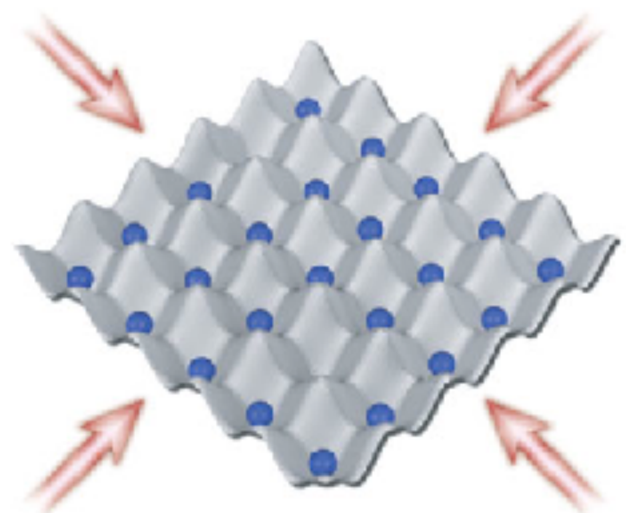
Nonequilibrium dynamics in ultra-cold atomic gases



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Wuhan Institute of Physics and Mathematics
April 16, 2018

GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

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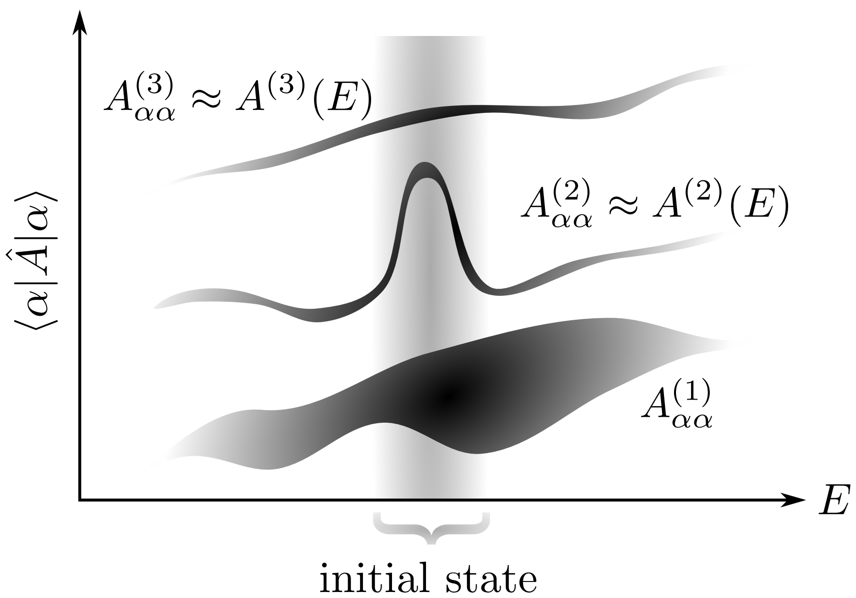
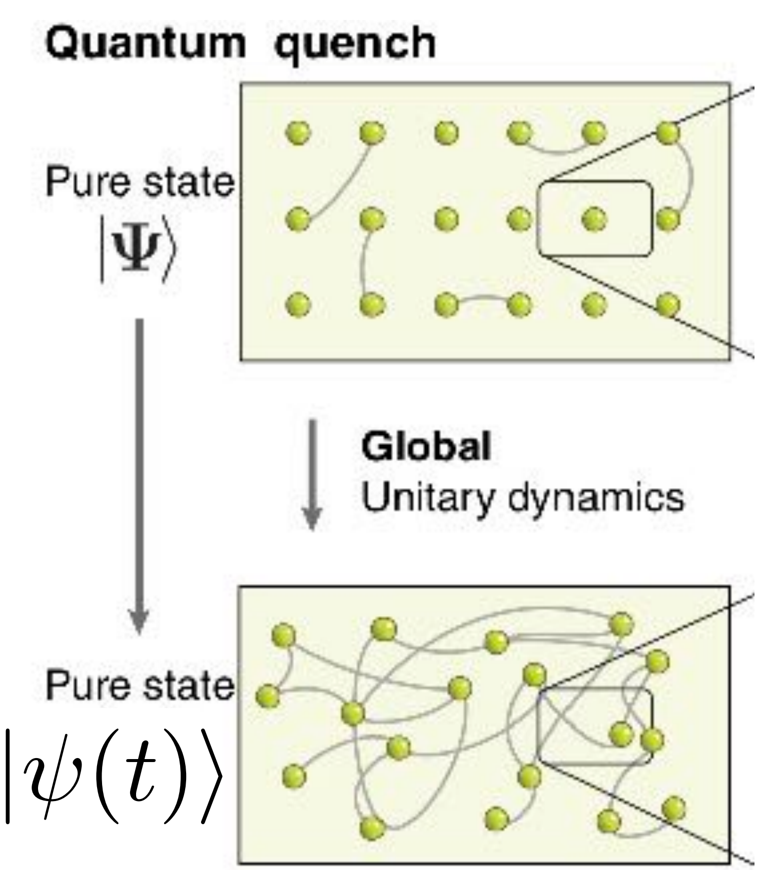
Main topics in this lecture

Thermalization in closed many-body systems

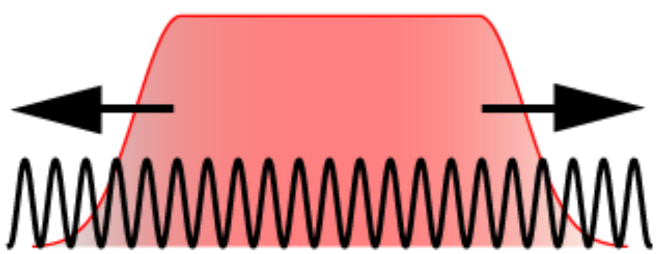




Experiments with quantum gases !

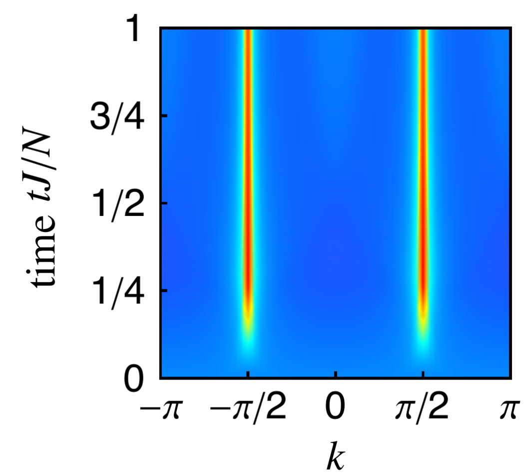
Kaufman et al. Science 353, 794 (2016)
Trotzky et al. Nat. Phys. 8, 325 (2012)
Cheneau et al. Nature 481, 484 (2012)

Nonequilibrium transport In optical lattices



Bloch group (LMU & MPQ), Weiss group (PSU)

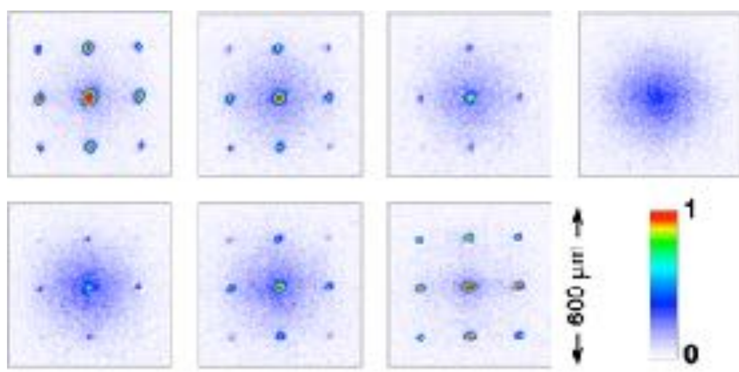
Transient dynamics & integrable 1d models



Ballistic transport
Nonequilibrium condensates
Measuring rapidity distributions!

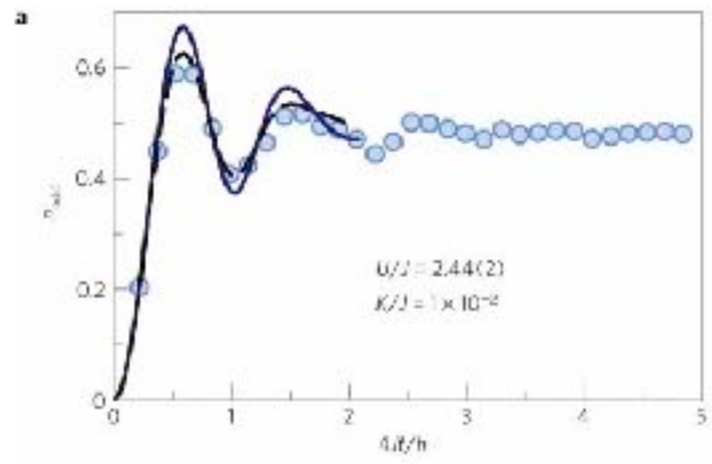
Non-equilibrium dynamics: Experiments

Collapse & Revival



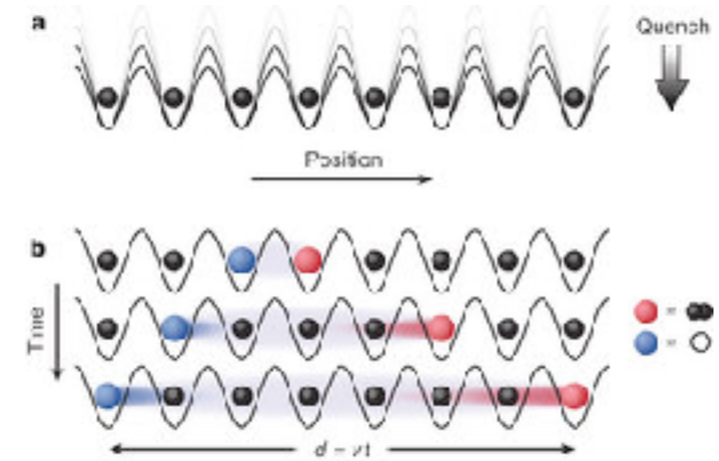
Greiner et al. Nature 419, 51 (2002)

Decay of ideal CDW state



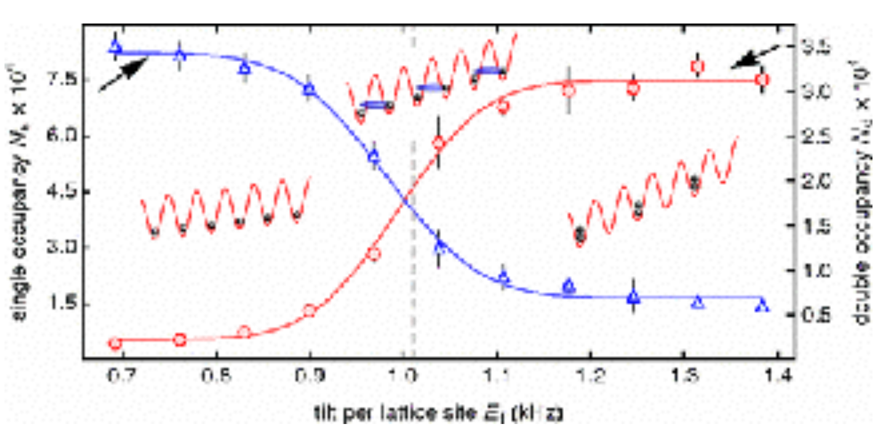
Trotzky et al. Nat. Phys. 8, 325 (2012)

Spreading of correlations



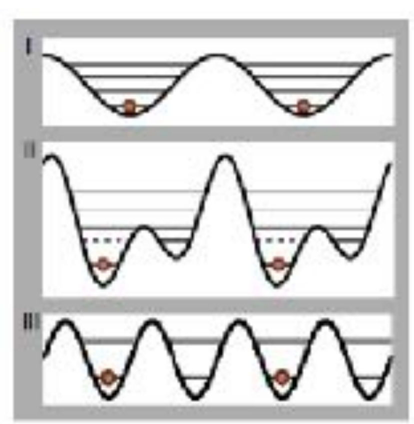
Cheneau et al. Nature 481, 484 (2012)

Tilted lattices



Meinert et al. PRL 111, 053003 (2013)

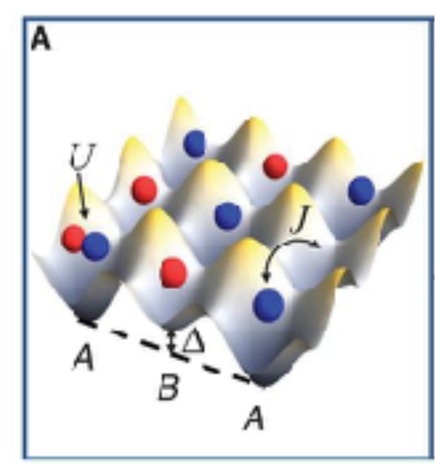
Fermionic relaxation



$$|1, 0, 1, 0, \ldots\rangle$$

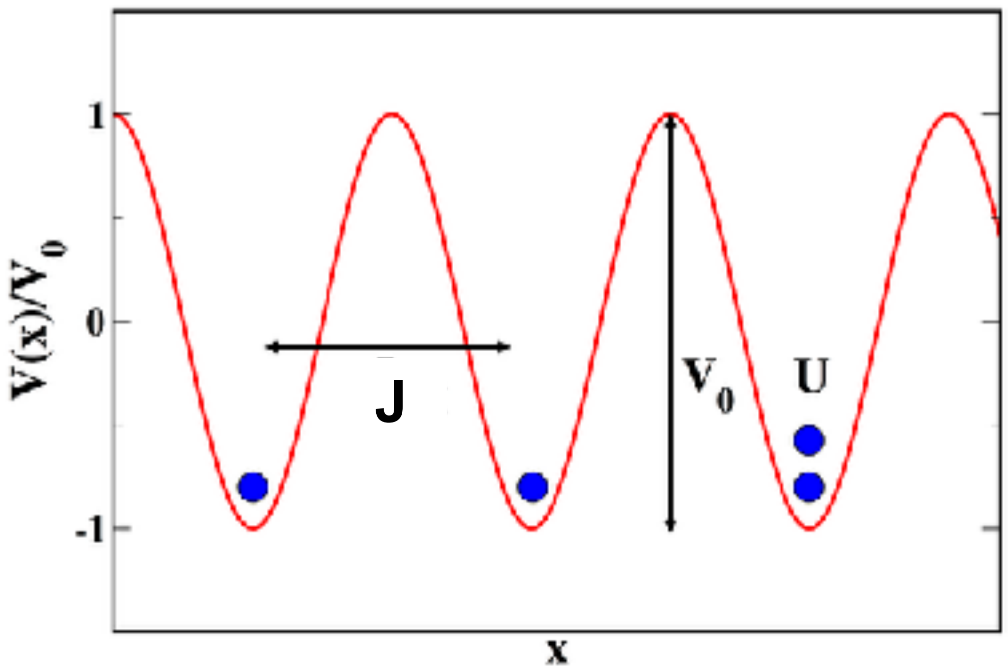
Pertot et al. PRL 113, 170403 (2014)

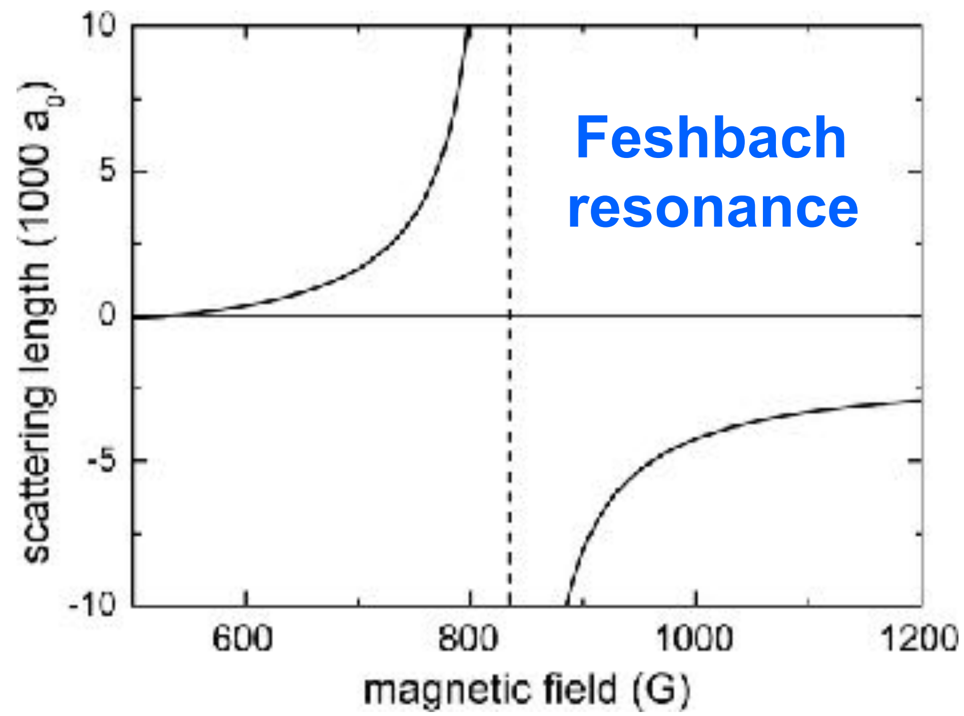
Decay of Néel order in 2D



Brown et al. Science 348, 540 (2015)

Realization of Hubbard models with quantum gases





Bosonic atoms: ^{87}Rb, ^{39}K

$$U/J = f(V_0, a_s) \quad U(\vec{r}) = U\delta(\vec{r}) \quad U \propto a_s$$

Bose-Hubbard model

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Tunability:

- → **Dimensionality**
- → **Interactions U/J**
- → **Time-dependence!**

Jaksch et al. PRL 1998

Bose-Hubbard model

$$H_0 = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu N$$

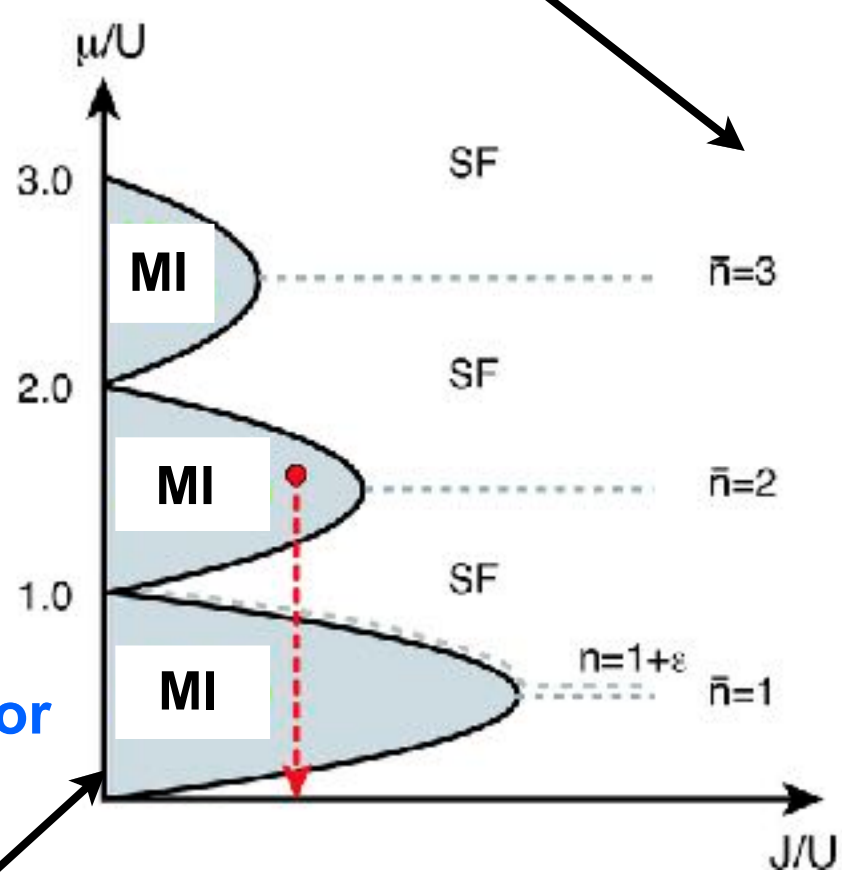
Superfluid

$$|\psi_0\rangle \propto \left(a_{k=0}^\dagger\right)^N |0\rangle$$

Dispersion:

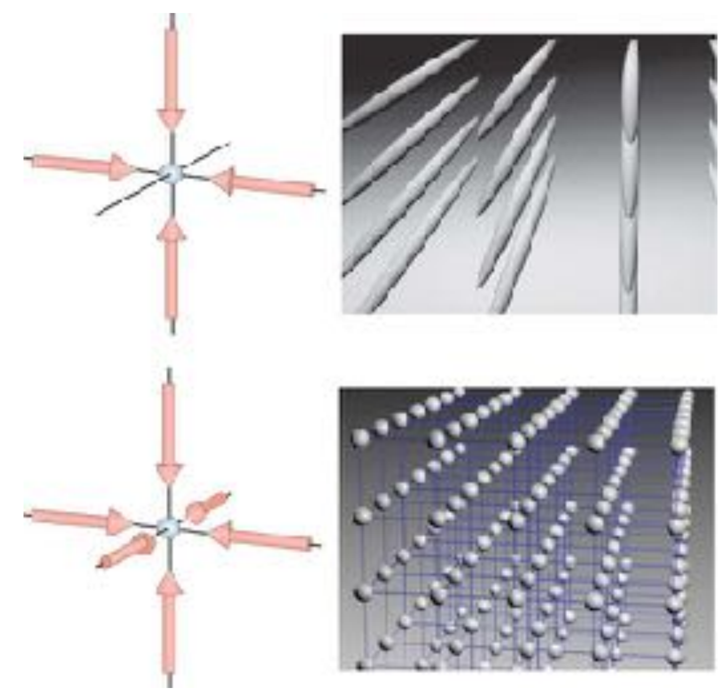
$$\epsilon_k = -2J\cos(k)$$

Optical lattices



Mott-Insulator

$$|\psi_0\rangle = \prod_i a_i^\dagger |0\rangle$$



Bloch, Dalibard, Zwerger Rev. Mod. Phys. 2008
Jaksch et al. Phys. Rev. Lett. 1998

Fisher et al. PRB 1989
Oosten, van der Straaten, Stoof PRA 2001

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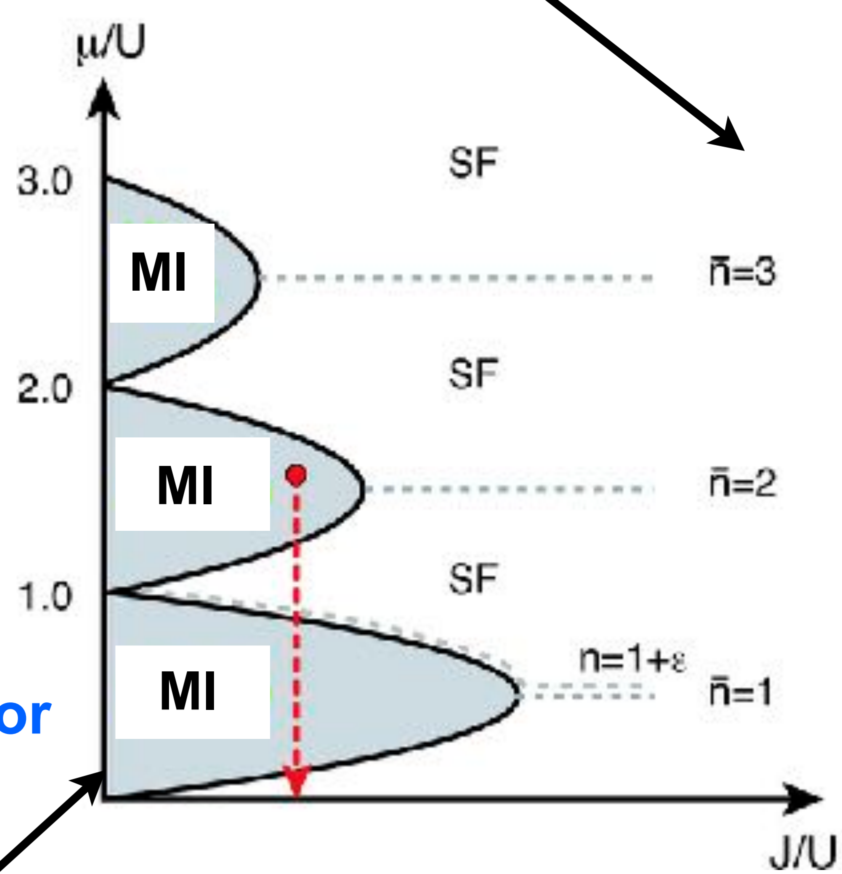
Superfluid

$$|\psi_0\rangle \propto \left(a_{k=0}^\dagger\right)^N |0\rangle$$

Density profiles



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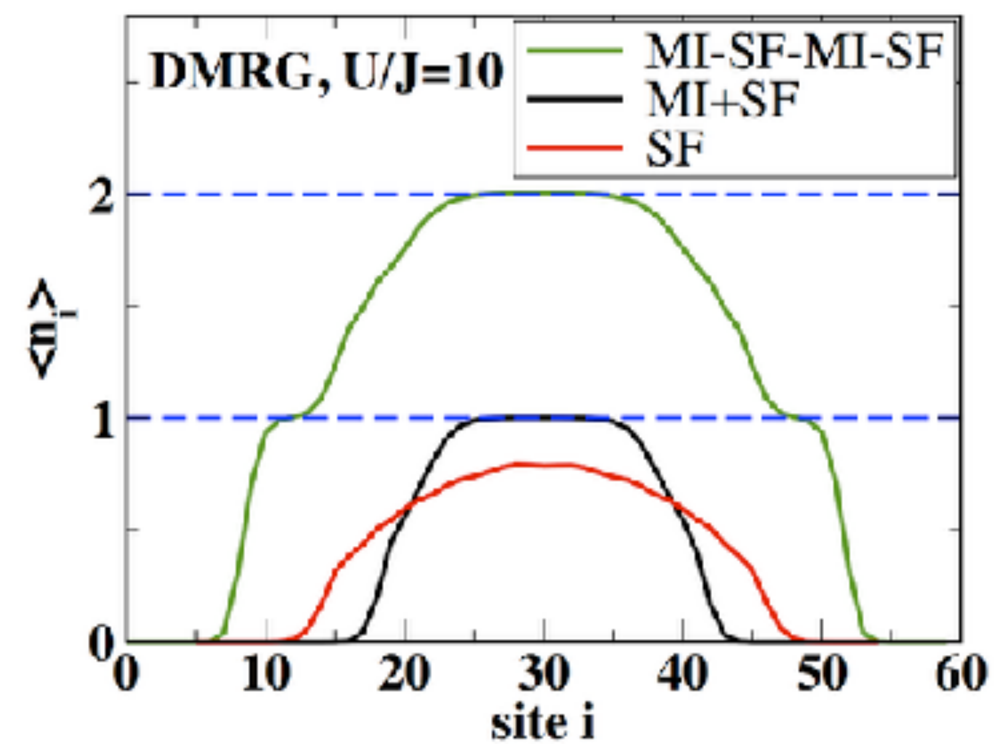


$$H = H_0 + V \sum_i n_i i^2$$

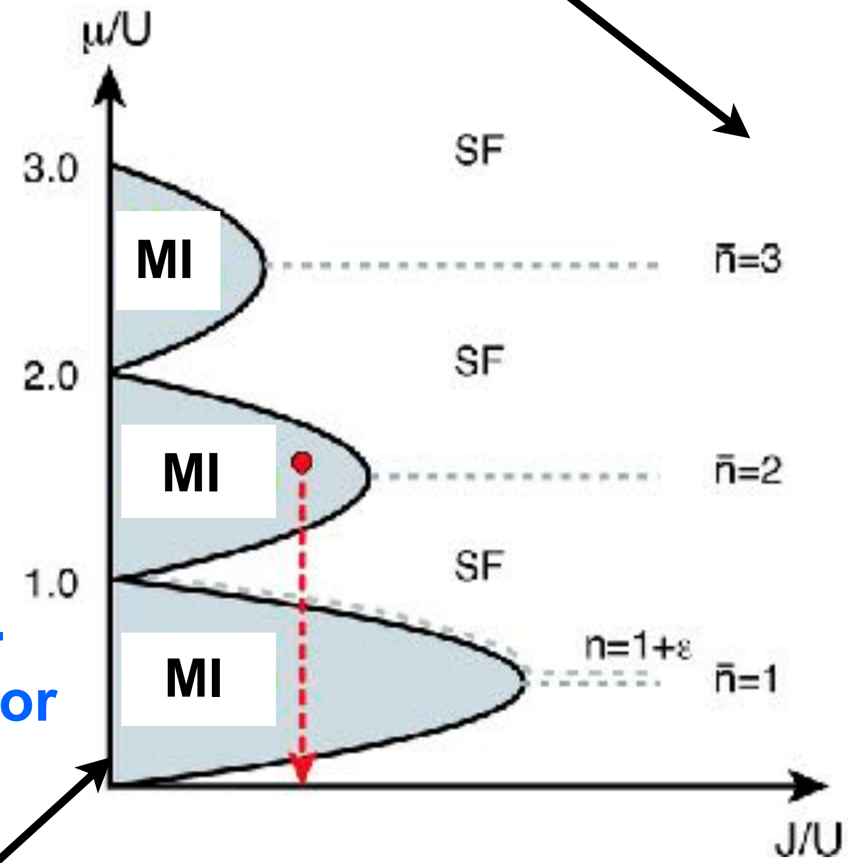
Bloch, Dalibard, Zwerger RMP 2008, Fisher et al. PRB 1989
Oosten, van der Straaten, Stoof PRA 2001

Bose-Hubbard model

$$H_0 = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu N$$

Superfluid

$$|\psi_0\rangle \propto \left(a_{k=0}^\dagger\right)^N |0\rangle$$

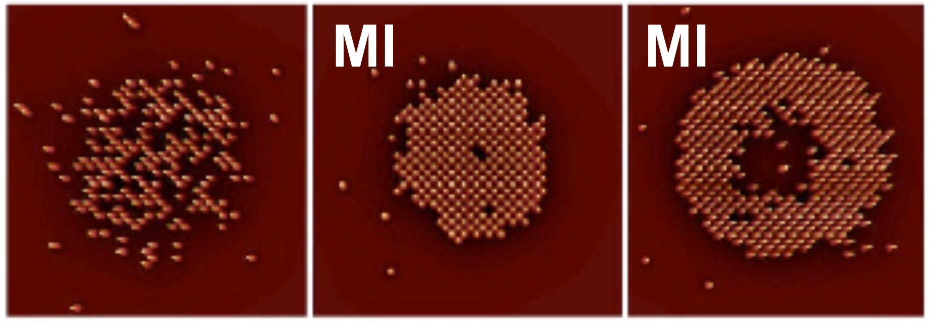


Mott-Insulator

$$|\psi_0\rangle = \prod_i a_i^\dagger |0\rangle$$

Density profiles

Single-site resolution



Measures parity of n_i

Bakr et al. Science 329, 547 (2010)
Weitenberg et al. Nature 471, 323 (2011)

$$H = H_0 + V \sum_i n_i i^2$$

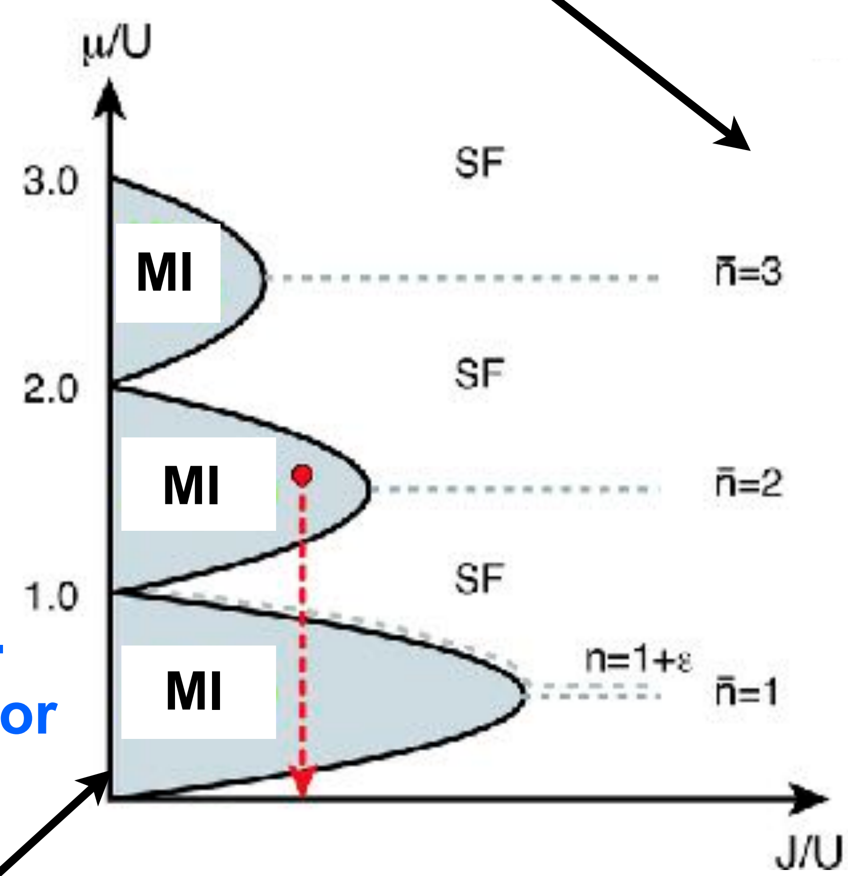
Bose-Hubbard model

$$H_0 = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu N$$

Superfluid

$$|\psi_0\rangle \propto \left(a_{k=0}^\dagger\right)^N |0\rangle$$

$$n_k = N\delta(k)$$



Mott-Insulator

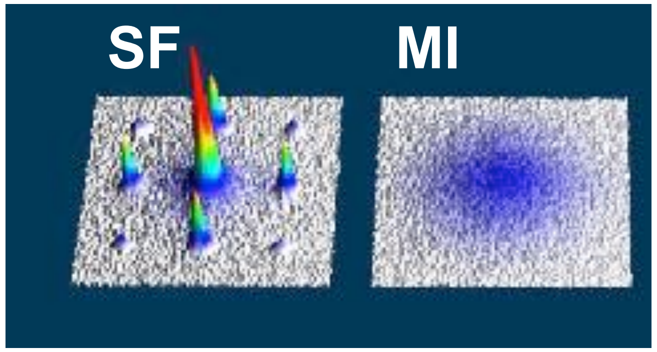
$$|\psi_0\rangle = \prod_i a_i^\dagger |0\rangle$$

$$n_k = const.$$

Quasi-momentum distribution (MDF)

$$n_k = \frac{1}{L} \sum_{lm} e^{i(l-m)k} \langle a_l^\dagger a_m \rangle$$

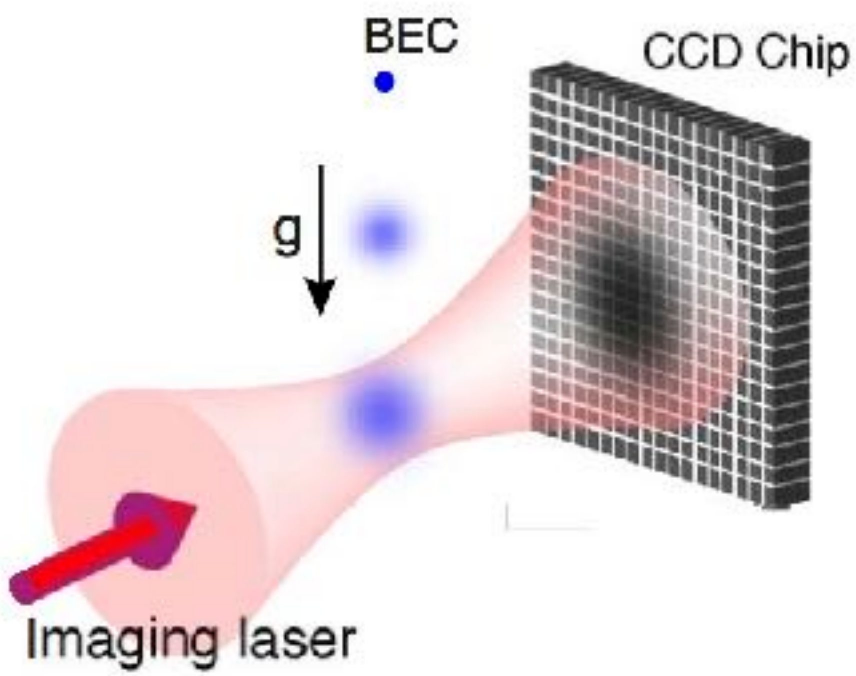
Time-of-flight → MDF



Greiner, Mandel, Esslinger, Hänsch, Bloch
Nature 415, 39 (2002)

Measuring the quasi-MDF

Time-of-flight
Removal of all potentials



Bloch, Dalibard, Zwerger RMP 2008

$$n(v) = \text{const.} \qquad n_k = \text{const.}$$

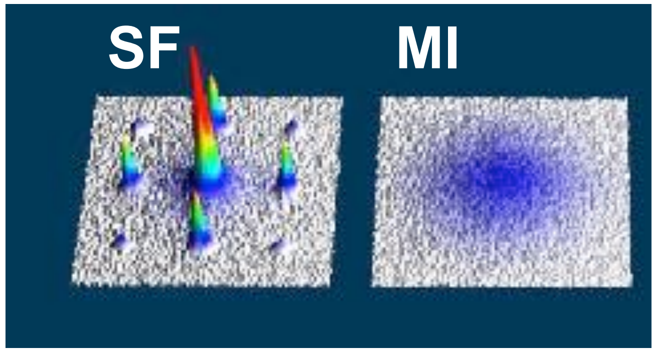
$$n(x, t = \infty) \propto n_k(t = 0)$$

Measurement of MDF n_k

Quasi-momentum distribution (MDF)

$$n_k = \frac{1}{L} \sum_{lm} e^{i(l-m)k} \langle a_l^\dagger a_m \rangle$$

Time-of-flight → MDF

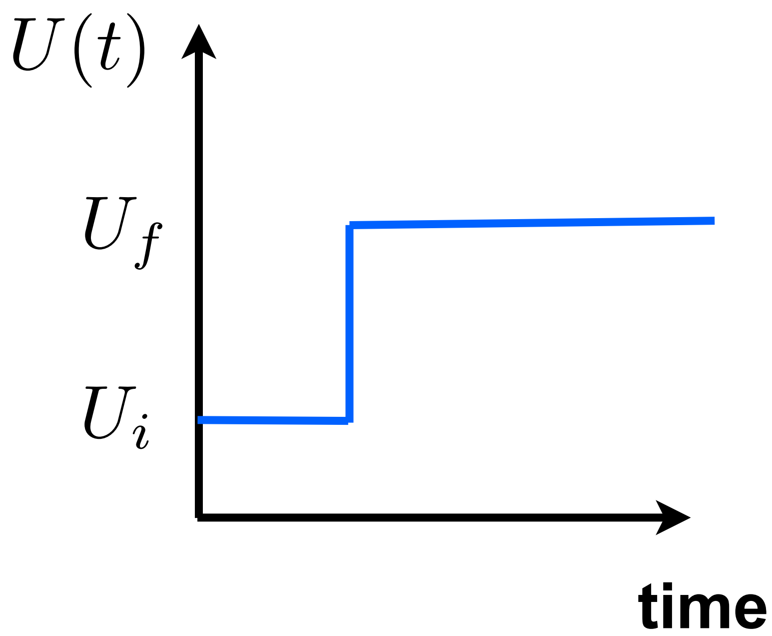


Greiner, Mandel, Esslinger, Hänsch, Bloch
Nature 415, 39 (2002)

Non-equilibrium dynamics: Experiments

Initial state: Superfluid, weak U/J

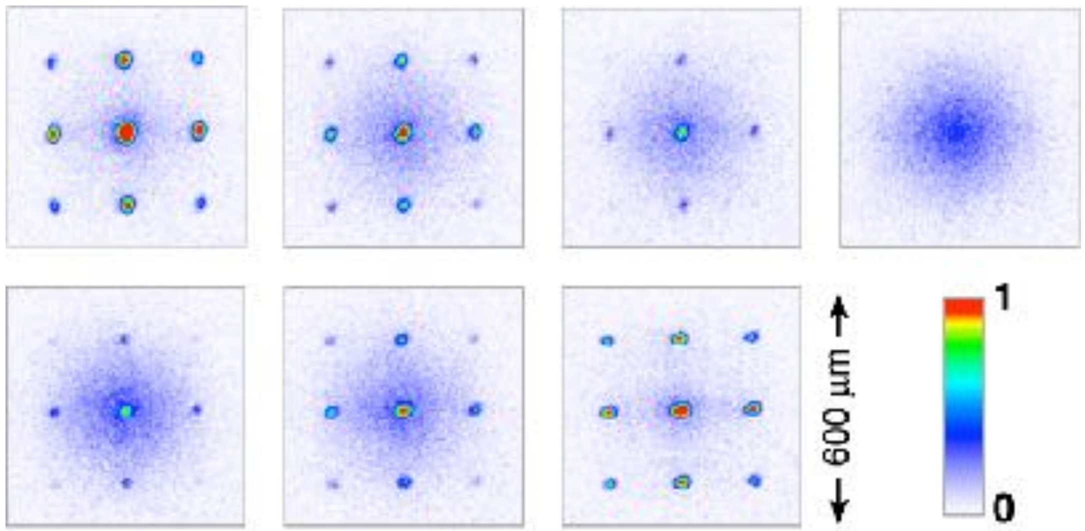
$$|\psi_{\text{initial}}\rangle \propto \prod_i \exp(-\gamma a_i^\dagger)|0\rangle_i$$



Quantum quench into deep MI

$$U_{\text{f}}/J \gg 1$$

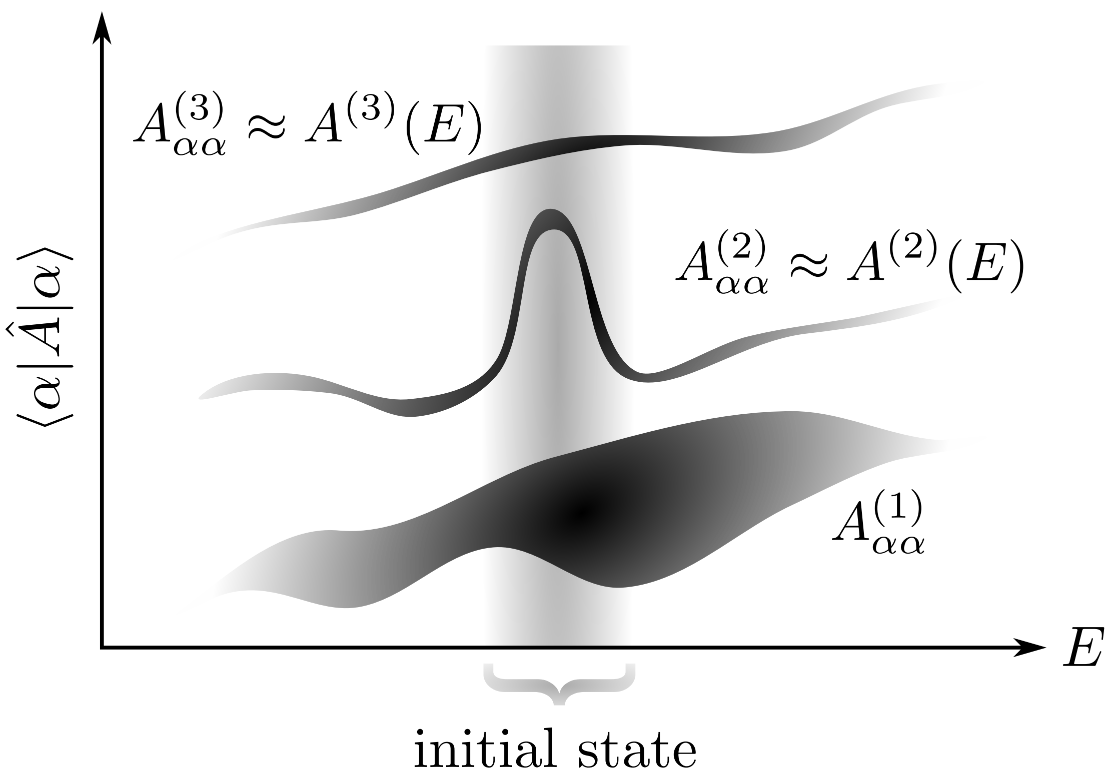
Collapse & Revival



Greiner, Mandel, Hänsch, Bloch Nature 419, 51 (2002) and subsequent work

$$|\psi(t)\rangle = e^{-itUn(n-1)/2}|\psi_{\text{init}}\rangle$$

Thermalization in closed quantum many-body systems



Eigenstate thermalization hypothesis

Random matrix theory

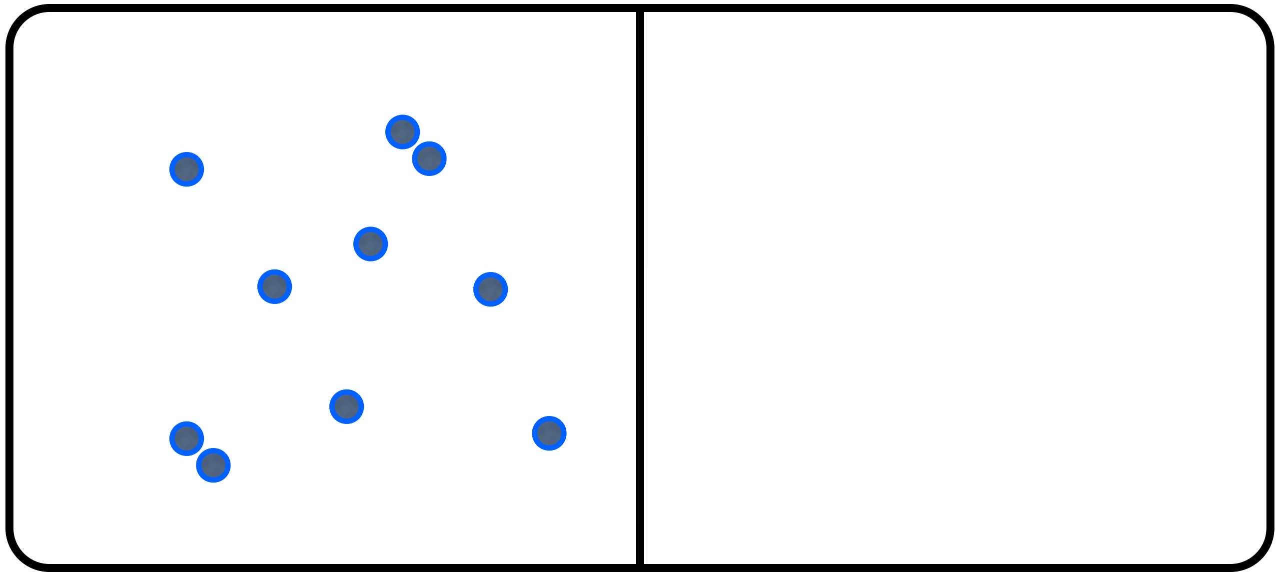
Reviews:

d'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. 65, 239 (2016)

Nandkishore, Huse, Annual Rev. Cond. Matt. Phys. 6, 15 (2015)

Relaxation & Thermalization

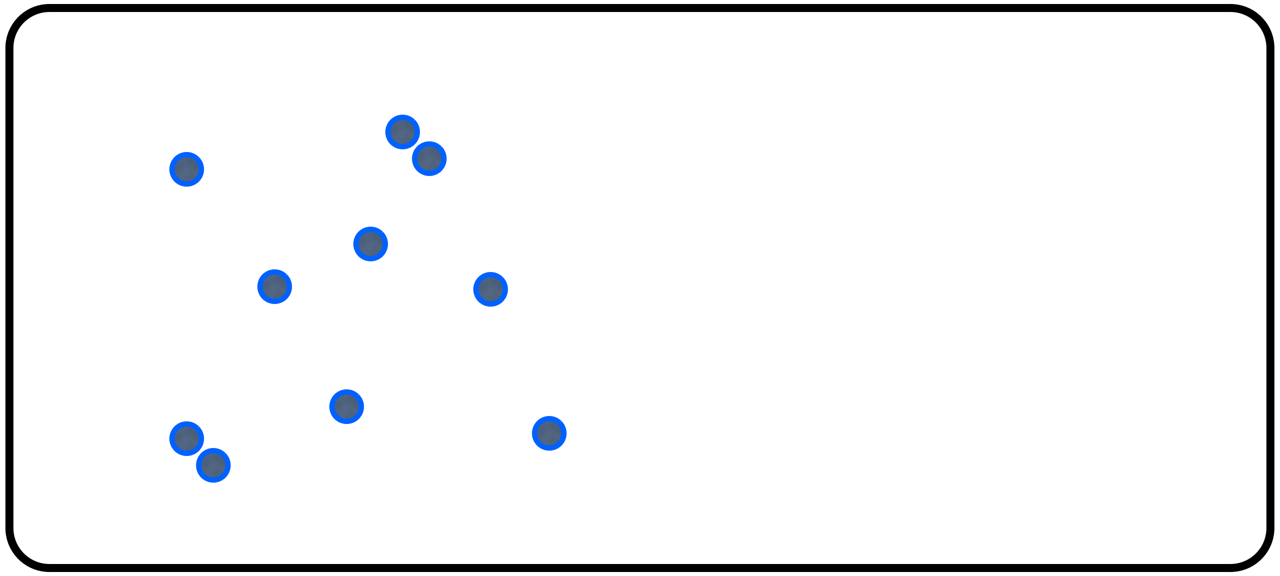
Standard example of a closed system: Ideal gas in a fixed volume V



Thermalization: Some interactions or coupling to a bath

Relaxation & Thermalization

Standard example of a closed system: Ideal gas in a fixed volume V

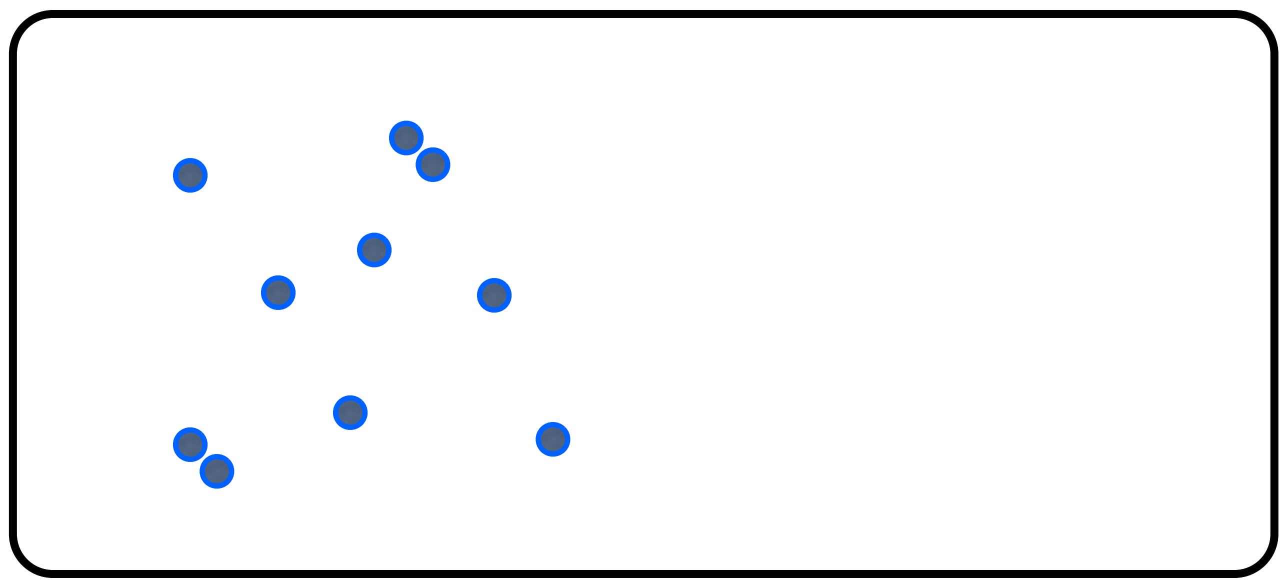


Thermalization: Some interactions or coupling to a bath

Relaxation & Thermalization

Standard example of a closed system: Ideal gas in a fixed volume V

Quantum realm! & Experiments!

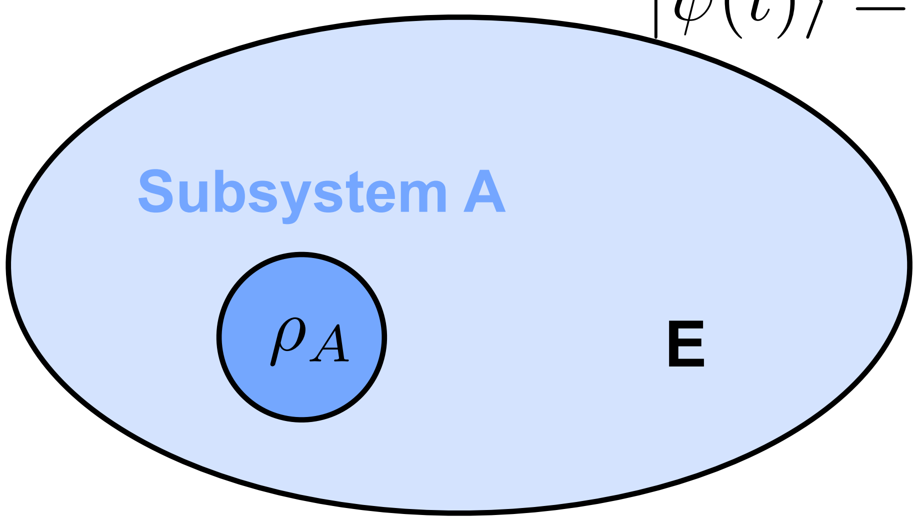


Thermalization: Some interactions or coupling to a bath

Thermalization in closed quantum systems

Closed quantum systems

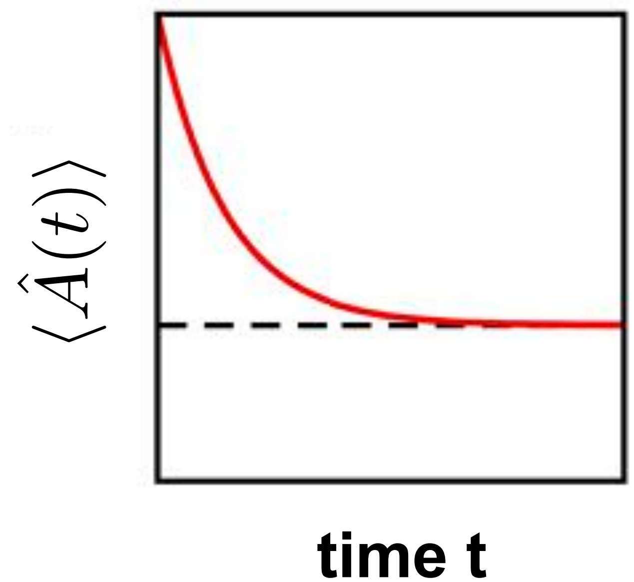
$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$$



Many-body system acts as its own bath:
Thermalization of subsystems

$$\rho_A = \mathrm{Tr}_E[|\psi(t)\rangle\langle\psi(t)|]$$

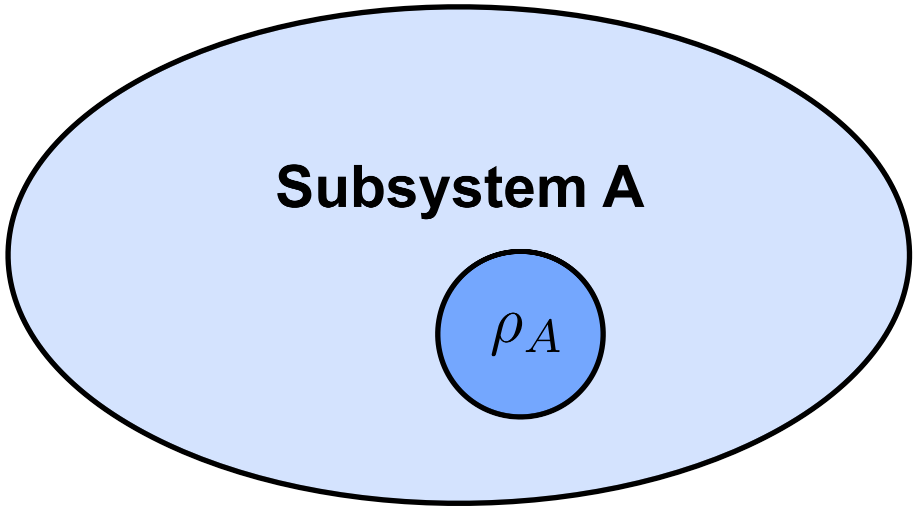
Non-equilibrium dynamics



? $= \mathrm{tr}[\rho_{\mathrm{eq.}} \hat{A}]$

$$\overline{\rho_A}^t \overset{?}{=} \rho_{\mathrm{eq.}}$$

Thermalization in closed quantum systems



$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$$

$$|\psi_0\rangle = \sum_n c_n|n\rangle \quad H|n\rangle = E_n\rangle$$

Observable acting on subsystem:

$$\langle \hat{A}(t)\rangle = \sum_n |c_n|^2 \langle n|\hat{A}|n\rangle + \sum_{n,m} c_n^* c_m e^{i(E_n - E_m)t}\langle n|\hat{A}|m\rangle$$

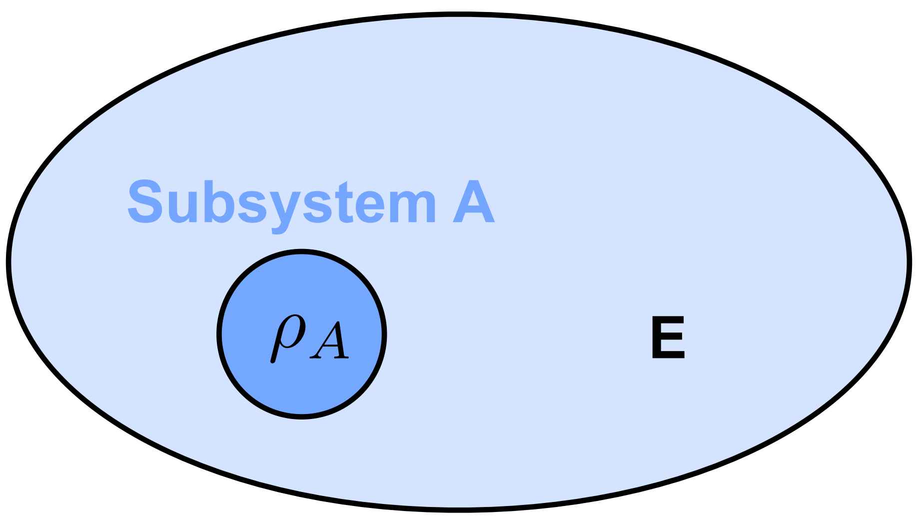
Initial state dependence!

Diagonal ensemble: $$\langle \hat{A}\rangle_{\mathrm{steady-state}} = \sum_n |c_n|^2 \langle n|\hat{A}|n\rangle$$

Eigenstate thermalization hypothesis (ETH)

Eigenstates of closed many-body system are thermal

Rigol, Dunjko, Olshanii Nature 2008; Prosen Phys. Rev. E 60, 3949 (1998), Deutsch Phys. Rev. A 43, 2046 (1991); Srednicki Phys. Rev. E 50, 888 (1994)



Many-body eigenstates:

$$H|n\rangle = E_n|n\rangle$$

Sub-system density matrices are thermal

$$\rho_A = \mathrm{tr}_E[|n\rangle\langle n|] = \rho_{\mathrm{thermal}}$$

Expectation values depend only on E:

$$E_n \approx E_m : \quad \langle n|\hat{A}|n\rangle \approx \langle m|\hat{A}|m\rangle$$

Other concepts: Typicality, quantum chaotic systems, …

Eigenstate thermalization hypothesis (ETH)

Eigenstates of closed many-body system are thermal

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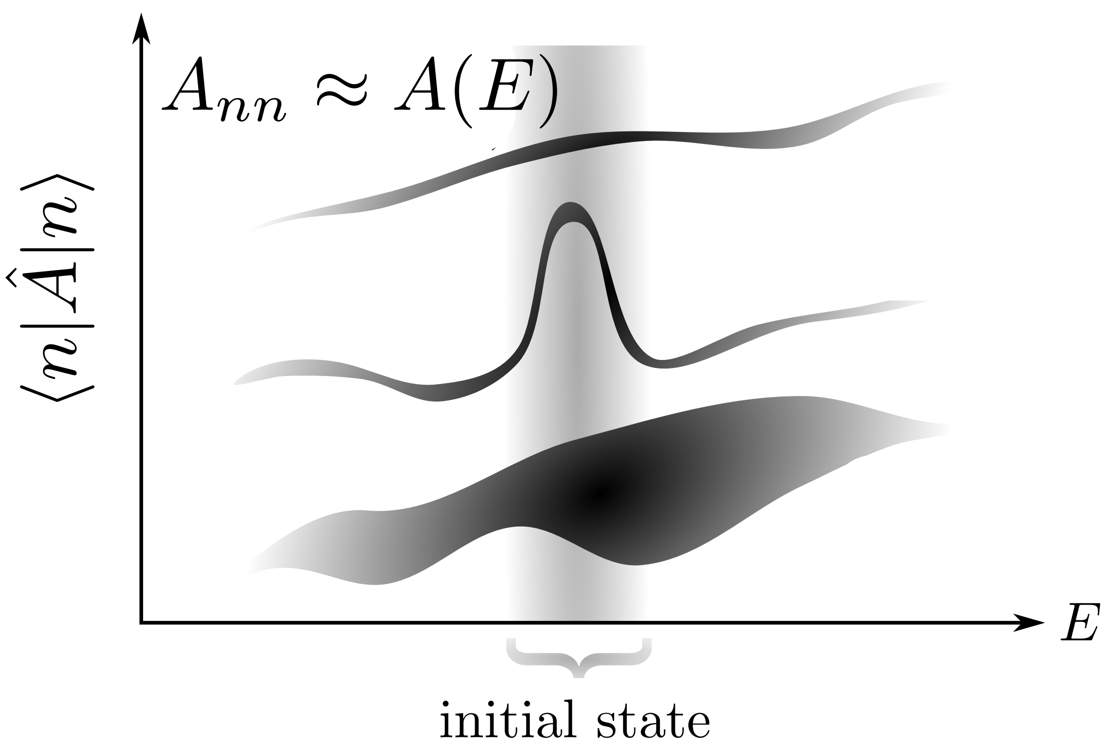

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Expectation values depend only on E:

$$E_n \approx E_m : \quad \langle n|\hat{A}|n\rangle \approx \langle m|\hat{A}|m\rangle$$

Initial-state independence !

$$\langle \hat{A}\rangle_{\mathrm{steady-state}} = \sum_n |c_n|^2 \langle n|\hat{A}|n\rangle = f(E)$$

Eigenstate thermalization hypothesis (ETH)

Eigenstates of closed many-body system are thermal

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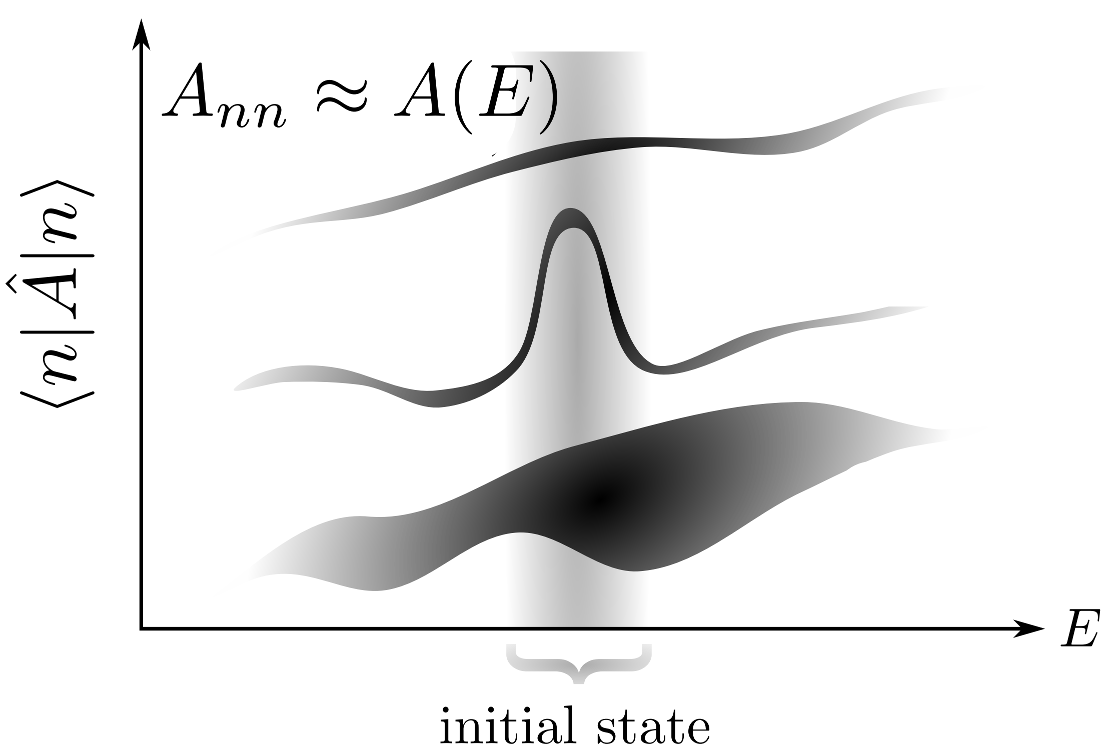

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Sub-system density matrices are thermal

$$\rho_A = \mathrm{tr}_E[|n\rangle\langle n|] = \rho_{\mathrm{thermal}}$$

Expectation values depend only on E:

$$E_n \approx E_m : \quad \langle n|\hat{A}|n\rangle \approx \langle m|\hat{A}|m\rangle$$

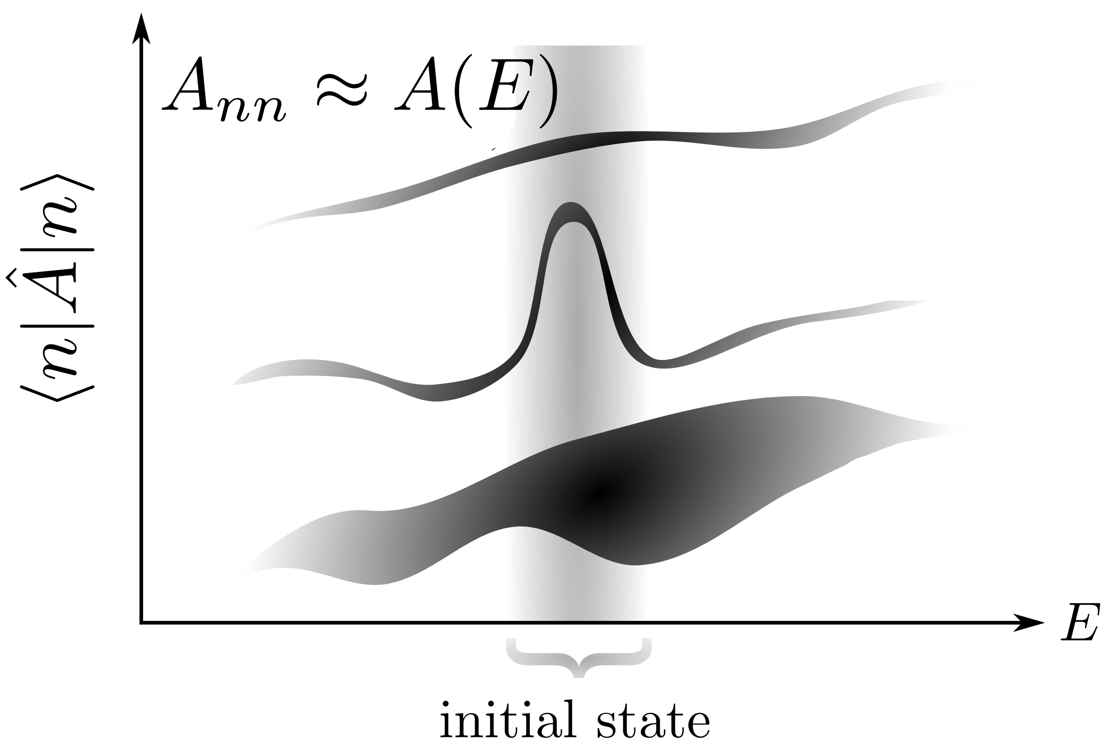
Micro-canonical ensemble

$$\langle \hat{A}\rangle_{\mathrm{steady-state}} = \langle \hat{A}\rangle_{\mathrm{mc}} = \frac{1}{\Delta}\sum_n \langle n|\hat{A}|n\rangle; \quad E + \Delta < E_n < E + \Delta$$

Eigenstate thermalization hypothesis (ETH)

Eigenstates of closed many-body system are thermal

Rigol, Dunjko, Olshanii Nature 2008; Prosen Phys. Rev. E 60, 3949 (1998), Deutsch Phys. Rev. A 43, 2046 (1991); Srednicki Phys. Rev. E 50, 888 (1994)



Many-body eigenstates:

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Sub-system density matrices are thermal

$$\rho_A = \mathrm{tr}_E[|n\rangle\langle n|] = \rho_{\mathrm{thermal}}$$

Expectation values depend only on E:

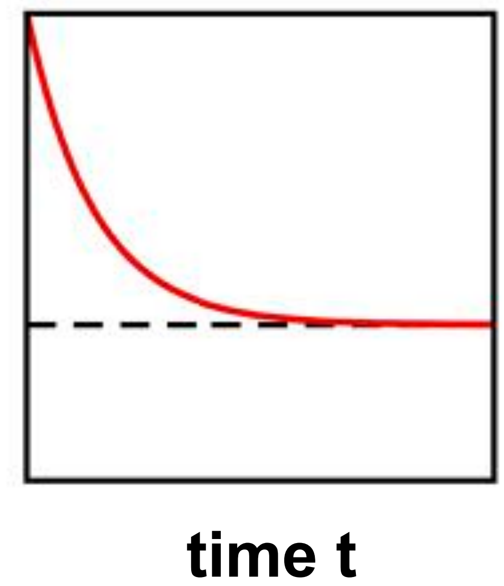
$$E_n \approx E_m : \quad \langle n|\hat{A}|n\rangle \approx \langle m|\hat{A}|m\rangle$$

Implication: "single-state" micro-canonical ensemble

$$\langle \hat{A}\rangle_{\mathrm{mc}}(E) = \langle n|\hat{A}|n\rangle; \quad E_n \approx E$$

Eigenstate thermalization hypothesis (ETH)

Off-diagonal elements are exponentially small:
Ensures relaxation to steady-state value



? $= \mathrm{tr}[\rho_{\mathrm{eq.}} \hat{A}]$

Dephasing!

$$\langle \hat{A}(t) \rangle = \sum_n |c_n|^2 \langle n|\hat{A}|n\rangle + \sum_{n,m} c_n^* c_m e^{i(E_n - E_m)t} \langle n|\hat{A}|m\rangle$$

Diagonal ensemble: $$\langle \hat{A} \rangle_{\mathrm{steady-state}} = \sum_n |c_n|^2 \langle n|\hat{A}|n\rangle$$

Eigenstate thermalization hypothesis (ETH)

Typical example: 1D Bose-Hubbard model

$$H_0 = -J \sum_i (a_i^\dagger a_{i+1} + h.c.) + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Initial state:

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$$

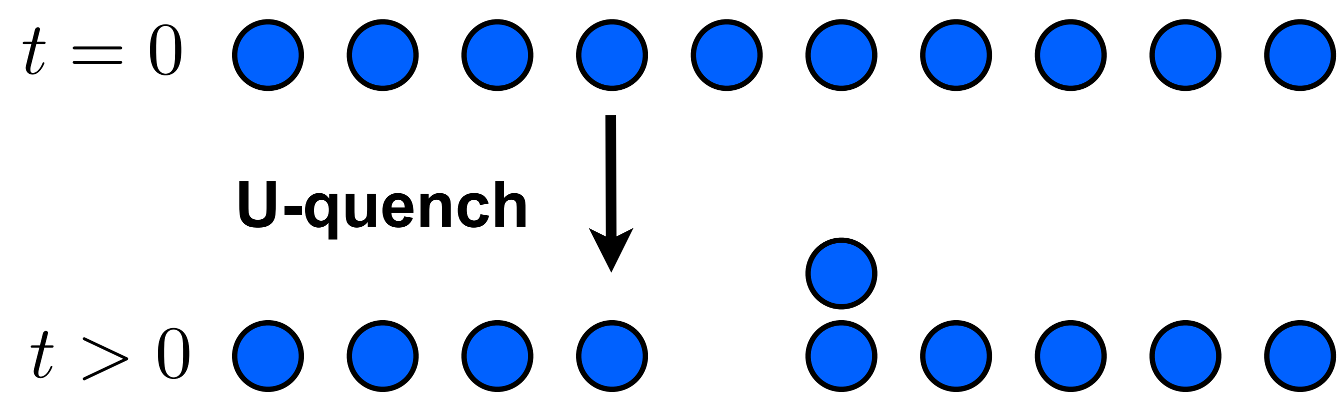
Quench from:

$$U/J = \infty \to U/J < \infty$$

Higher occupancy:

$$\hat{A} = \sum_{\nu > 1} \hat{P}_i(\nu) = \sum_{\nu > 1} |\nu\rangle_{ii}\langle\nu|$$

$$P(\nu = 0) = P(\nu = 2) = 0$$



$$P(\nu = 0), P(\nu = 2) > 0$$

Sorg, Vidmar, Pollet, FHM Phys. Rev. A 90, 033606 (2014)

Eigenstate thermalization hypothesis (ETH)

Typical example: 1D Bose-Hubbard model

$$H_0 = -J\sum_i (a_i^\dagger a_{i+1} + h.c.) + \frac{U}{2}\sum_i n_i(n_i - 1)$$

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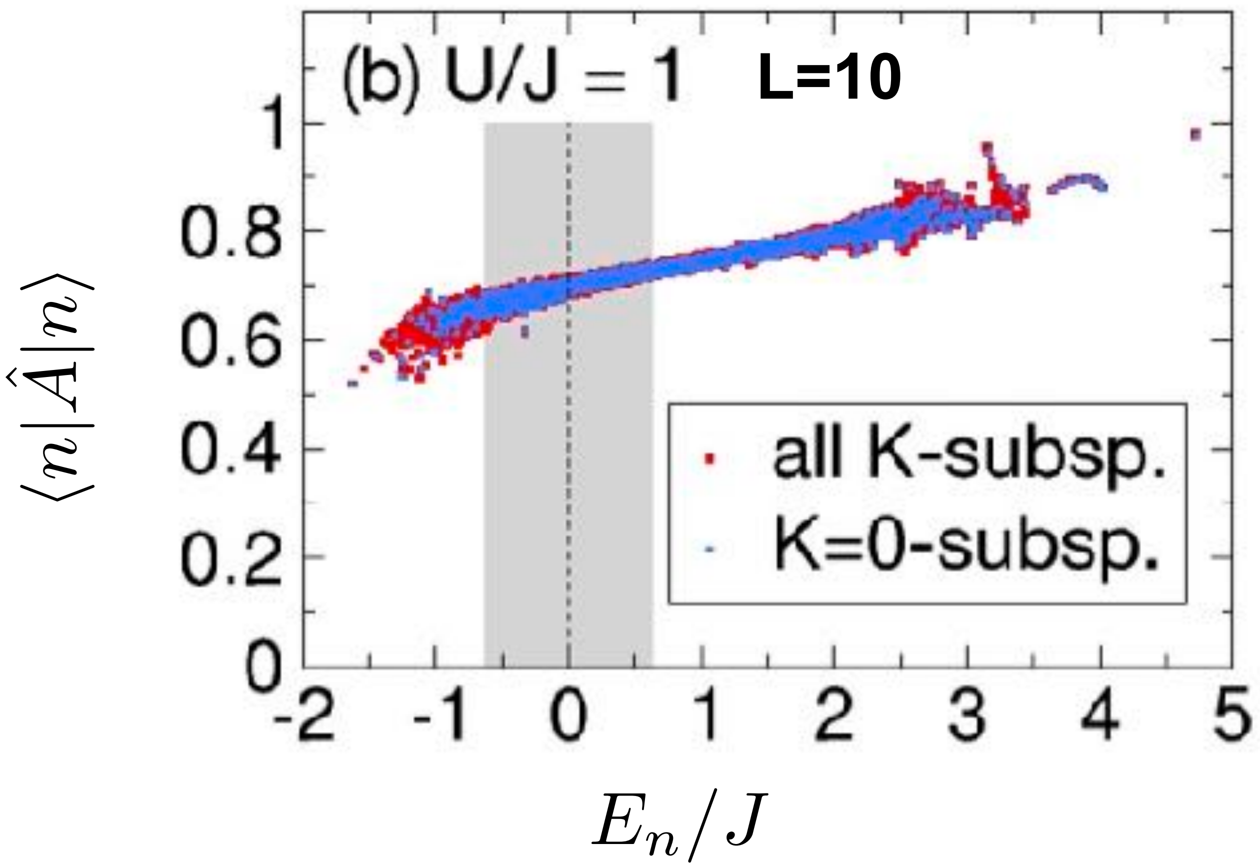

$$\langle n|\hat{A}|n\rangle \approx \langle n'|\hat{A}|n'\rangle \quad E_n \approx E_{n'}$$

Sorg, Vidmar, Pollet, FHM Phys. Rev. A 90, 033606 (2014)

Many other numerical verifications of ETH: Rigol, Santos, …

Eigenstate thermalization hypothesis (ETH)

Typical example: 1D Bose-Hubbard model

$$H_0 = -J\sum_i (a_i^\dagger a_{i+1} + h.c.) + \frac{U}{2}\sum_i n_i(n_i - 1)$$

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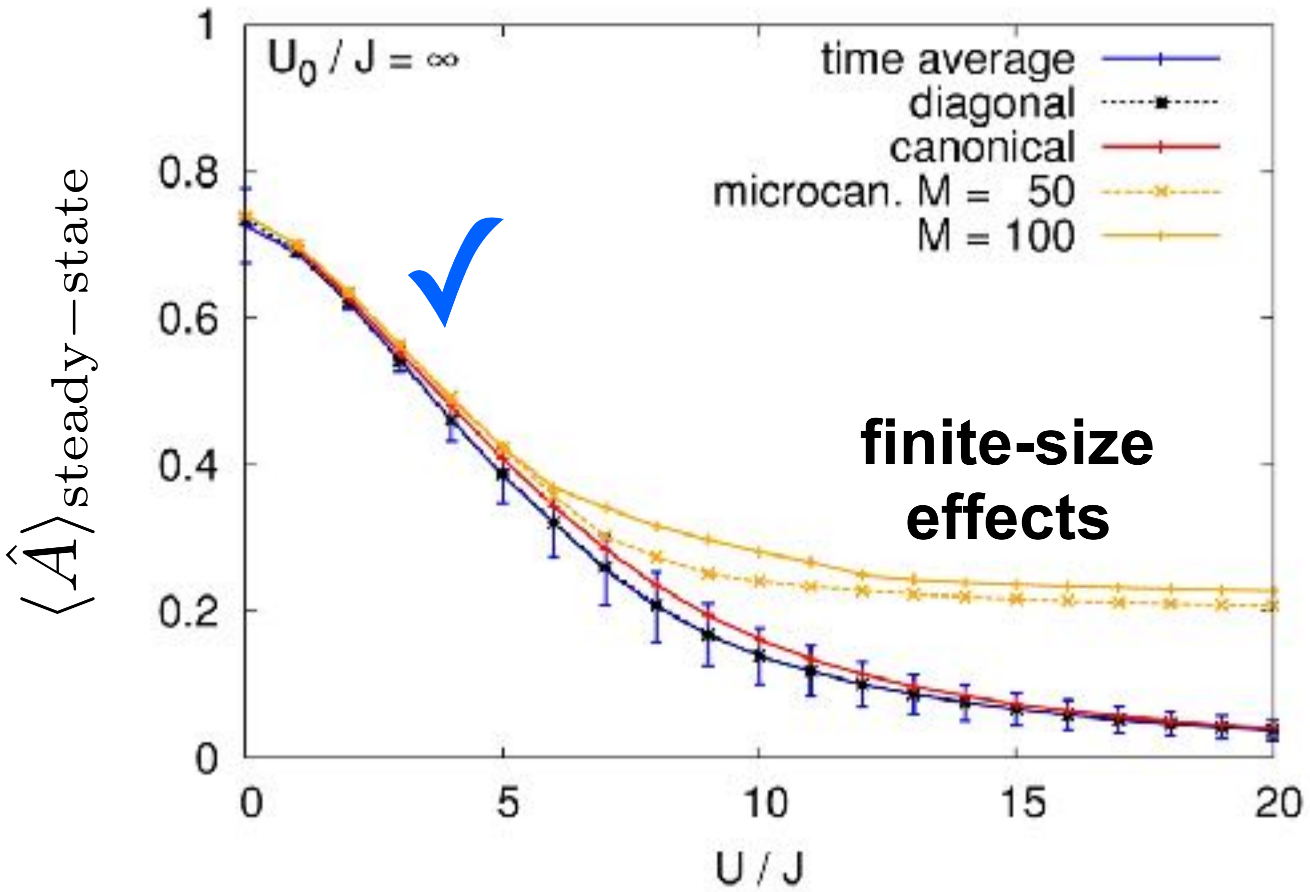
Higher occupancy:

$$\hat{A} = \sum_{\nu>1} \hat{P}_i(\nu) = \sum_{\nu>1} |\nu\rangle_{ii}\langle\nu|$$



Steady state is thermal!

Sorg, Vidmar, Pollet, FHM Phys. Rev. A 90, 033606 (2014)

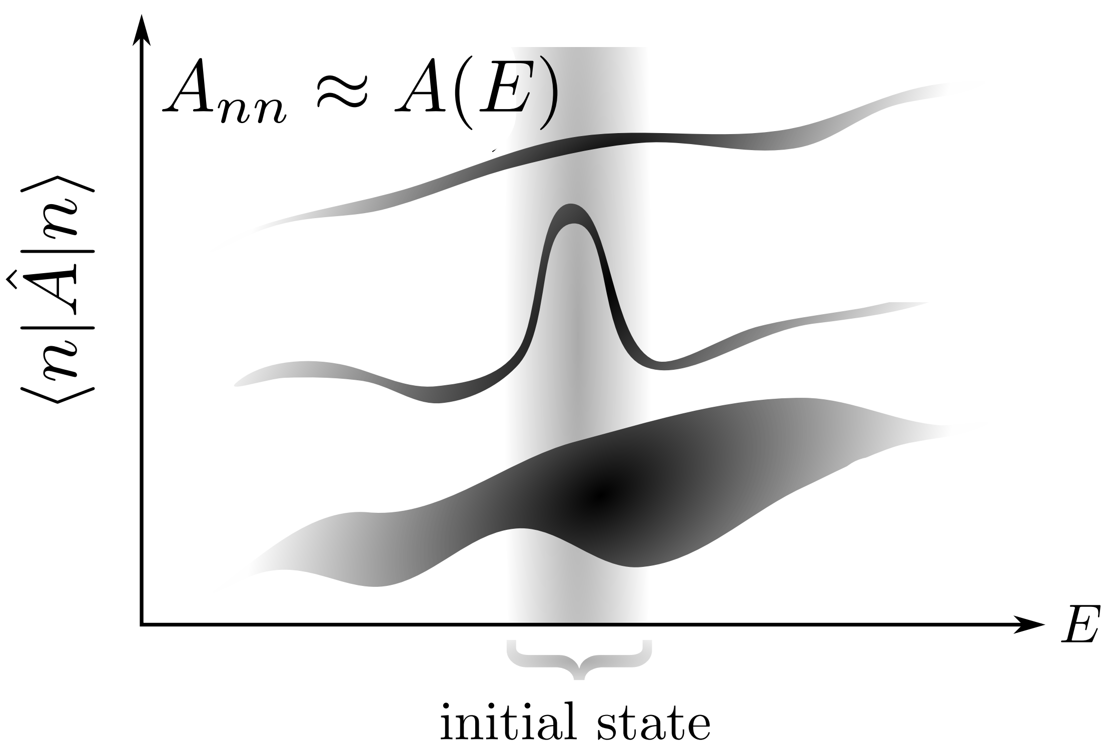
Eigenstate thermalization hypothesis (ETH)

Summary

(1) Eigenstate expectation values of local observables function of energy

(2) Off-diagonal elements are exponentially small: Ensures relaxation!

(3) Initial states are narrow in energy



Energy spread of initial state

$$\sigma^2_{\rm diag} = \langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2$$

Physically relevant states:

$$\frac{\sigma_{\rm diag}}{LJ} = \frac{2}{\sqrt{L}}$$

Rigol, Dunjko, Olshanii Nature 2008 Deutsch Phys. Rev. A 43, 2046 (1991) Srednicki Phys. Rev. E 50, 888 (1994)

Random matrix theory

Assume: Hamiltonian is a random matrix

Eigenvectors are essentially random (but orthogonal) In any basis

$$\overline{(\psi_i^m)^*(\psi_j^n)} = \frac{1}{\mathcal{D}}\delta_{mn}\delta_{ij}$$

Matrix elements of local observables

$$A_{mn} = \overline{A}\delta_{nm} + \sqrt{\frac{\overline{A}}{\mathcal{D}}}R_{nm}$$

Random number (R_{nm})

Hilbert space dimension ($\mathcal{D}$)

This fulfils ETH (at infinite T)!

Review: d'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. 65, 239 (2016)

Generalization of Random Matrix Theory

$$A_{mn} = \overline{A}\delta_{nm} + \sqrt{\frac{\overline{A}}{\mathcal{D}}} R_{nm}$$

Alternative formulation of ETH: *M. Srednicki, J. Phys. A 32 , 1163 (1999)*

$$A_{nm} = A(\overline{E})\delta_{nm} + e^{-S(\overline{E})/2} f_A(\overline{E},\omega) R_{n,m}$$

Smooth functions

$$\overline{E} = (E_n + E_m)/2; \quad \omega = E_n - E_m \quad S(\overline{E}) : \text{entropy}$$

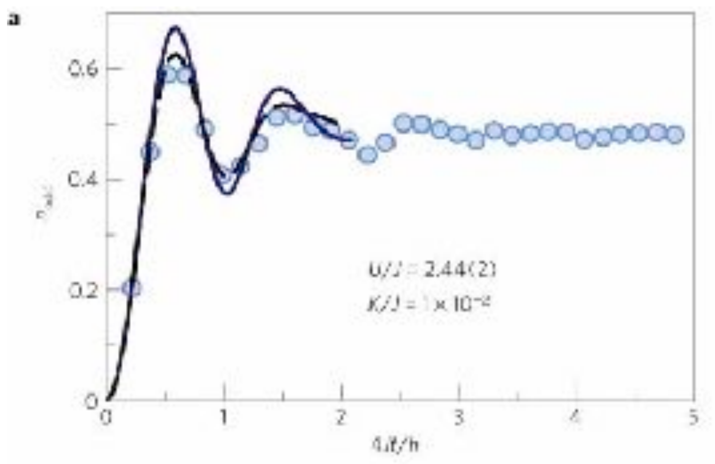
In what sense can many-body Hamiltomians be interpreted as random matrix?

'cause they are not, either in their eigenbasis or computational basis

Review: d'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. 65, 239 (2016)

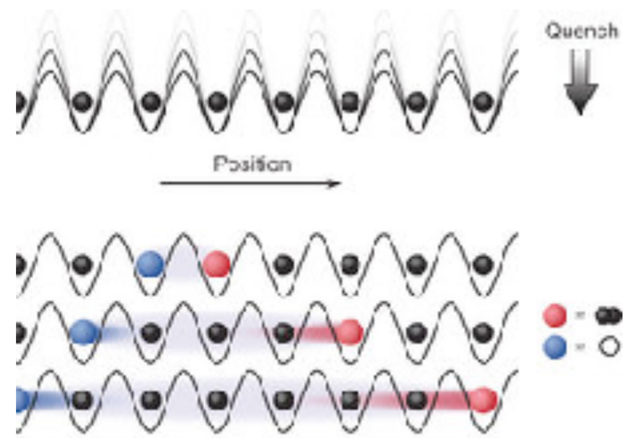
Thermalization: Experiments

Decay of ideal CDW state



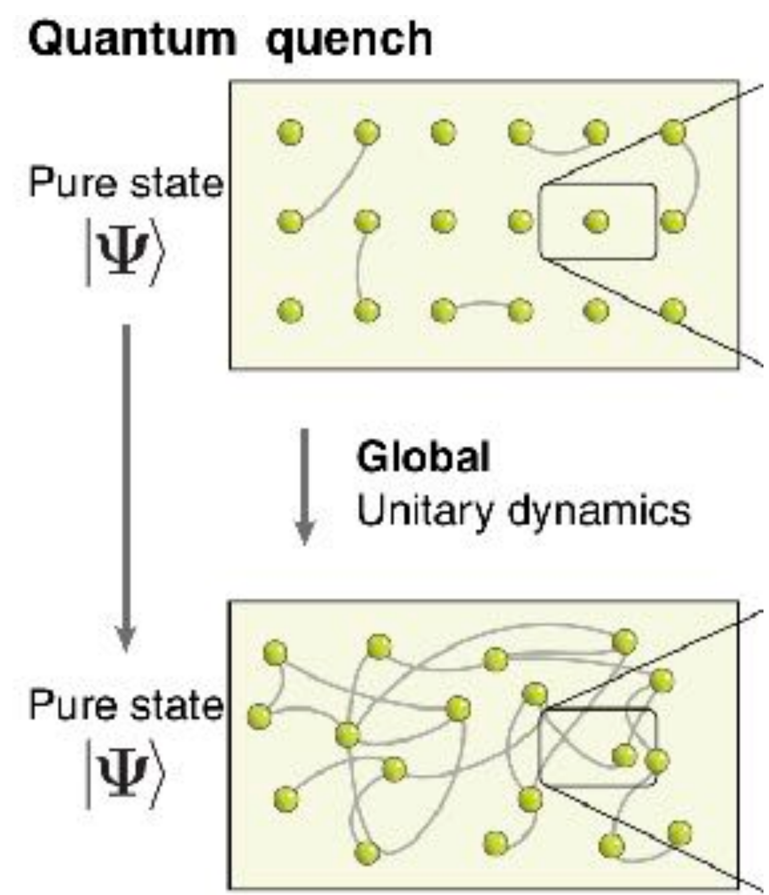
Trotzky et al. Nat. Phys. 8, 325 (2012)

Spreading of correlations



Cheneau et al. Nature 481, 484 (2012)

Probing thermalisation



Kaufman et al. Science 353, 794 (2016)

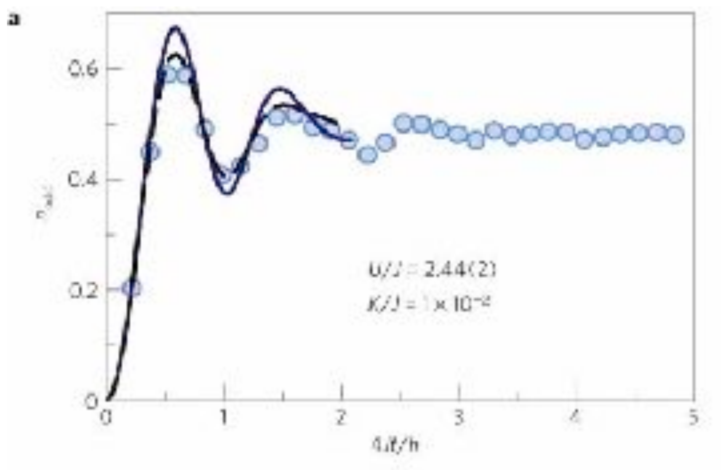
$$|\psi(t=0)\rangle = |1,0,1,0,1,0,\ldots\rangle$$

$$|\psi(t=0)\rangle = |1,1,1,1,1,1,\ldots\rangle$$

1D Bose-Hubbard model $$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i-1)$$

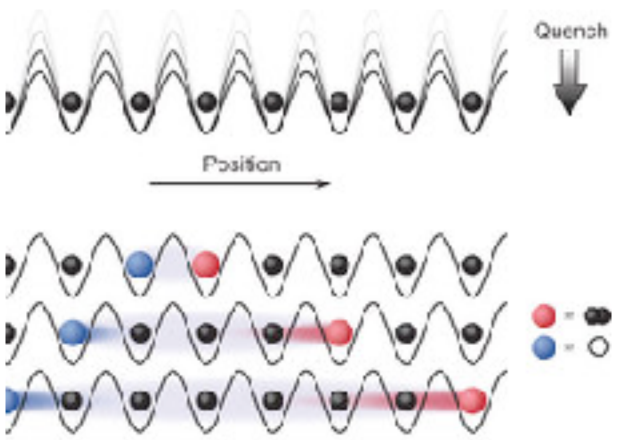
Thermalization: Experiments

Decay of ideal CDW state



Trotzky et al. Nat. Phys. 8, 325 (2012)

Spreading of correlations



Cheneau et al. Nature 481, 484 (2012)

$$|\psi(t=0)\rangle = |1,0,1,0,1,0,\ldots\rangle$$

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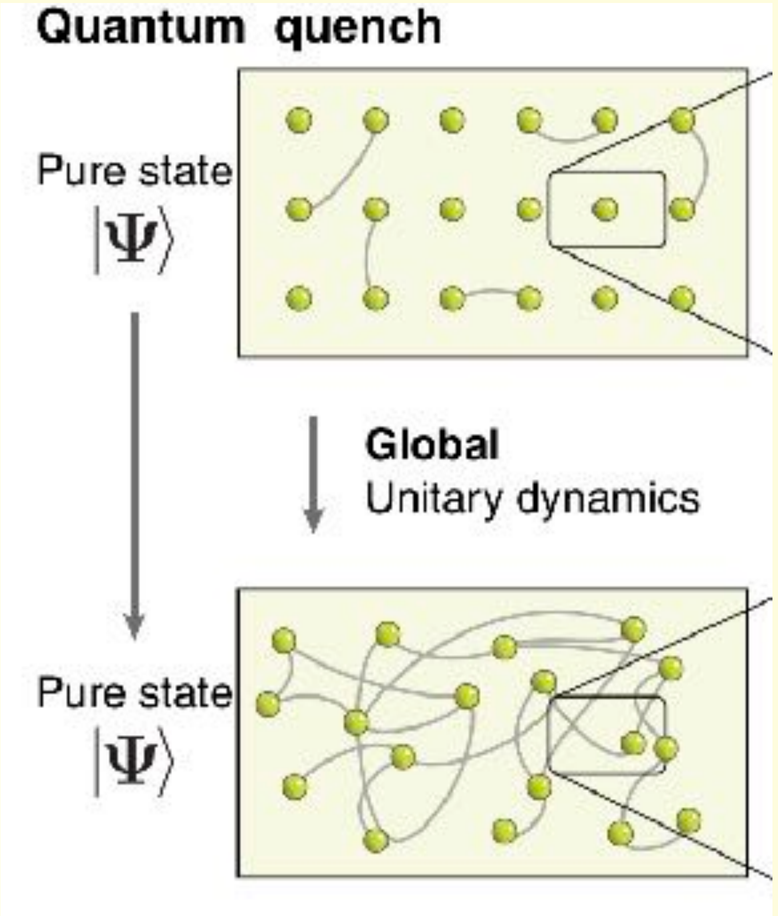
Probing thermalisation



Kaufman et al. Science 353, 794 (2016)

Thermalization in experiments

Quantum gas microscope:
Projective (parity) measurement

Measures:

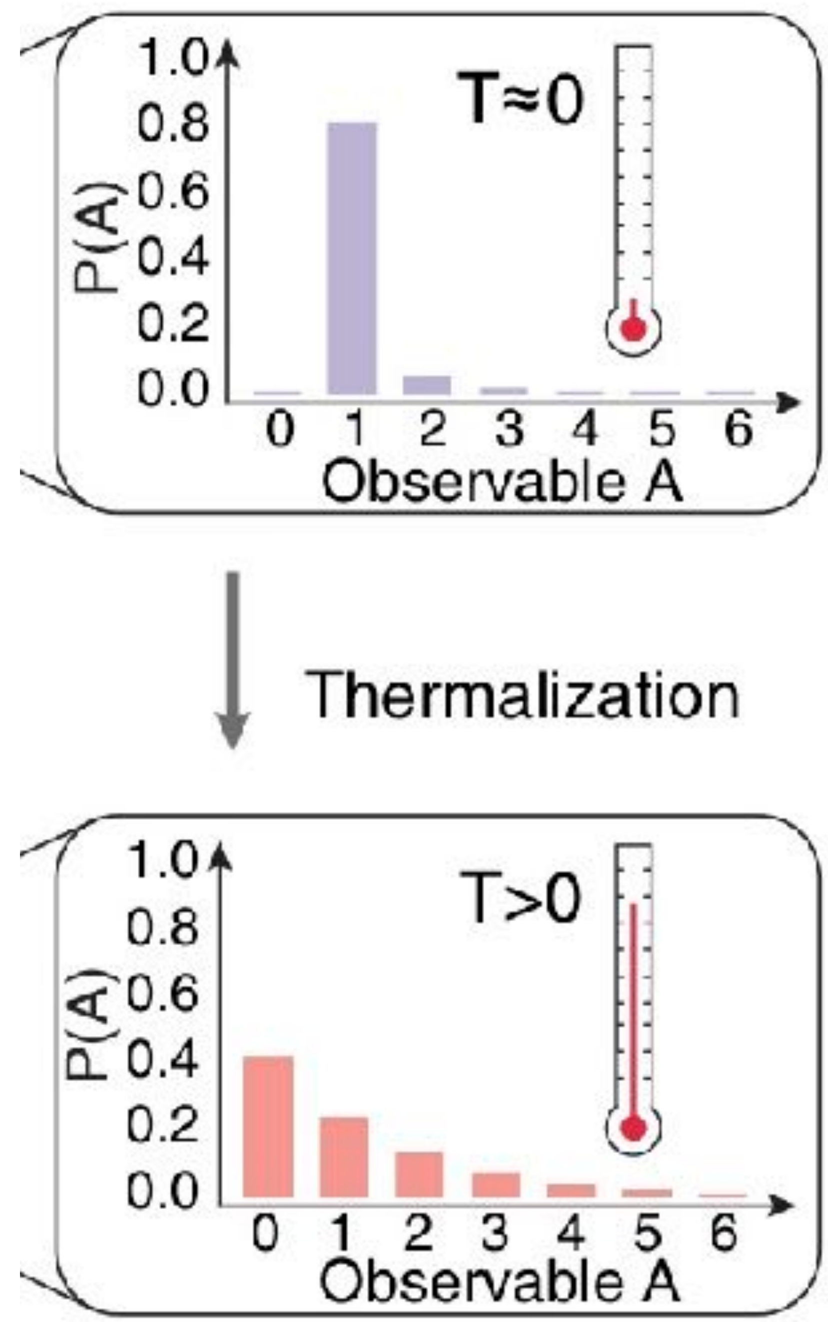
Projection on local particle numbers

$$\langle \hat{n}_i \rangle \rightarrow \nu = 0, 1, 2, 3, \ldots$$

$P(\nu)$ **Probability to measure nu particles/site**

Entanglement entropy via Interference between two copies
→ **More maybe in Hui Zhai's lecture**

Initial state

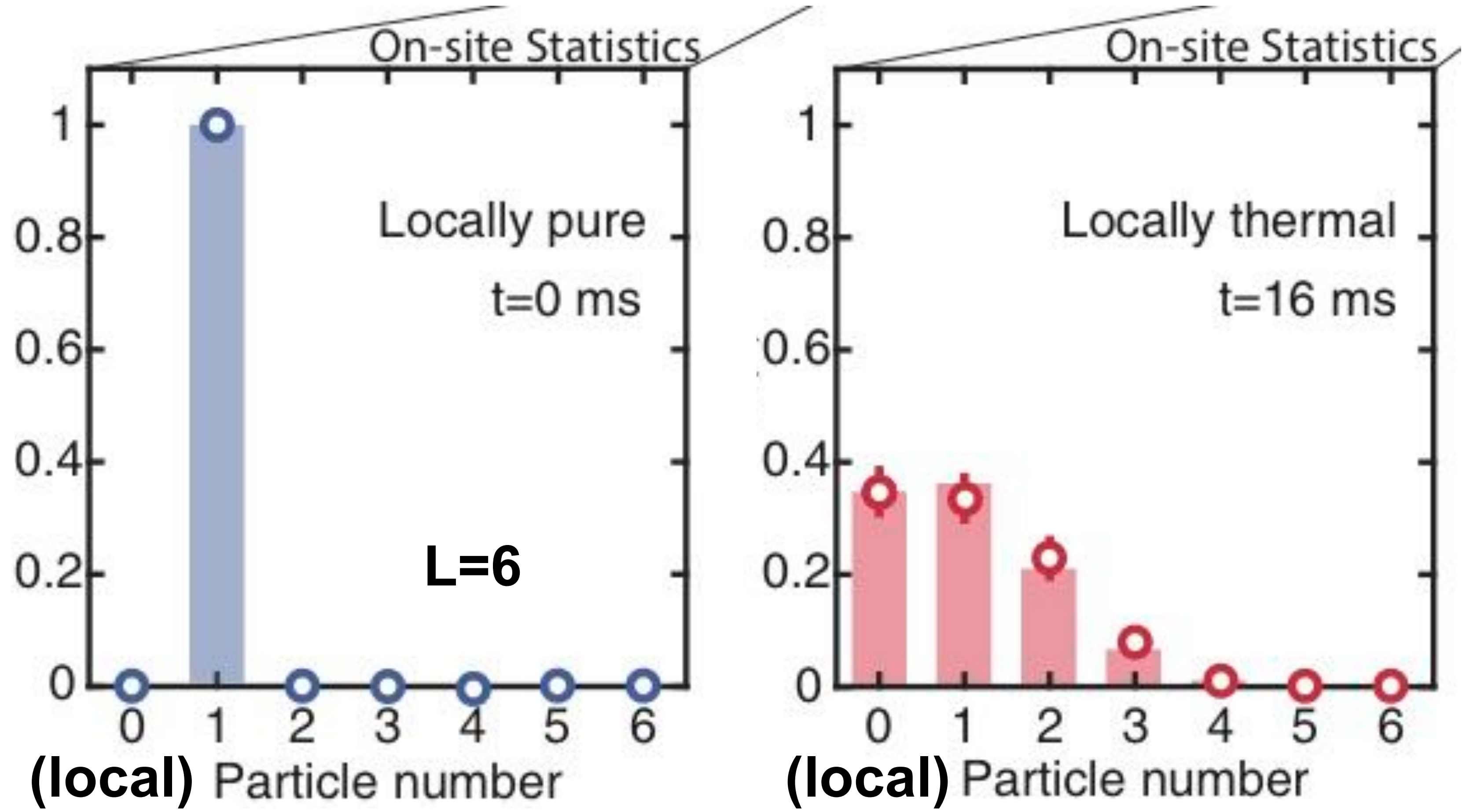


Thermalization in experiments

1D Bose-Hubbard model $|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$

Experimental data quench to: $U/J \approx 1.56$

$P(\nu)$



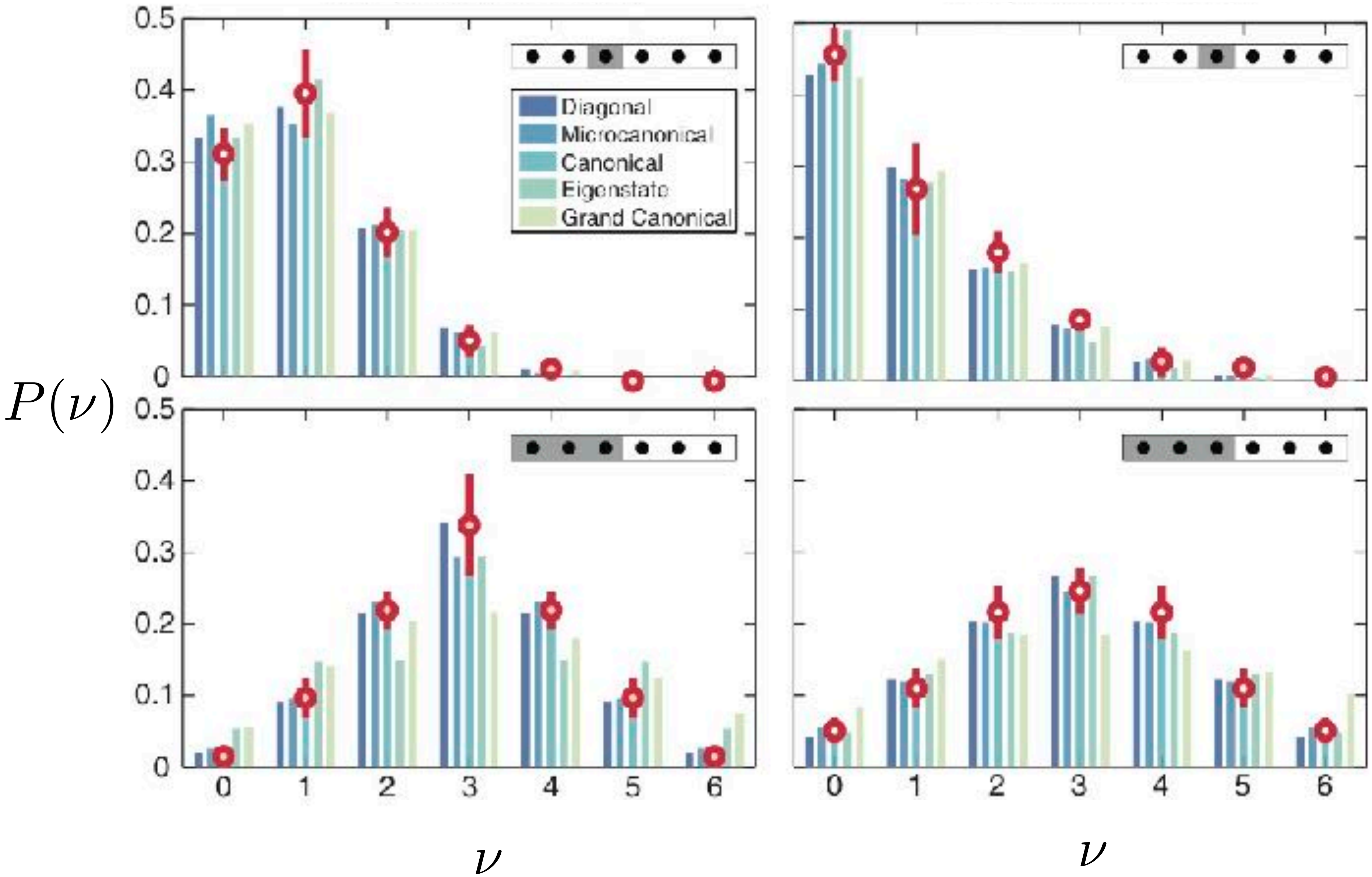
(local) **(local)**

Kaufman et al. Science 353, 794 (2016) (Greiner Lab, Harvard)

Thermalization in experiments?

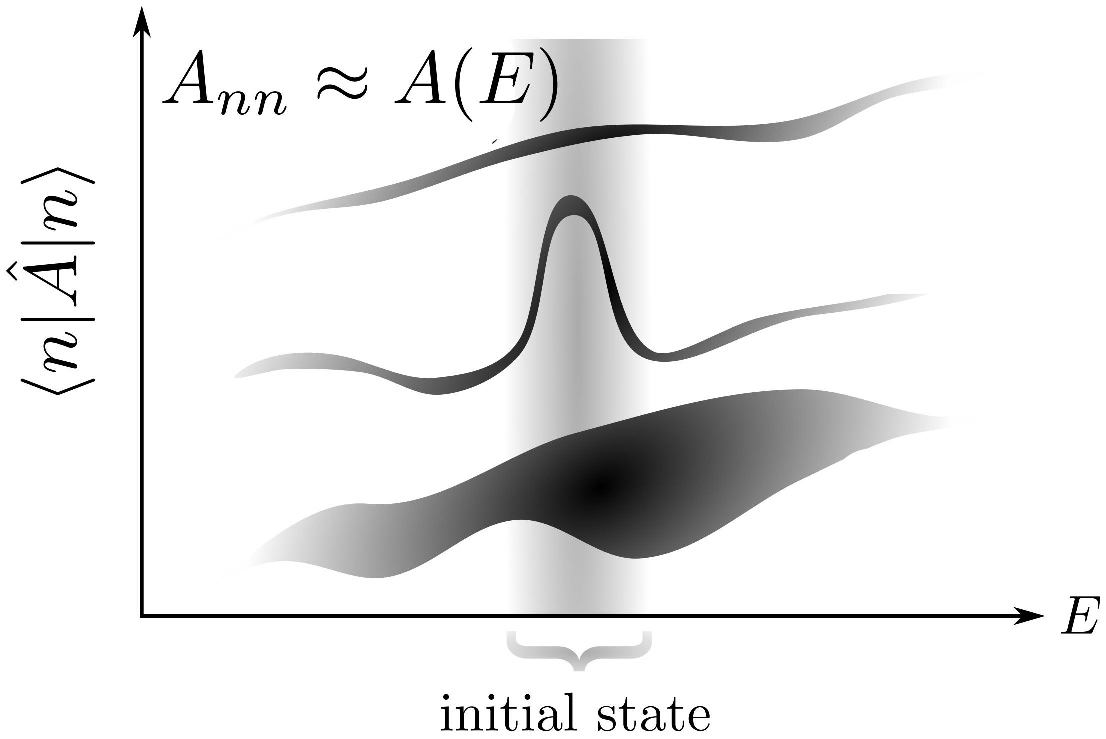
1D Bose-Hubbard model $|\psi_{\mathrm{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$

$U/J \approx 1.56$ $U/J \approx 0.4$



Kaufman et al. Science 353, 794 (2016) (Greiner Lab, Harvard)

Eigenstate thermalization hypothesis (ETH)



Take home message I:

Generic many-body systems thermalize

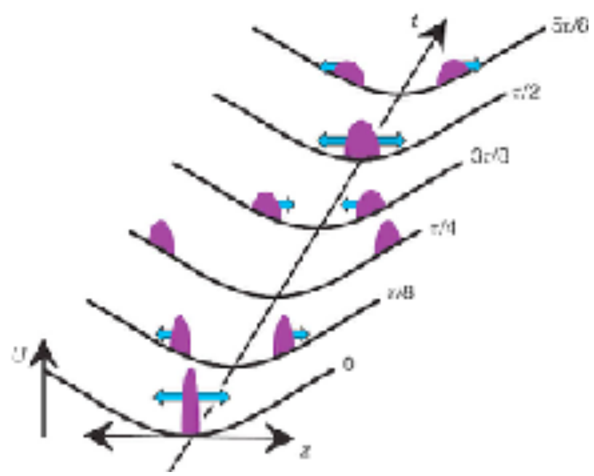
In the ETH sense

Rigol, Dunjko, Olshanii Nature 452, 854 (2008);
Deutsch Phys. Rev. A 43, 2046 (1991); Srednicki Phys. Rev. E 50, 888 (1994)

Thermalization: Exceptions

Integrable 1D systems
(Natan Andrei's lecture)

$$[H, I_i] = 0$$

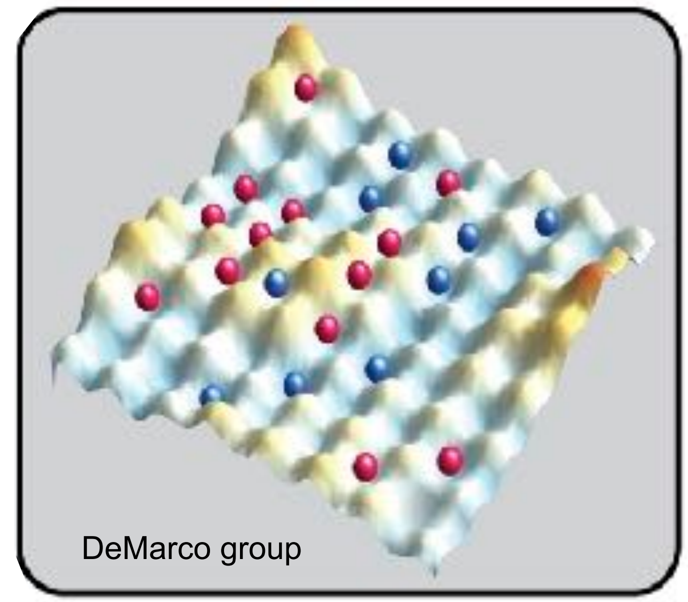


Lieb-Liniger model: 1D Bose gas

Kinoshita, Wenger, Weiss Nature, 440, 900 (2006)
Schmiedmayer group:
Langen et al. Science 348, 207 (2015)

Integrable systems (can) avoid thermalization

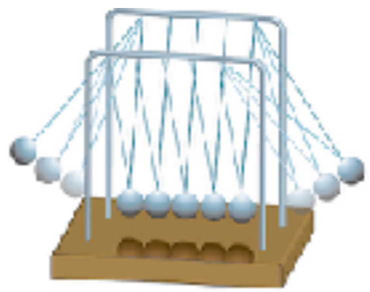
Many-body localization
(Wednesday)



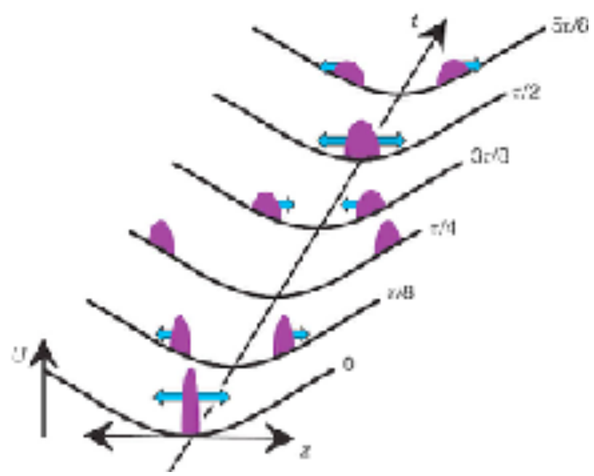
MBL systems don't thermalize, no fine-tuning

Thermalization: Exceptions

Integrable 1D systems
(Natan Andrei's lecture)



$$[H, I_i] = 0$$

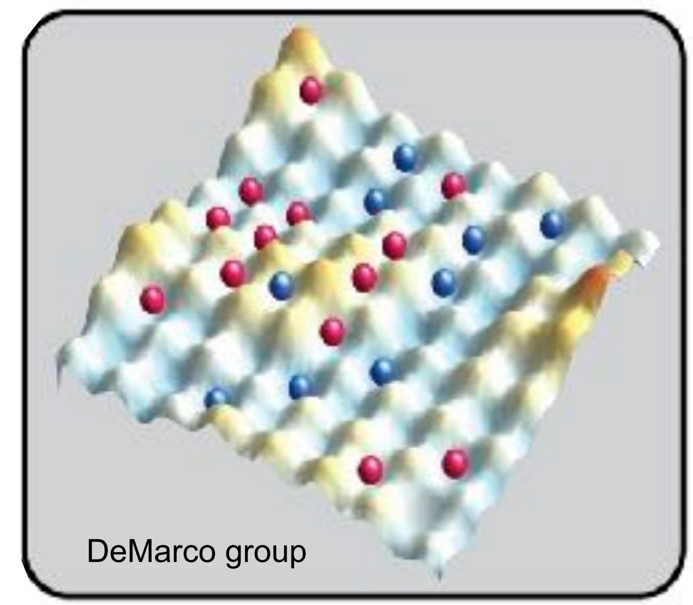


Lieb-Liniger model: 1D Bose gas

Kinoshita, Wenger, Weiss Nature, 440, 900 (2006)
Schmiedmayer group:
Langen et al. Science 348, 207 (2015)

Integrable systems are also unusual conductors

Many-body localization
(Wednesday)



MBL systems don't thermalize, no fine-tuning

Transport in strongly correlated 1D systems

Ballistic transport in integrable 1D systems

$$\mathrm{Re}\,\sigma(\omega) = D(T)\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

Conserved currents:

$$[H, J] = 0 \quad \rightarrow \quad \mathrm{Re}\ \sigma(\omega) = D(T)\delta(\omega)$$

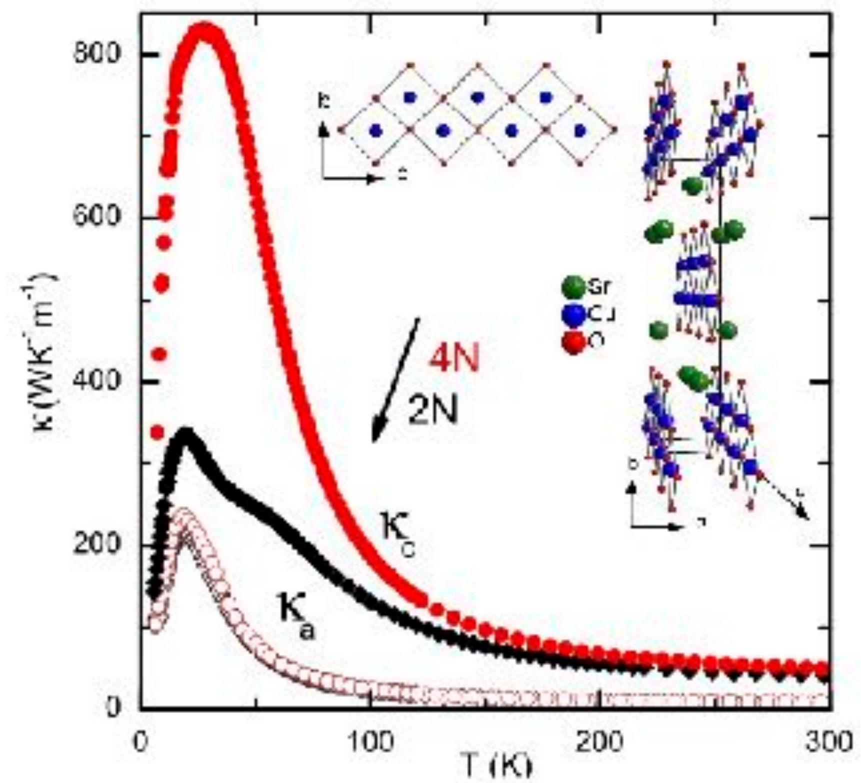
(local) Conservation laws/Mazur inequality:

$$[H, J] \neq 0 \quad \text{but} \quad [H, I_\alpha] = 0$$

$$D(T) \geq \mathrm{const}\frac{|\langle J I_\alpha \rangle|^2}{\langle I_\alpha^2 \rangle} > 0$$

Zotos, Naef, Prelovsek PRB 55, 11027 (1997)
FHM, Honecker, Brenig EPJST (2007)
Prosen PRL 106, 217206 (2011)

Giant thermal conductivity in quantum magnets



Hlubek et al., Phys. Rev. B 81, 020405(R) (2010)

1D Heisenberg model

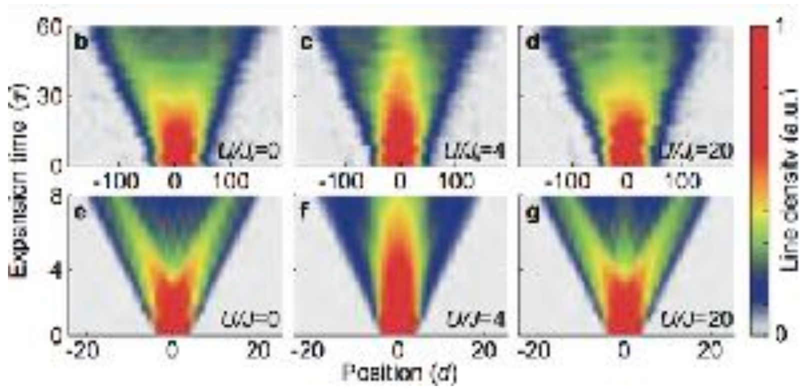
$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

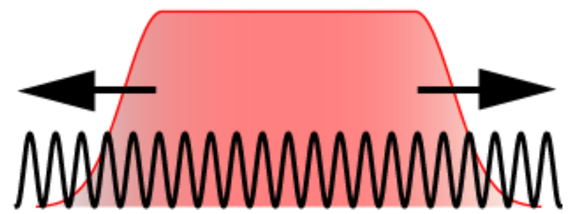
$$[H, J_E] = 0$$

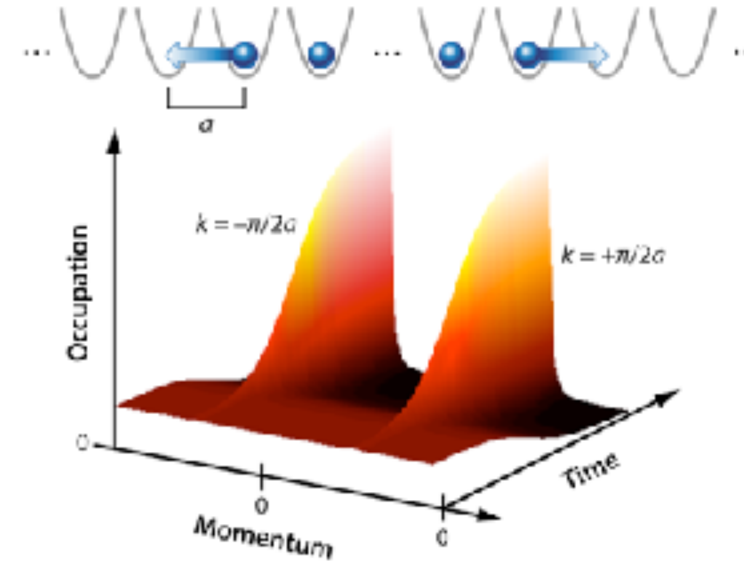
Ballistic thermal conductor!

Nonequilibrium transport: Overview

Fun physics with integrable 1d hardcore bosons







1D Hardcore bosons: Ballistic transport

Ronzheimer et al. Phys. Rev. Lett. 110, 205301 (2013)

Dynamical quasi-condensation

Vidmar et al. Phys. Rev. Lett. 115, 175301 (2015)

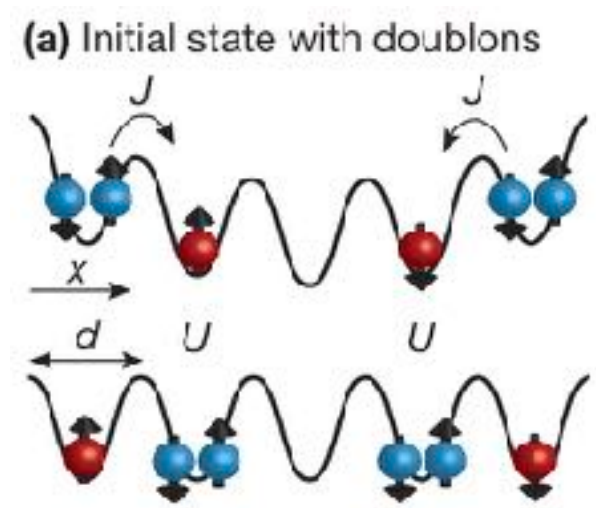
Clark Physics 8, 99 (2015)

Bonus material:

Asymptotic/dilute limit of integrable models

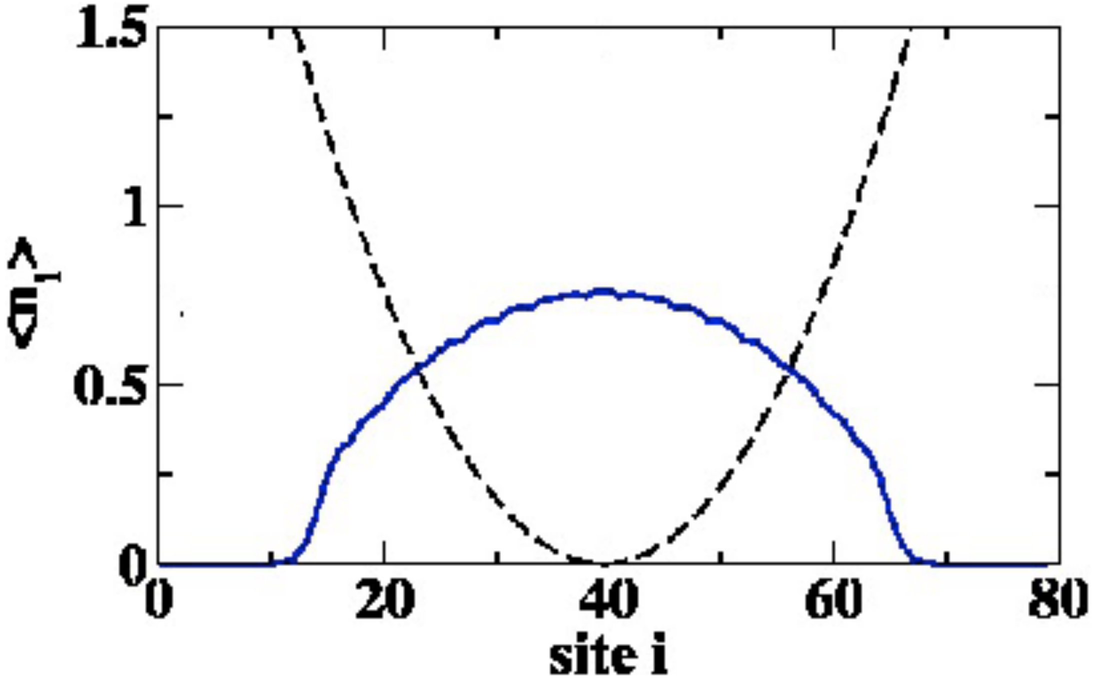
$$n_k^{\text{physical}}(t \to \infty) \to n_\rho$$

Quantum distillation



Nonequilibrium transport in optical lattices

$$H_0 = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + \boxed{V_{\text{trap}}} \sum_i n_i \vec{r}_i^{\,2}$$



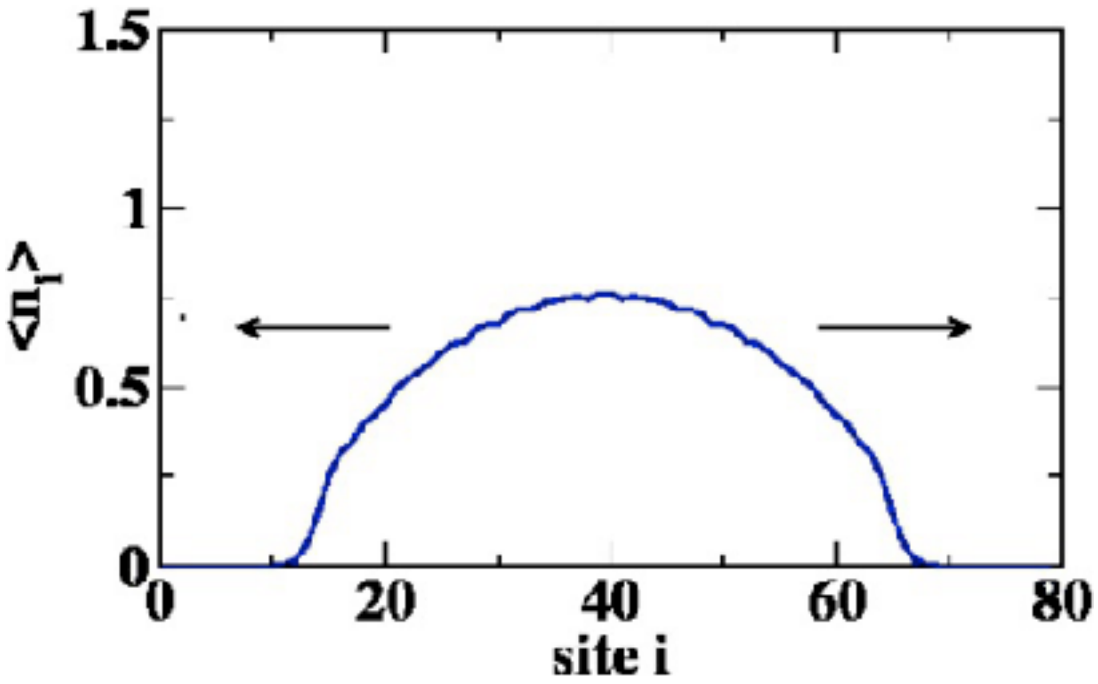
"Sudden expansion"

Other experimental approaches:
digital mirror devices (Esslinger/ ETH and Greiner/Harvard)

Nonequilibrium transport in optical lattices

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

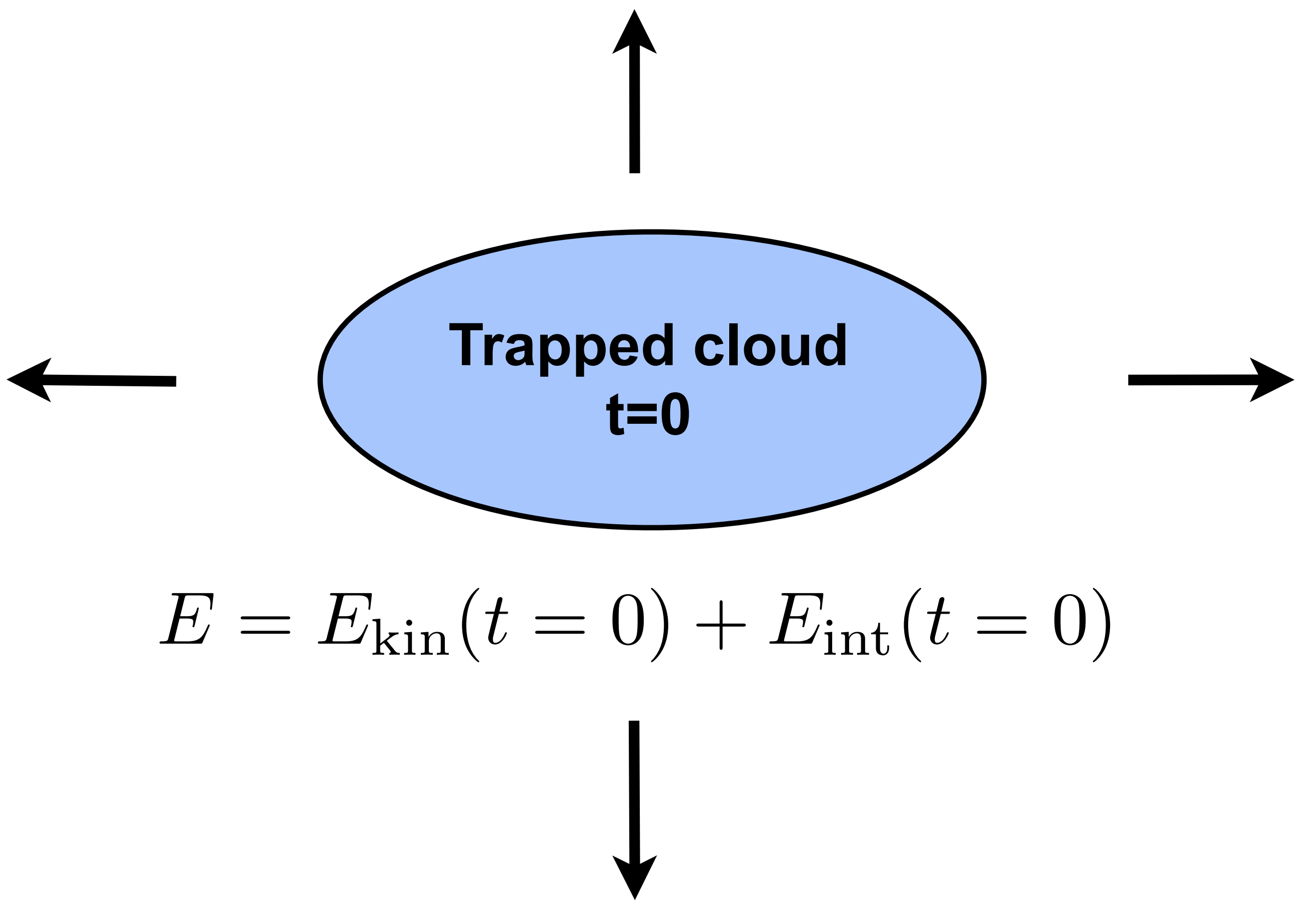
Quantum quench of the trapping potential $|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$



Induces finite mass currents in homogeneous lattice

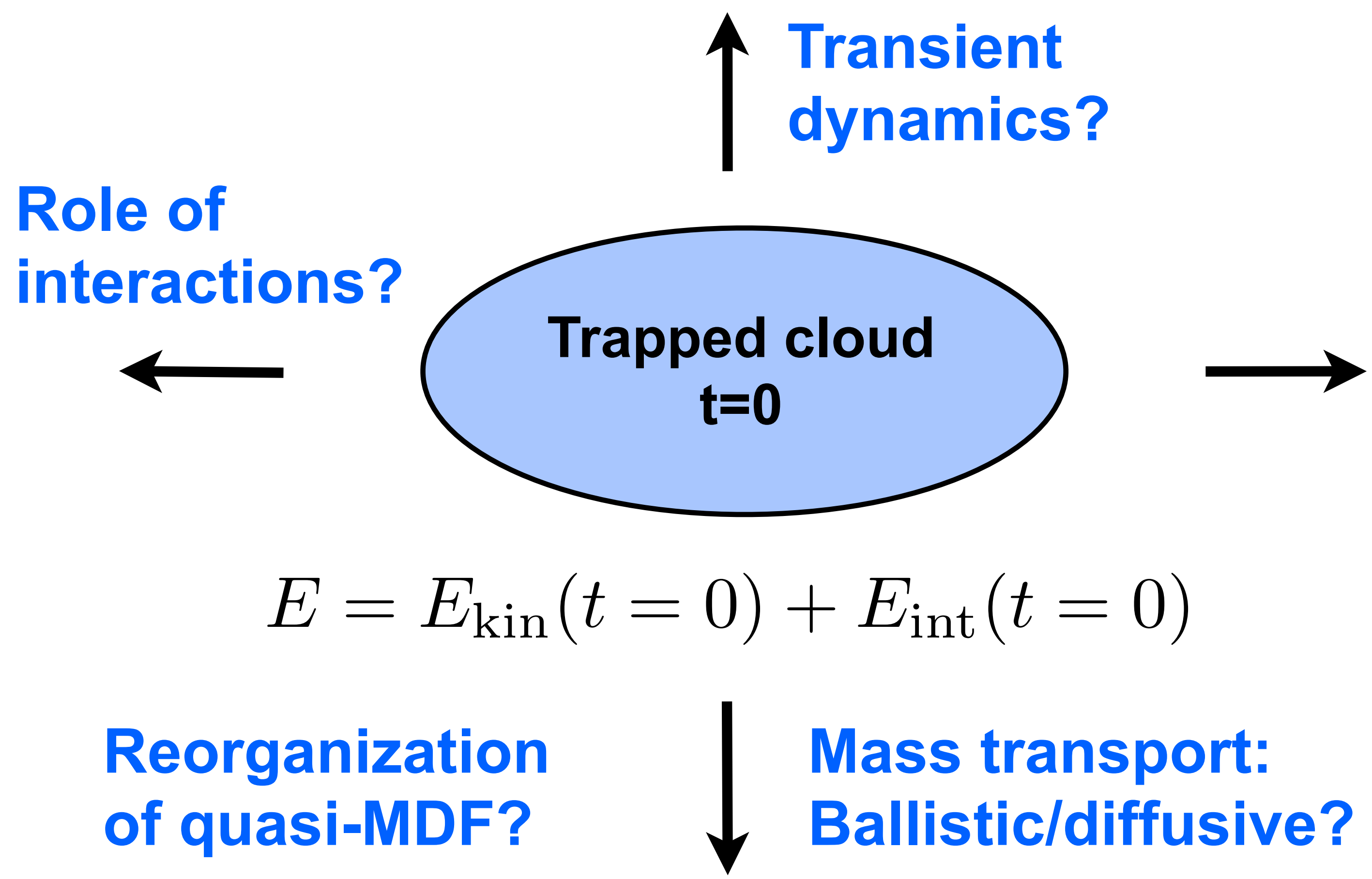
Sudden expansion

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + \boxed{V(t)} \sum_i n_i \vec{r}_i^{\ 2}$$



$$E = E_{\mathrm{kin}}(t=0) + E_{\mathrm{int}}(t=0)$$

Sudden expansion



$$E = E_{\rm kin}(t=0) + E_{\rm int}(t=0)$$

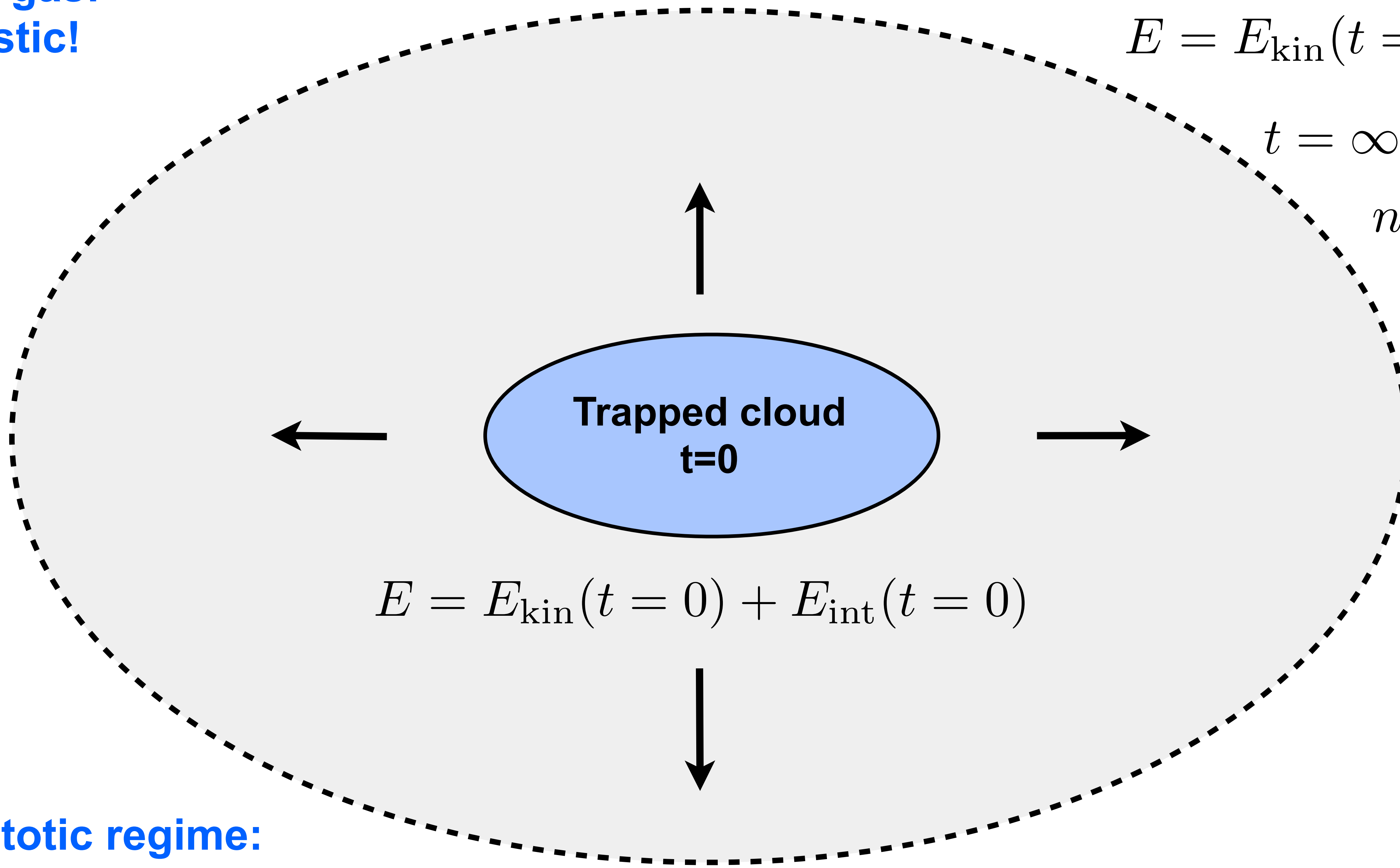
Sudden expansion

Dilute gas!
Ballistic!

$$E = E_{\text{kin}}(t=\infty)$$

$$t = \infty$$

$$n \to 0$$



Asymptotic regime:

$$H \to \sum_k \epsilon_k n_k(t=\infty)$$

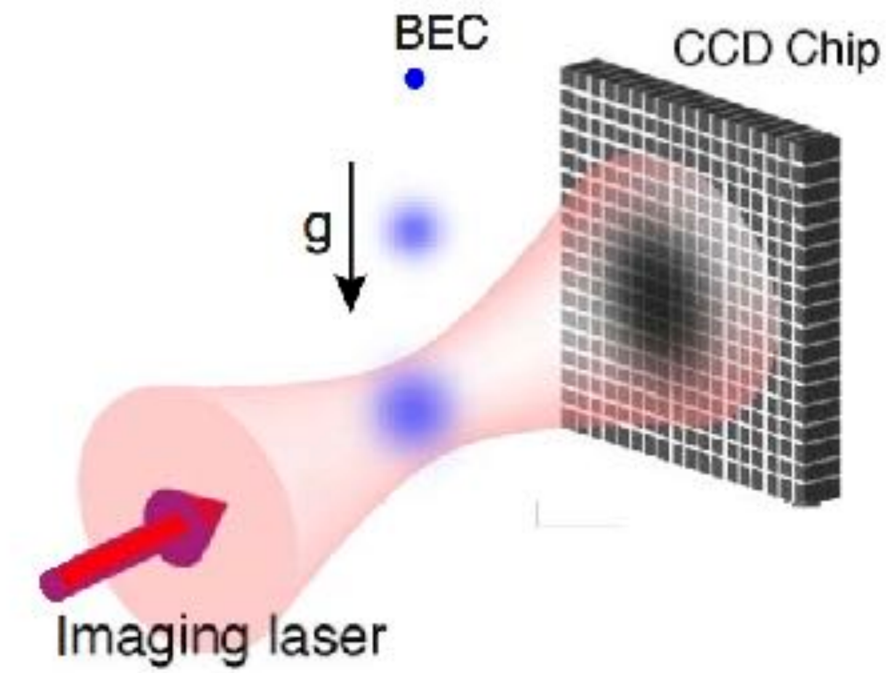
$$n_k(t=\infty) = f(E/N, \dots)$$

Role of conservation laws/ integrability?

Time-of-flight versus sudden expansion

Time-of-flight (Tof)

Removal of all potentials



Bloch, Dalibard, Zwerger RMP 2008

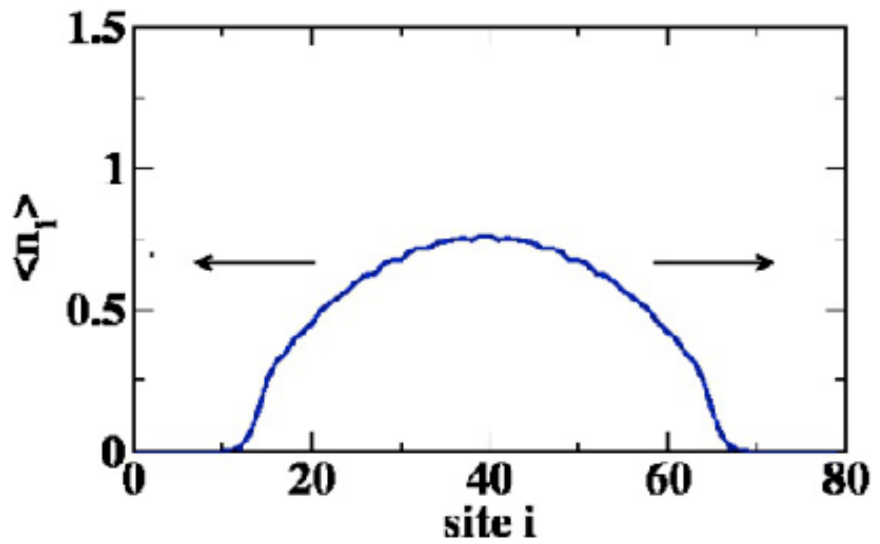
$$n(v) = \text{const.} \qquad n_k = \text{const.}$$

$$n(x, t = \infty) \propto n_k(t = 0)$$

Measurement of momentum distribution n_k

Sudden expansion

Removal of trapping potential



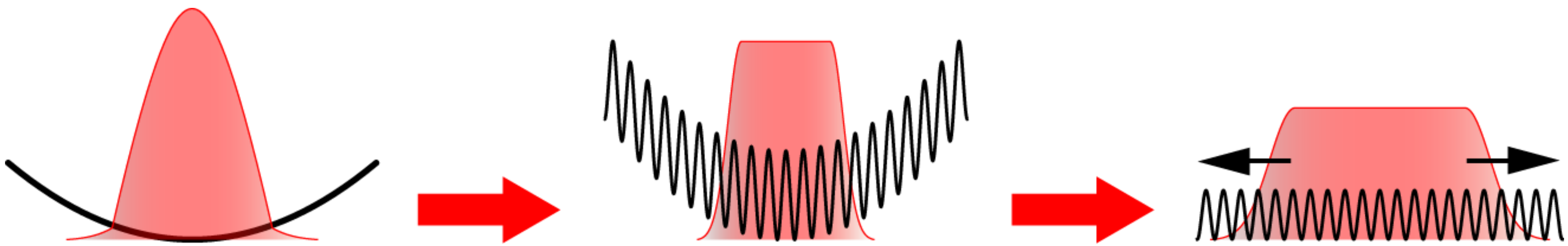
In general: Interactions matter (and give rise to interesting stuff)

$$n_k = n_k(t)$$

$$n(x, t = \infty) \not\propto n_k(t = 0)$$

$$n_k = \frac{1}{L} \sum_{lm} e^{i(l-m)k} \langle a_l^\dagger a_m \rangle$$

Sudden expansion: Experimental state preparation



Load ^{39}K atoms into trap

Ramp up deep optical lattice: One boson/site

Remove trap V → 0, Go to desired U/J

Initial state: g.s. of U/J=∞

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle \rightarrow n_k = \text{const.}$$

Initial quasi-momentum distribution function

$$n_k = const.$$

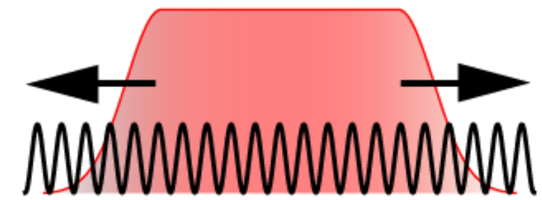
Removal of trap & interaction quench

Ronzheimer, Schreiber, Braun, Hodgman, Langer, McCulloch, FHM, Bloch, Schneider, Phys. Rev. Lett. 110, 205301 (2013)
Schneider et al. Nature Phys. (2012)

Expansion of bosons in 1D

Initial state:
g.s. of U/J=∞

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle \rightarrow n_k = \text{const.}$$



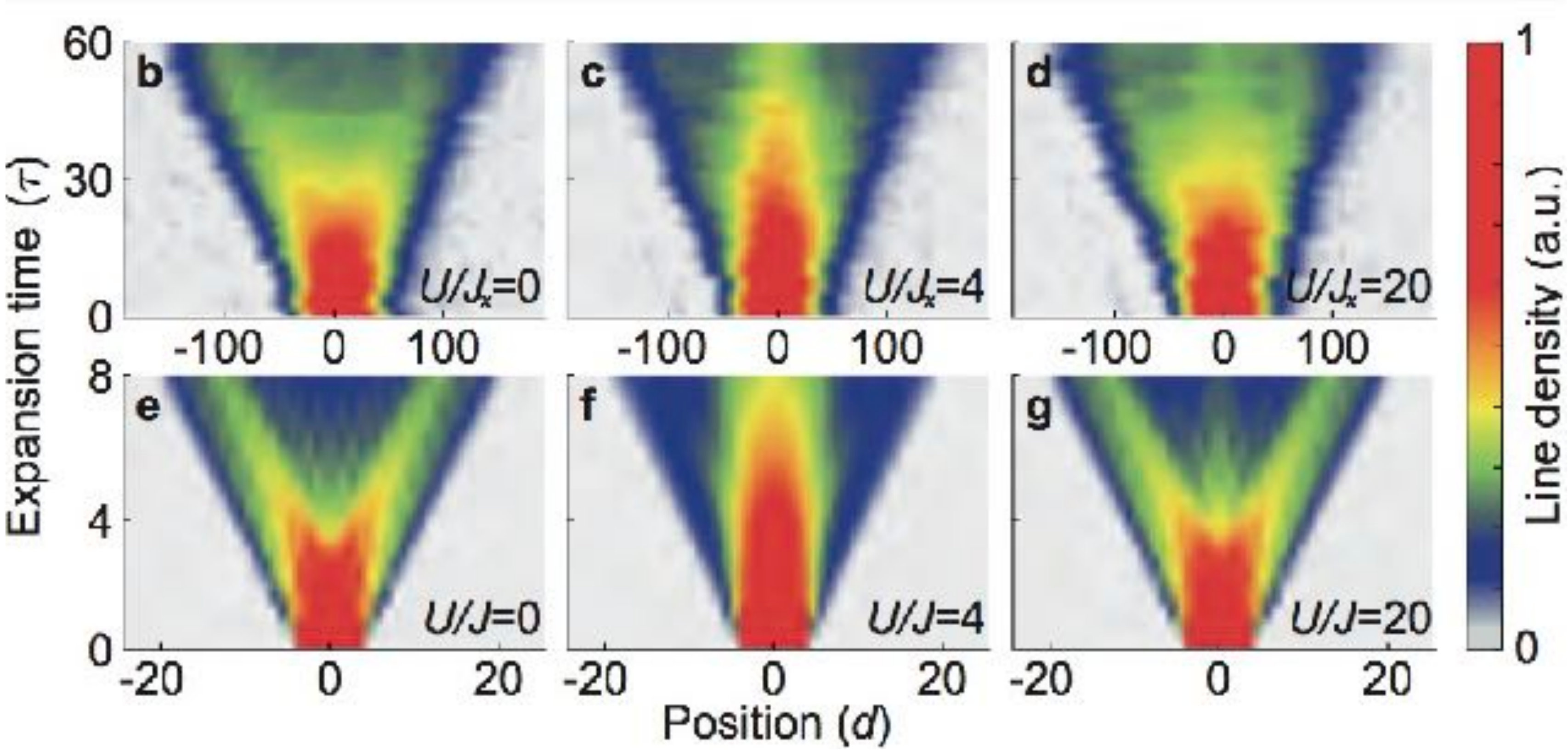
Initial quasi-momentum distribution function

$$n_k = const.$$

Removal of trap & interaction quench

Experimental data

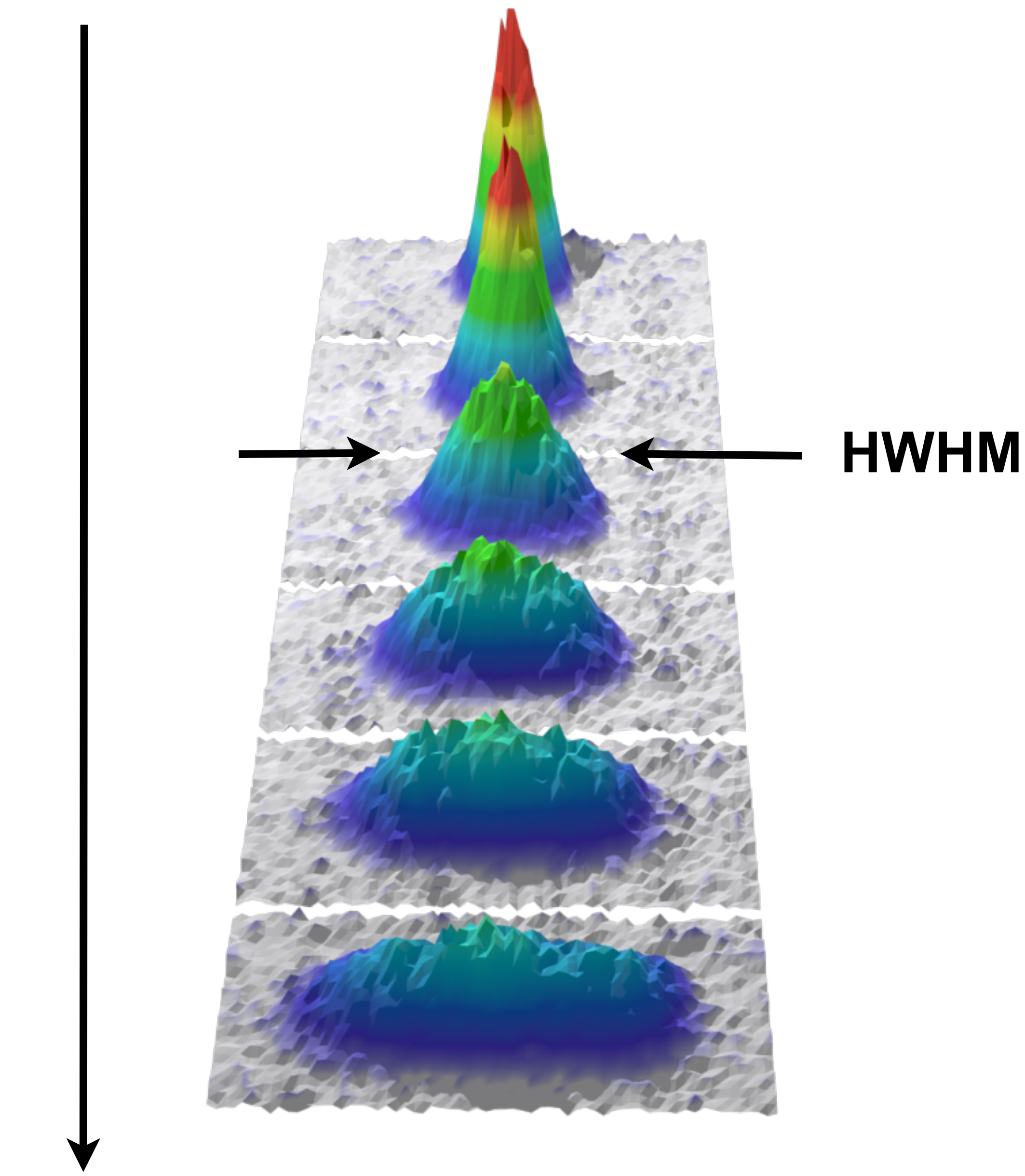
DMRG



Profiles for *non-interacting* and *strongly* interacting bosons are identical!

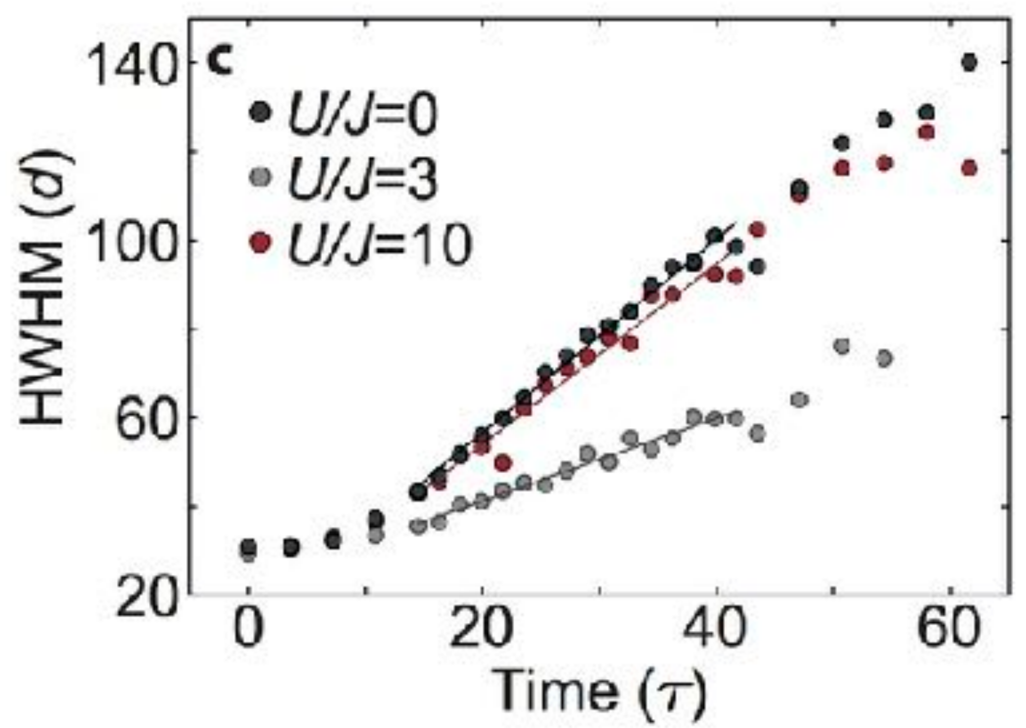
Ronzheimer, Schreiber, Braun, Hodgman, Langer, McCulloch, FHM, Bloch, Schneider, PRL 110, 205301 (2013)

Expansion of bosons in 1D & 2D



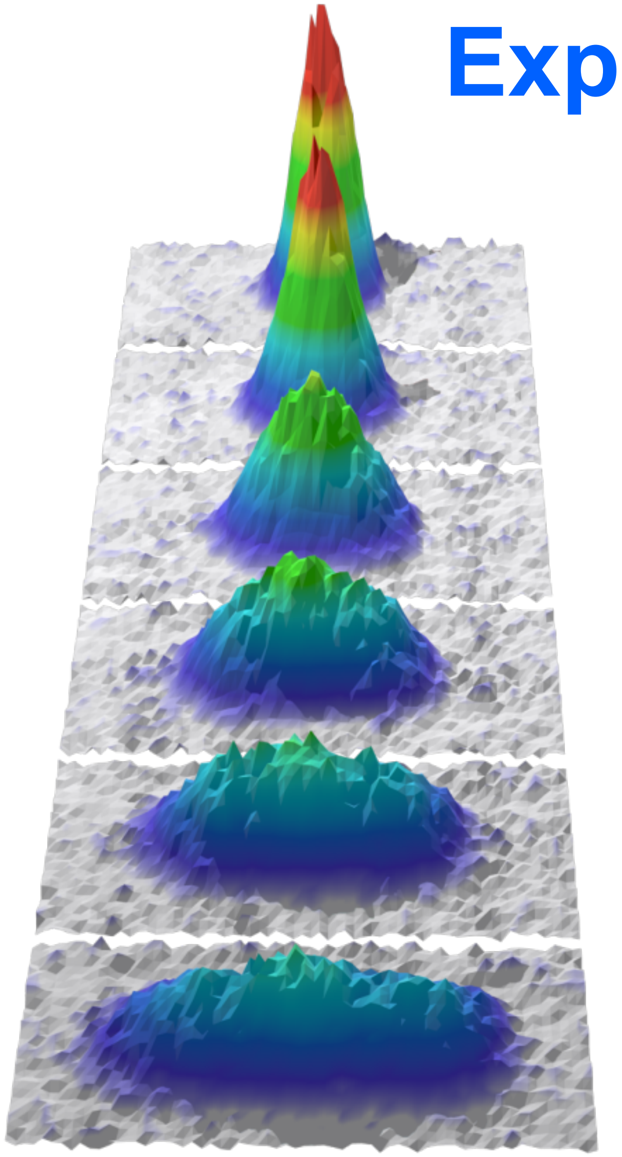
Averaged over many 1D systems

Time-evolution of half-width-at-half maximum HWHM=f(t)

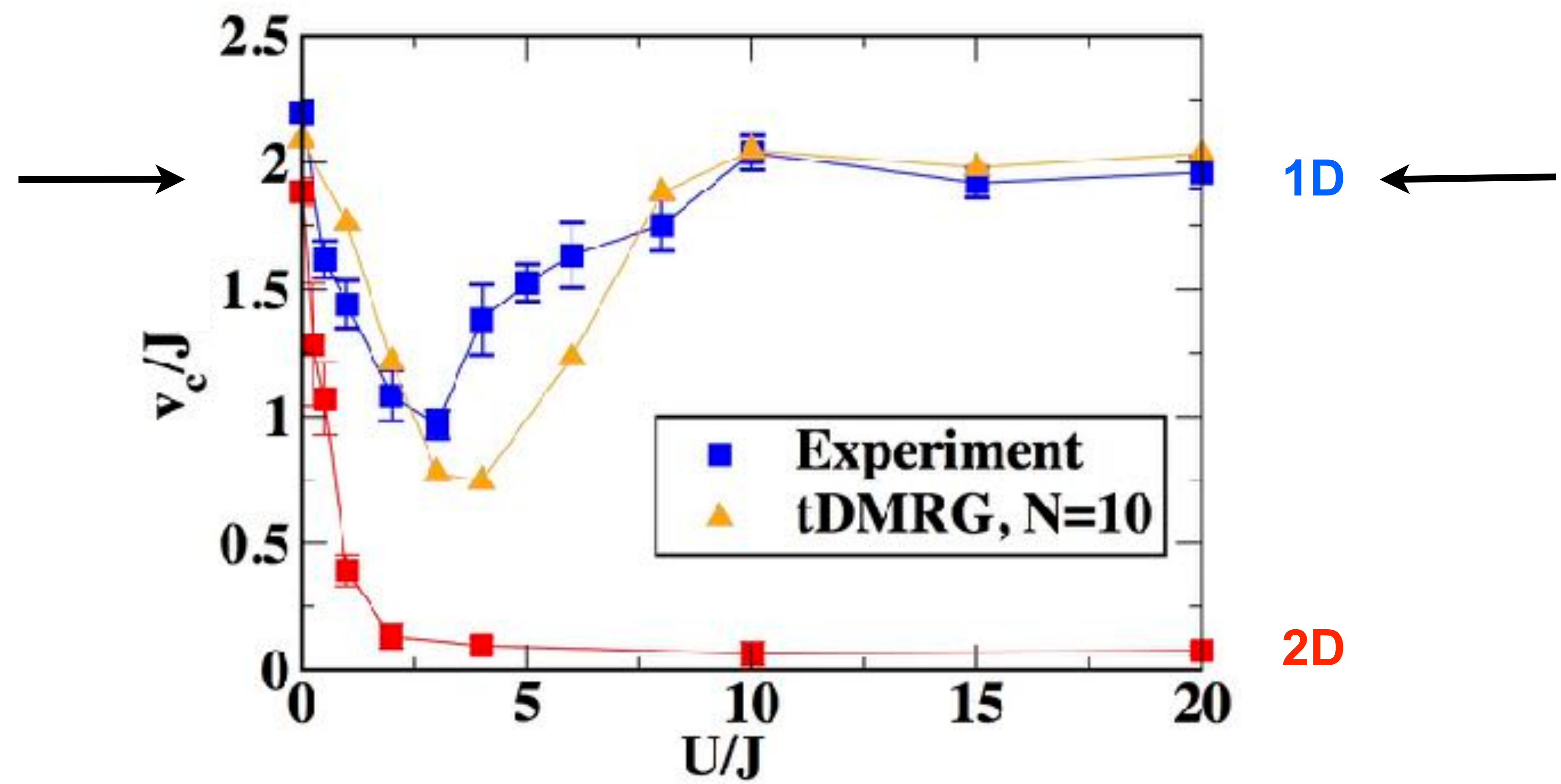


Core expansion velocity v_c

Expansion velocity: 1D versus 2D



half-width-half-maximum



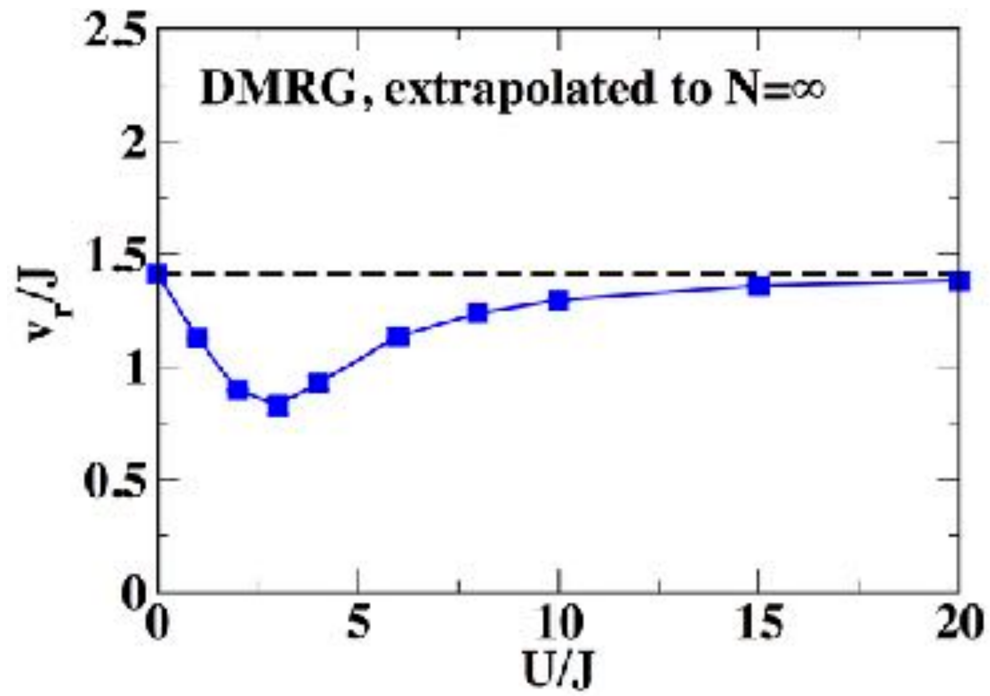
Dispersion: $\epsilon_k = -2J\cos(k)$

Fast & ballistic expansion for non-interacting and strongly interacting bosons in 1D

Expansion velocity

Radius of expanding cloud

$$R^2(t) = \frac{1}{N}\sum_i \langle n_i(t)\rangle (i - i_0)^2$$



Non-interacting particles:

$$H = \sum_k \epsilon_k a_k^\dagger a_k = \sum_k \epsilon_k n_k$$

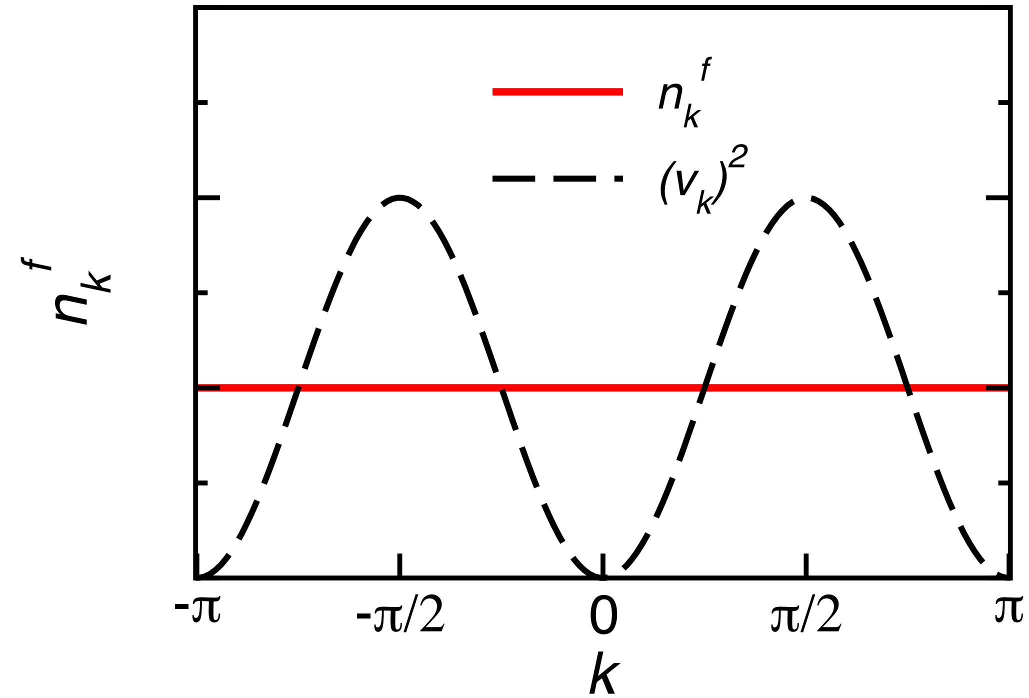
Ballistic & fast expansion:

$$R(t) = v_r t = \sqrt{2} J t$$

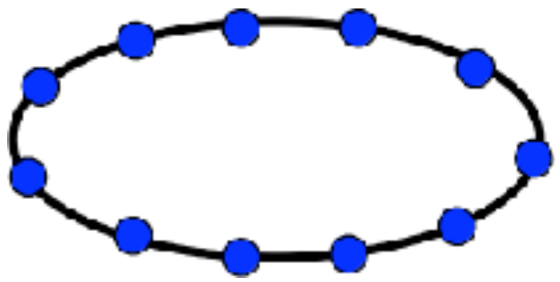
$$v_r^2 = v_{\text{av}}^2 = \frac{1}{N}\sum_k v_k^2 n_k$$

$$v_k = 2J\sin(k)$$

Quasi-Momentum distribution function



Hard-core bosons in 1D: Integrable!



$$H = -J\sum_i (a^\dagger_{i+1}a_i + h.c.) + \frac{U}{2}\sum_i n_i(n_i - 1) \quad (a_i^\dagger)^2 = 0$$

U/J=∞: Hard-core bosons map to spinless non-interacting fermions!

$$\rightarrow H = -J\sum_i (f^\dagger_{i+1}f_i + h.c.) = \sum_k \epsilon_k n_k^f$$

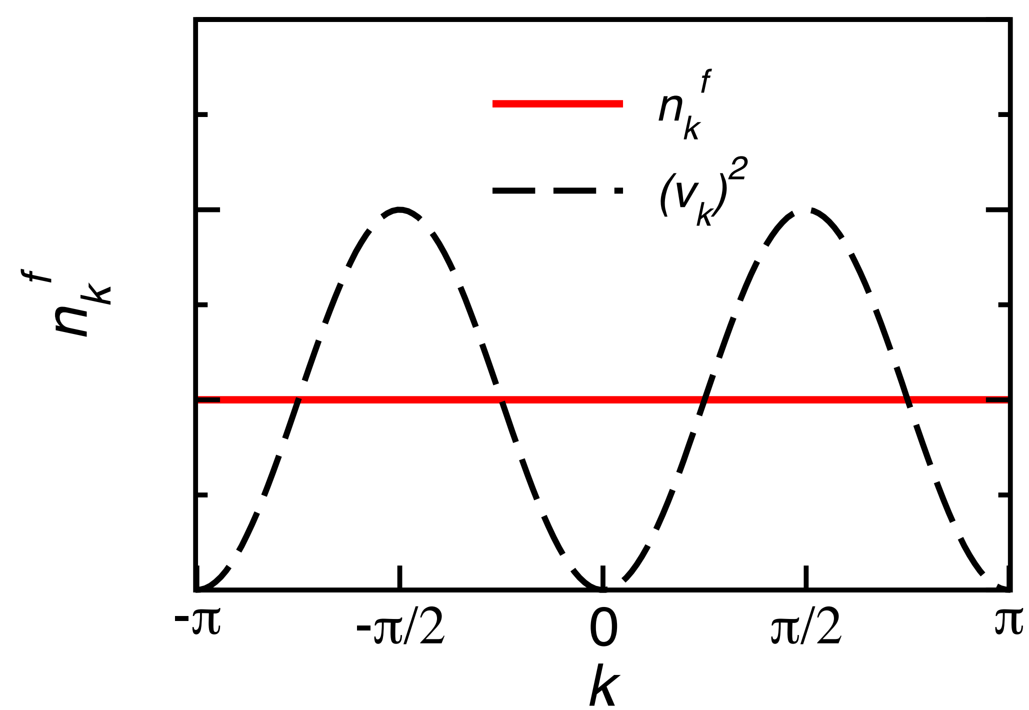
Conserved charges

$$\epsilon_k = -2J\cos(k)$$

$$R^2(t) = \frac{1}{N}\sum_i \langle n_i(t)\rangle (i - i_0)^2$$

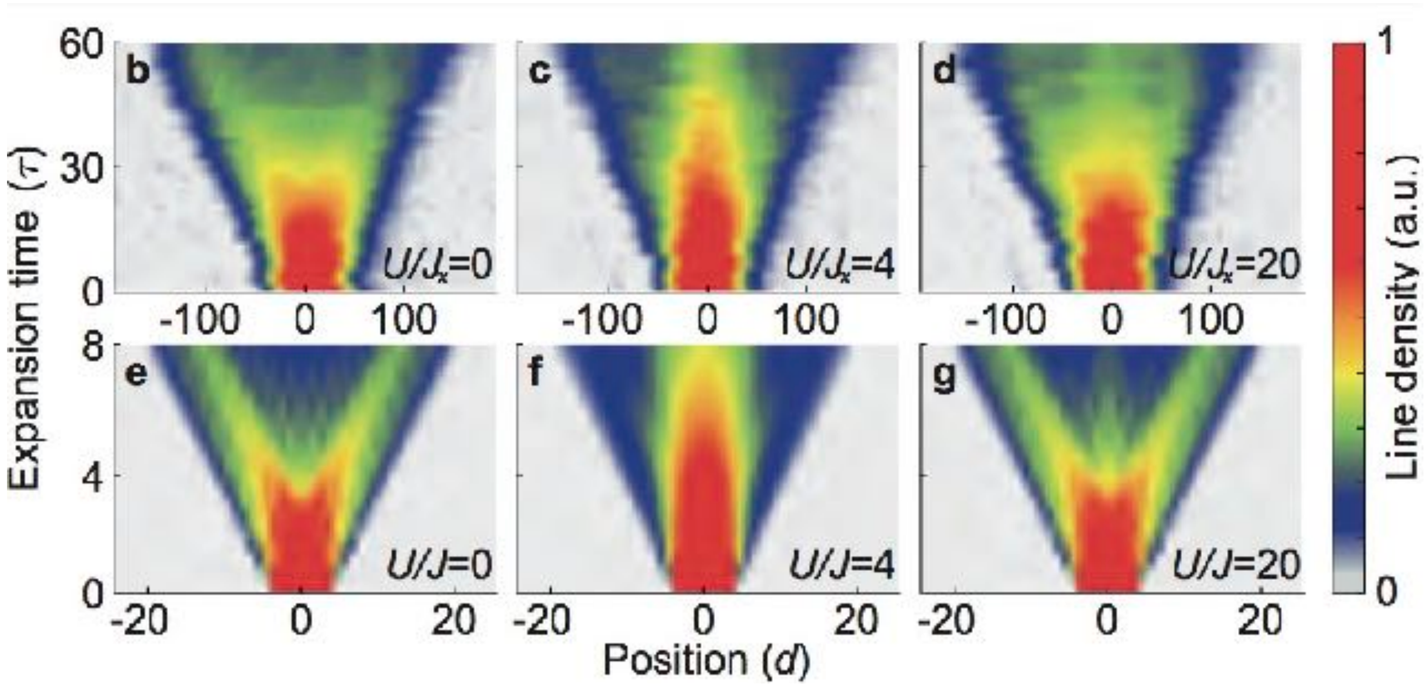
$$n_i^{HCB} = n_i^f \rightarrow R = \sqrt{2}Jt$$

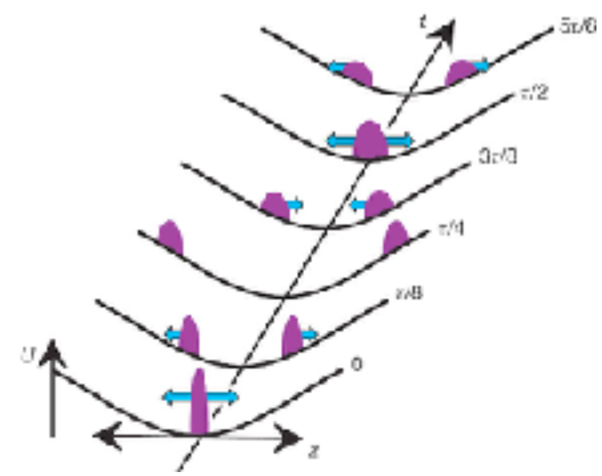
Ballistic mass transport despite strong interactions: Indistinguishable from free particles



Take home message II:

Integrable systems: many conservation laws $[H, I_i] = 0$





Ballistic nonequilibrium mass transport in a strongly interacting 1D system due to integrability

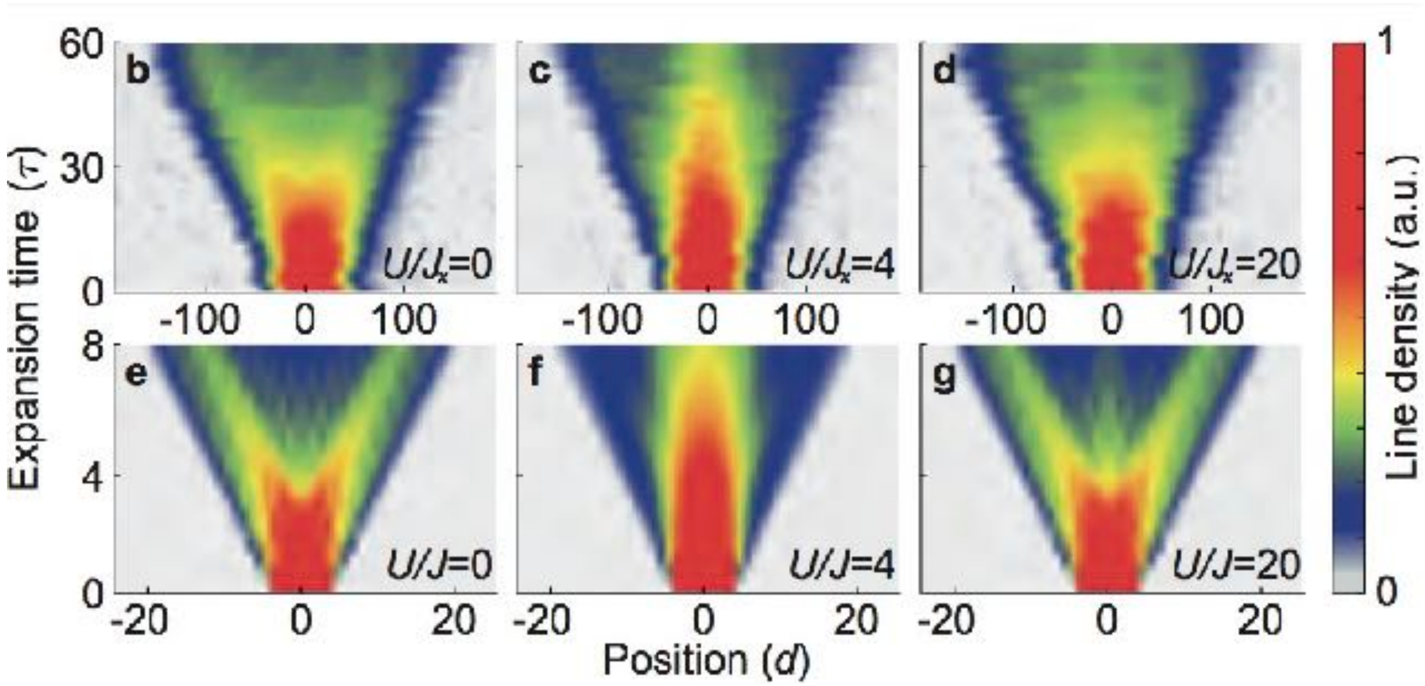
Analogy to non-ergodicity in integrable systems in quantum quenches

Ronzheimer, Schreiber, Braun, Hodgman, Langer, McCulloch, FHM, Bloch, Schneider, PRL 110, 205301 (2013)

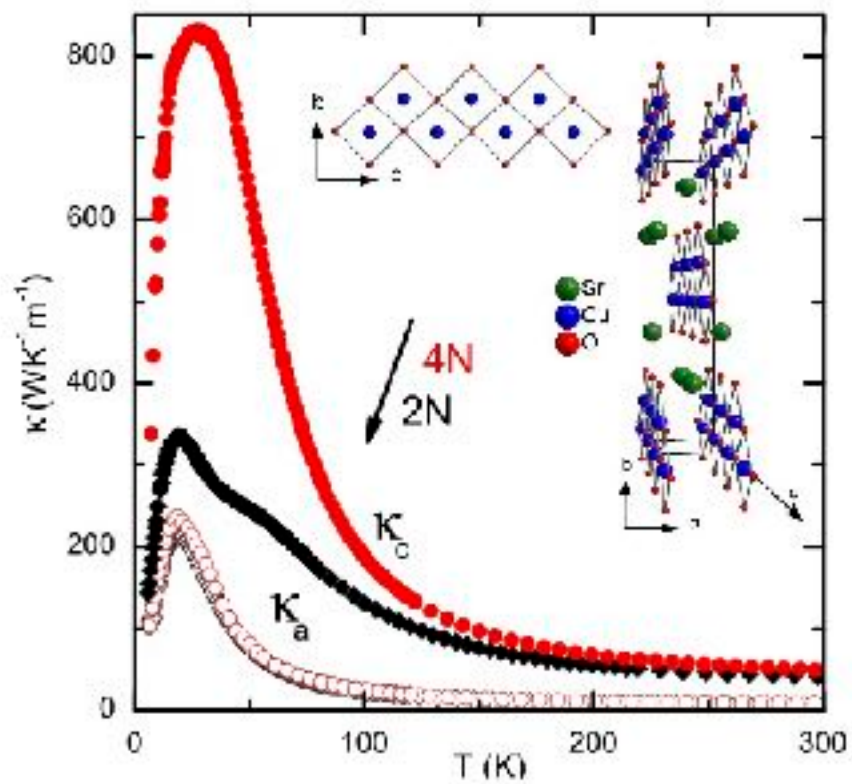
Kinoshita, Wenger, Weiss Nature, 440, 900 (2006)
Schmiedmayer group:
Langen et al. Science 348, 207 (2015)

Take home message II:

Integrable systems: many conservation laws $[H, I_i] = 0$



Giant thermal conductivity in quantum magnets



Hlubek et al., PRB 81, 020405(R) (2010)

Ballistic nonequilibrium mass transport in a strongly interacting 1D system due to integrability

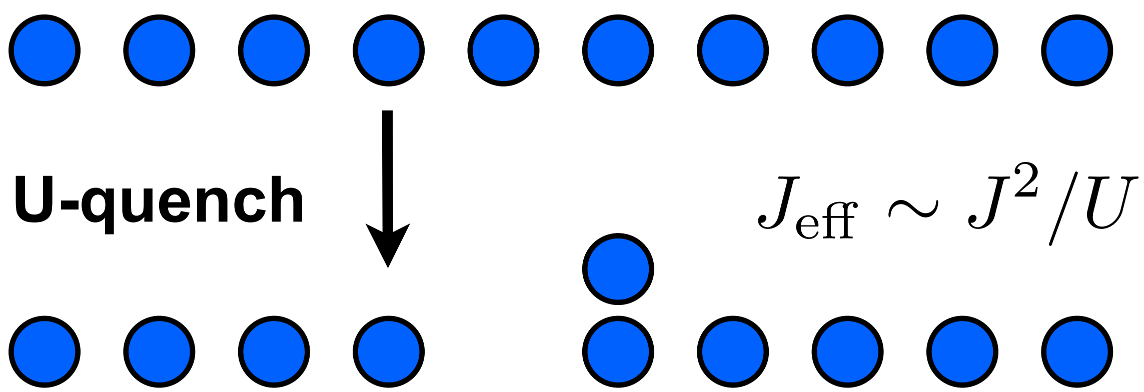
Ronzheimer, Schreiber, Braun, Hodgman, Langer, McCulloch, FHM, Bloch, Schneider, PRL 110, 205301 (2013)

Nonequilibrium version of ballistic transport in integrable spin chains!

Zotos, Naef, Prelovsek PRB 1997
FHM et al. EPJST 2007

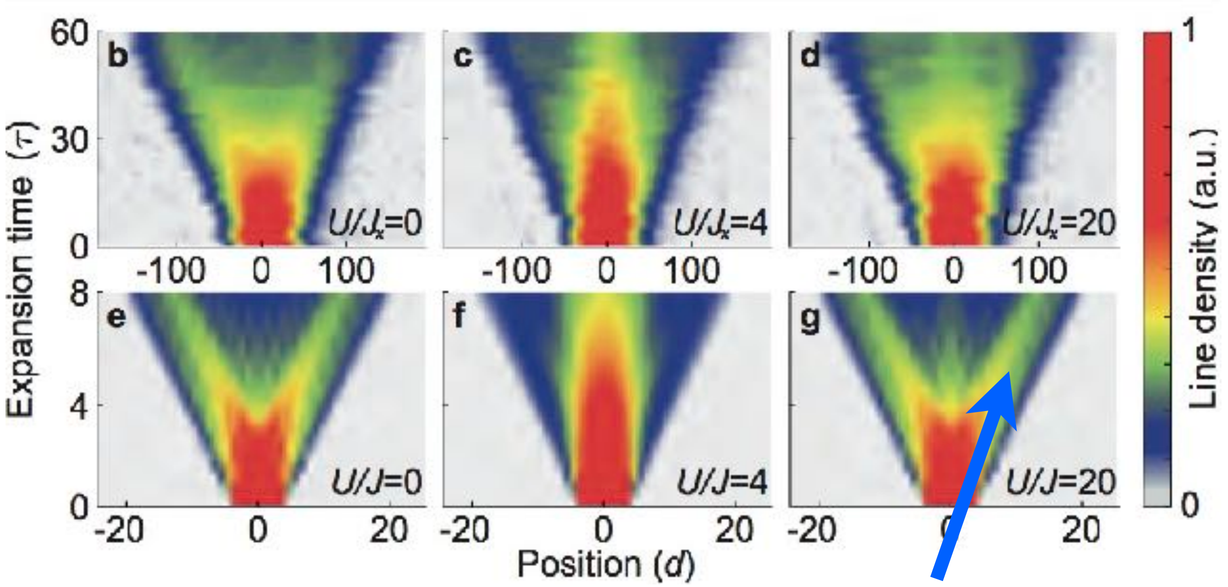
Various ways of breaking integrability

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$$



$J_{\text{eff}} \sim J^2/U$

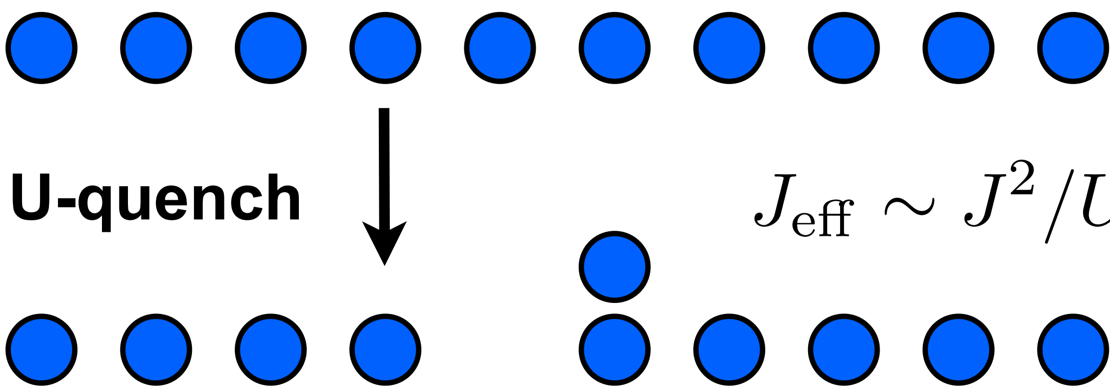
Dynamical formation of doublons on fast time scale ~ 1/J



Doublons cluster in the center! Quantum distillation)

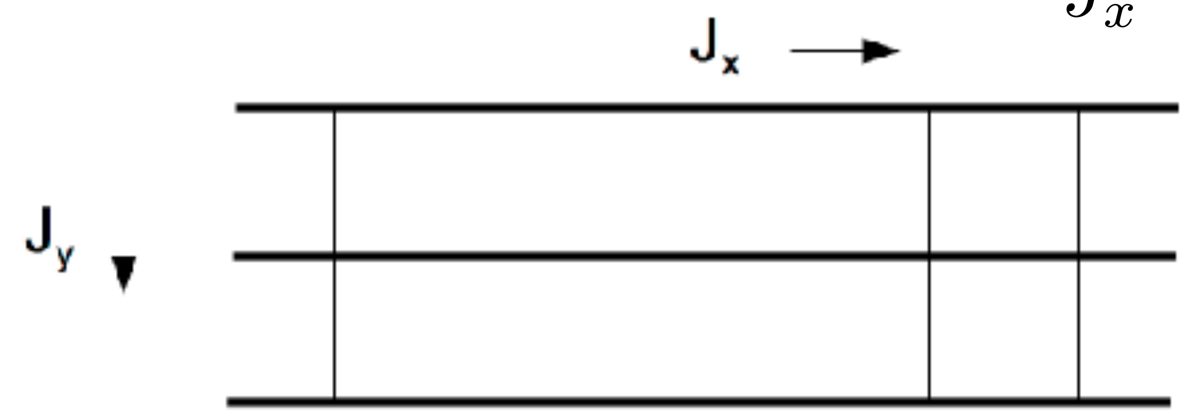
Various ways of breaking integrability

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$$

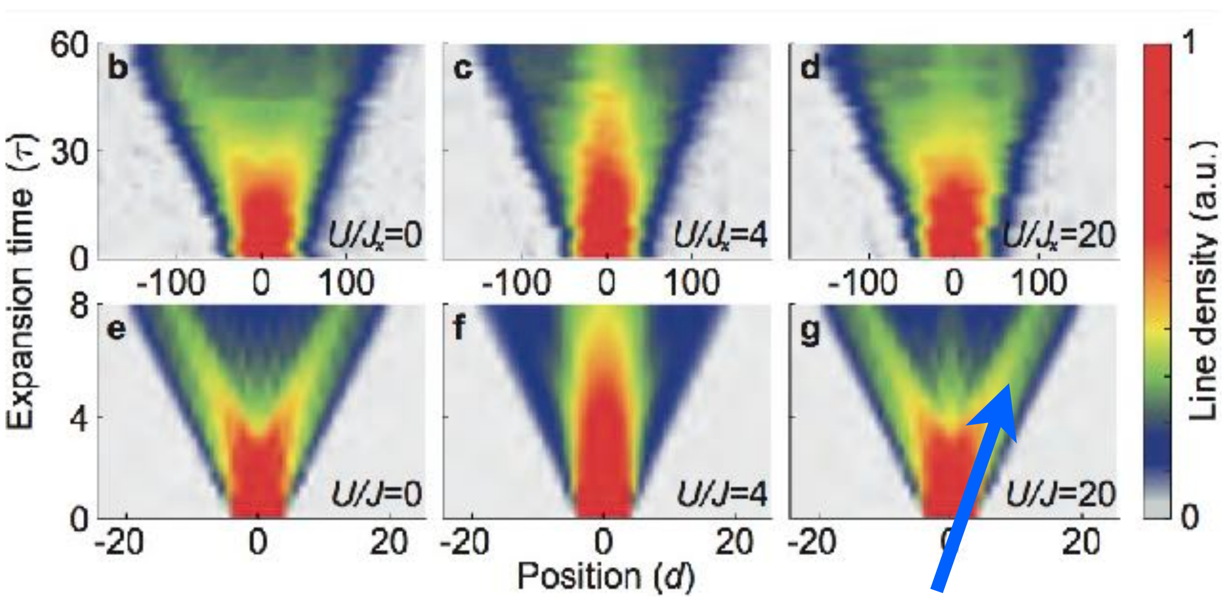


$$J_{\text{eff}} \sim J^2/U$$

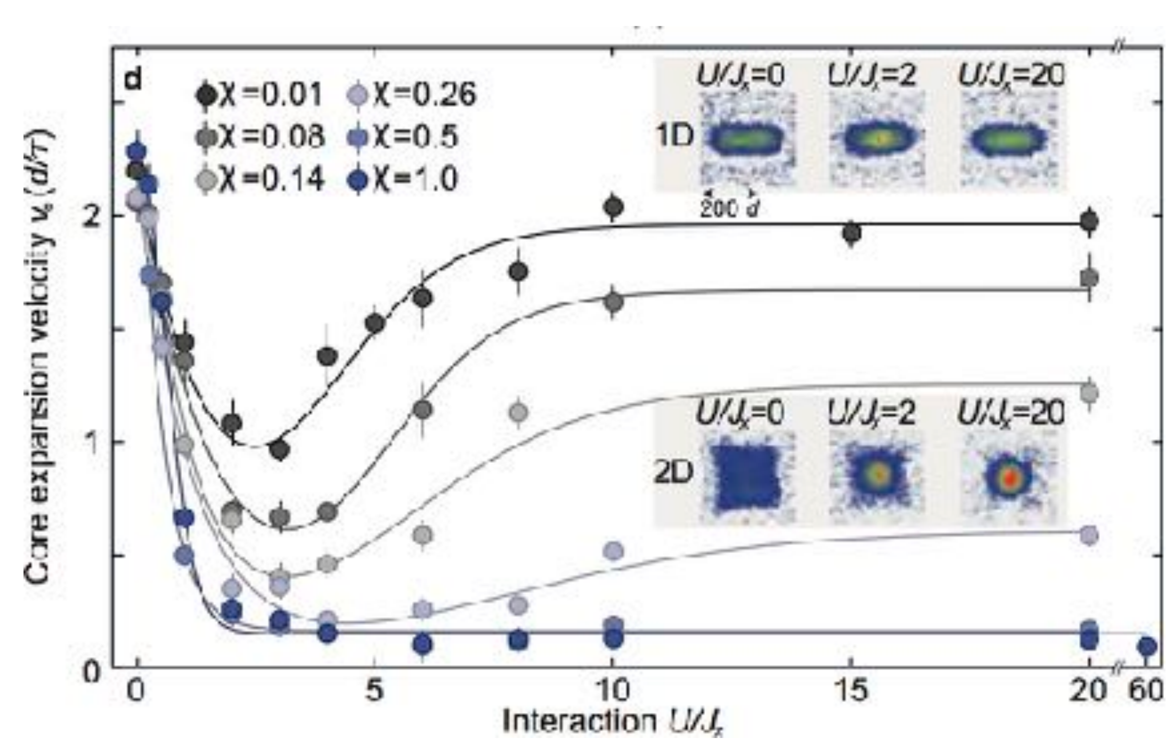
Towards 2D $\chi = \frac{J_y}{J_x}$



Dynamical formation of doublons on fast time scale ~ 1/J



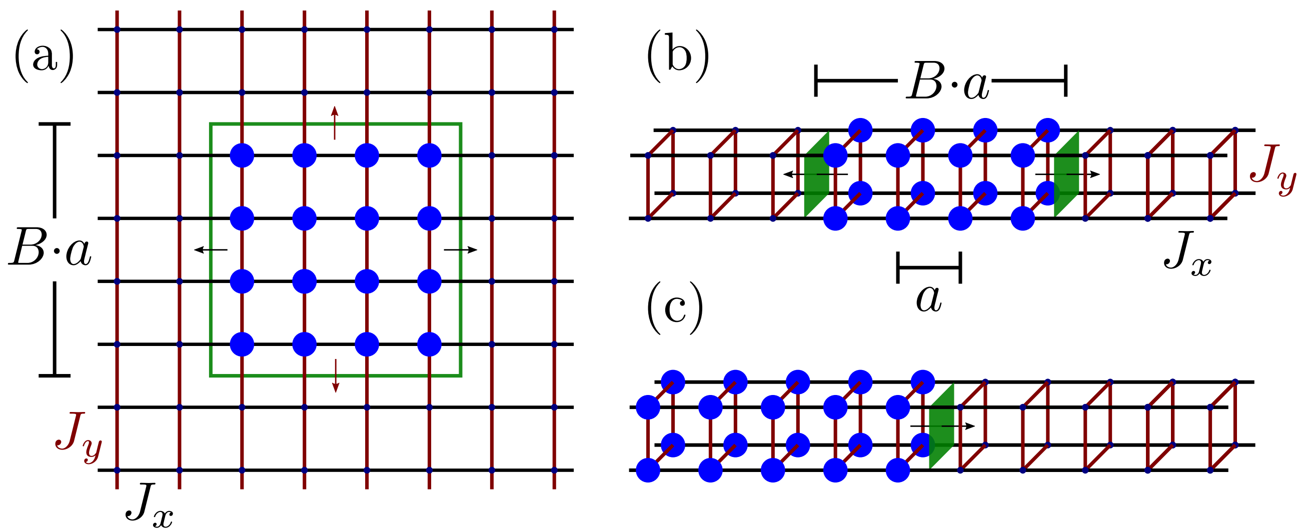
Doublons cluster in the center! Quantum distillation)



Finite J_y suppresses expansion rapidly!

Ronzheimer et al. Phys. Rev. Lett. 110, 205301 (2013)
Schneider et al. Nat. Phys. 8, 213 (2012)

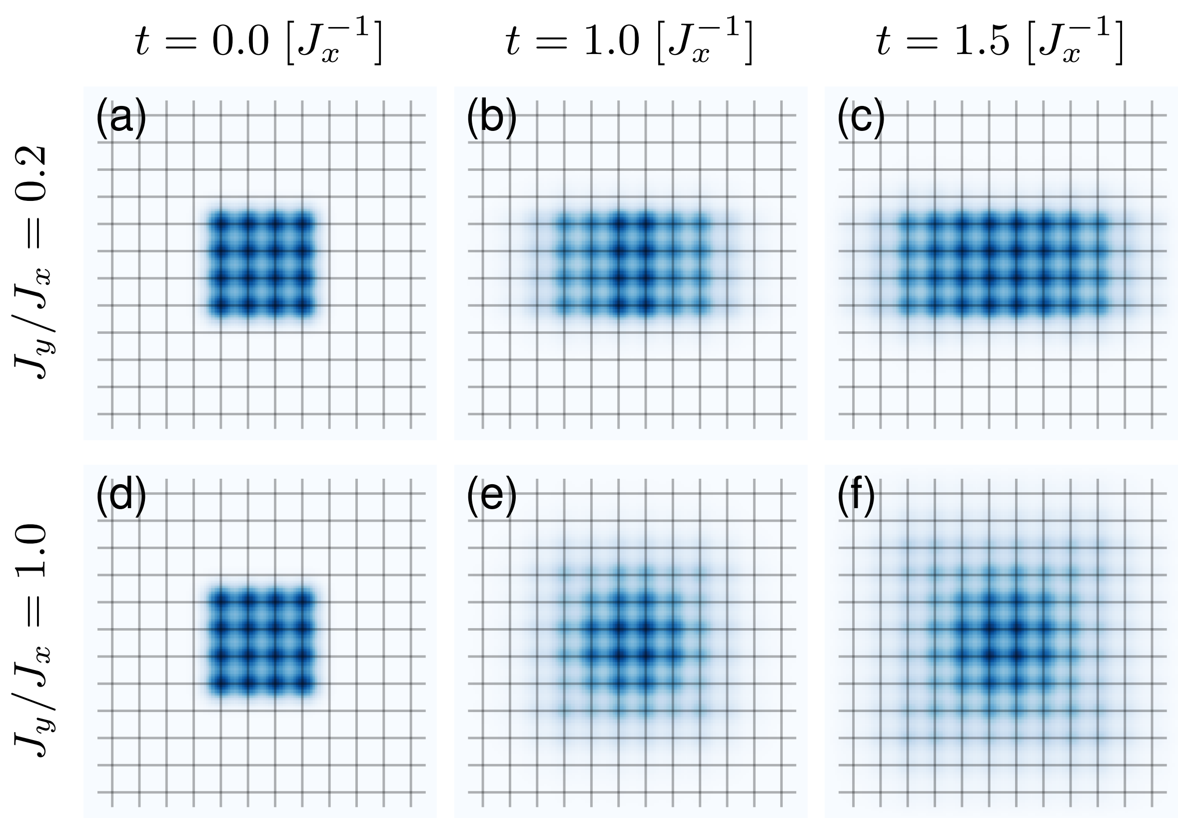
Outlook: Sudden expansion in 2D



2D lattices

Multi-leg ladders/cylinders

Sudden expansion/ Domain-wall melting



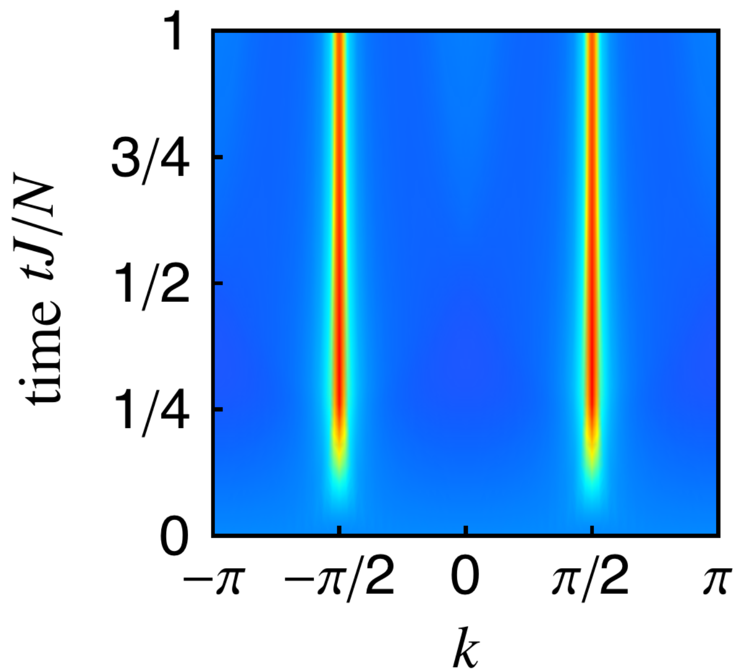
Hard-core bosons U/J=∞:

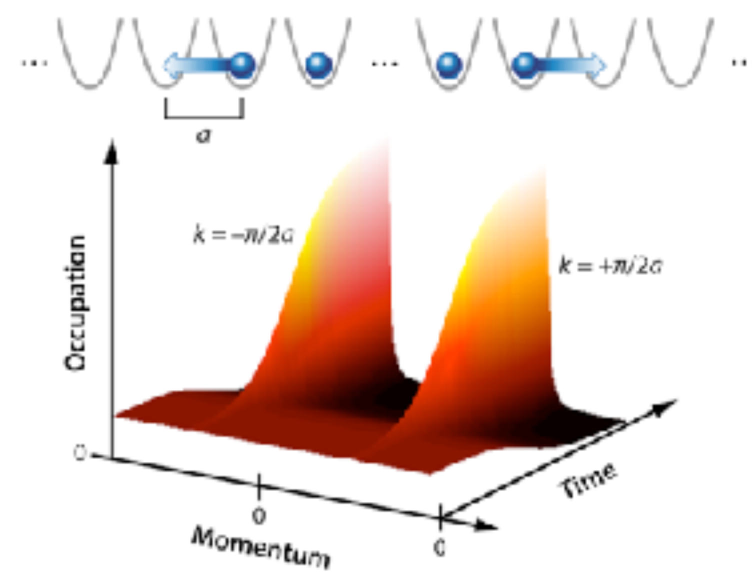
Spherical symmetry emerges!

J. Hauschild, F. Pollmann
TU Munich

Hauschild, Pollmann, FHM PRA 92, 053629 (2015)
Vidmar et al. PRB 88, 235117 (2013)

Back to 1d Hard-core bosons: Momentum space information





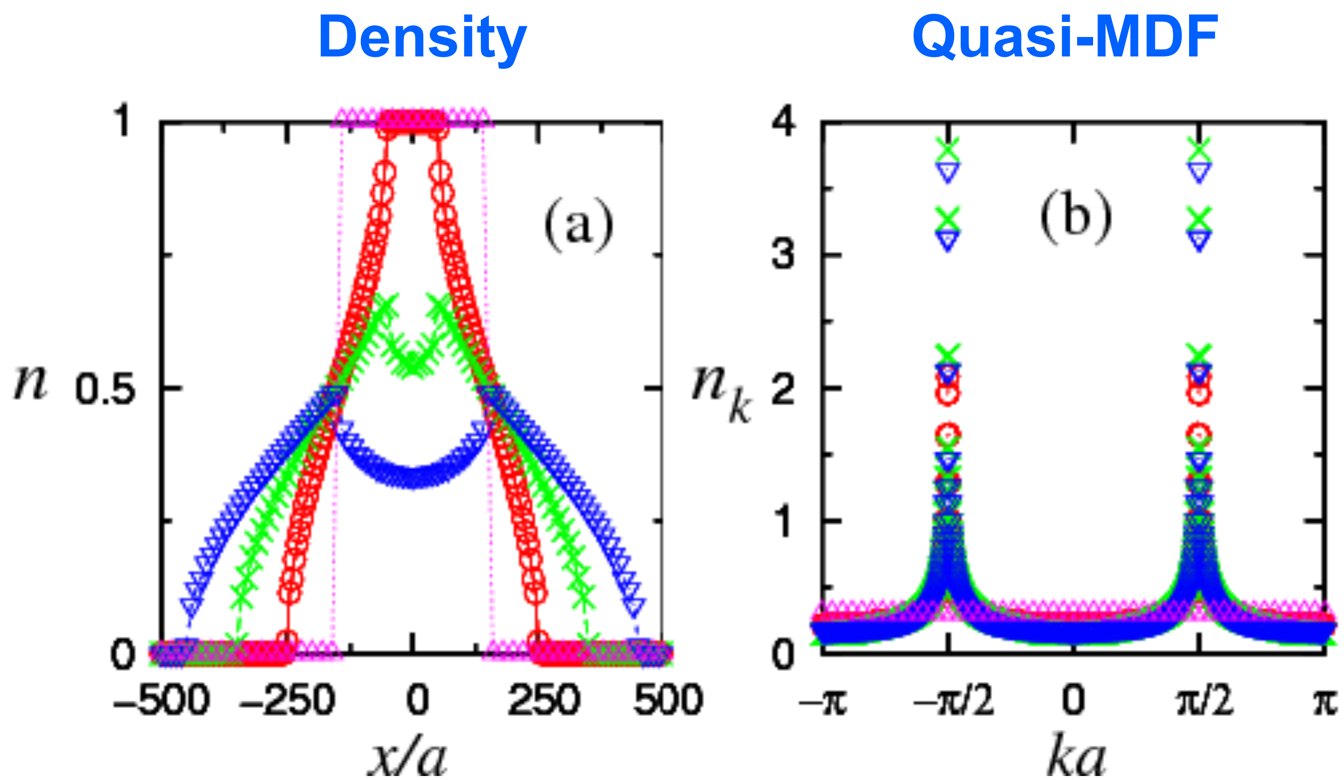
Clark, Physics **8, 99 (2015)**

Dynamical emergence of coherence

Dynamical quasi-condensation in 1D at finite momenta

Vidmar, Ronzheimer, Schreiber, Hodgman, Braun, Langer, FHM, Bloch, Schneider
PRL 115, 175301 (2015)

Dynamical quasi-condensation at finite momenta



$$n_k^{HCB} = n_k^{HCB}(t)$$

$$n_k^f \neq n_k^f(t)$$

← flat initial MDF

Rigol, Muramatsu, PRL 93, 230404 (2004)

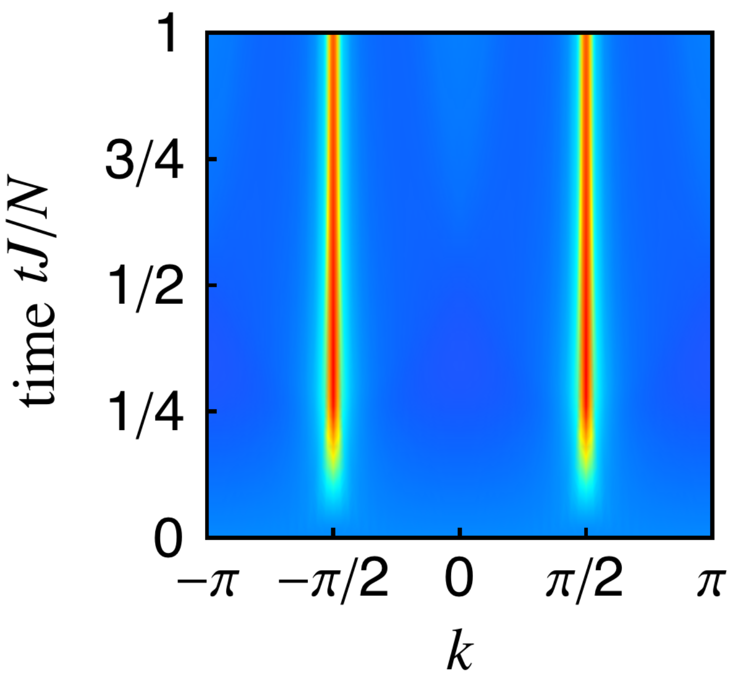
$$H = -J\sum_i (a_{i+1}^\dagger a_i + h.c.) = \sum_k \epsilon_k n_k^f$$

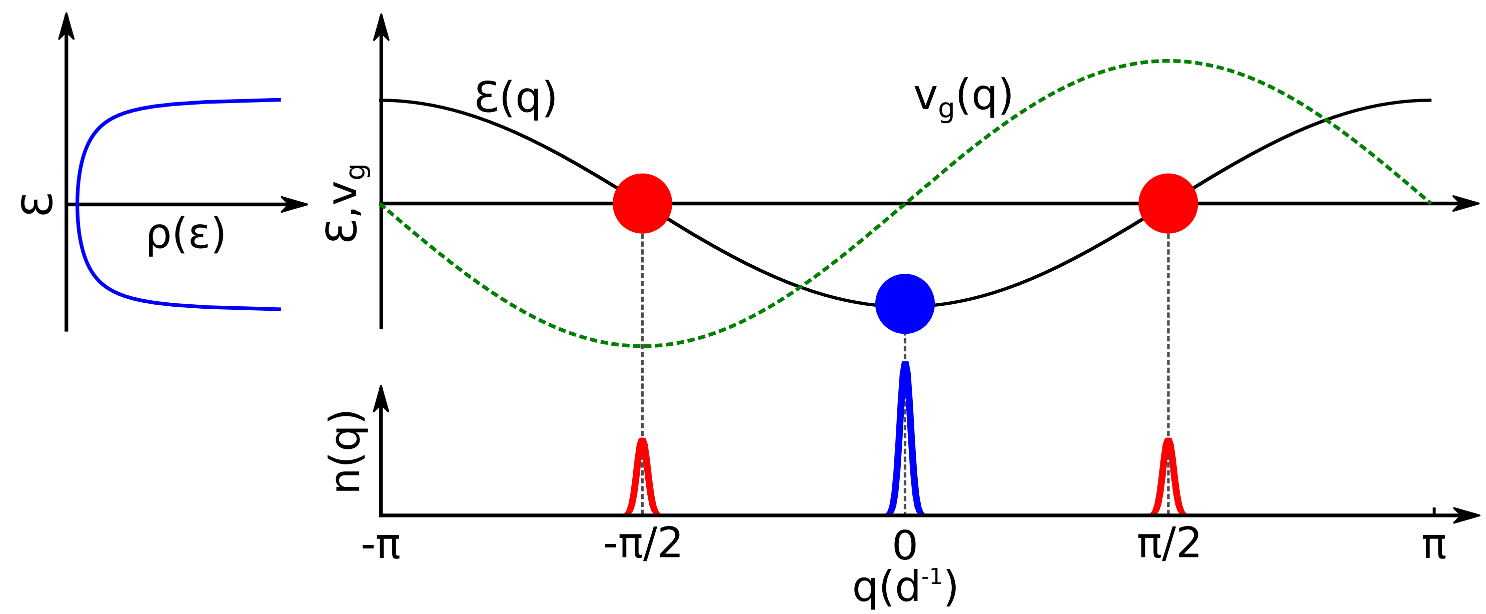
Hard-core bosons are strongly interacting:
Bosonic MDF undergoes transient dynamics,
particles quasi-condense at finite momenta

See also Micheli et al. PRA 93, 140408 (2004)

Dynamical quasi-condensation at finite momenta

Quasi-MDF





→ **Position of quasi-condensate: Average energy E/N=0**

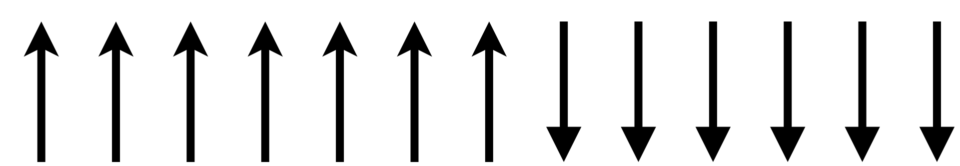
→ **Power-law correlations in expanding cloud with *ground-state* exponents**

→ **Mapping to spin-1/2 XX chains**
Lancaster, Mitra PRE 81, 061134 (2010)

→ **Quasi-condensation in co-moving frame!**

→ **Similar behavior for interacting fermions!**

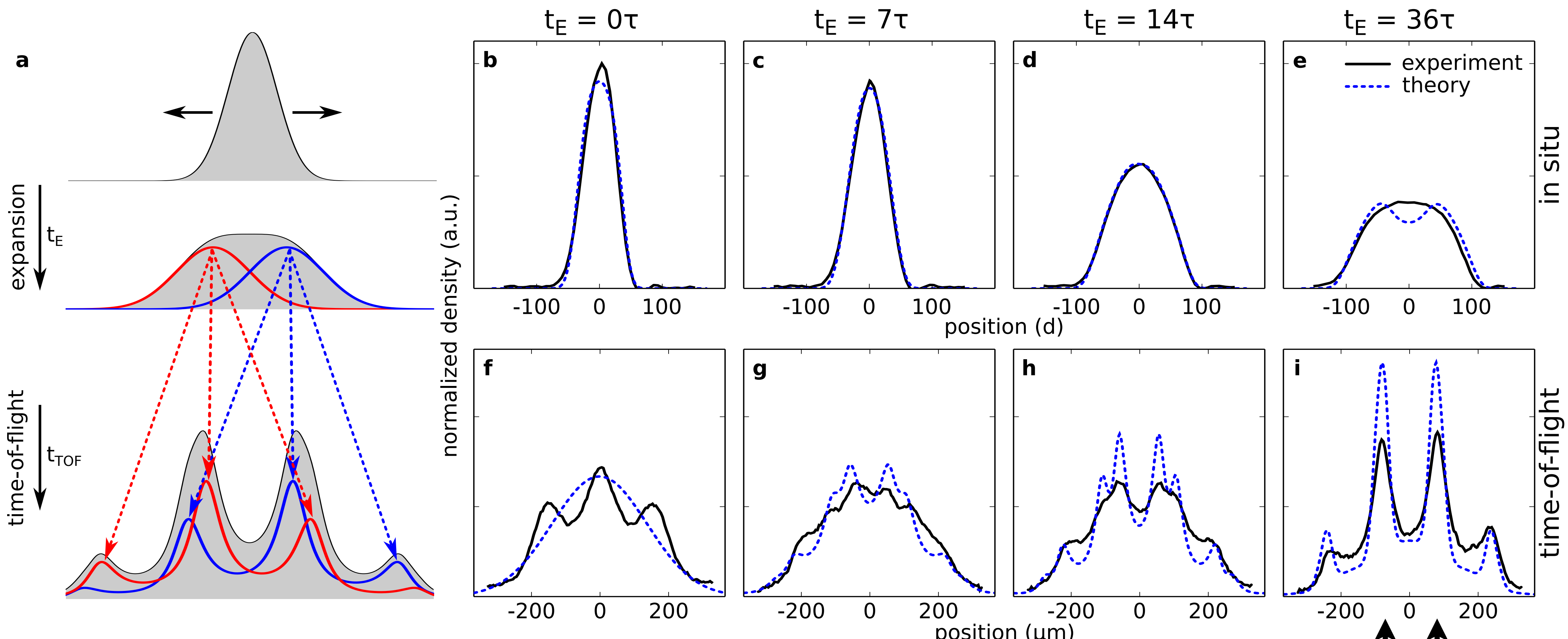
FHM, Rigol, Muramatsu, Feiguin, Dagotto PRA 78, 013620 (2008)



Domain-wall melting

Yes! This is also seen in experimental data

Density profiles during 1D expansion



Time-of-flight profiles

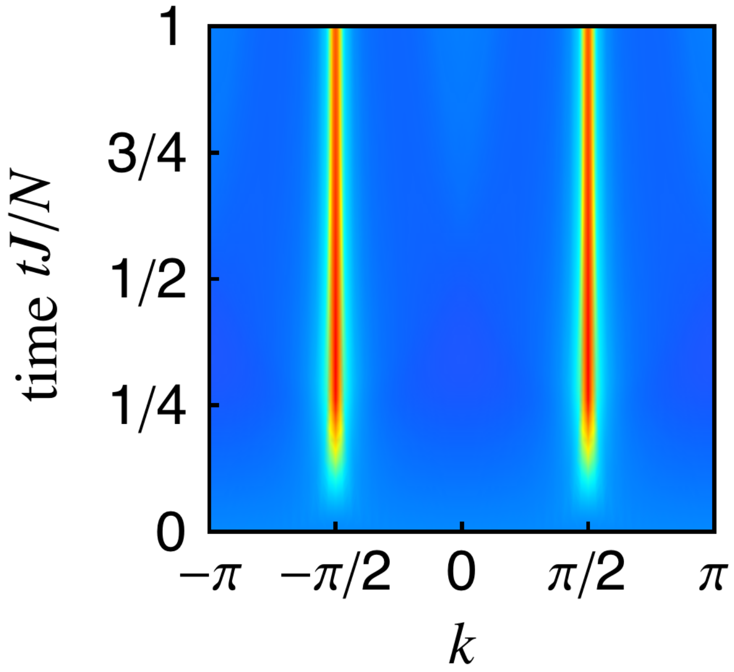
Our numerical simulations (hard-core bosons):

3D cloud, harmonic trap, finite initial entropy (holes!), short-field time-of-flight profiles

Vidmar, Ronzheimer, Schreiber, Hodgman, Braun, Langer, FHM, Bloch, Schneider
PRL 115, 175301 (2015)

Take home message III:

Quasi-MDF



Transient dynamcis:
Quasi-condensation in non-equilibrium!
Observed in experiments!

Vidmar et al. PRL 115, 175301 (2015)
Rigol, Muramatsu, PRL 93, 230404 (2004)

Open questions:

What about 2D?

Time-dep. Gutzwiller: Yes
Jreissaty,et al. PRA 84, 043610 (2011)

DMRG: Inconclusive
Hauschild, Pollmann, FHM PRA 92, 053629 (2015)

Underlying principle?
Correlations atop current-carrying states?

Equilibrium
Antal et al. PRE 57, 5184 (1998)

$$H = H_0 - \lambda j$$

Ground-state reference systems
FHM et al. PRA 78, 013620 (2008)

$$H_t^{\mathrm{ref}}|\psi(t)\rangle = E_t|\psi(t)\rangle$$

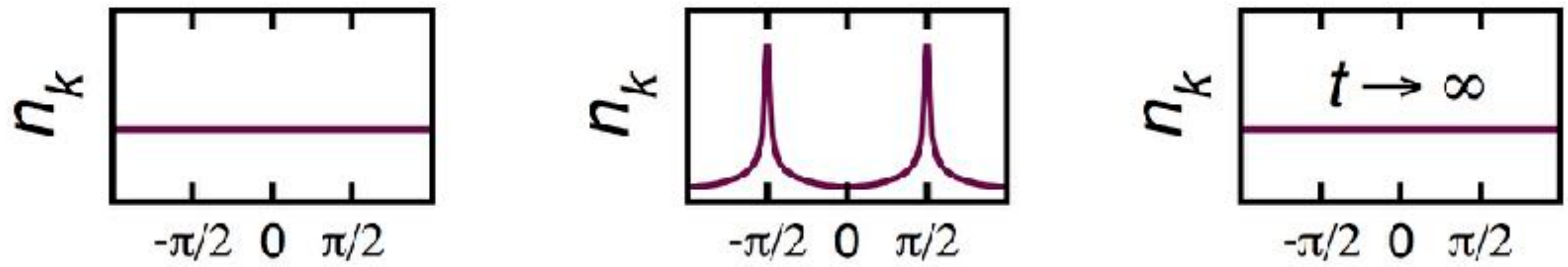
Emergent eigenstate solution
Vidmar, Iyer, Rigol Phys. Rev. X 7, 021012 (2017)

$$H_t^{\mathrm{local}}|\psi(t)\rangle = 0$$

Asymptotic regime: Hard-core bosons

1D Bose-Hubbard model at: $U/J = \infty \quad H = -2J \sum_k \cos(k) n_k^f$

Initial state | **Dynamical quasi-condensation** | **Dynamical fermionization**



Momentum distribution of *physical* particles becomes identical to the one of *underlying free fermions*

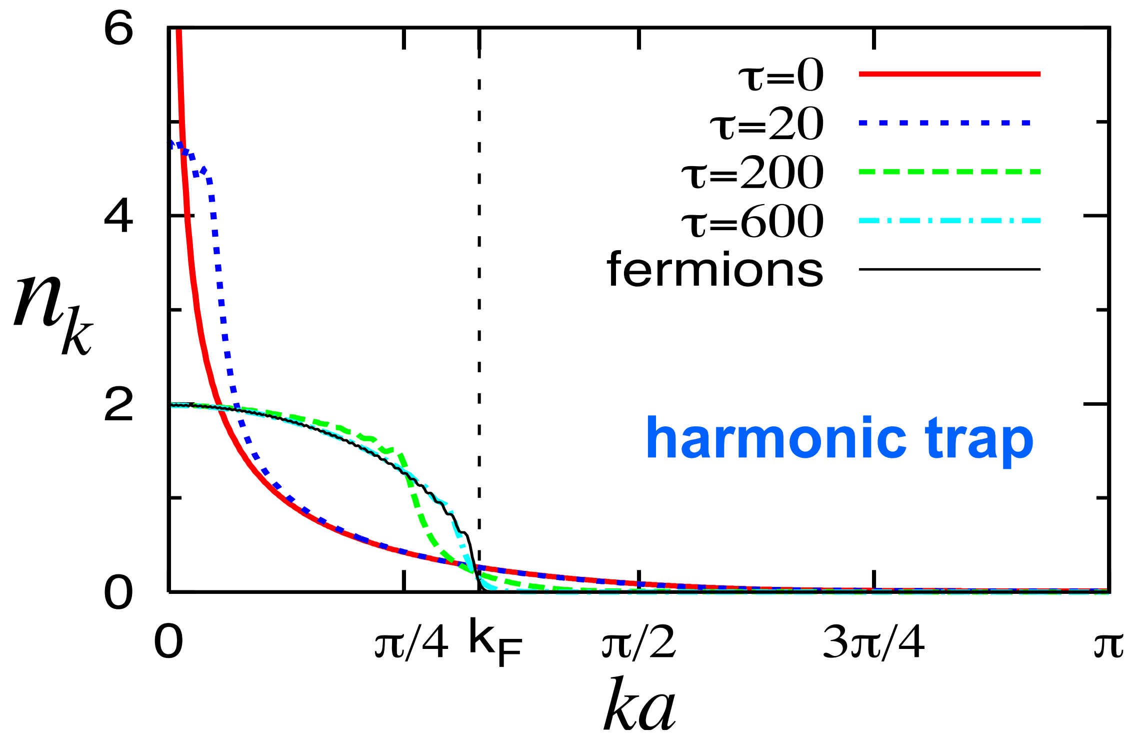
What about bosons at finite U/J?
Interacting Fermions?

$$n_k^{HCB}(t \to \infty) \to n_k^f$$

Sufficiently dilute gas: $\langle H \rangle \to \sum_k \epsilon_k n_k (t = \infty)$

Rigol, Muramatsu, PRL 93, 230403 (2005)
Minguzzi, Gangardt PRL 94, 240404 (2005)

Asymptotic dynamics: Dynamical fermionization



Rigol, Muramatsu, PRL 93, 230403 (2005)

Asymptotic MDF of hard-core bosons given by integrals of motion!

$$n_k^{HCB}(t \to \infty) \to n_k^f$$

$$H = -2J \sum_k \cos(k) n_k^f$$

Analytical solution: Tonks-Girardeau gas
Minguzzi, Gangardt PRL 94, 240404 (2005)
Jukic, Pezer, Gasenzer, Buljan PRA 78 053602 (2008)

Experimental realization: Ongoing at PSU (D. Weiss'group)

What about bosons at finite U/J?
Interacting Fermions?

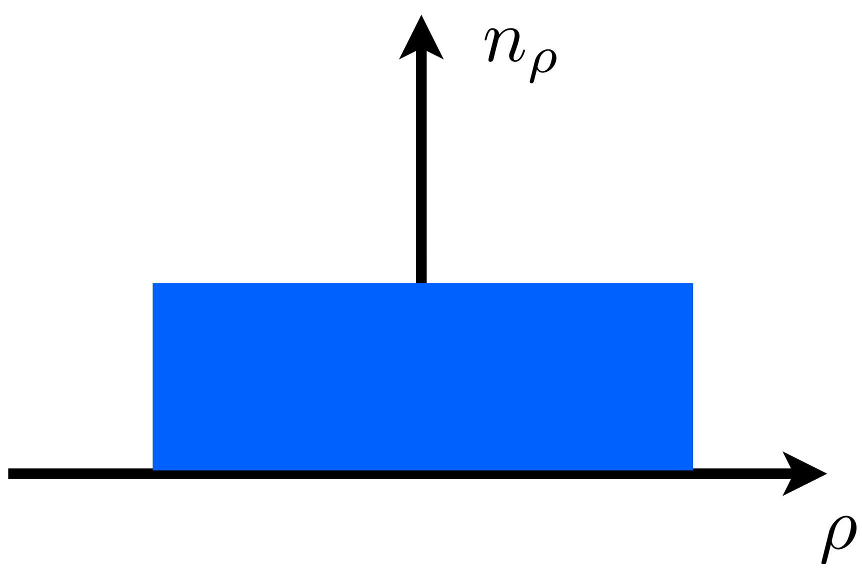
Predicting the asymptotic MDF from "first principles"

Generalization of dynamical fermionization of HCBs for other *integrable* 1D models

Distribution of *rapidities:*

Quantum numbers in Bethe ansatz
Defined by initial state
Time invariant

$$E = \int d\rho \, n_\rho \epsilon_\rho$$



$$\rho = \rho(N, U, \dots)$$

Sutherland's interpretation:
Rapidities = Asymptotic momenta

$$n_k^{\text{physical}}(t \to \infty) \to n_\rho$$

Sutherland PRL 80, 3678 (1988)
Sutherland: "Beautiful models"

Predicting the asymptotic MDF from "first principles"

Here: Fermi-Hubbard model, U<0

$$U < 0; \quad N_\uparrow > N_\downarrow$$

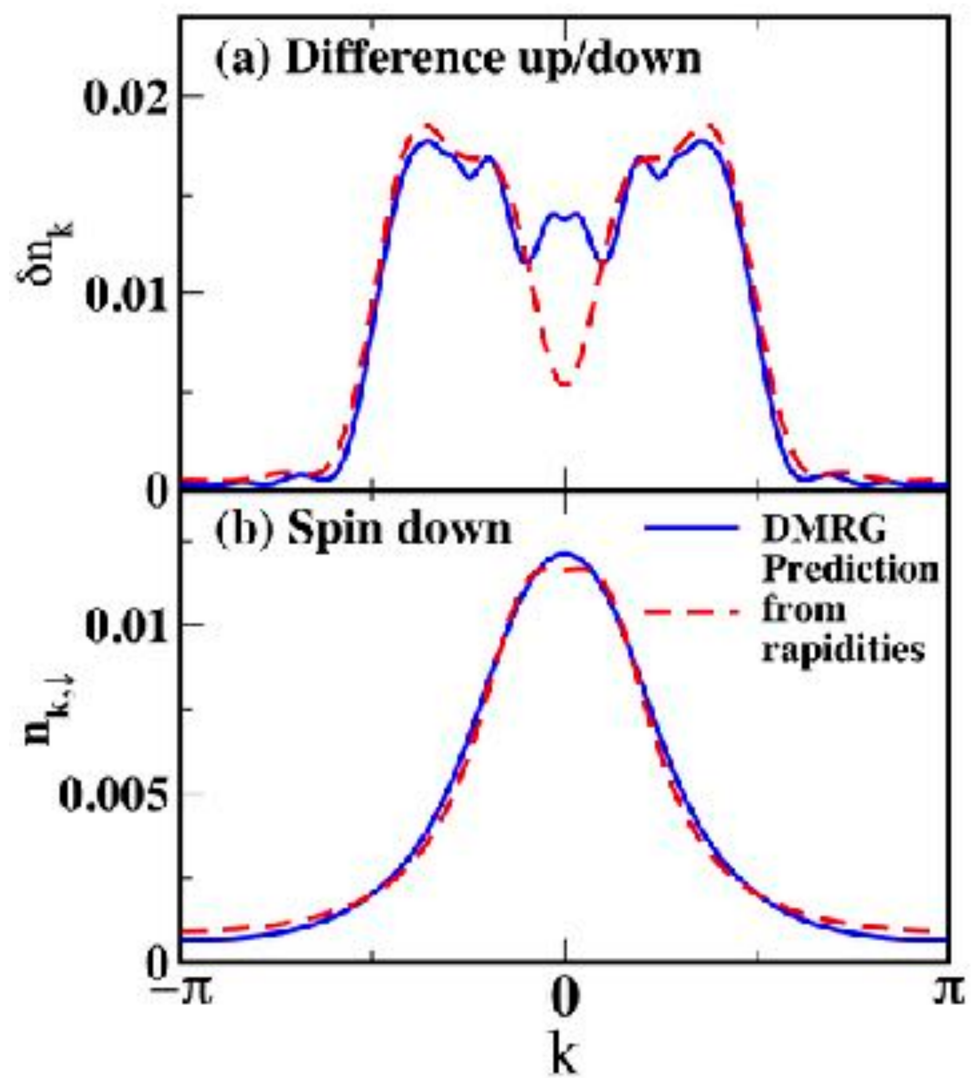
$$N_\downarrow \text{ pairs}$$

$$N_\uparrow - N_\downarrow \text{ unpaired fermions}$$

$$\delta n_k = n_{k,\uparrow} - n_{k,\downarrow} \to n_{\rho_{\text{unpaired}}}$$

$$n_{k,\downarrow}(t \to \infty) \to n_{\rho_{\text{pair}}}$$

Asymptotic form of MDF
U=-8J



Long-time limit of MDFs: Determined by distribution of Bethe-ansatz rapidities of initial state

Bolech, FHM, Langer, McCulloch, Orso, Rigol PRL 109, 110602 (2012)
For the continuum: Campbell, Gangardt, Kheruntsyan PRL 114, 125302 (2015)

Predicting expansion velocities

Repulsive interactions: Slow approach of MDF to asymptotic regime

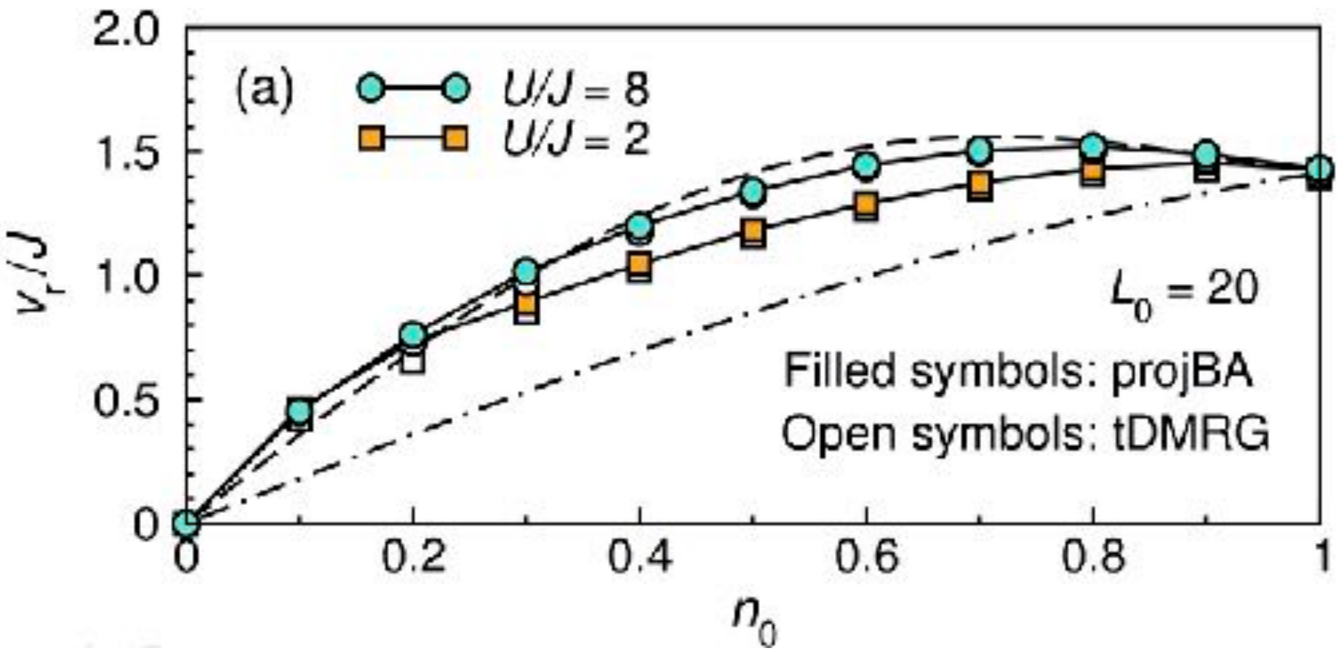
Expansion velocities converge fast !

Consequence $R = v_r t$

$$v_r^2 = \frac{1}{N} \sum_\rho v_\rho^2 n_\rho$$

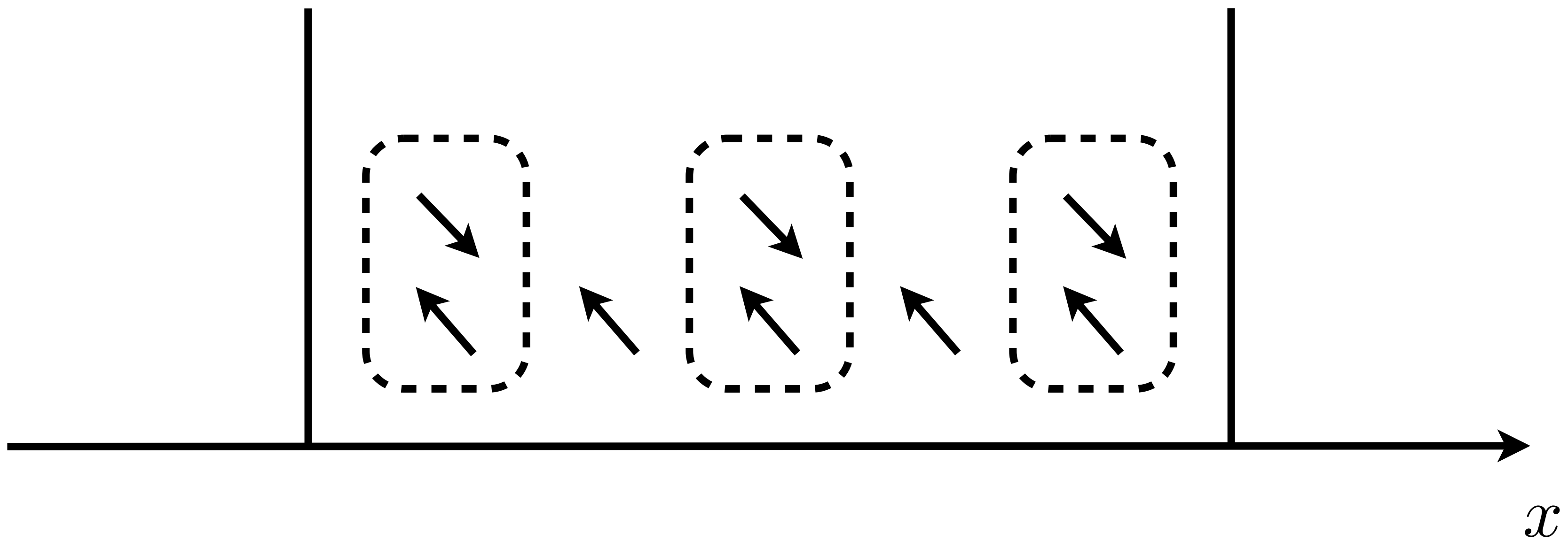
Defined by initial condition
Obtained from Bethe ansatz

DMRG vs Bethe ansatz - fermions
Expansion from ground states



Mei, Vidmar, FHM, Bolech PRA 93, 021607(R) (2016)
Schuetz, Langer, McCulloch, Schollwöck, FHM PRA 85, 043618 (2012)

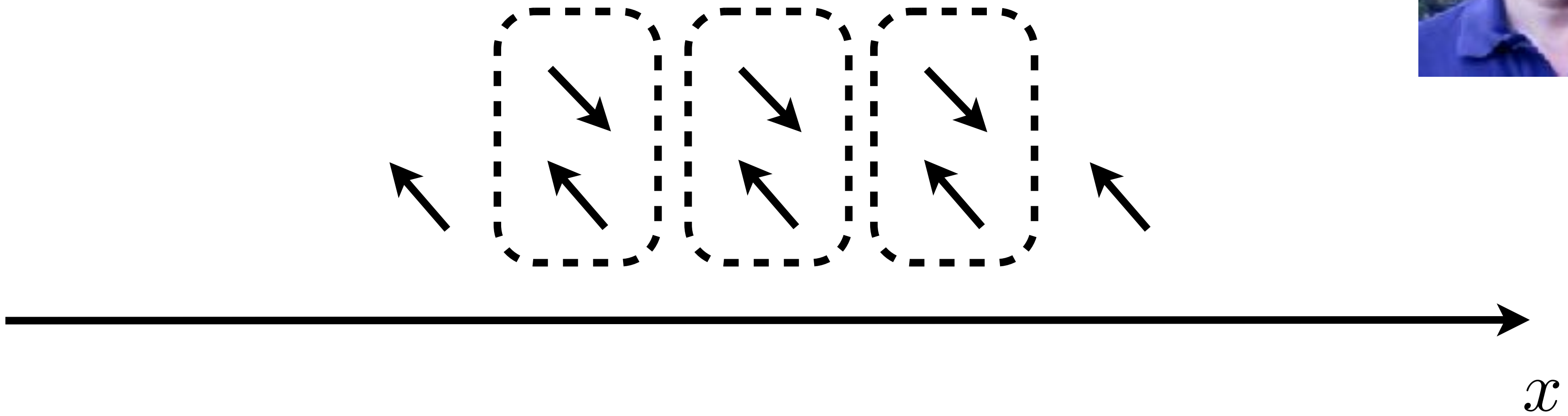
Dynamics of doublons at higher densities: Quantum distillation



$$E = \text{const} \qquad J_{\text{pair}} \sim \frac{J^2}{U} \ll J$$

Expansion blocked by slow pairs/doublons?

Dynamics of doublons at higher densities: Quantum distillation



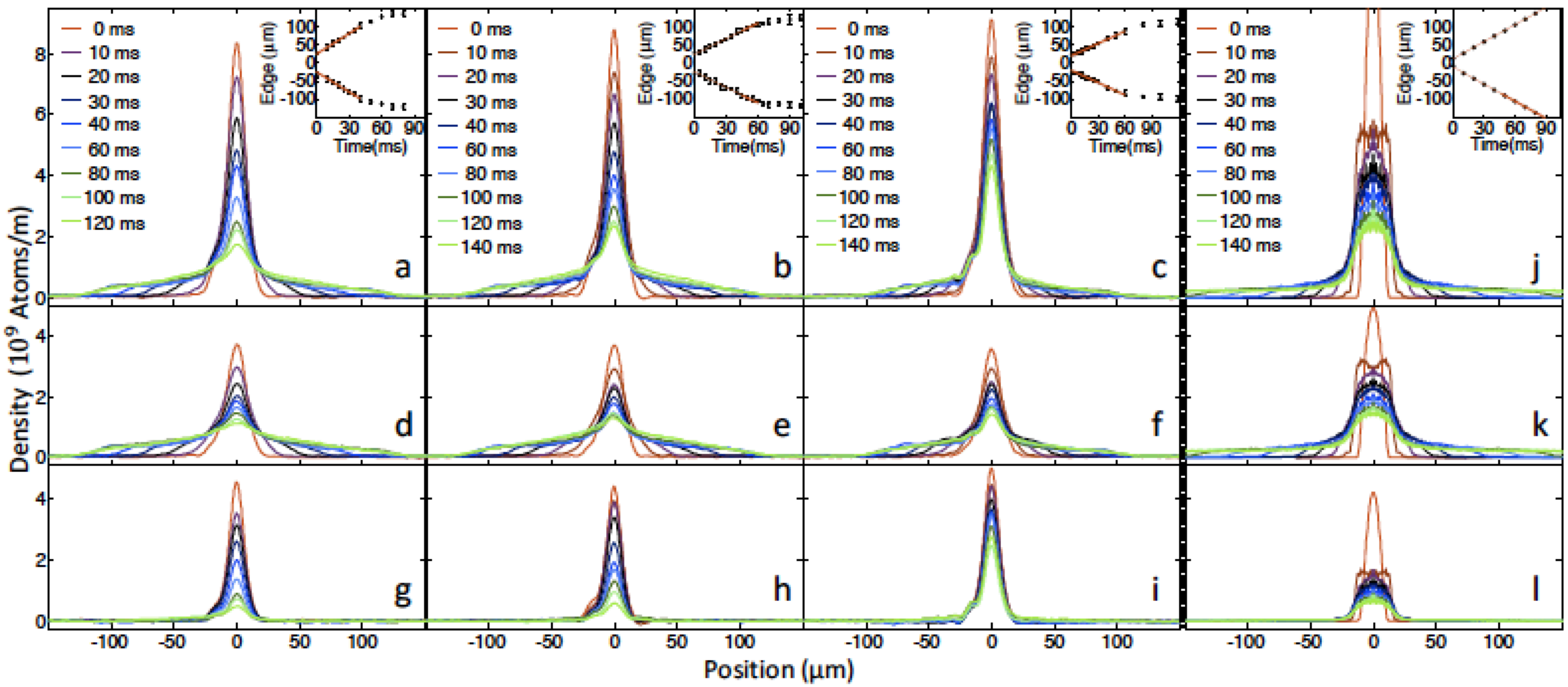
$$E = \text{const} \qquad J_{\text{pair}} \sim \frac{J^2}{U} \ll J$$

Doublons/ Pairs move towards center,
"single" atoms evaporate:"Quantum distillation"

Purification of an imperfect fermionic band insulator!

FHM, Manmana, Rigol, Muramatsu, Feiguin, Dagotto PRA 80, 041603(R) (2009)
Bosons: Muth, Petroysan, Fleischhauer PRA 85, 013615 (2012)

Quantum distillation: Observed for bosons!



Bosons cluster in the center!

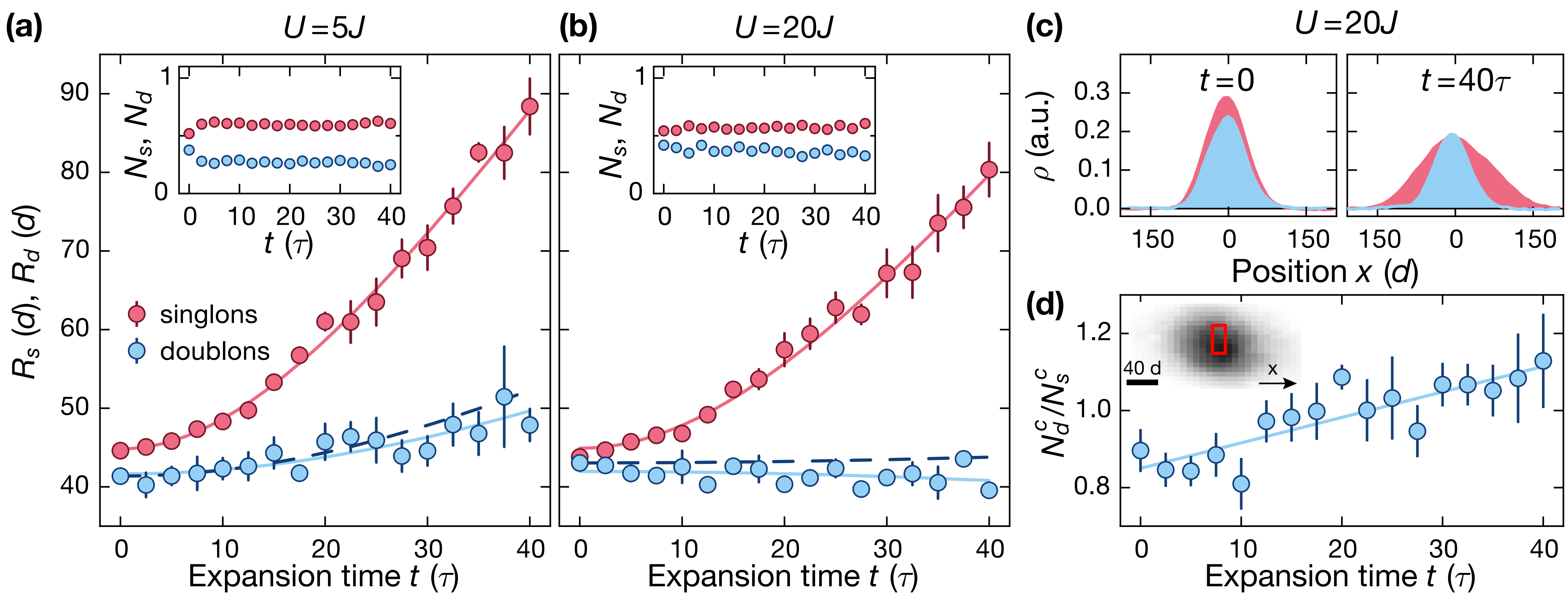
Experiment from PSU (D. Weiss' group)

Xia et al. Nature Physics 11, 316 (2015)

Open: Does this work in 2d? Experiments for fermions?

New LMU experiment: Quantum distillation with fermions

1d Fermi-Hubbard $$H = -J \sum_{\langle i,j \rangle} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, in preparation

In collaboration with

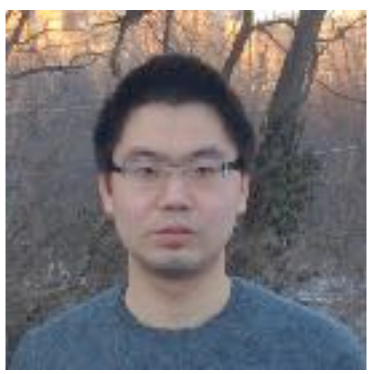
Lev Vidmar, Ljubljana

Stefan Sorg
Lode Pollet
Ulrich Schollwöck
LMU Munich

Ian McCulloch
U Queensland

Sebastian Scherg
Thomas Kohlert
Simon Braun
Michael Schreiber
Sean Hodgman
Monika Aidelsburger
LMU Munich &
MPQ Garching

Philipp Ronzheimer



Zhangtao Mei & Carlos Bolech
U Cinncinati

Immanuel Bloch
LMU & MPQ

Ulrich Schneider
Cambridge, UK

Stephan Langer
U Pittsburgh

Giuliano Orso
U Paris-Diderot

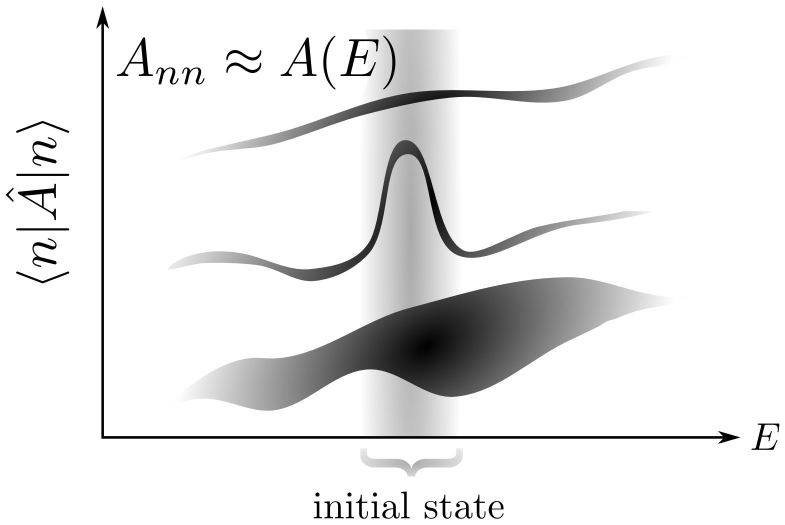
Marcos Rigol
PSU

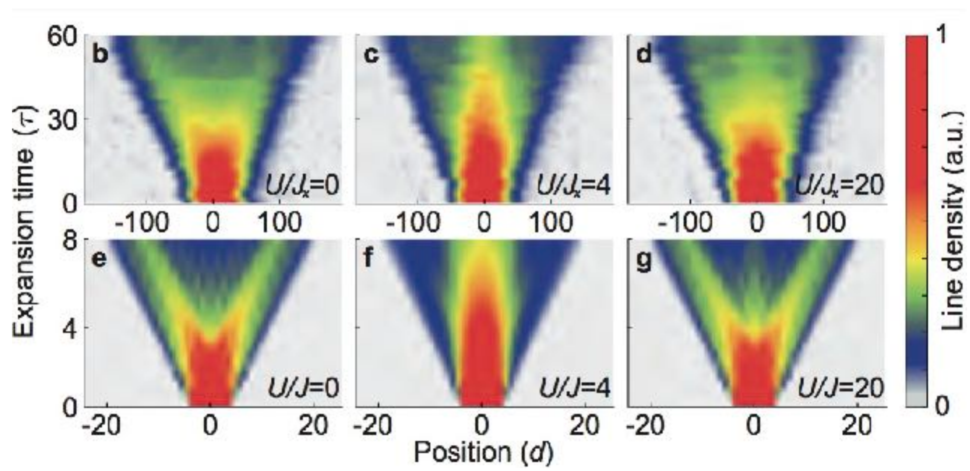
Deutsche Forschungsgemeinschaft DFG

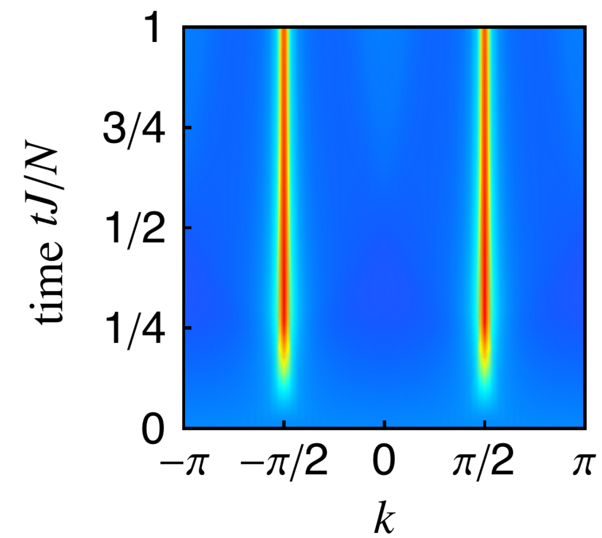
Exciting physics in non-equilibrium!

I. Generic many-body systems thermalise in the ETH sense!

III. Quasi-condensation of 1D bosons in non-equilibrium!







II. Ballistic nonequilibrium Transport of 1d hardcore bosons

IV. Asymptotic properties
Hidden structure of many-body wave functions of integrable models

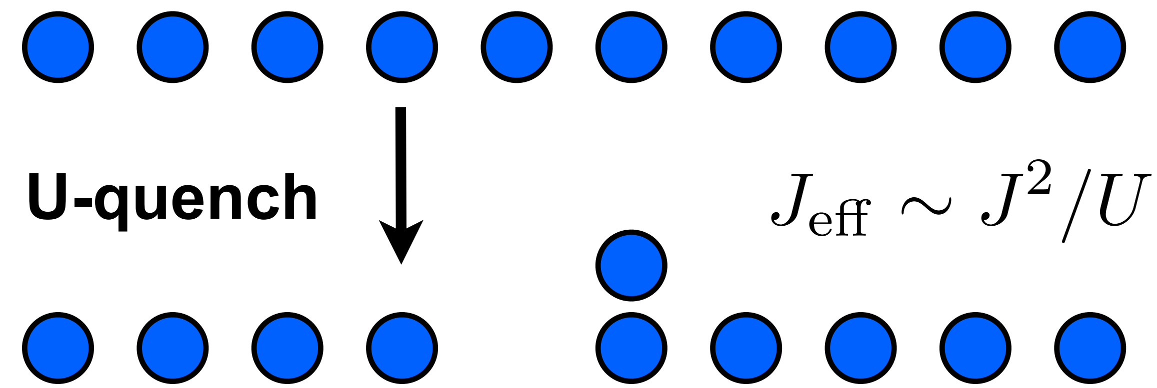
Thank you!

Breaking integrability: Beyond 1D hard-core bosons

$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$$

Dynamics at U/J<∞

Initial state not eigenstate in trap:
Local dynamics due to
interaction quench

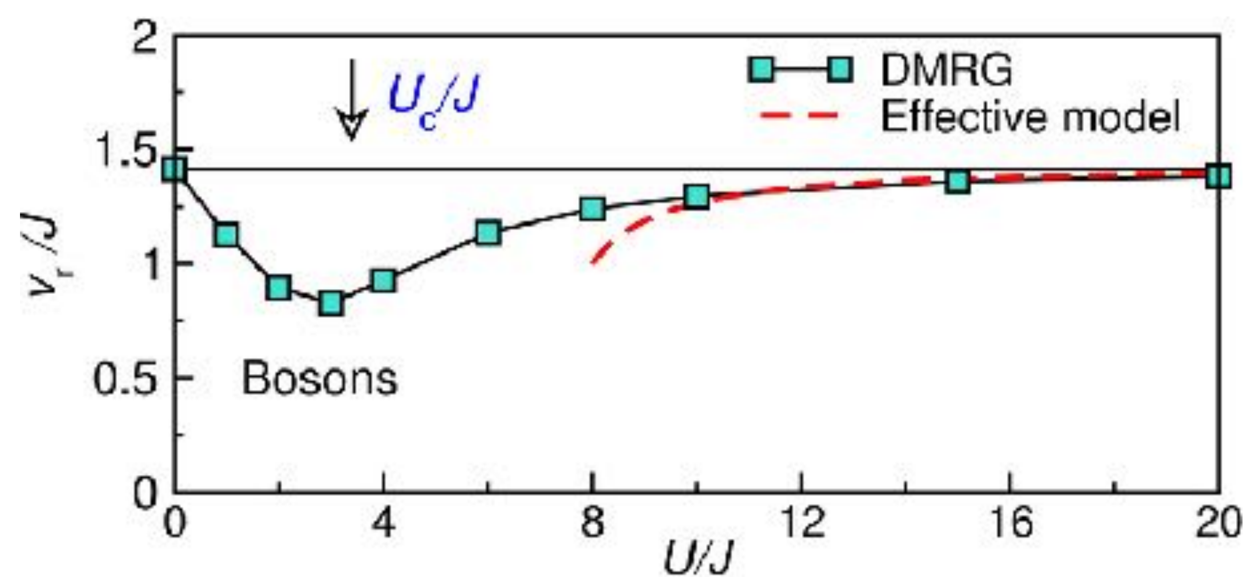


Dynamical formation of doublons
on fast time scale ~ 1/J

Expansion velocity

Large U/J:
“Ballistic” expansion of “single” atoms
but no expansion of doublons

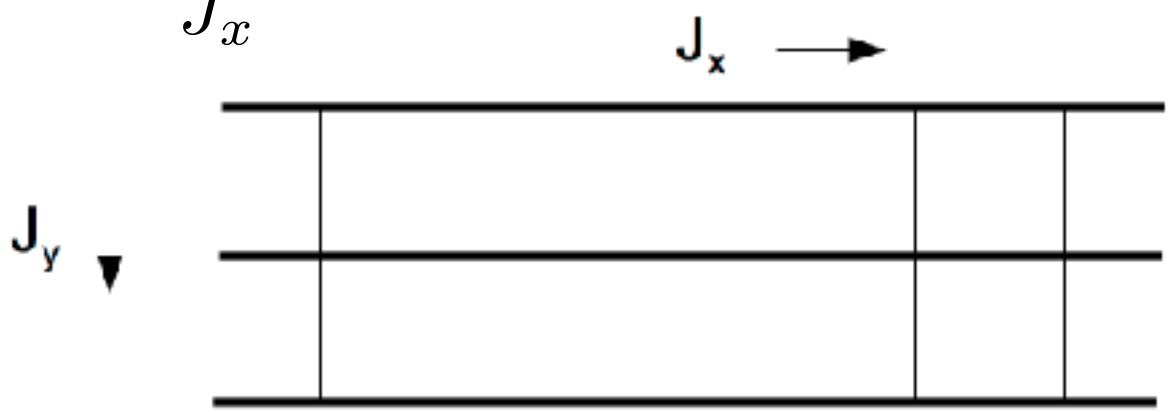
$$v_r = v_r^{HCB}\sqrt{\frac{N - 2N_{\text{doublons}}}{N}}$$

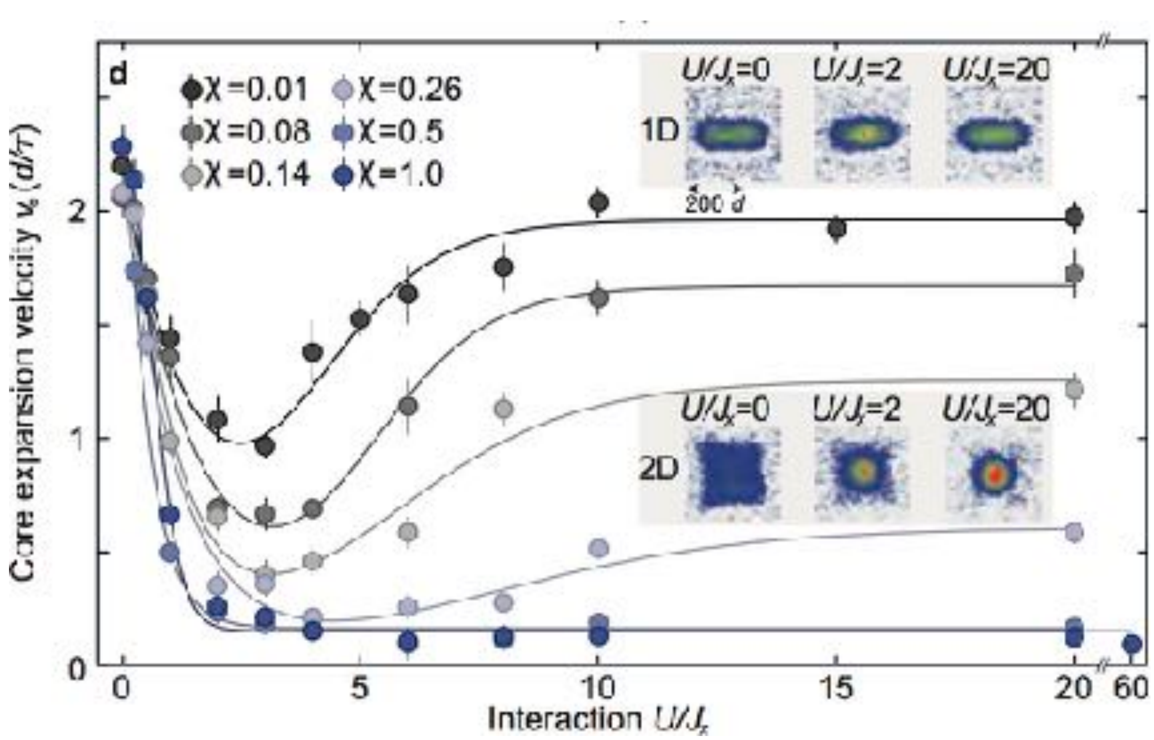


Sorg, Vidmar, Pollet, FHM, PRA 90, 033606 (2014)
Boschi et al. PRA 90, 043606 (2014)
Kajala et al. PRL 106, 206401 (2011)

Outlook: Non-equilibrium transport in 2D?

$$\chi = \frac{J_y}{J_x}$$





Finite J_y suppresses expansion rapidly!

Ronzheimer et al. Phys. Rev. Lett. 110, 205301 (2013)
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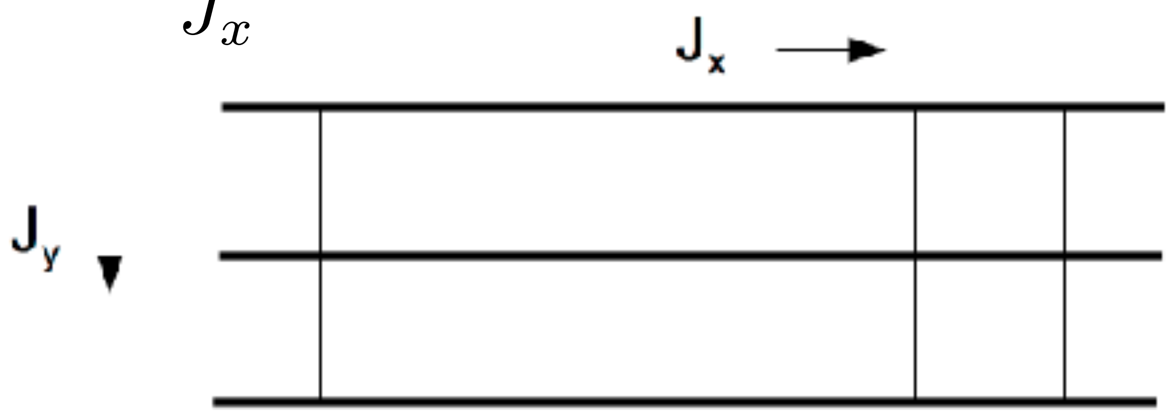

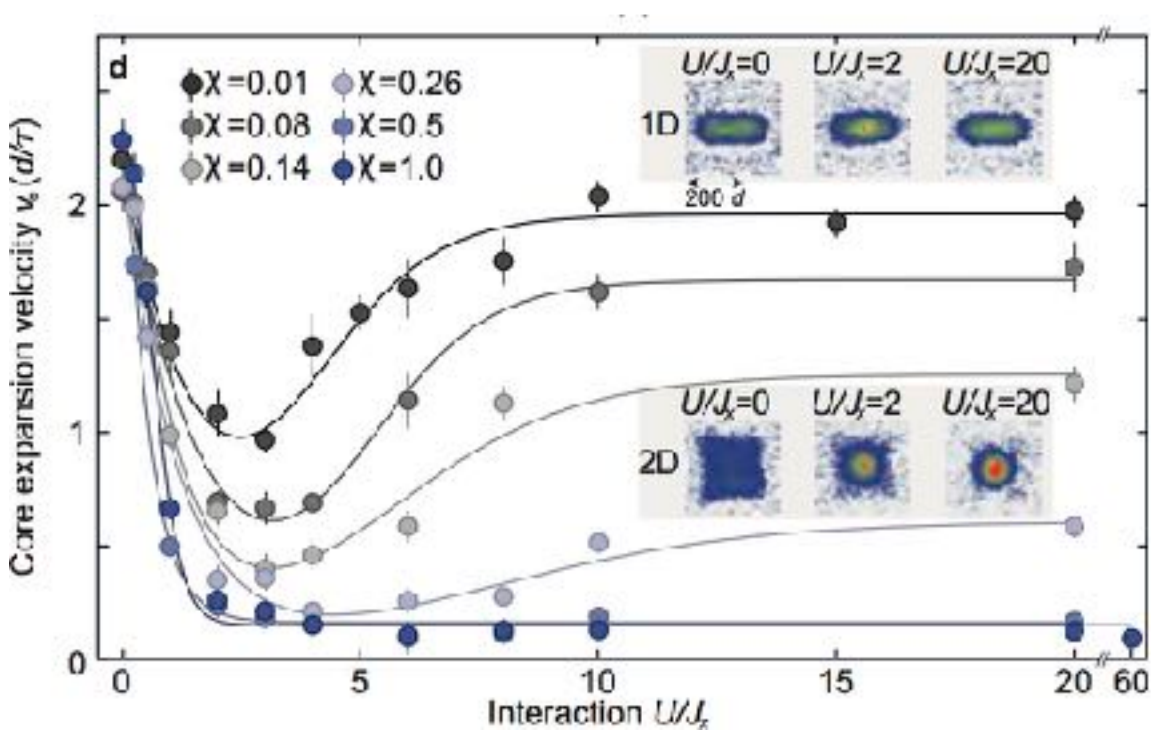

Finite J_y suppresses expansion rapidly!

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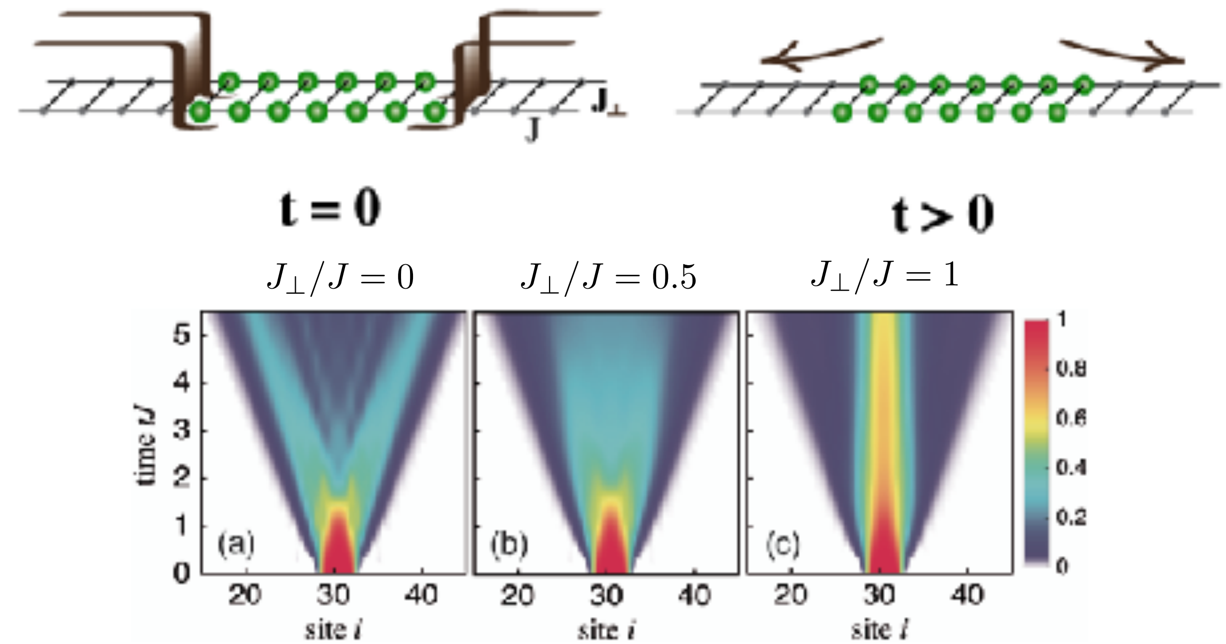
Schneider et al. Nat. Phys. 8, 213 (2012)

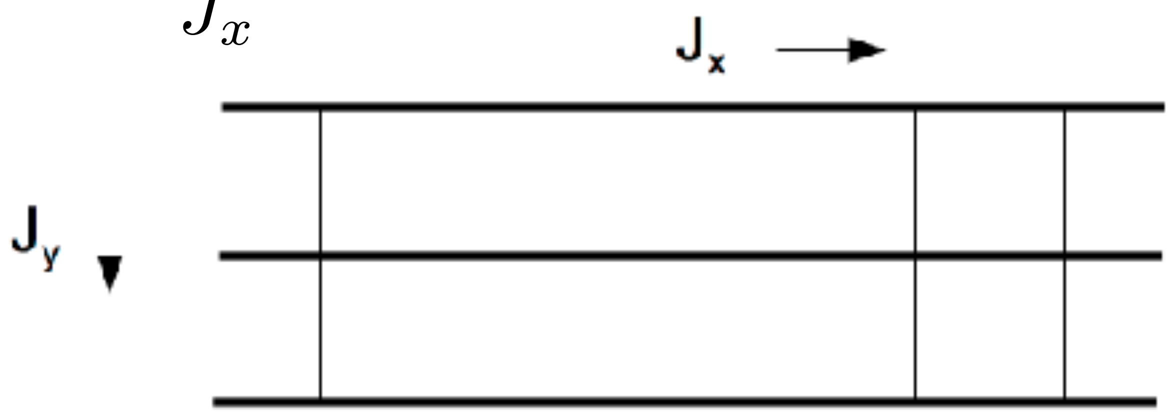
DMRG: Coupled chains, ladders

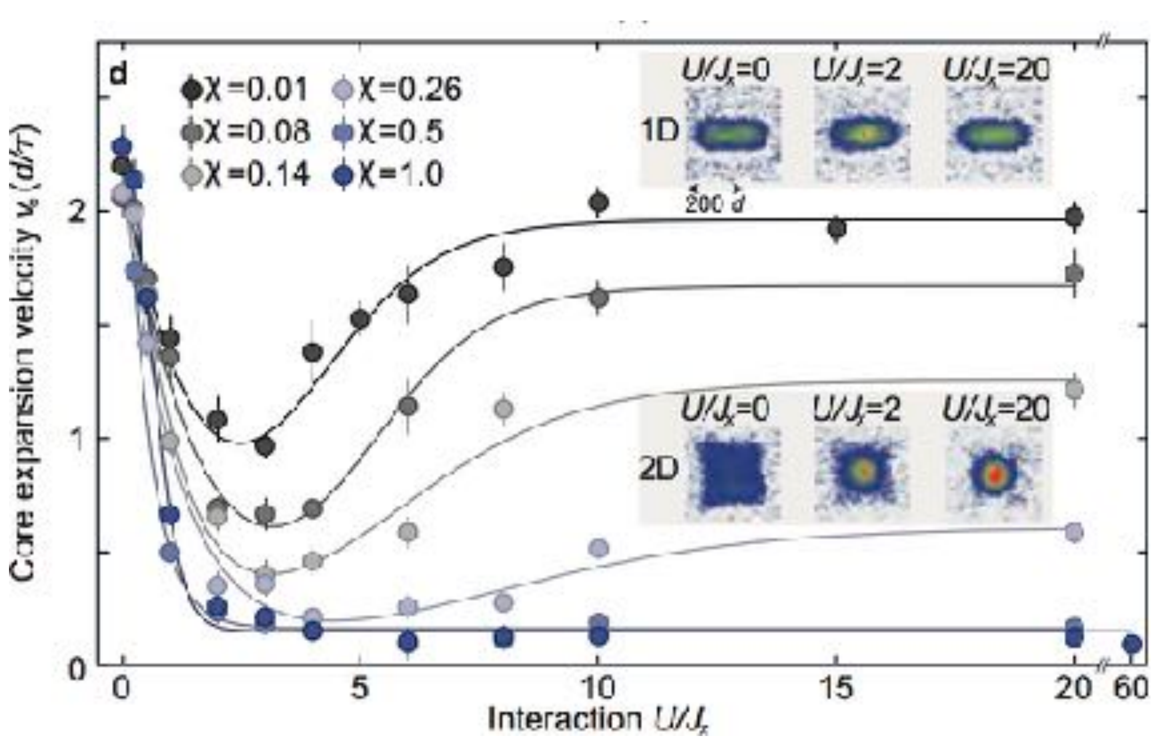


Vidmar et al. PRB 88, 235117 (2013)
Steinigeweg, FHM, et al. PRB 90, 094417 (2014)

Outlook: Non-equilibrium transport in 2D?

$$\chi = \frac{J_y}{J_x}$$

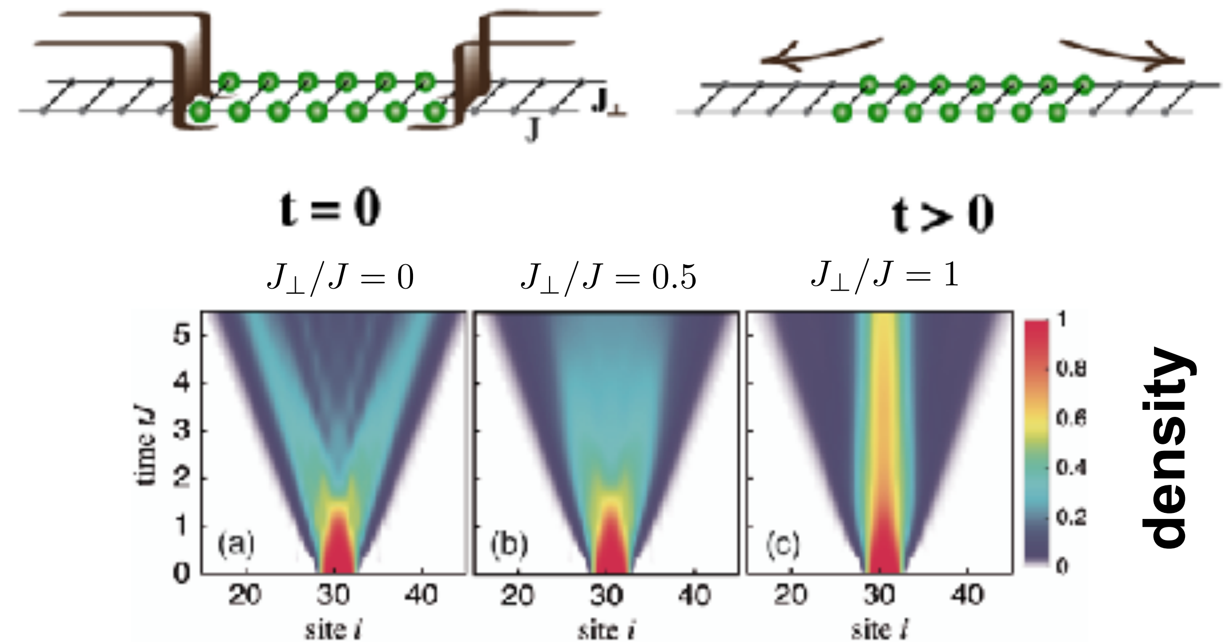




Finite J_y suppresses expansion rapidly!

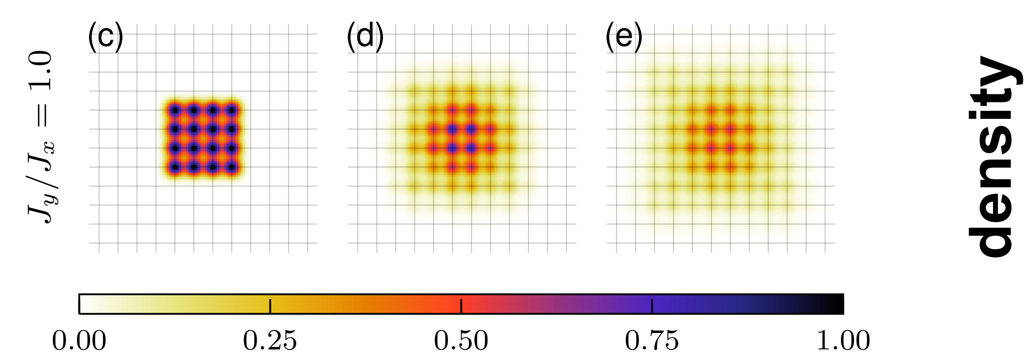
Ronzheimer et al. Phys. Rev. Lett. 110, 205301 (2013)
Schneider et al. Nat. Phys. 8, 213 (2012)

DMRG: Coupled chains, ladders



Vidmar et al. PRB 88, 235117 (2013)
Steinigeweg, FHM, et al. PRB 90, 094417 (2014)

DMRG: Hard-core bosons in 2D



Hauschild, Pollmann, FHM PRA 92, 053629 (2015)
Method: Zaletel et al. PRB 91, 165112 (2015)