might be interesting from a many-body point of view is that by applying a strong 1D optical lattice, they can selectively populate p-wave molecules with $m = 0$ and $m = \pm 1$ states. This would be interesting if furthermore $m = \pm 1$ can be distinguished by, say, rotating the harmonic trap. This would be an interesting problem even to consider the normal state of a system with angular momentum and see how it evolves across the resonance. Incidentally the conversion rate to molecules is generally in the level of 20%.

A Summary for ^{40}K.

I will not dwell on the details of ^{40}K p-wave resonance as most of the physics are close (a bit less explored than ^{6}Li it seems to me), except to point out the most salient features.

(1) There is a significantly larger splitting of the three angular momentum projection states with $m = \pm$ at higher magnetic field by 0.5G) due to dipole-dipole interaction).

(2) As mentioned in Lecture One, the p-wave barrier is smaller than ^{6}Li. There is an asymmetry for the life-time of the bound and quasi-bound state (Cf. Li lifetime measured using magneto-association spectroscopy). (3) There is a nonlinearity in the molecular bind-

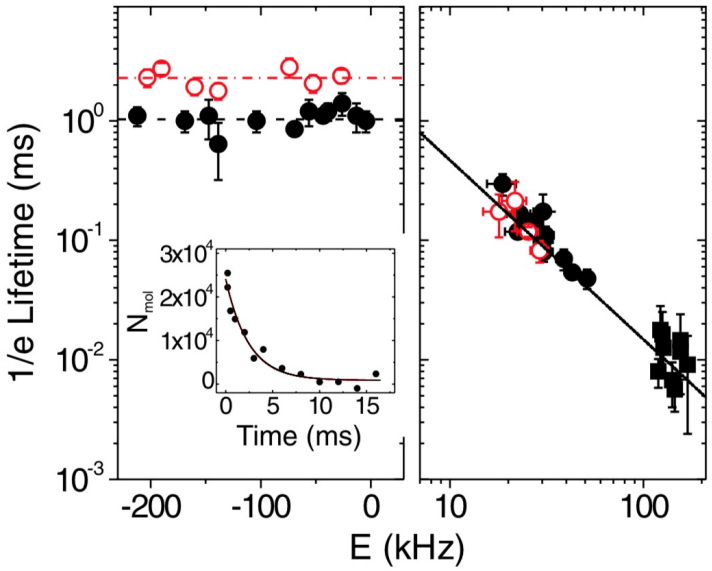

Figure 8: Asymmetry in the molecular lifetime across resonance.

ing energy close to resonance which is not seen in ^{6}Li (at least not

commented upon as far as I know).

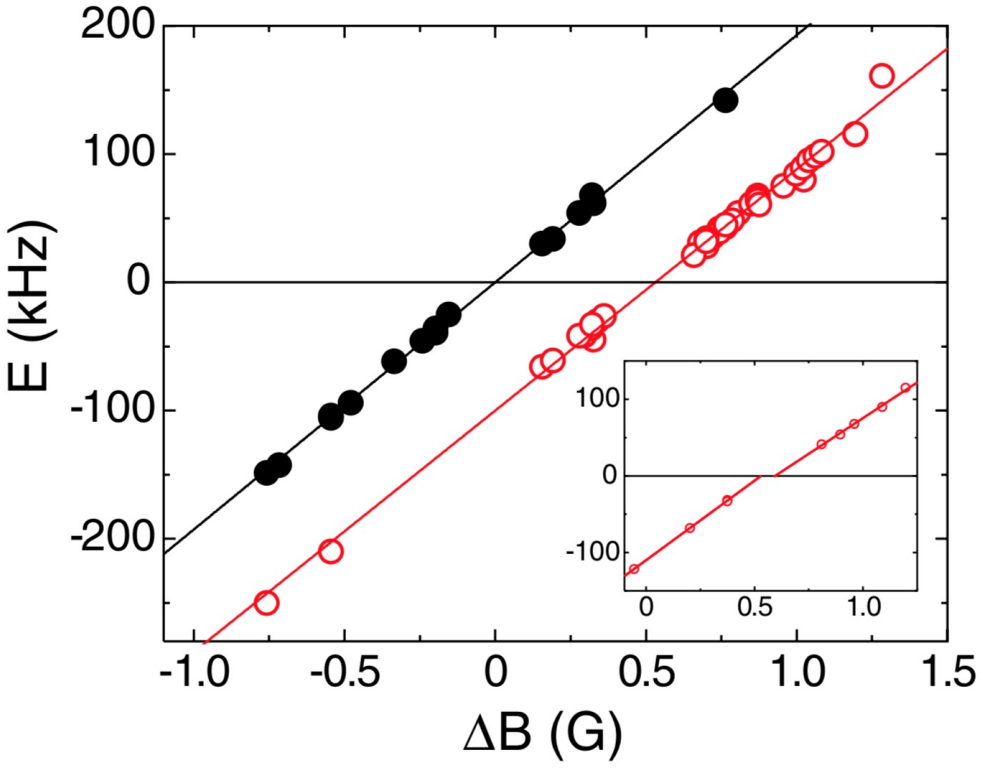

Figure 9: Binding energy of ^{40}K.

The only many-body experiment on p-wave Fermi gas ^{40}K is done by Toronto in which they measured the universal relations in this system.
(1) momentum tails
(2) tails of the radio-frequency response
(3) dynamics of molecule-atom pair conversion