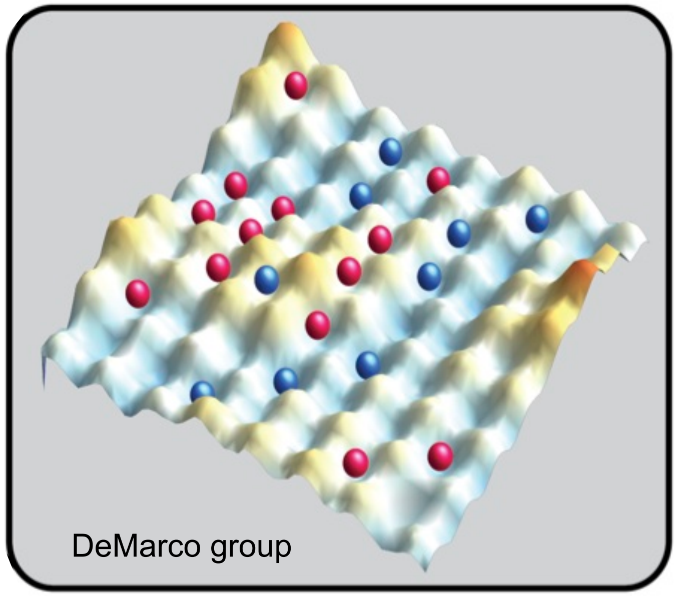
Many-body localization



Fabian Heidrich-Meisner
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April 18, 2018

GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

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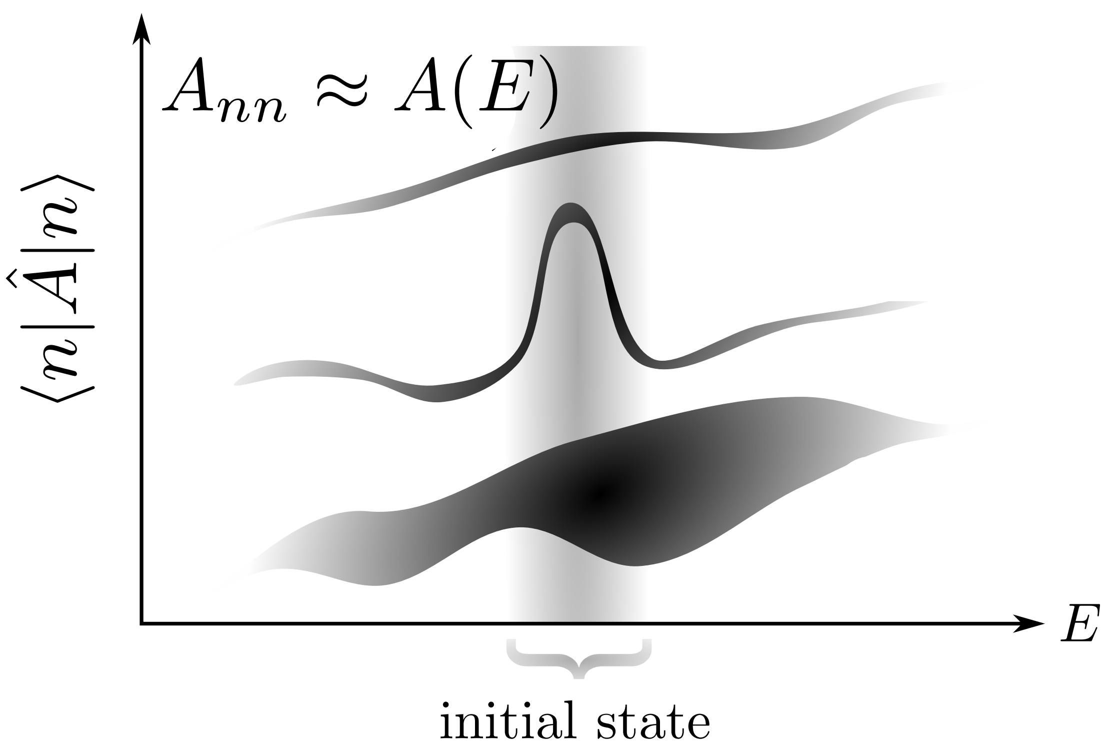
For details: Bera, Schomerus, FHM, Bardarson, Phys. Rev. Lett. 115, 046603 (2015)
Bera, Martynec, Schomerus, FHM, Bardarson Annalen der Physik (2017)

Eigenstate thermalization hypothesis

Local observable in closed many-body system:
Expectation values in eigenstates are thermal

Rigol, Dunjko, Olshanii Nature 2008; Prosen Phys. Rev. E 60, 3949 (1998), Deutsch Phys. Rev. A 43, 2046 (1991); Srednicki Phys. Rev. E 50, 888 (1994)



Sub-system density matrices are thermal

$$\rho_A = \mathrm{tr}_E[|n\rangle\langle n|] = \rho_{\mathrm{thermal}}$$

Typical many-body eigenstates:

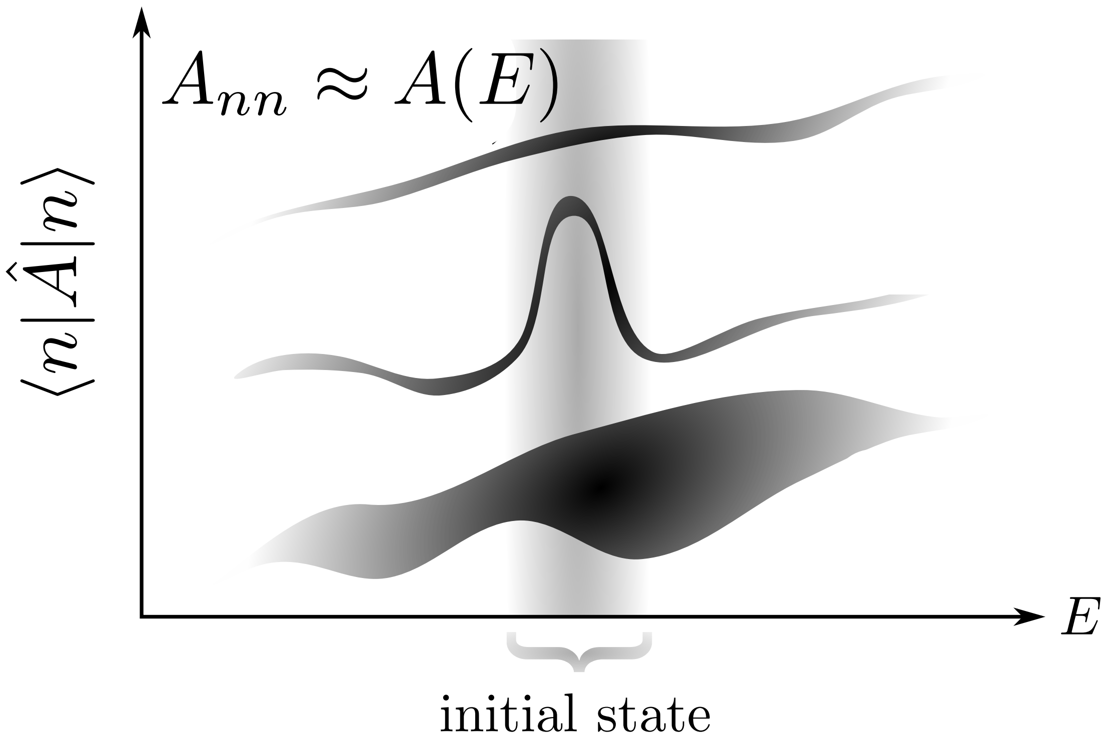
$$H|n\rangle = E_n|n\rangle$$

$$E_n \approx E_m : \quad \langle n|\hat{A}|n\rangle \approx \langle m|\hat{A}|m\rangle$$

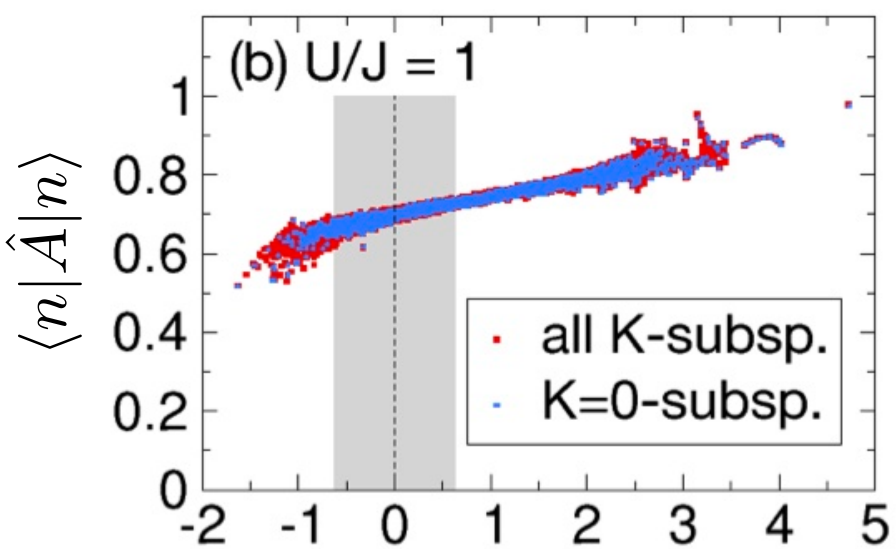
Eigenstate thermalization hypothesis

**Local observable in closed many-body system:
Expectation values in eigenstates are thermal**

Rigol, Dunjko, Olshanii Nature 2008; Prosen Phys. Rev. E 60, 3949 (1998), Deutsch Phys. Rev. A 43, 2046 (1991); Srednicki Phys. Rev. E 50, 888 (1994)



**Typical example:
1D Bose-Hubbard model**

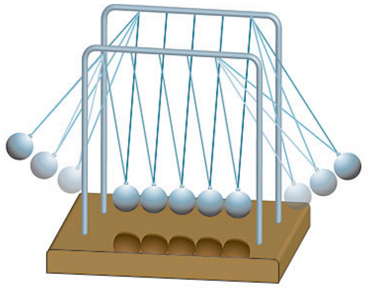


eigen-energy

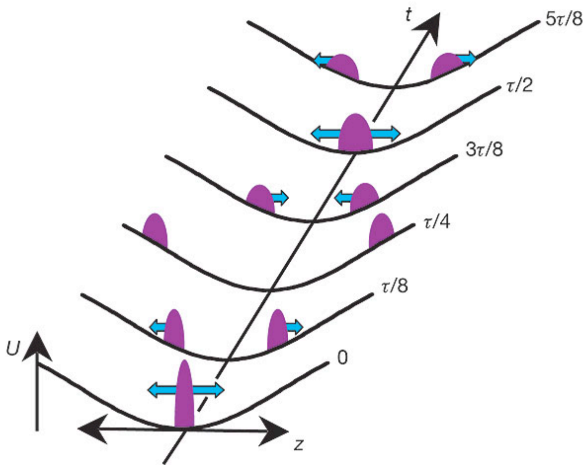
Sorg, Vidmar, Pollet, FHM PRA (2014)

Thermalization: Exceptions

Integrable 1D systems
(Natan Andrei's lecture)



$$[H, I_i] = 0$$

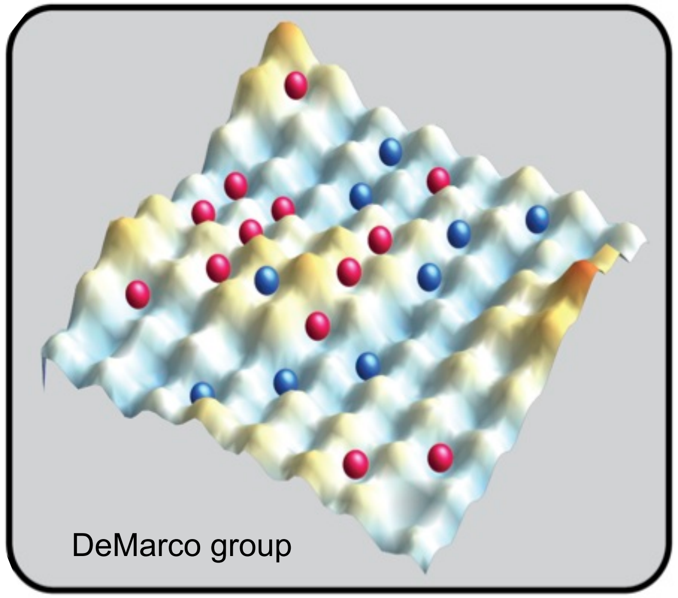


Lieb-Liniger model: 1D Bose gas

Kinoshita, Wenger, Weiss Nature, 440, 900 (2006)
Schmiedmayer group:
Langen et al. Science 348, 207 (2015)

Integrable systems (can) avoid thermalization

Many-body localization
(today)



MBL systems don't thermalize, no fine-tuning

Outline

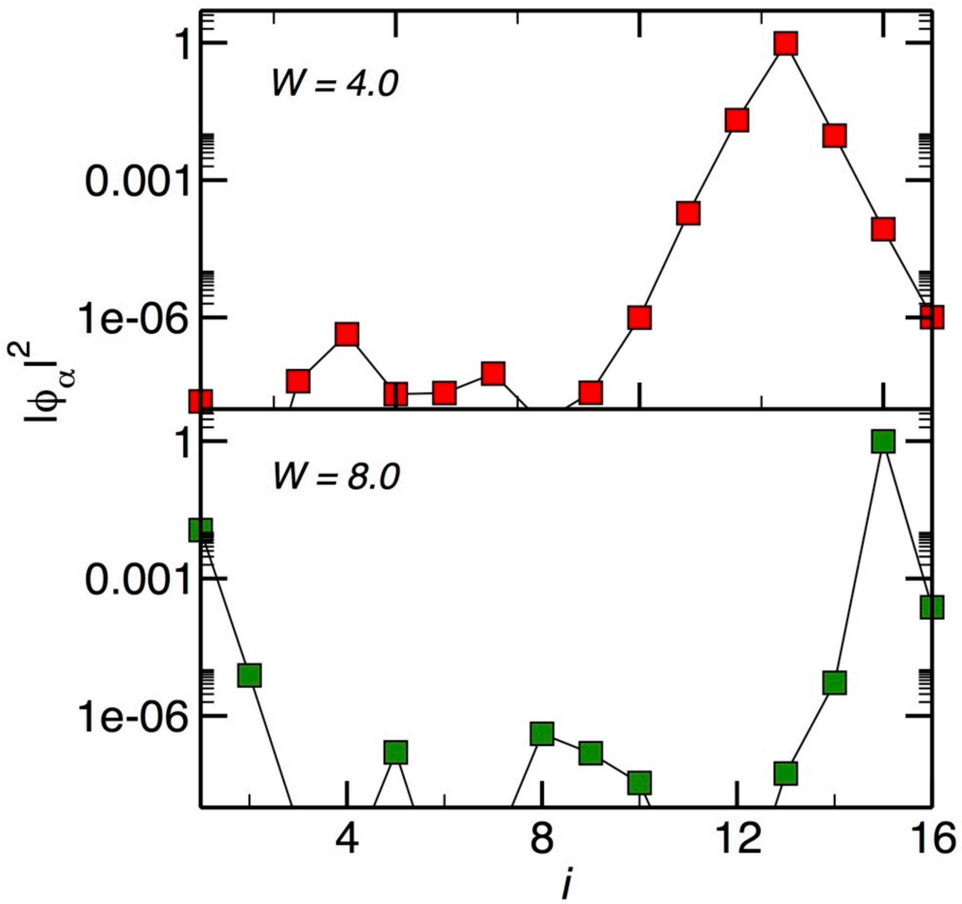
1) Anderson localization

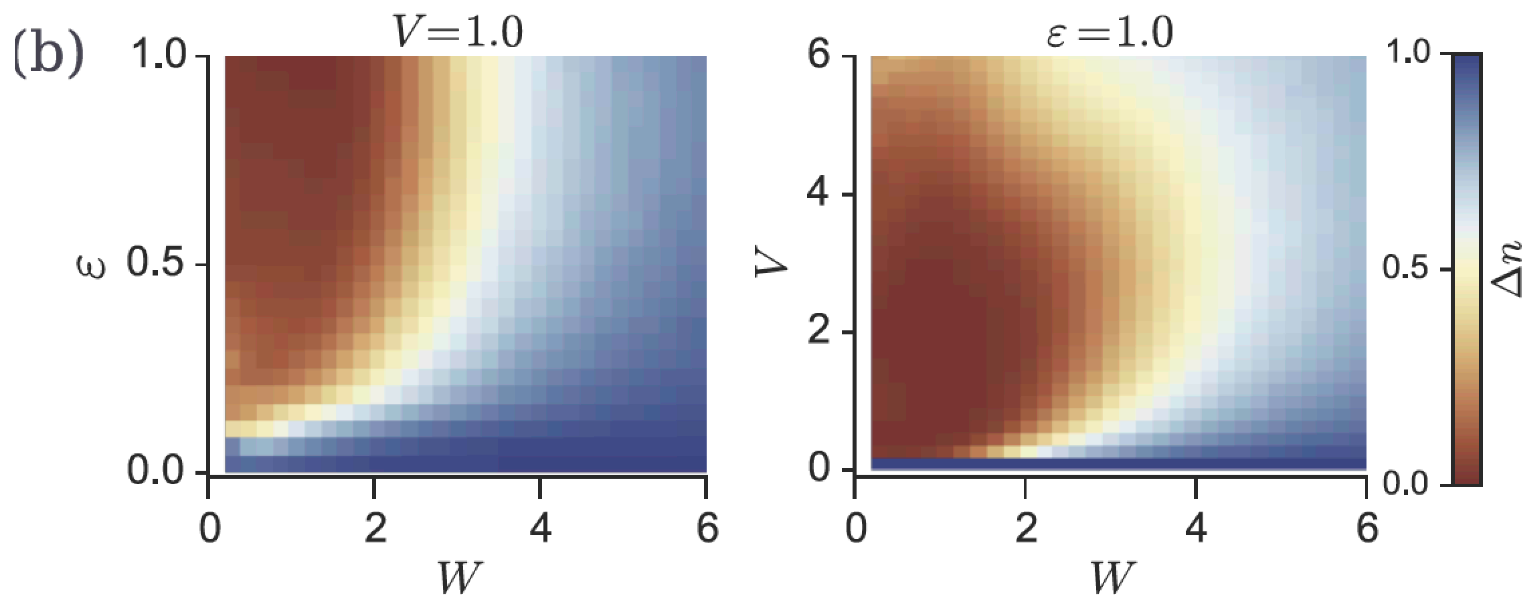
2) Overview: Many-body localization in 1D

3) Experiments

4) One-particle characterization:
   One-particle density matrix (OPDM)

5) Fermi-liquid analogy

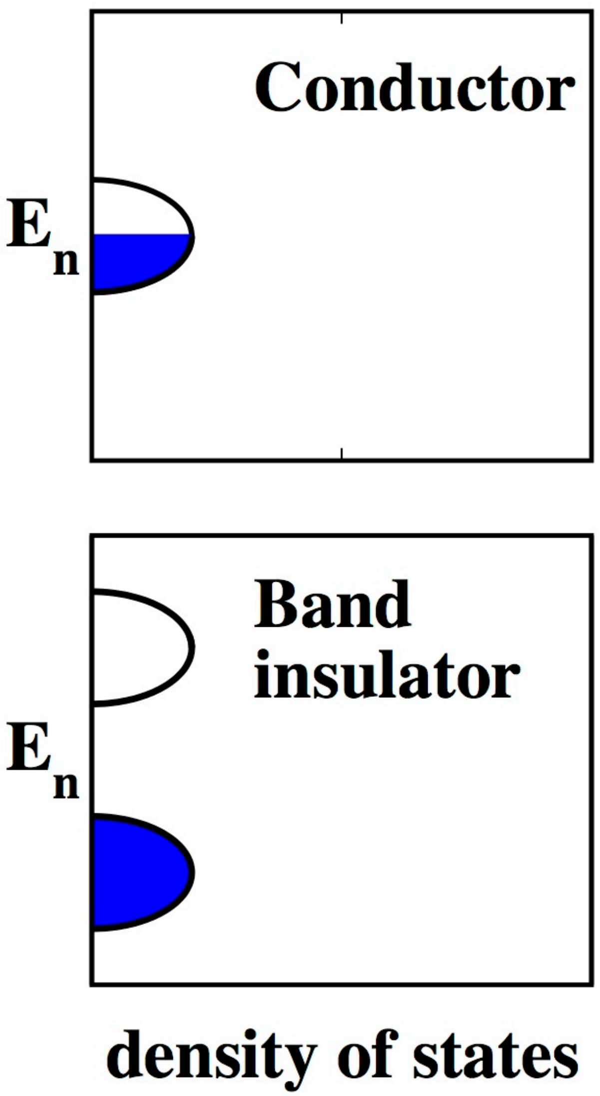


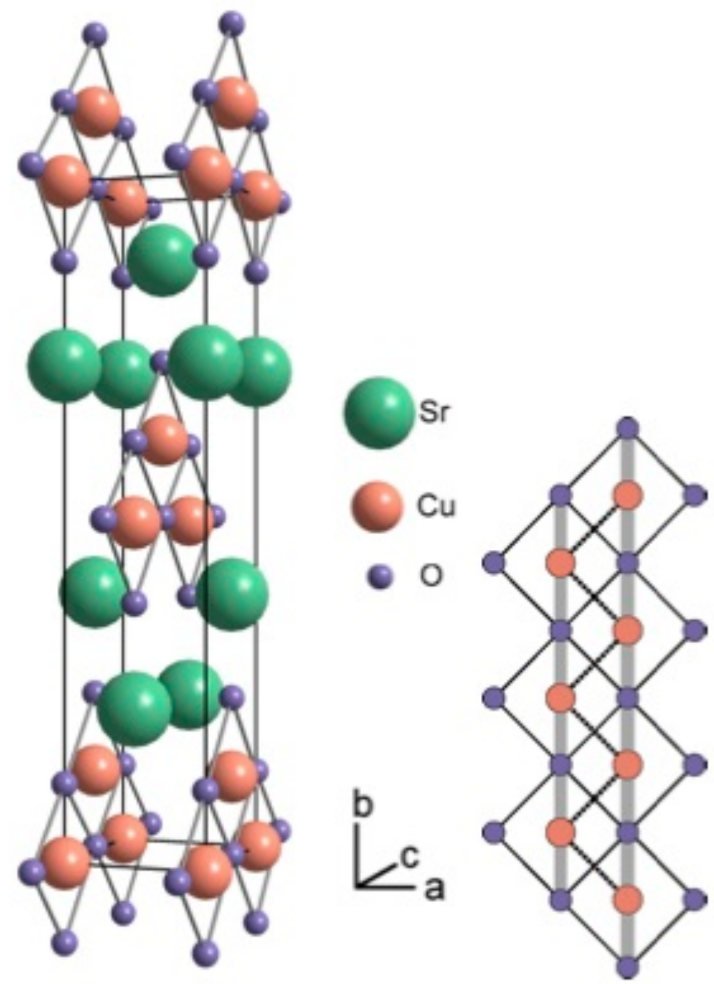


Anderson localization

Insulators in condensed matter physics

Band-insulator | **Mott-insulator**





$SrCuO_2$

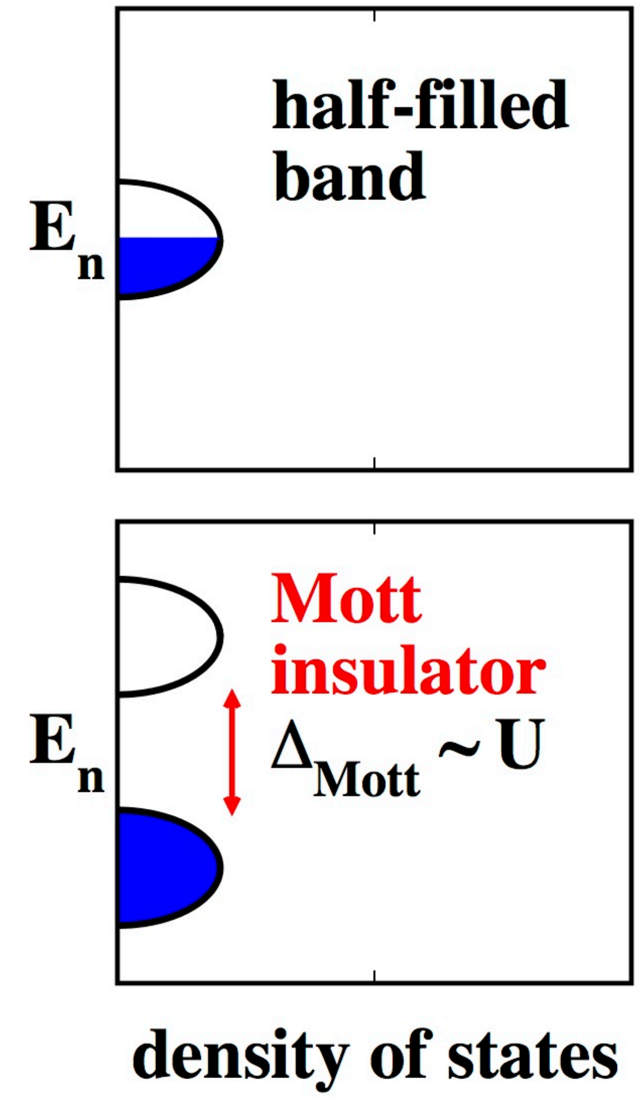


Figure courtesy of N. Hlubek

$$\sigma_{\rm dc}(T=0)=0 \quad \sigma_{\rm dc}(T>0)>0$$

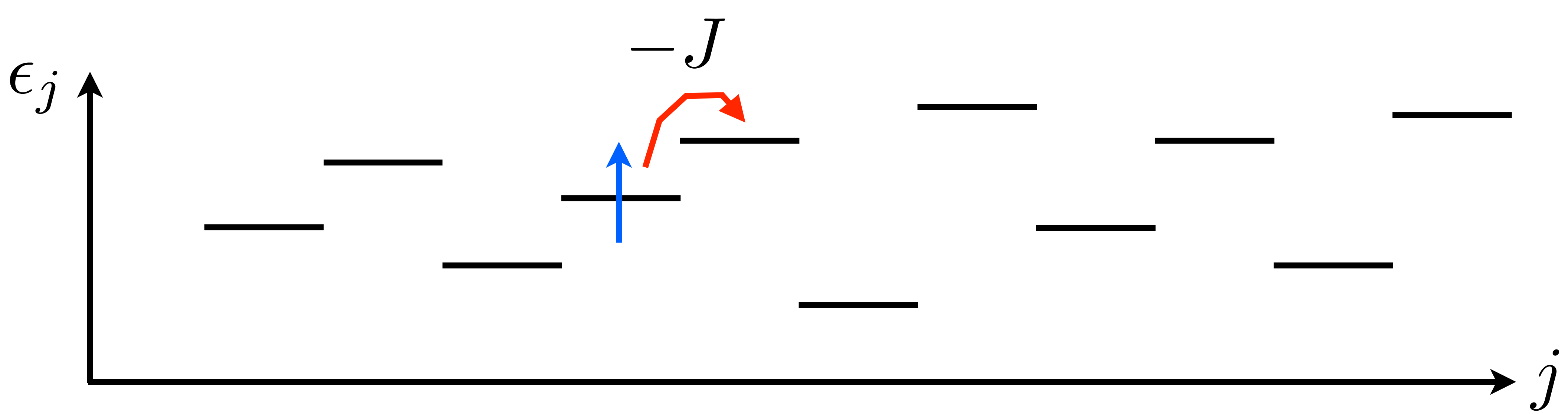
Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



Spectrum of fully localized single-particle states possible:

$$\sigma_{\mathrm{dc}}(T \geq 0) = 0$$

Anderson localization

Electrons in periodic potential: Bloch states

$$\psi_{\vec{k}}(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\psi_{\vec{k}}(\vec{r})$$

$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.)$$

Electrons in the presence of disorder:
full localization of all single-particle eigenstates possible

Asymptotic form of eigenstates:
localization length

$$\psi(r) = f(r)e^{-r/\xi}$$

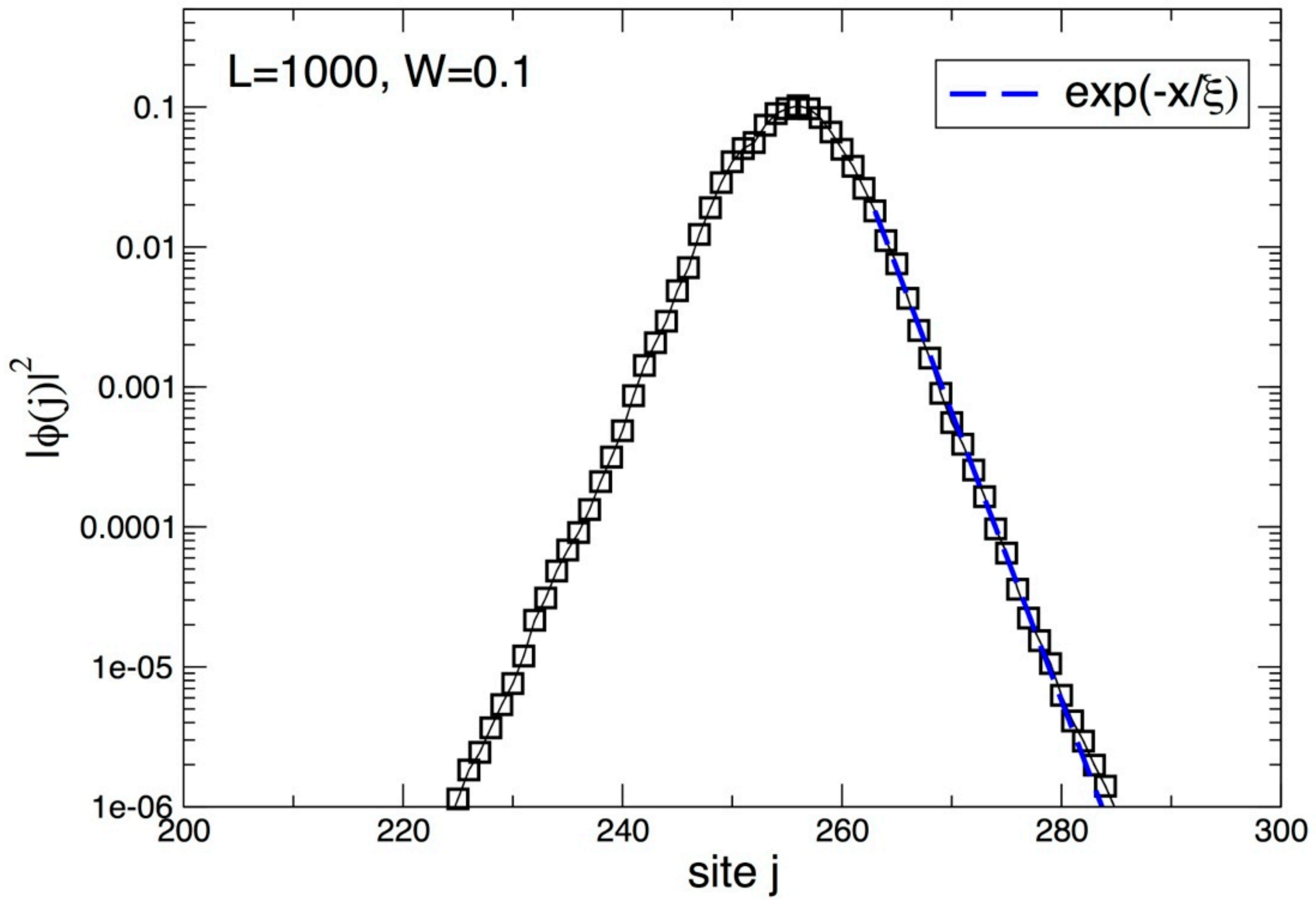
$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i$$

$$\epsilon_i \in [-W, W]$$

Anderson Phys. Rev. 109, 1492 (1958)

Kramer, McKinnon Rep. Prog. Phys. 1993, Evers, Mirlin Rev. Mod. Phys. 2008

Typical localized single-particle state



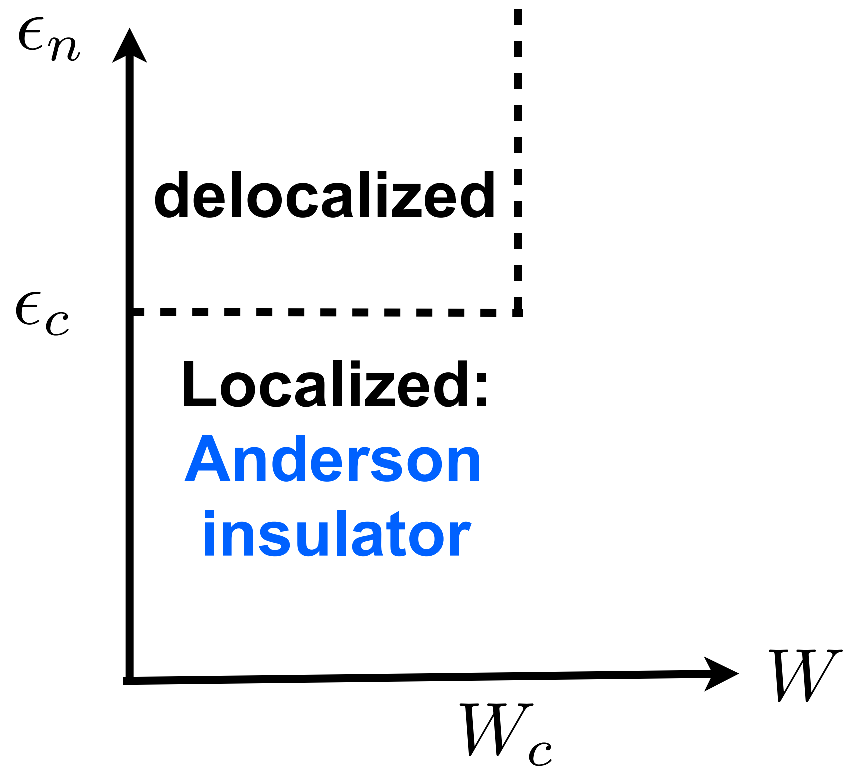
1D model of spinless fermions:

$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i \qquad \epsilon_i \in [-W, W]$$

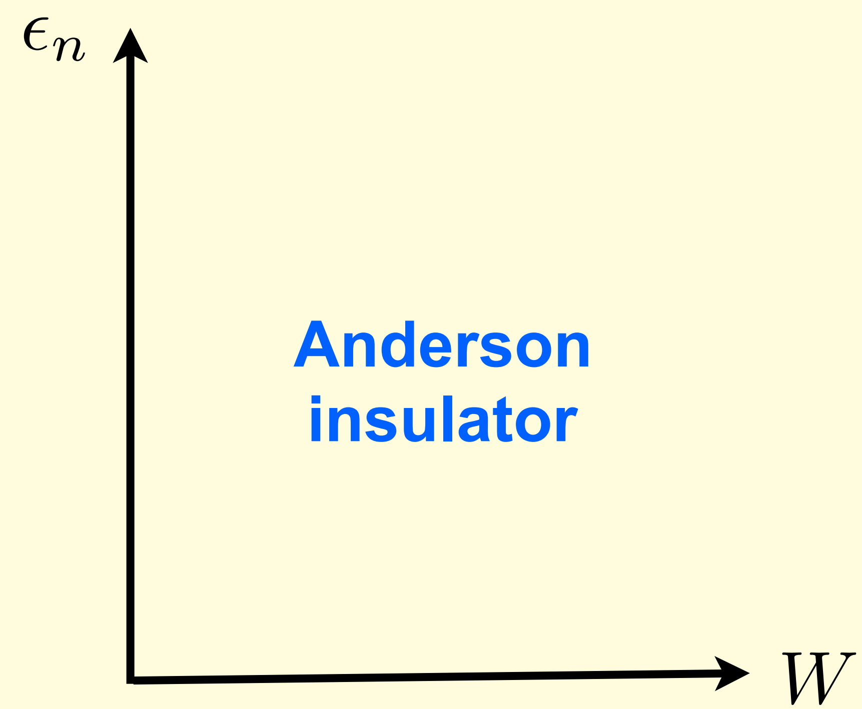
Anderson localization: Key results

$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i \qquad \epsilon_i \in [-W, W]$$

Single-particle eigenstates: $H|n\rangle = \epsilon_n |n\rangle$



3D: Mobility edge, critical W_c



1D: all single-particle eigenstates are localized

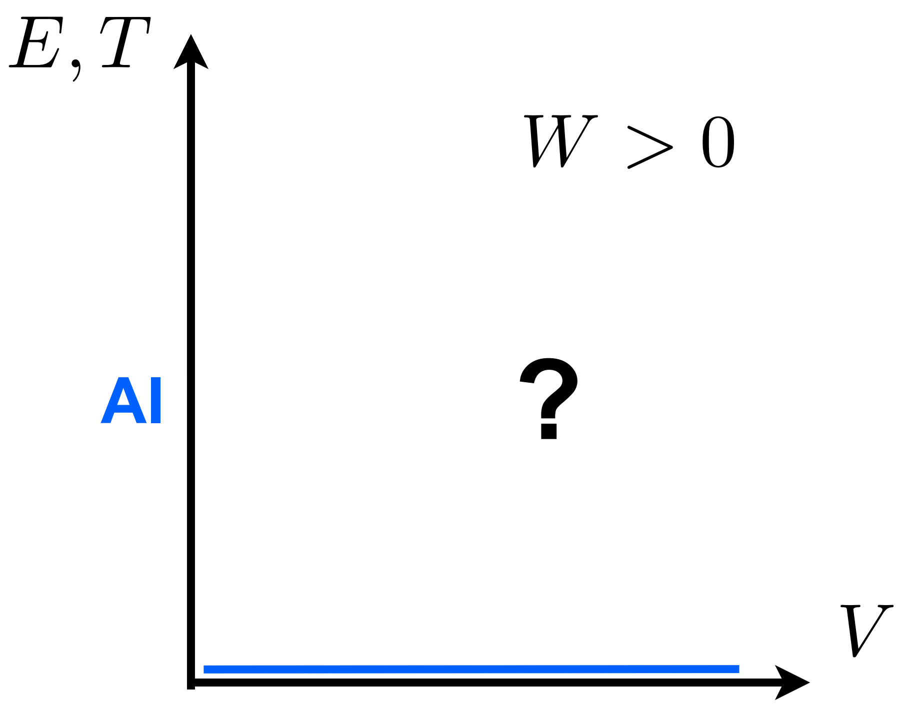
Kramer, McKinnon Rep. Prog. Phys. 1993, Evers, Mirlin Rev. Mod. Phys. 2008

Disorder & interactions

$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.) + V\sum_i n_i n_{i+1} - \sum_i \epsilon_i n_i \qquad \epsilon_i \in [-W, W]$$

Single-particle eigenstates: $H|n\rangle = \epsilon_n |n\rangle$

1D: all single-particle eigenstates are localized



1) Can interactions cause delocalization?

2) Is a perfect T>0 insulator possible in the presence of interactions?

3) Mobility edge? Metal-insulator transition line?

Kramer, McKinnon Rep. Prog. Phys. 1993, Evers, Mirlin Rev. Mod. Phys. 2008

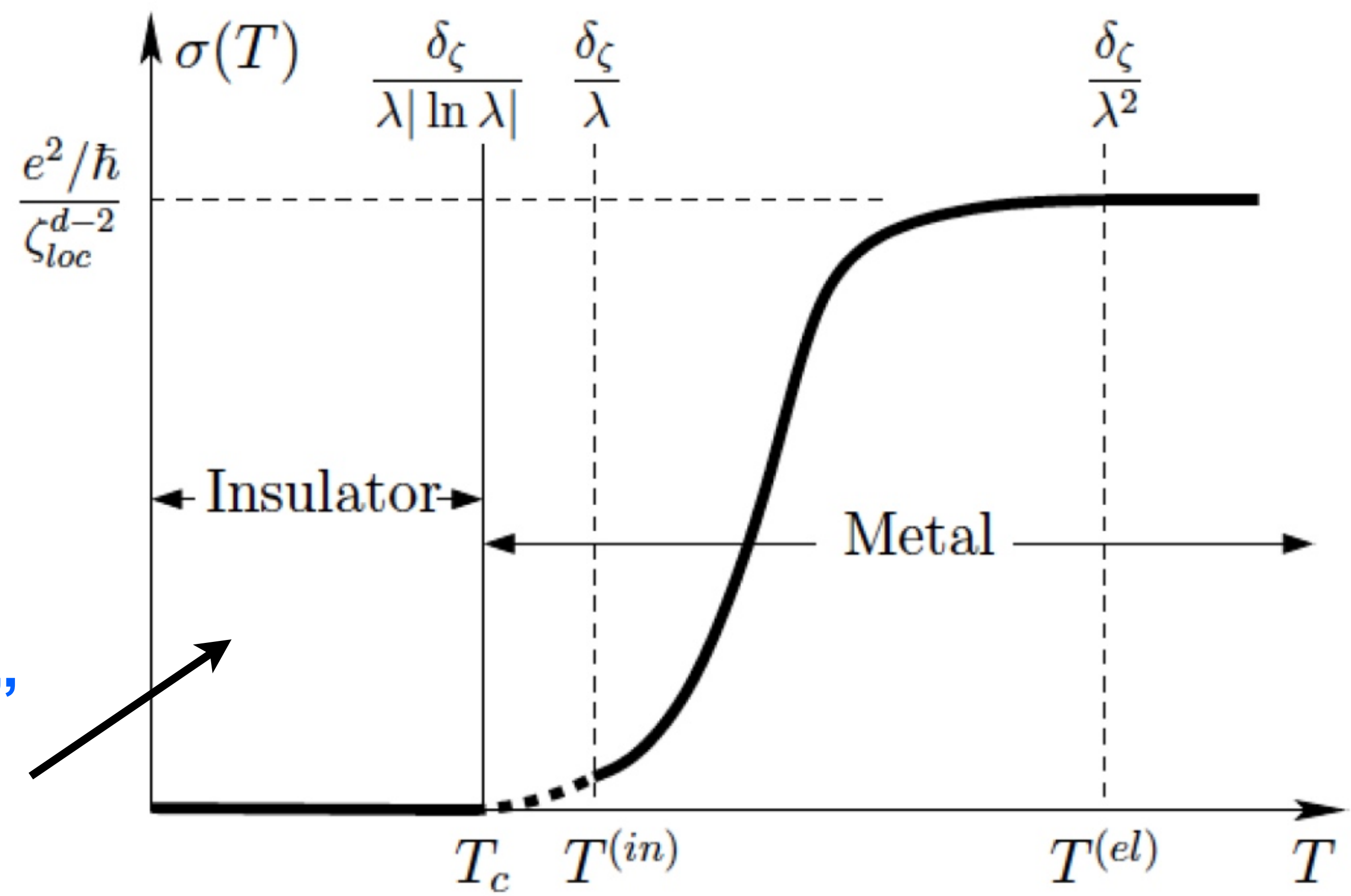
Many-body localization

Interactions & disorder: Many-body localization (MBL)

Is perfectly insulating behavior at finite temperatures possible? **Yes!**

Can interactions give rise to delocalization at T>0? **Yes!**

"Many-body" localized



Perturbative analysis

Basko, Aleiner, Altshuler Annals of Physics (2006)
Gornyi, Mirlin, Polyakov PRL (2005)

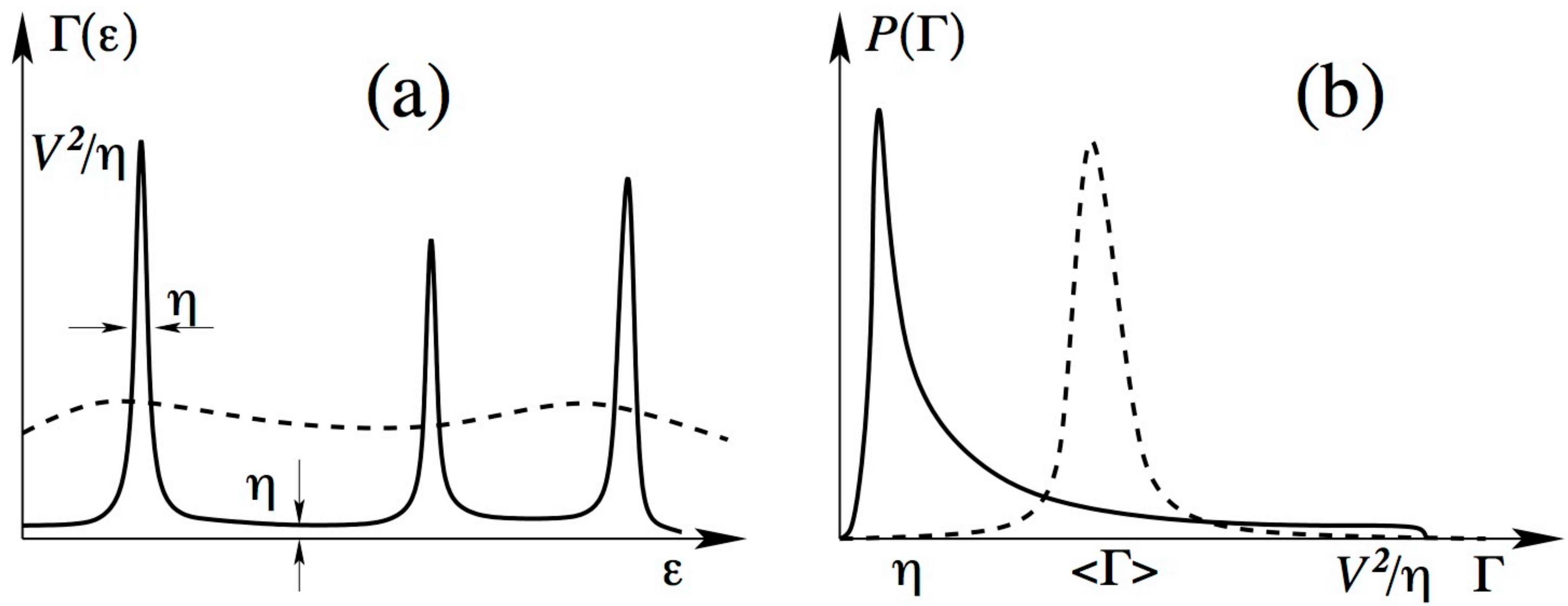
Quasi-particle life times

$$G(t) = -i\Theta(t)\langle[c_\alpha(t), c_\alpha^\dagger]\rangle$$

Anderson eigenbasis

quasi-particle decay rate

distribution of decay rates



Basko, Aleiner, Altshuler Annals of Physics 321, 1126 (2006)

MBL phase has (local) quasi-particles!

Many-body localization in 1D

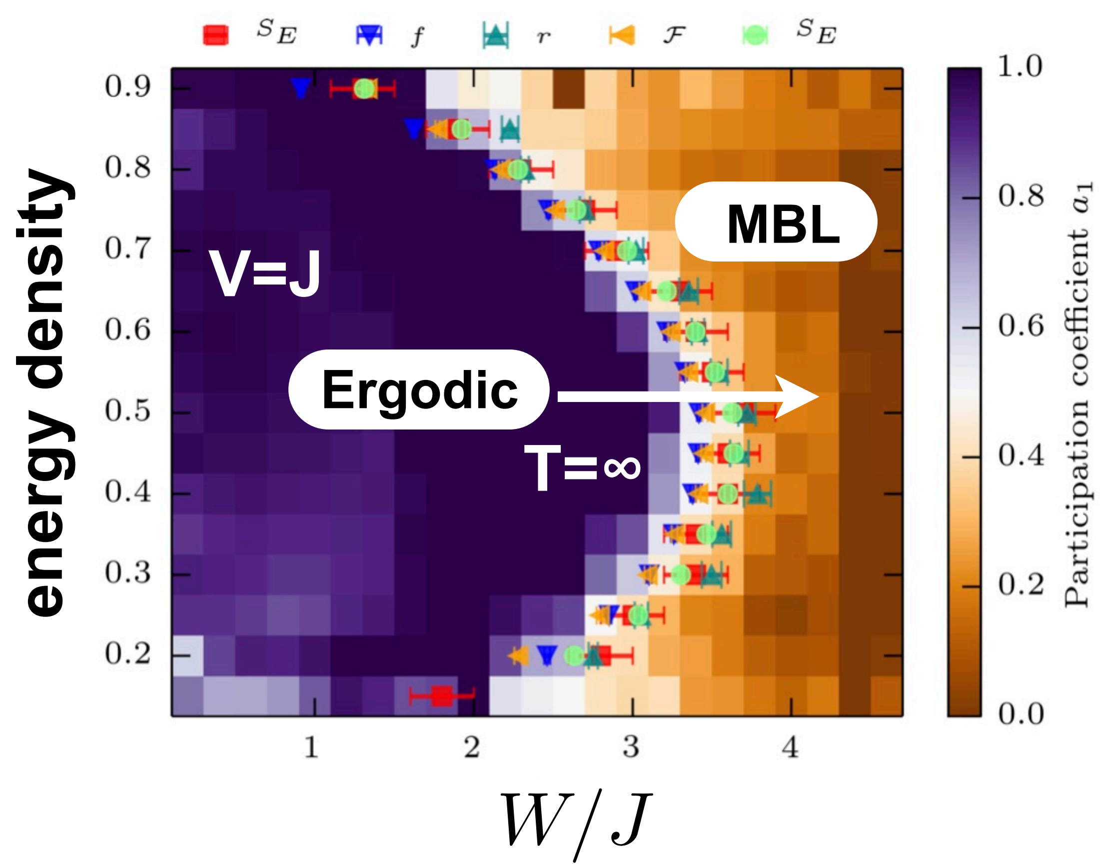
Spinless fermions in 1D

$$H = \sum_{j=1}^{L} [-\frac{J}{2}(c_{j+1}^\dagger c_j + h.c.) + V n_j n_{j+1}] - \sum_j \epsilon_j n_j \qquad \epsilon_j \in [-W, W]$$

energy density

$$\epsilon = \frac{E - E_{\min}}{E_{\max} - E_{\min}}$$

Phase diagram:



Tool: Exact diagonalization

Luitz, Laflorencie, Alet PRB (2015)

Properties:

No transport, no thermalization

Memory of initial conditions

"Eigenstate quantum phase transition"

$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

Fock-space localization

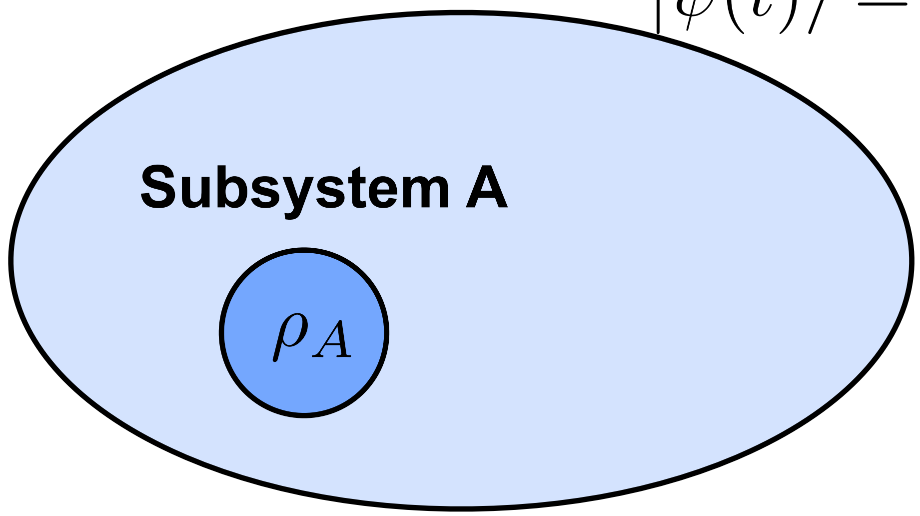
Entanglement scaling

Review: Nandkishore, Huse, Annual Rev. CMP (2015)
Altman, Vosk, Annual Rev. CMP (2015)

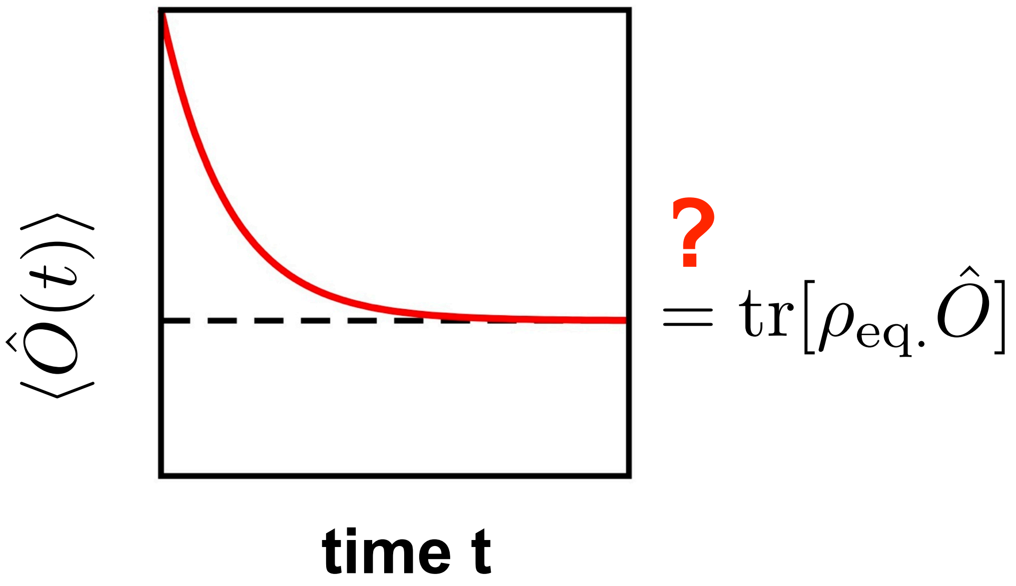
Thermalization in closed quantum systems

Closed quantum systems

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$$



Nonequilibrium dynamics



Many-body system acts as its own bath:

$$\overline{\rho_A}^t \overset{!}{=} \rho_{\mathrm{eq.}}$$

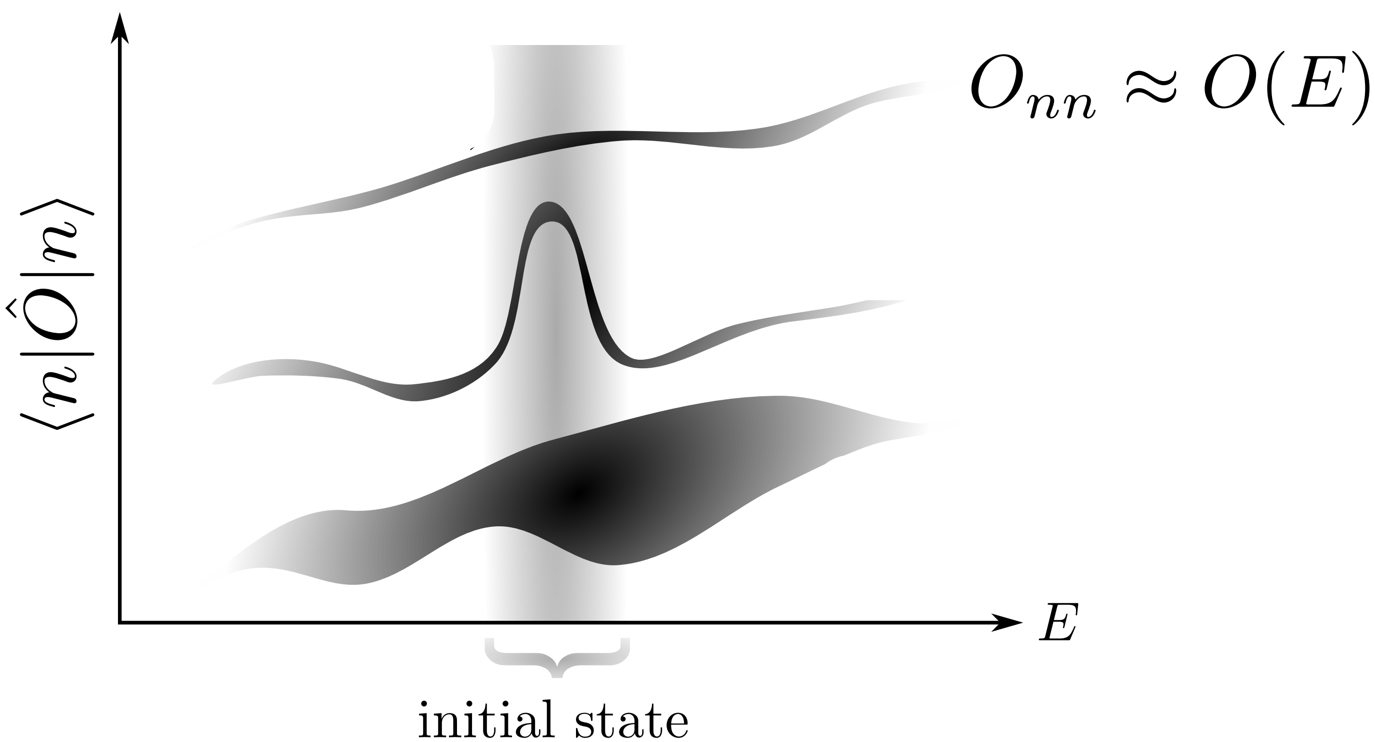
Most interacting systems do just that !
Eigenstate thermalization hypothesis

Rigol, Dunjko, Olshanii Nature (2008); Prosen PRE (1998),
Deutsch PRA (1991); Srednicki PRE (1994), ..., Sorg, Vidmar, Pollet, FHM PRA (2014), ...

Eigenstate thermalization hypothesis

Apparent initial state dependence:

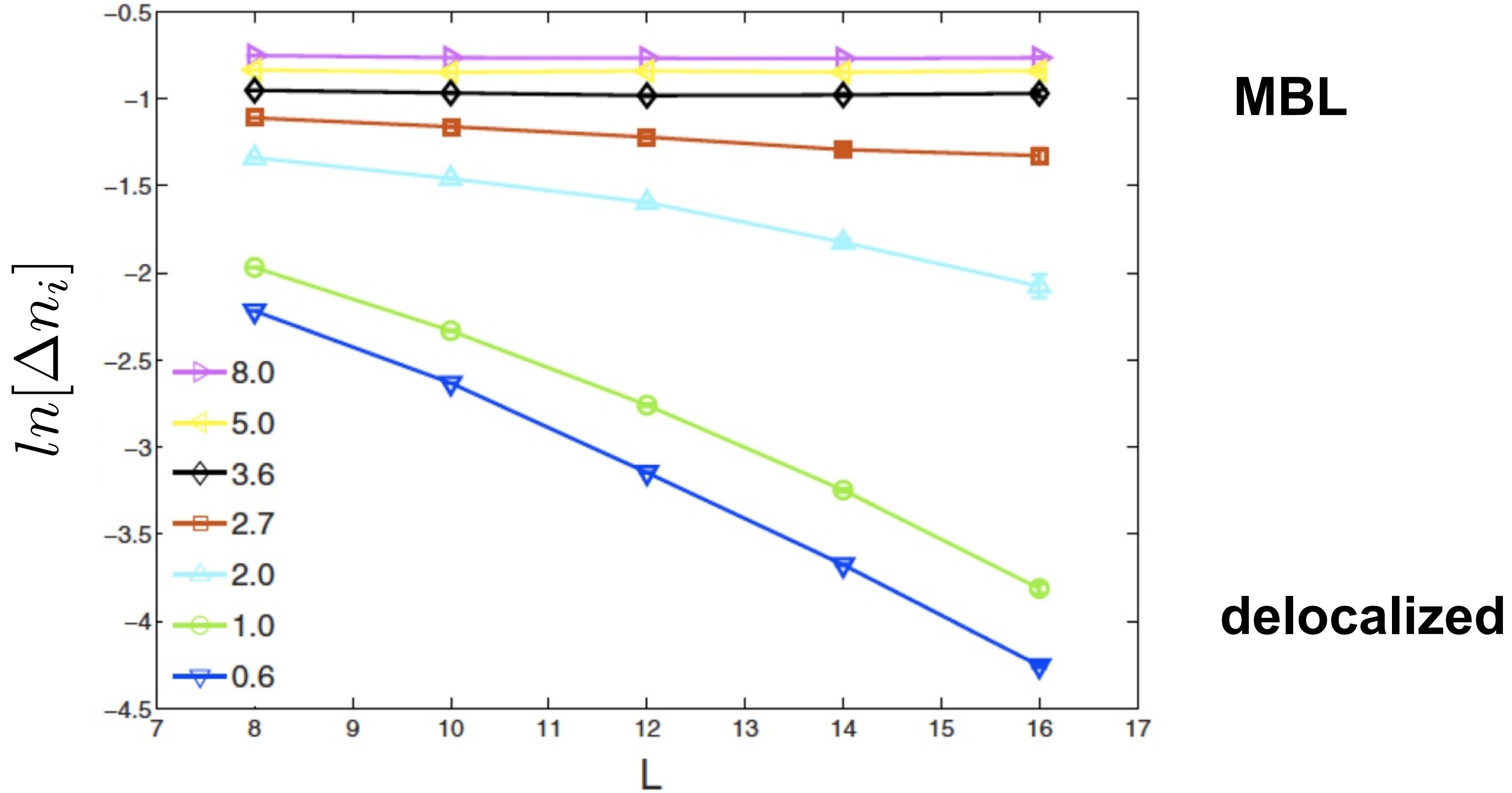
$$|\psi(t)\rangle = \sum_n c_n |n\rangle : \quad \langle \hat{O} \rangle_{t\to\infty} = \sum_n |c_n|^2 \langle n|\hat{O}|n\rangle$$



Rigol, Dunjko, Olshanii Nature (2008); Prosen PRE (1998), Deutsch PRA (1991); Srednicki PRE (1994), ..., Sorg, Vidmar, Pollet, FHM PRA (2014), ...

$$E_n \approx E_m : \langle n|\hat{O}|n\rangle \approx \langle m|\hat{O}|m\rangle : \quad \langle \hat{O} \rangle_{t\to\infty} = \frac{1}{\Delta} \sum_{E-\Delta<E_n<E} \langle n|\hat{O}|n\rangle$$

Failure of ETH in MBL phase



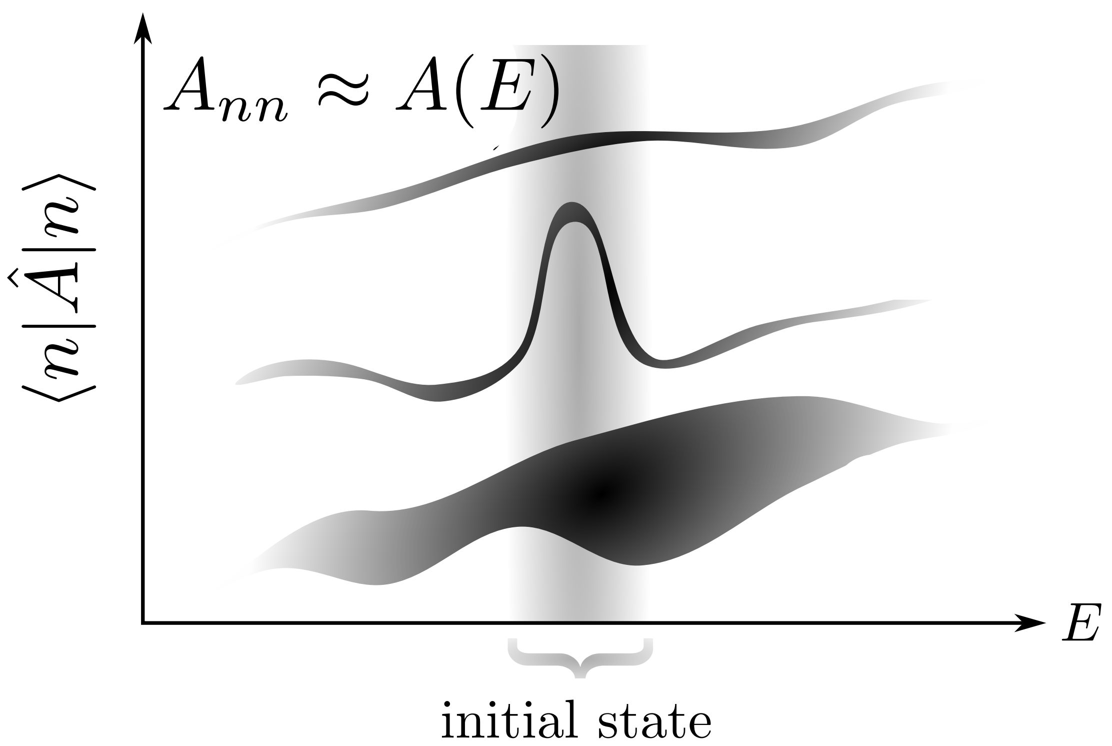
Difference in local density between adjacent eigenstates

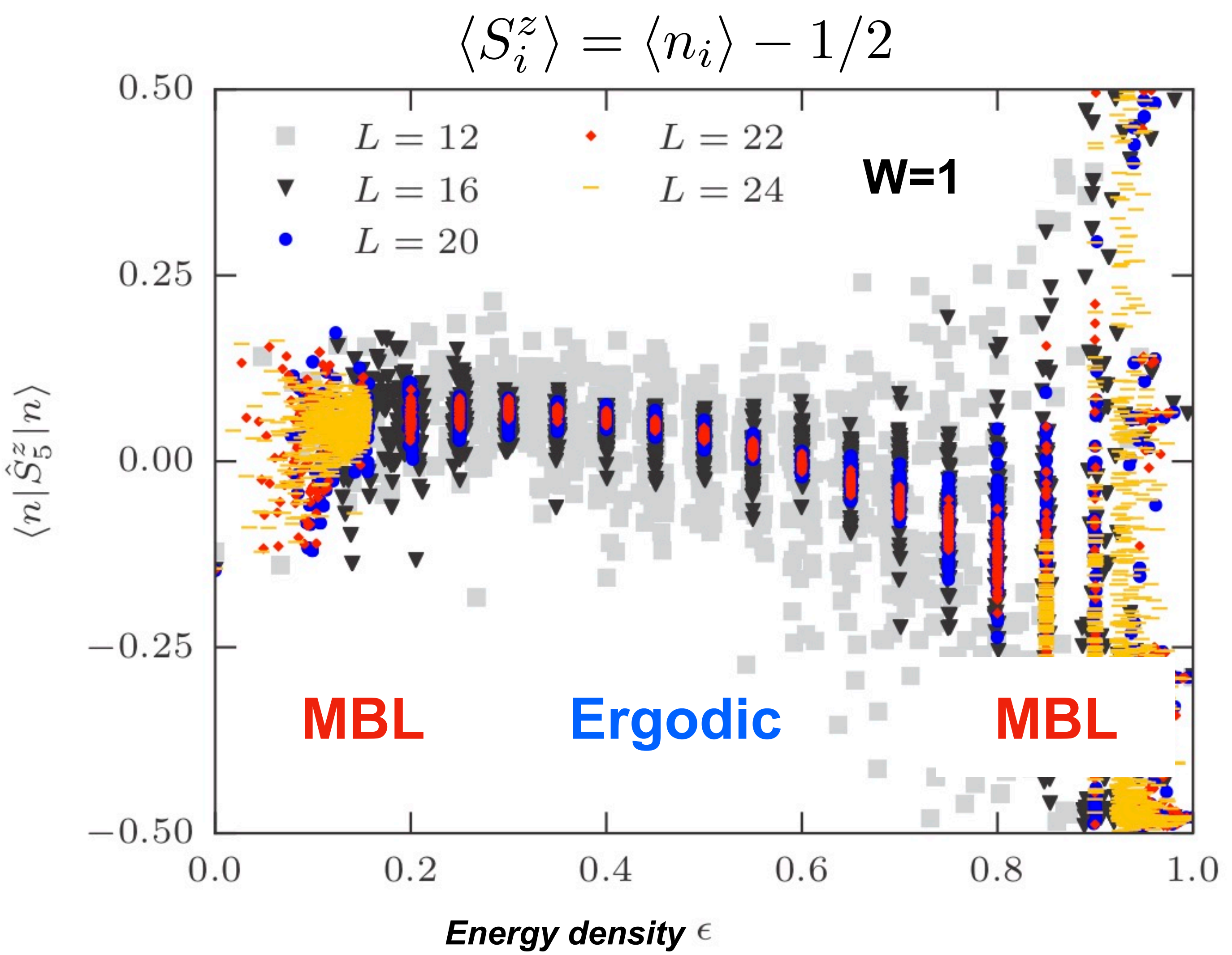
$$\Delta n_i = \langle E_n | n_i | E_n \rangle - \langle E_m | n_i | E_m \rangle$$

$$E_n \approx E_m$$

Pal, Huse, Phys. Rev. B 82, 174411 (2010)

Failure of ETH in MBL phase

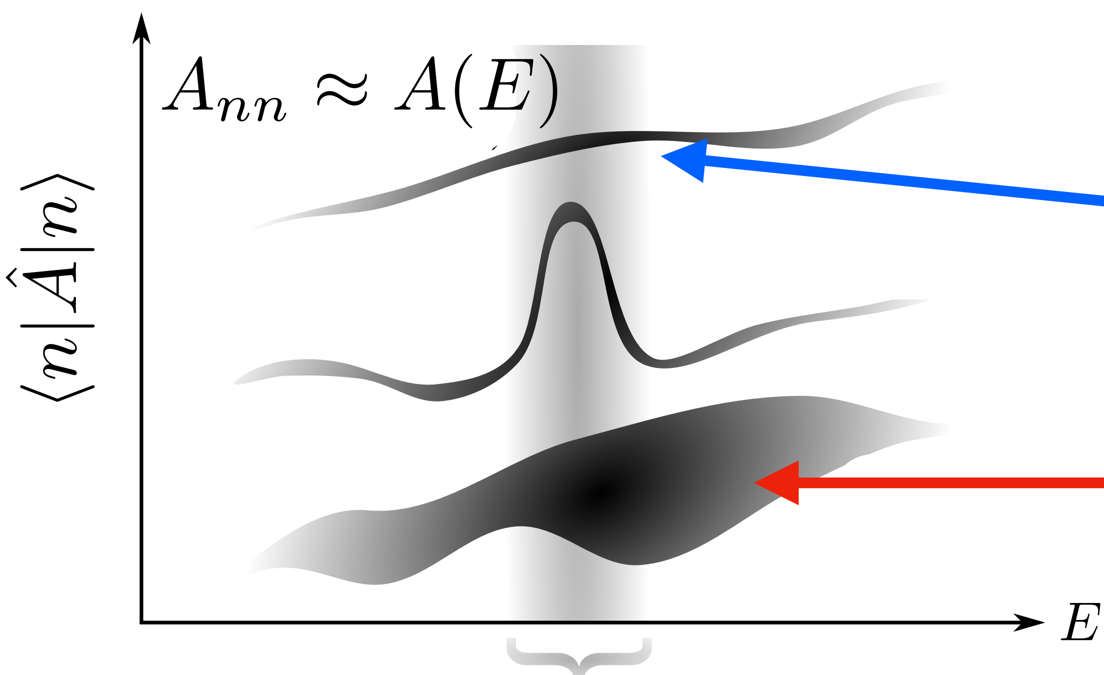


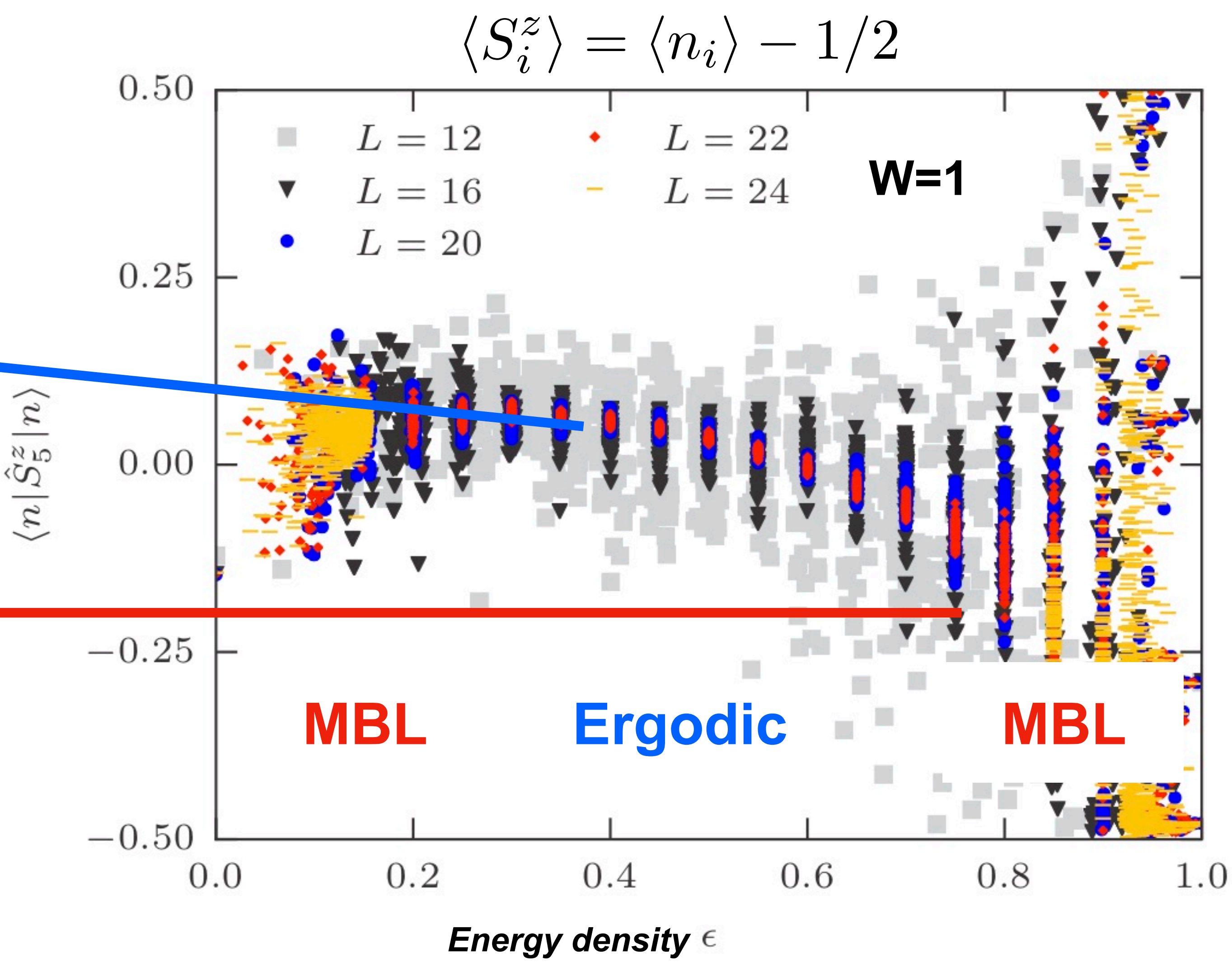


Luitz, Phys. Rev. B 93, 134201 (2016)

ETH violated already for onsite densities

Failure of ETH in MBL phase

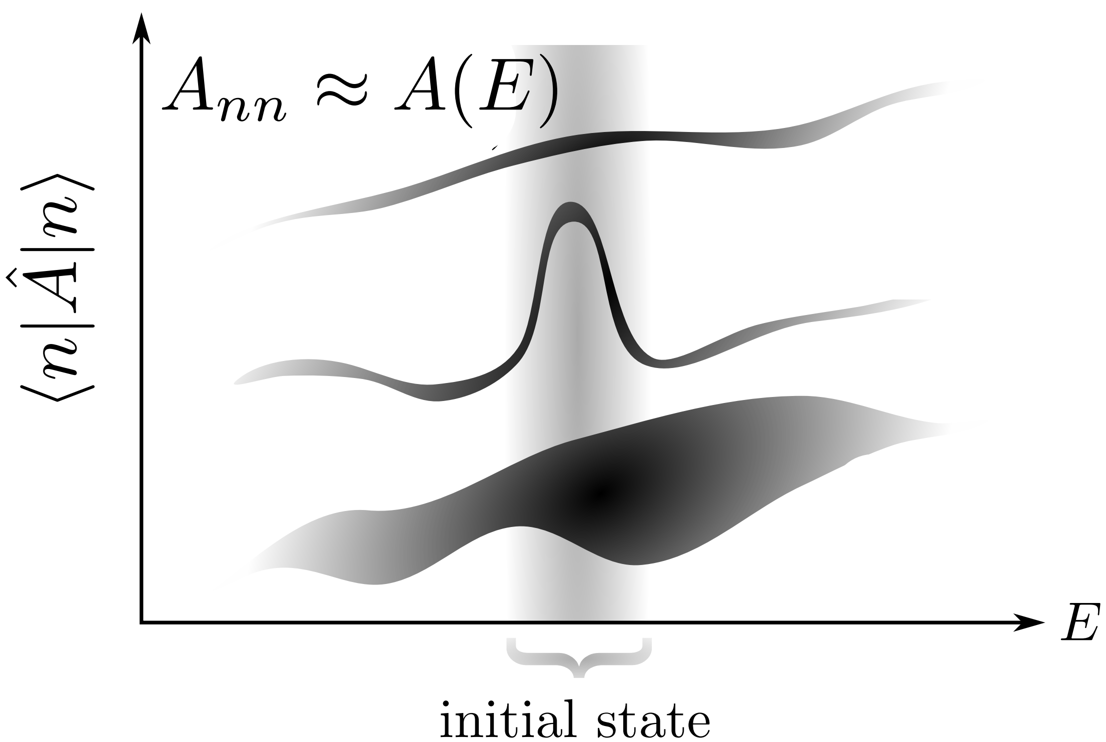


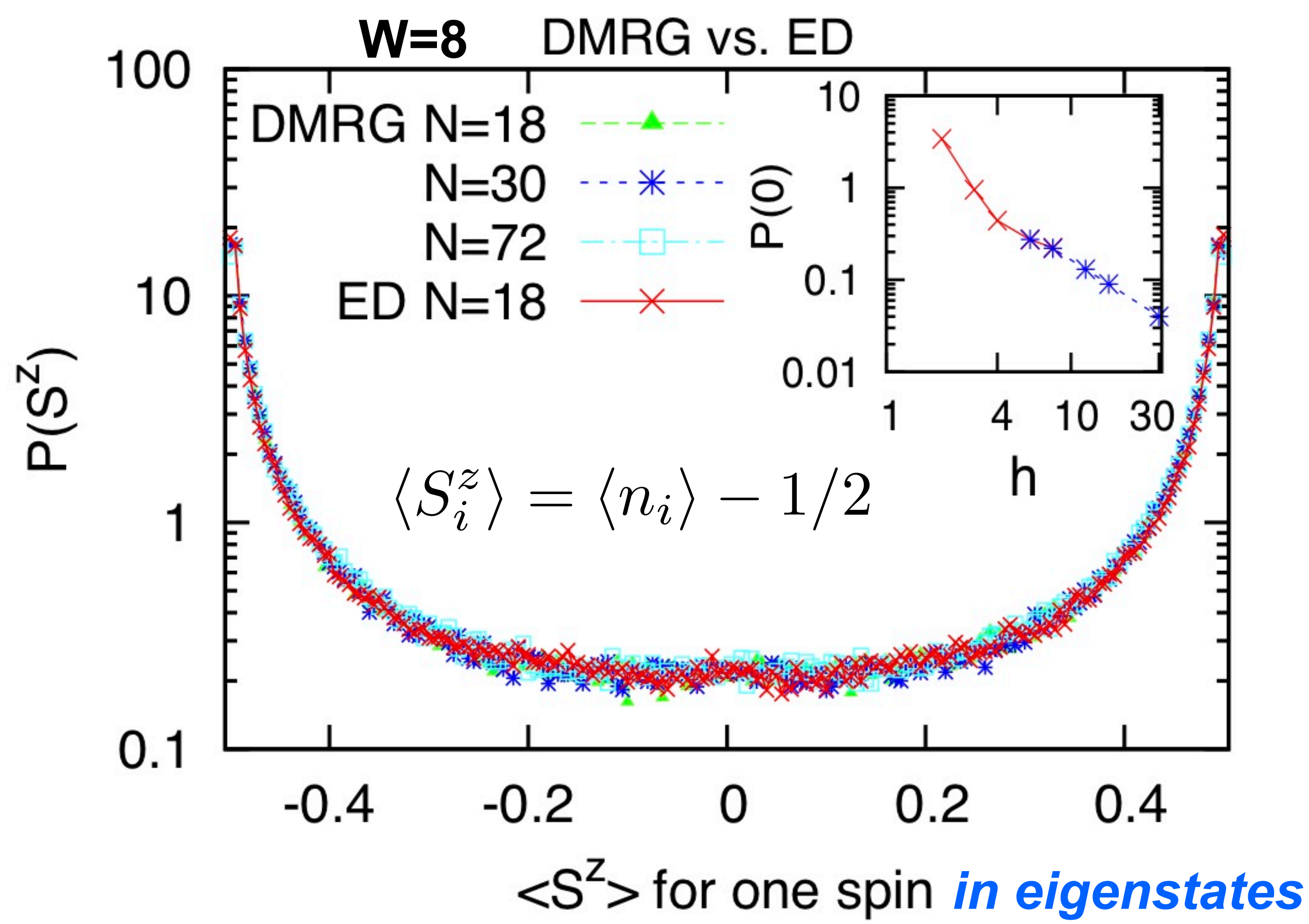


Luitz, Phys. Rev. B 93, 134201 (2016)

ETH violated already for onsite densities

Failure of ETH in MBL phase



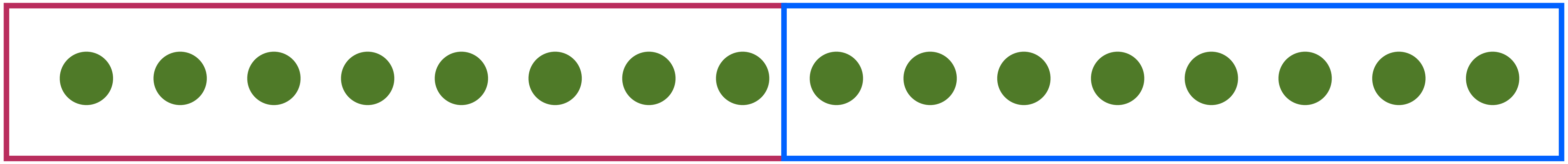


Lim, Sheng, Phys. Rev. B 94, 045111 (2016)

Bimodal structure of eigenstate expectation-value distribution: survives in thermodynamic limit

Kennes, Karrasch Phys. Rev. B 93, 245129 (2016)

Entanglement entropy



Part A: "System", length L_A

Part B: "System", length L_B

Reduced density matrix in Schmidt basis:

$$\rho_A = \sum_\alpha s_\alpha^2 |\alpha\rangle_{A\,A}\langle\alpha|$$

Van-Neumann entropy:

$$S_{\mathrm{vN}} = -\mathrm{tr}[\rho_A \log_2 \rho_A] = -\sum_\alpha s_\alpha^2 \log_2 s_\alpha^2$$

Generic many-body eigenstate: volume law

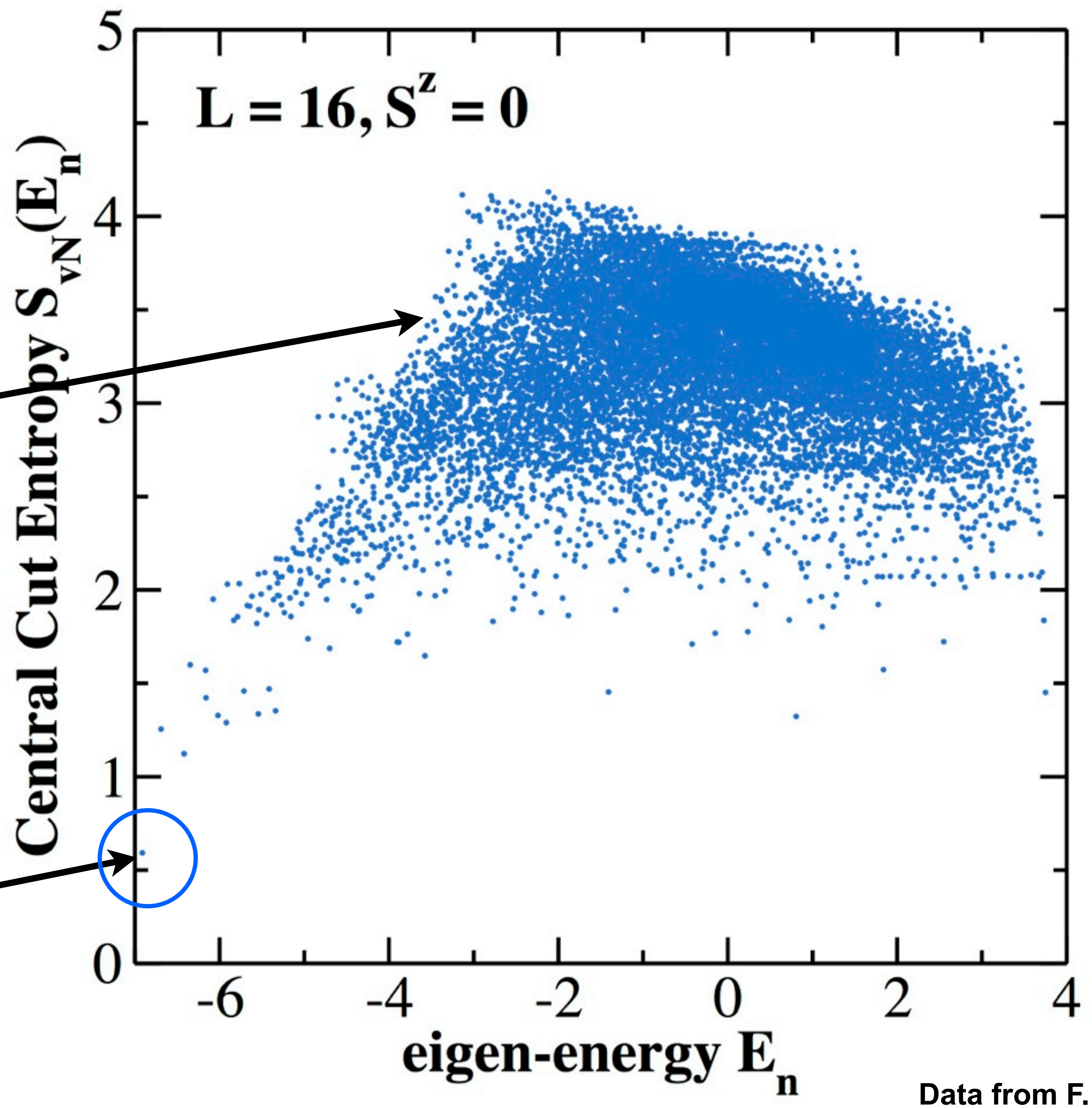
$$S_{\mathrm{vN}} \propto L^d$$

Clean systems: Heisenberg chain

$$H = J \sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1}$$

Typical eigenstates in the bulk: "large" entanglement
volume law (ETH!)

$$S_{vN} \sim L^d$$



Data from F. Dorfner

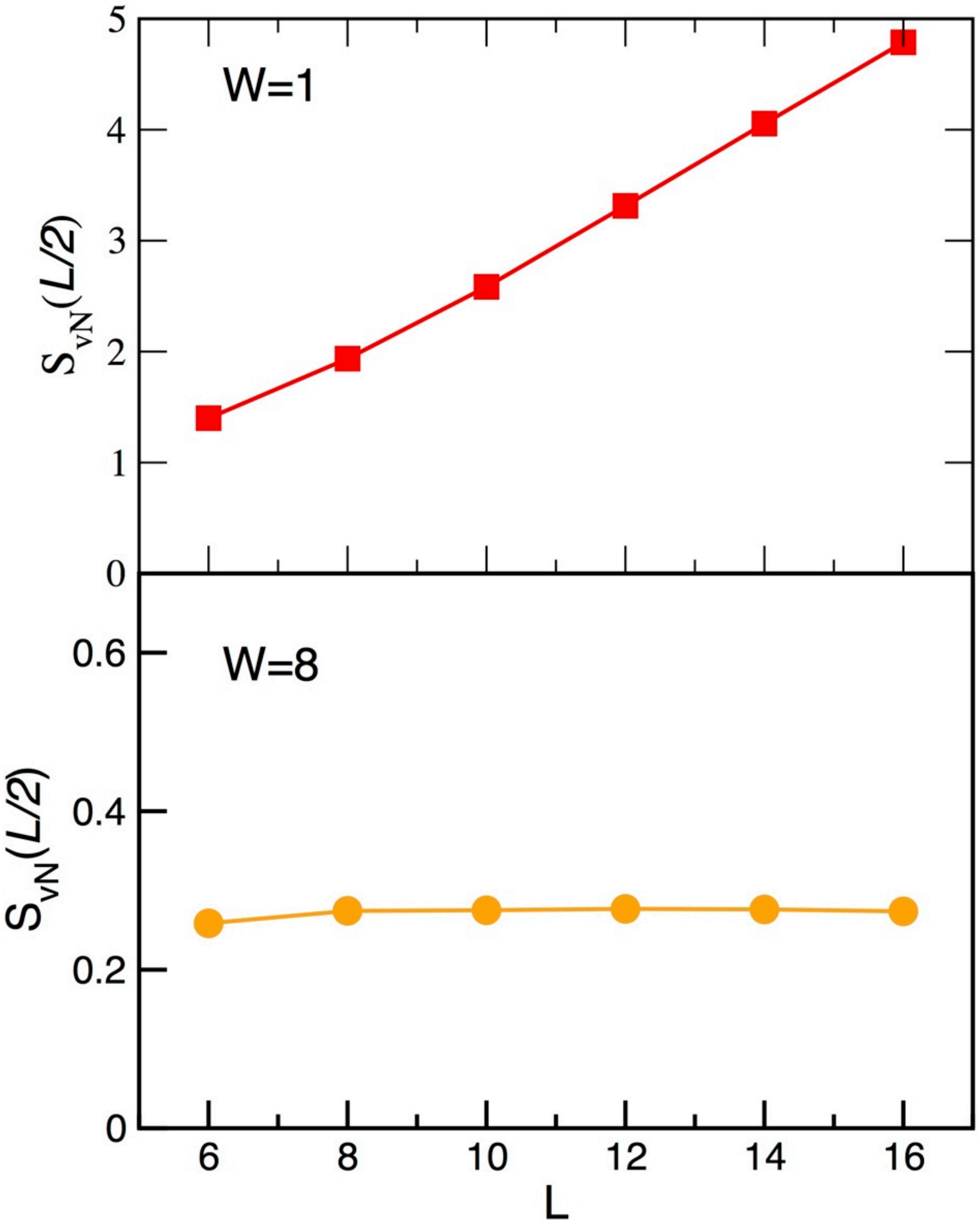
Ground-state: Smallest S_{vn} possibly area law

$$S_{vN} \sim L^{d-1} + c \ln(L)$$

Eisert, Cramer, Plenio Rev. Mod. Phys. 82, 277 (2010)

Area law in MBL phase

$E = 0$



Consequence of ETH:

volume law

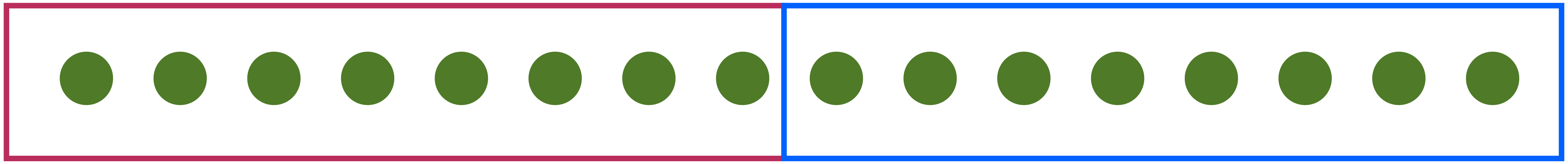
$S_{\mathrm{vN}} \propto L$

area law:

$S_{\mathrm{vN}} \sim const$

Bauer, Nayak J. Stat. Mech. (2013) P09005; Kjall, Bardarson, Pollmann Phys. Rev. Lett 113, 107204 (2014)

Key idea of DMRG/MPS methods



Part A: “System”, length L_A Part B: “System”, length L_B

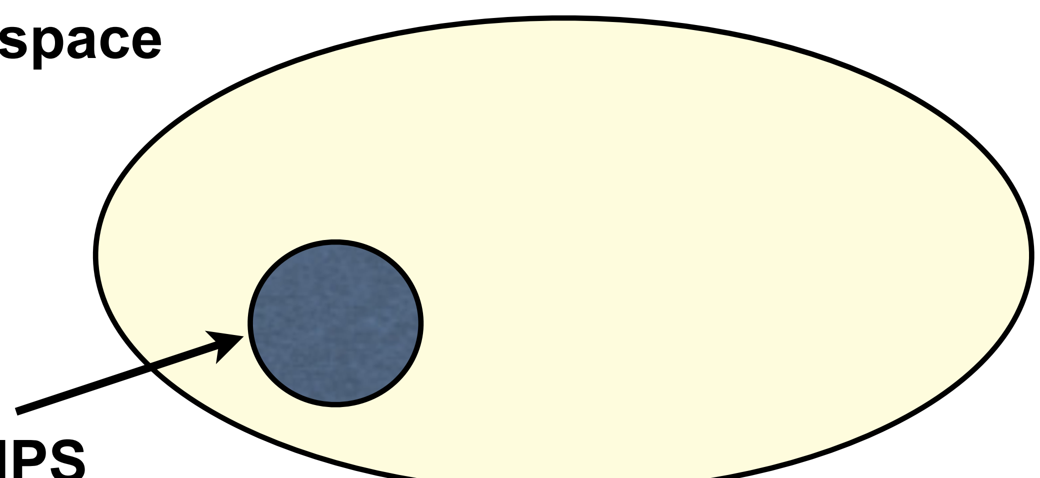
$$|\psi\rangle = \sum_{i,j}^{N_A, N_B} \psi_{ij}\, |i\rangle_A \otimes |j\rangle_B$$

$$\rightarrow |\psi\rangle \approx |\psi_m\rangle = \sum_{\alpha}^{m \ll r} s_\alpha\, |\alpha\rangle_A |\alpha\rangle_B$$

Also diagonalizes reduced density matrix: $\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi| = \sum_\alpha s_\alpha^2 |\alpha\rangle\langle\alpha|$

Ground states in 1D (simple 2D models), time evolution, T>0 in 1D, L~100
for fermions, bosons, spins, mixtures, any density

Hilbert-space



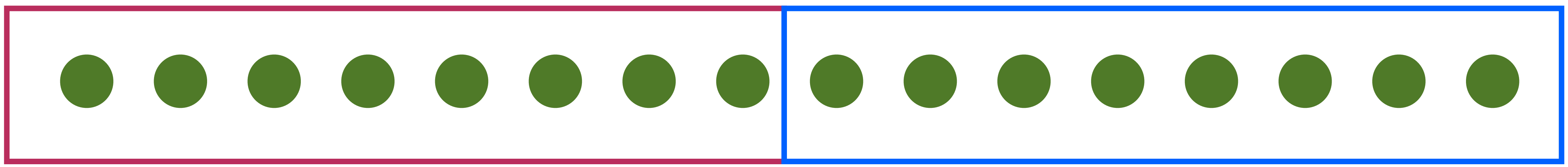
sub-space of
m-dimensional MPS

Matrix product states (MPS):
finite entanglement

$$|\psi\rangle \approx |\psi_{\mathrm{MPS}}\rangle$$

White, Phys. Rev. Lett. 69, 2863 (1992)
Schollwöck, Annals of Physics 326, 96 (2011)

Key idea of DMRG/MPS methods



Part A: “System”, length L_A Part B: “System”, length L_B

$$|\psi\rangle = \sum_{i,j}^{N_A,N_B} \psi_{ij}\, |i\rangle_A \otimes |j\rangle_B \qquad \rightarrow |\psi\rangle \approx |\psi_m\rangle = \sum_{\alpha}^{m \ll r} s_\alpha\, |\alpha\rangle_A |\alpha\rangle_B$$

Also diagonalizes reduced density matrix: $\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi| = \sum_\alpha s_\alpha^2 |\alpha\rangle\langle\alpha|$

“finite entanglement”

$$S_{\mathrm{vN}} = -\mathrm{tr}[\rho_A \log_2 \rho_A] = -\sum_\alpha s_\alpha^2 \log_2 s_\alpha^2$$

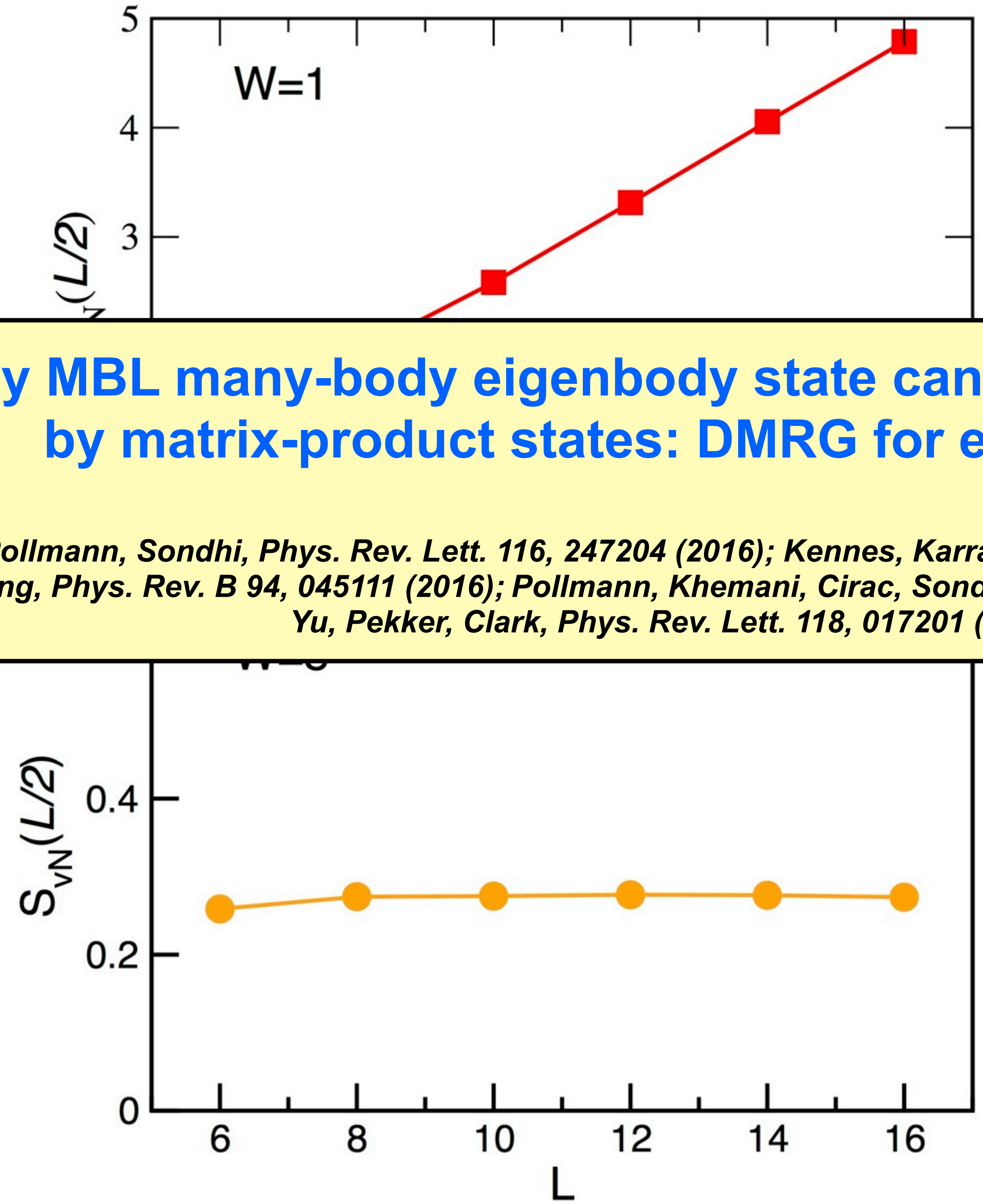
Area laws (ground states, gap):

$$S_{\mathrm{vN}} \propto L^{d-1}$$

Review: Eisert et al. Rev. Mod. Phys.

Area law in MBL phase

$E = 0$



Consequence of ETH:

volume law

Any MBL many-body eigenbody state can be approximated by matrix-product states: DMRG for excited states

Khemani, Pollmann, Sondhi, Phys. Rev. Lett. 116, 247204 (2016); Kennes, Karrasch, Phys. Rev. B 93, 245129 (2016); Lim, Sheng, Phys. Rev. B 94, 045111 (2016); Pollmann, Khemani, Cirac, Sondhi, Phys. Rev. B 94, 041116 (2016). Yu, Pekker, Clark, Phys. Rev. Lett. 118, 017201 (2017)

area law:

$S_{\rm vN} \sim const$

Bauer, Nayak J. Stat. Mech. (2013) P09005; Kjall, Bardarson, Pollmann Phys. Rev. Lett 113, 107204 (2014)

Level-spacing statistics

Important measure for quantum chaos

$$\delta_n = E_{n+1} - E_n$$

Poisson statistics

$$P(\delta) = \exp(-\delta)$$

Wigner-Dyson statistics (GOE)

$$P(\delta) = \frac{\pi}{2}\,\delta\exp\left(-\frac{\pi^2}{2}\delta^2\right)$$

See: d'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. 65, 239 (2016)
For original references on level-spacing statistics

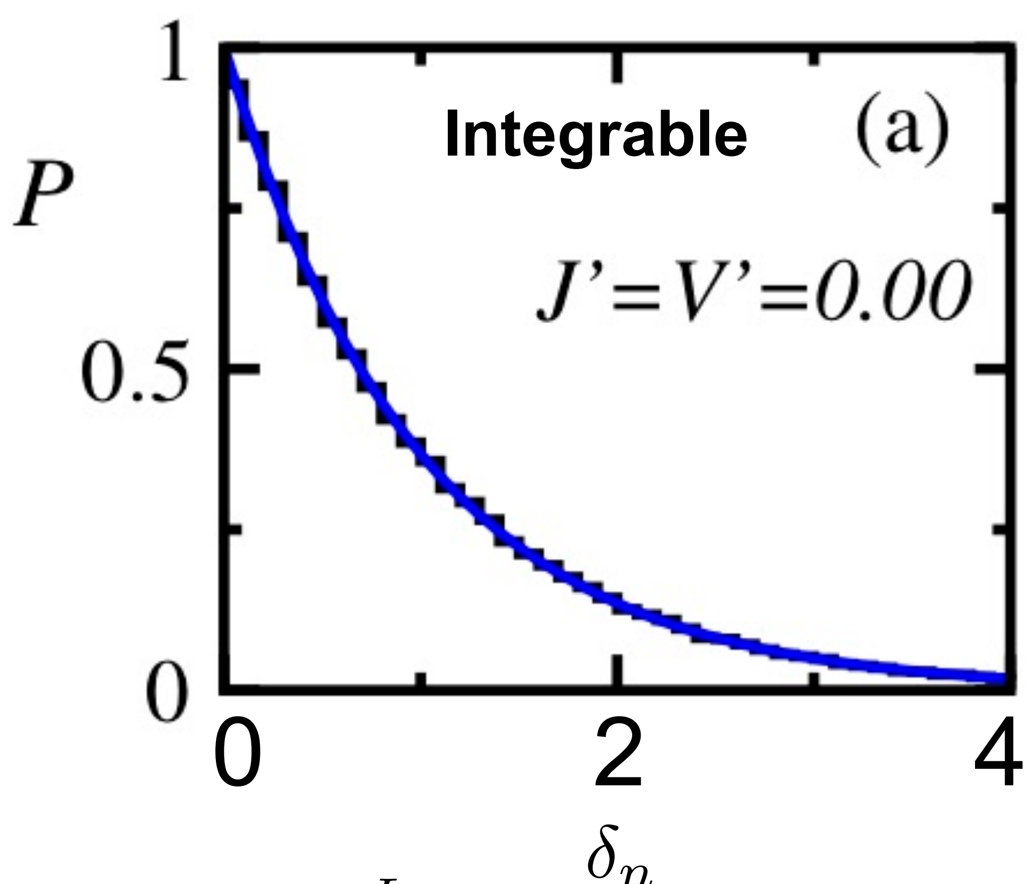
Level-spacing statistics

Important measure for quantum chaos

$$\delta_n = E_{n+1} - E_n$$

Poisson statistics

$$P(\delta) = \exp(-\delta)$$

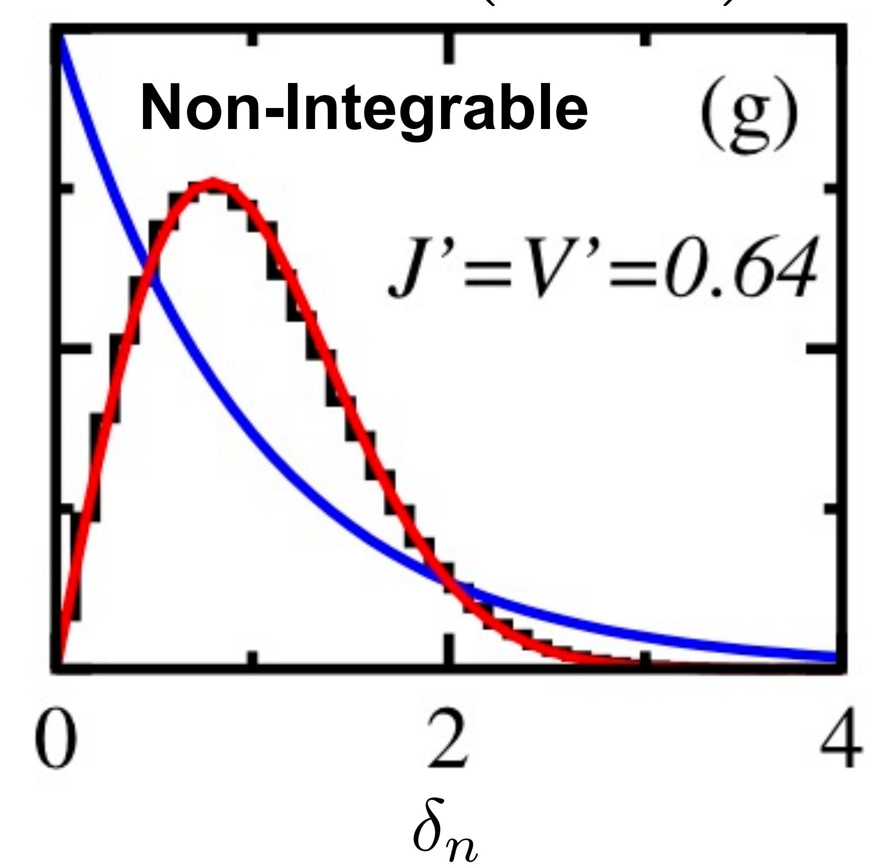


Blue: Poisson,
Red: GOE

From: Santos, Rigol
PRE 81, 036206 (2010)

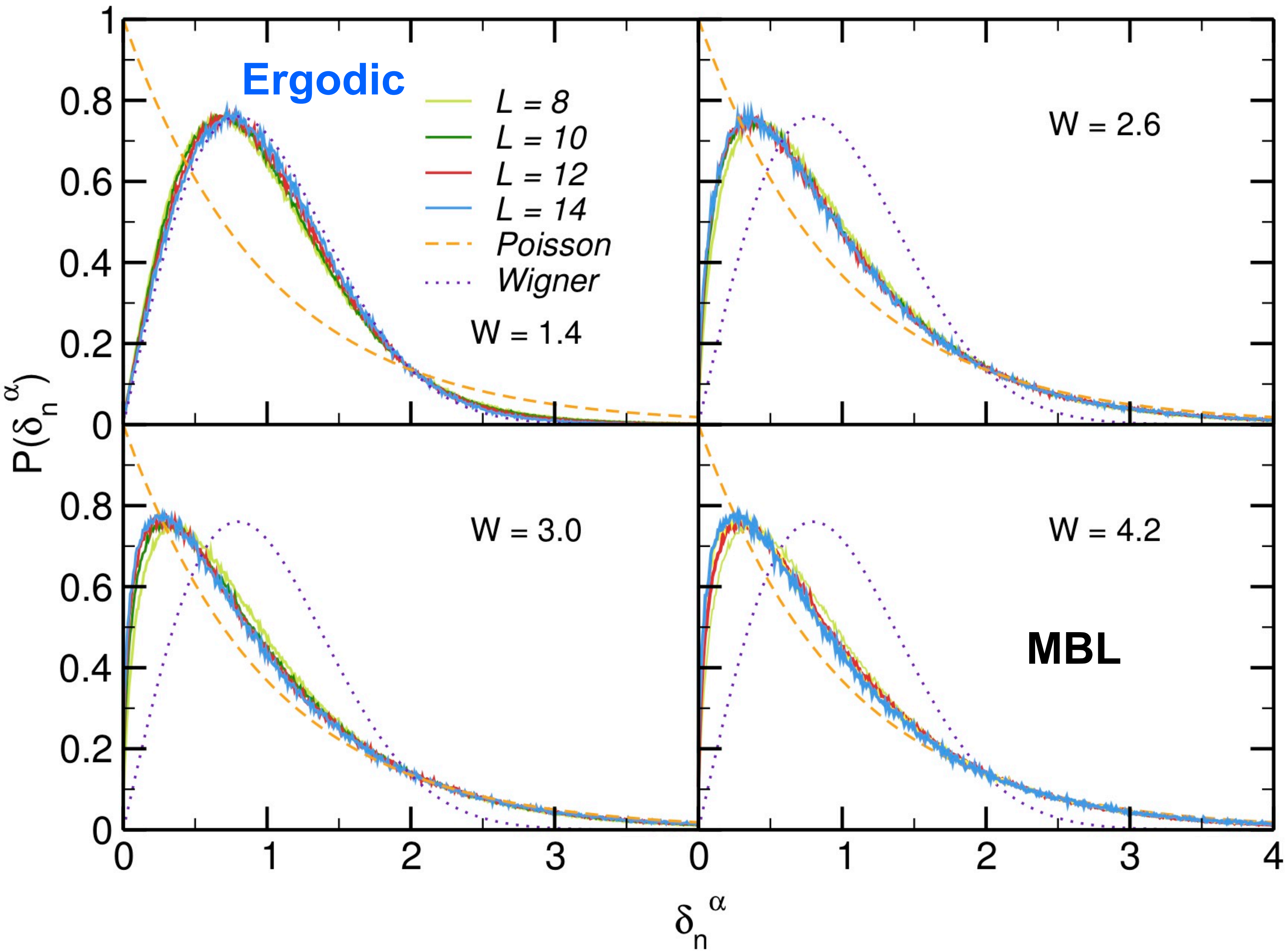
Wigner-Dyson statistics (GOE)

$$P(\delta) = \frac{\pi}{2}\,\delta \exp\left(-\frac{\pi^2}{2}\delta^2\right)$$



$$H = \sum_{j=1}^{L} \left[-J(c_j^\dagger c_{j+1} + h.c.) + V n_j n_{j+1} - J'(c_j^\dagger c_{j+1} + h.c.) + V' n_j n_{j+1} \right]$$

Level-spacing statistics



Oganesyan, Huse Phys. Rev. B 75, 155111 (2007)
Figure from: T. Martynec, Mastes thesis TU Munich 2016

Adjacent gap ratio

$$r_{\text{gap}} = \frac{\min(\delta^{(n)}, \delta^{(n+1)})}{\max(\delta^{(n)}, \delta^{(n+1)})} \qquad \delta^{(n)} = E_n - E_{n+1}$$

Prediction:

For ETH systems: $[r_{\text{gap}}] = r_{\text{GOE}} \approx 0.5307$

For integrable models: $[r_{\text{gap}}] = r_{\text{Poisson}} \approx 0.3863$

Oganesyan, Huse Phys. Rev. B 75, 155111 (2007)
Atas, Bogomolny, Giraud, Roux, Phys. Rev. Lett. 110, 084101 (2013)

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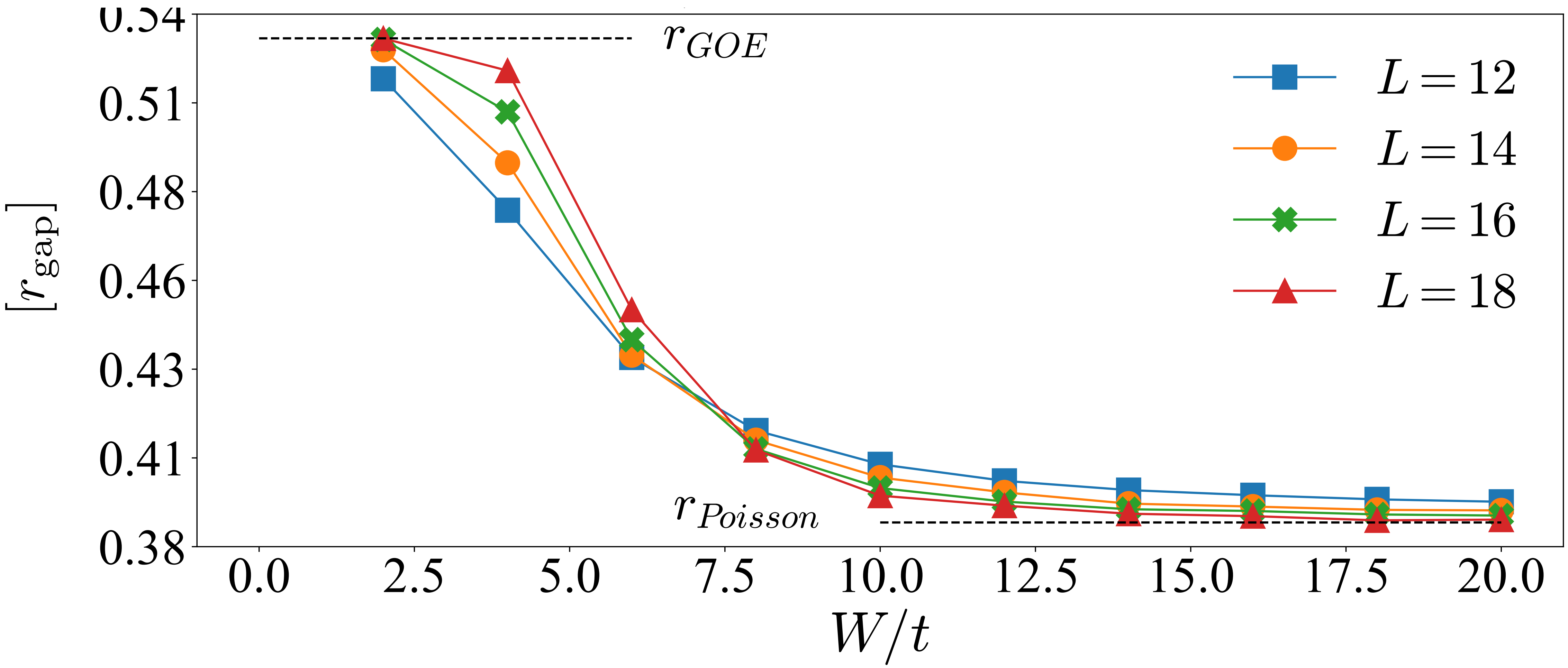


Oganesyan, Huse Phys. Rev. B 75, 155111 (2007)
Figure from Lin, Sbierski, Dorfner, Karrasch, FHM Sci. Post. Phys. 4 002 (2018)

Emergent conserved charges

$$H = \sum_{j=1}^{L}[-\frac{J}{2}(c^\dagger_{j+1}c_j + h.c.) + V n_j n_{j+1}] - \sum_j \epsilon_j n_j$$

Idea: (effective ?) Hamiltonian in MBL phase:

Huse, Nandishkore, Oganesyan PRB(2014)
Serbyn, Papic, and Abanin, PRL (2013)
Vosk, Altman PRL (2013)

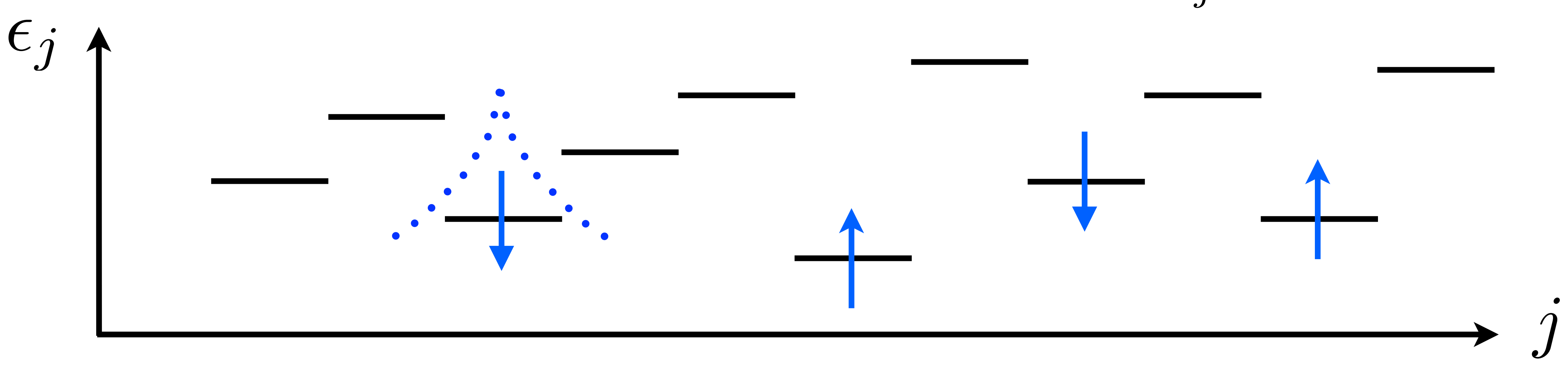
$$H = \sum_i h_i n_i^{qp} + \sum_{i,j} J_{i,j} n_i^{qp} n_j^{qp} + \sum_{i,j,\{k\}} K^n_{i,\{k\},j} n_i^{qp} n_{k_i}^{qp} \dots n_j^{qp}$$

$[H, n_i^{qp}] = 0 \quad [n_j^{qp}, n_i^{qp}] = 0 \quad n_i^{qp} = 0, 1$

L l-bits: all 2^L many-body states!

Anderson localization:
Exponentially localized particle

$$n_i^{qp} = \sum_j a^i_j n_j + \dots$$



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Serbyn, Papic, and Abanin, PRL (2013)
Vosk, Altman PRL (2013)

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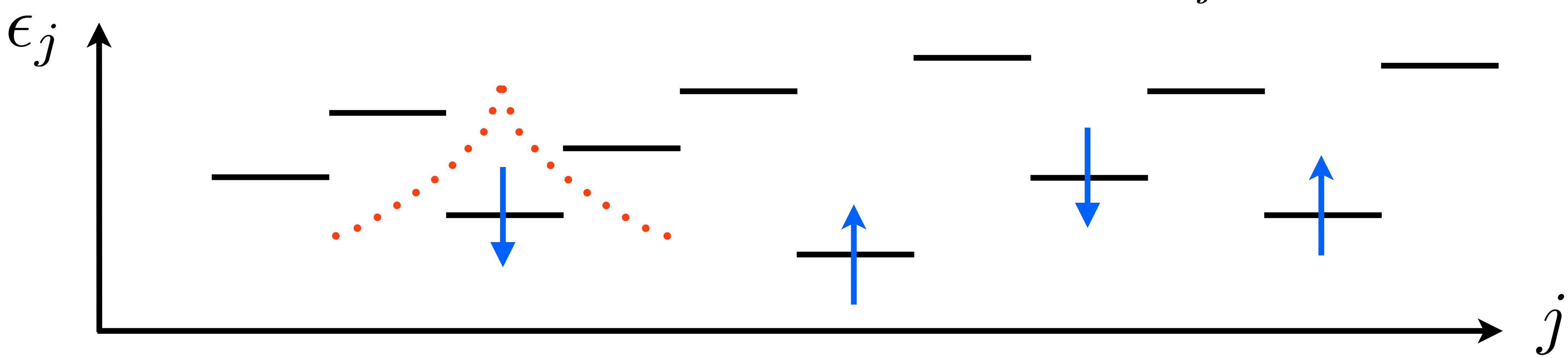
$[H, n_i^{qp}] = 0 \quad [n_j^{qp}, n_i^{qp}] = 0 \quad n_i^{qp} = 0, 1$

L l-bits: all 2^L many-body states!

Quasi-particle
Exp. localized + particle-hole pairs

$$n_i^{qp} = \sum_j a^i_j n_j + \dots$$



Analogy to Fermi-liquids!

Emergent conserved charges

$$H = \sum_{j=1}^{L} [-\frac{J}{2}(c_{j+1}^{\dagger} c_j + h.c.) + V n_j n_{j+1}] - \sum_j \epsilon_j n_j$$

Idea: (effective ?) Hamiltonian in MBL phase:

Huse, Nandishkore, Oganesyan PRB(2014)
Serbyn, Papic, and Abanin, PRL (2013)
Vosk, Altman PRL (2013)

$$H = \sum_i h_i n_i^{qp} + \sum_{i,j} J_{i,j} n_i^{qp} n_j^{qp} + \sum_{i,j,\{k\}} K_{i,\{k\},j}^{n} n_i^{qp} n_{k_i}^{qp} \ldots n_j^{qp}$$

$[H, n_i^{qp}] = 0 \quad [n_j^{qp}, n_i^{qp}] = 0 \quad n_i^{qp} = 0, 1$

L l-bits: all 2^L many-body states!

Existence of these quasilocal objects (l-bits) explains:

(i) Vanishing dc transport
(ii) Failure of ETH
(iii) Entanglement scaling: Area law
(iv) Level-spacing statistics
(v) Log-increase in global quantum quenches (Hui Zhai's 3. talk!)

MBL: Open questions

(i) Does MBL exist in d>1?

(ii) Characterization of the MBL-to-ergodic transition/ divergent length scale?

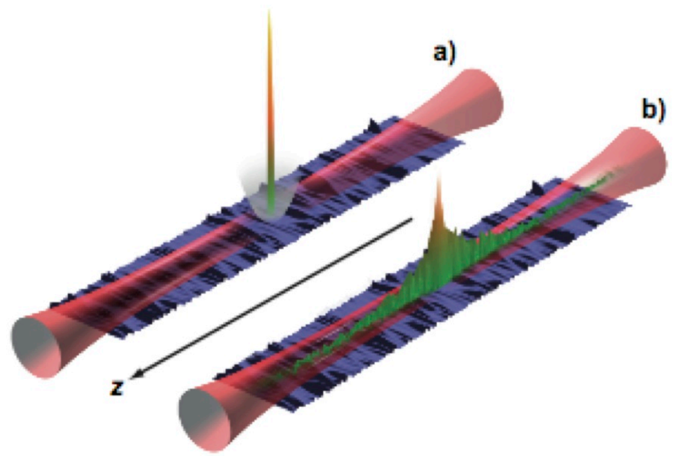
(iii) Existence of the many-body mobility edge

(iv) Subdiffusive dynamics in the ergodic phase

(v) Signatures of MBL in solid-state systems?

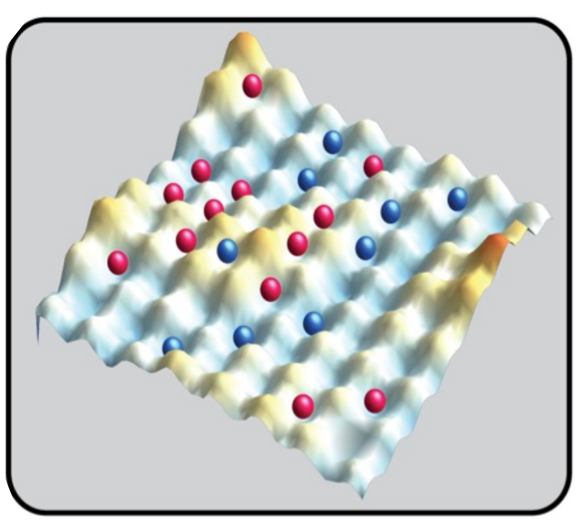
MBL experiments with quantum gases

Anderson localization



Billy, Aspect et al. Nature (2008)
Roati, Inguscio et al. Nature (2008)

3D Bose-Hubbard

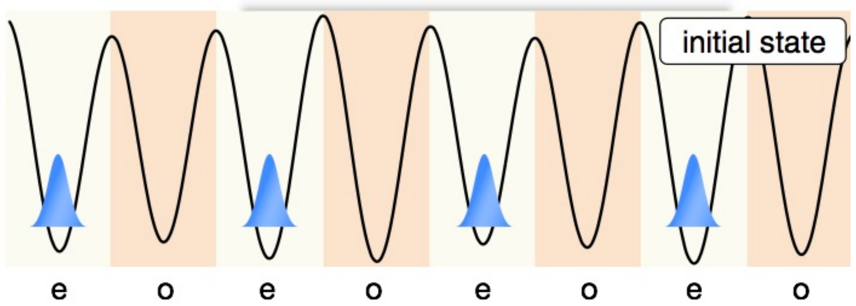


Kondov, de Marco et al. PRL (2015)

2D Bose-Hubbard

Choi, Bloch, Gross et al. Science (2016)

1D fermions, quasi-periodicity

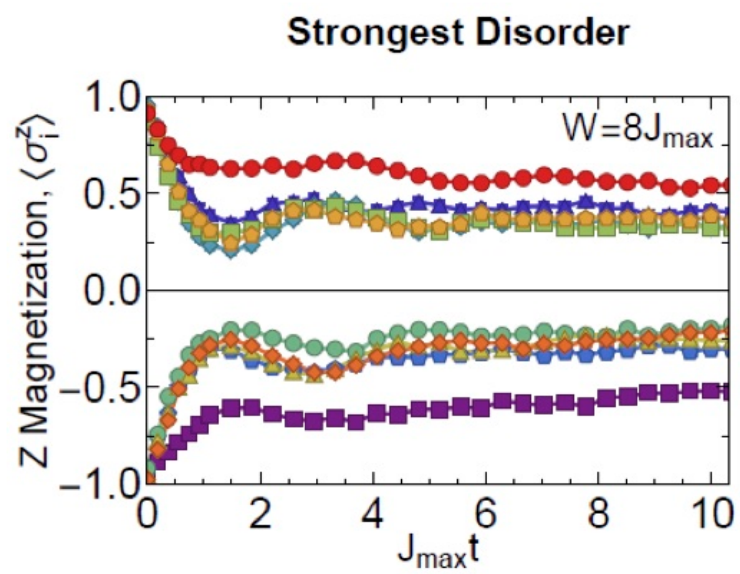


MBL: No decay of CDW!

…neider et al. Science (2015)
… al. PRL (2016)

Good for experiments:
no cooling to low T necessary!

…erse Ising model



Smith, Monroe et al. Nature Phys. (2016)

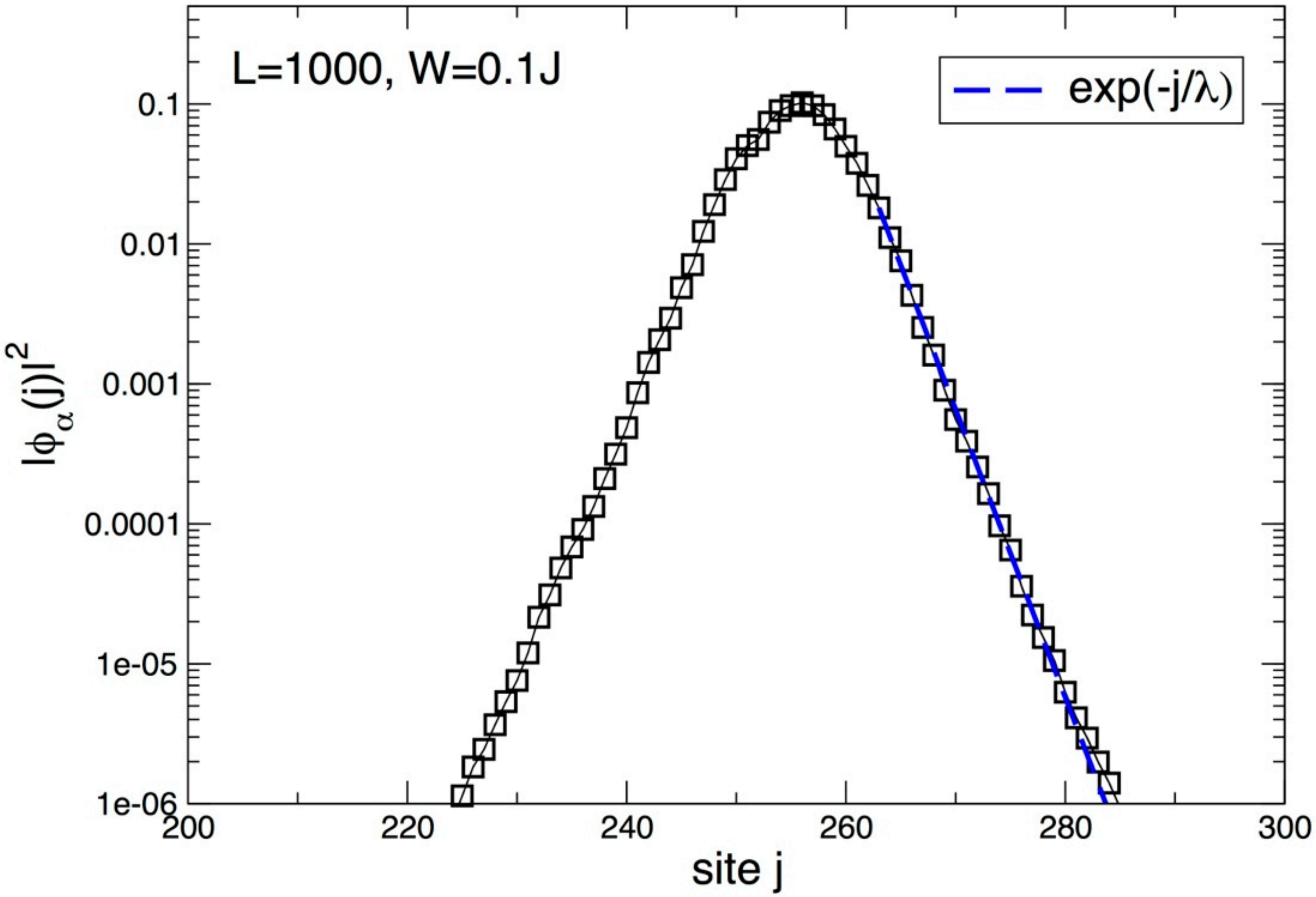
One-Particle Density Matrix Characterization of Many-Body Localization

Bera, Schomerus, FHM, Bardarson, Phys. Rev. Lett. 115, 046603 (2015)

Non-interacting case: 1D Anderson model

$$H = -J \sum_{j=1}^{L} (c_j^\dagger c_{j+1} + h.c.) - \sum_j \epsilon_j n_j \qquad \epsilon_j \in [-W, W]$$

Typical localized single-particle state



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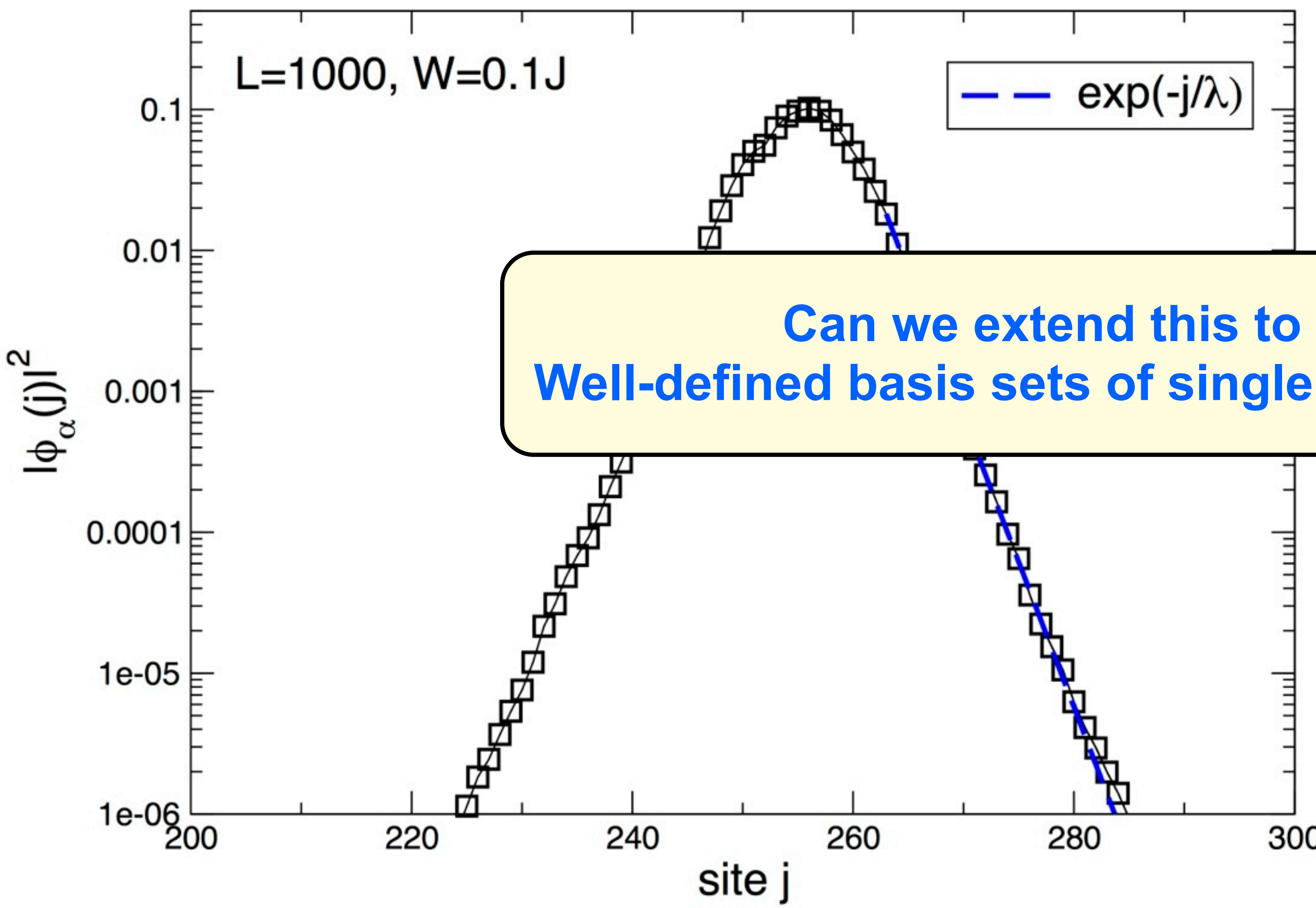


Can we extend this to MBL?
Well-defined basis sets of single-particle states?

One-particle density matrix & MBL

One-particle density matrix (OPDM)

$$H|\psi_n\rangle = E_n|\psi_n\rangle \quad \rightarrow \quad \rho^{(1)}_{ij} = \langle\psi_n|c_i^\dagger c_j|\psi_n\rangle$$ **(exact diagonalization)**

$$\rho^{(1)}|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle$$

Natural orbitals

Occupation spectrum

Bose-Einstein condensation in many-body systems

$$\rho^{(1)}_{ij} = n_0\phi_0^*(i)\phi_0(j) + \sum_{\alpha\neq 0} n_\alpha\phi_\alpha^*(i)\phi_\alpha(j)$$

$$n_0 \sim \mathcal{O}(N)$$

Penrose, Onsager, C.H. Yang

Bloch theorem in many-body systems

$$\phi_\alpha(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\phi_\alpha(\vec{r})$$

Koch, Goedecker, Solid State Communications

One-particle density matrix & MBL

One-particle density matrix (OPDM)

$$H|\psi_n\rangle = E_n|\psi_n\rangle \quad \rightarrow \quad \rho^{(1)}_{ij} = \langle\psi_n|c_i^\dagger c_j|\psi_n\rangle$$ **(exact diagonalization)**

$$\rho^{(1)}|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle$$

Natural orbitals

Occupation spectrum

V=0, W>0: Natural orbitals = single-particle energy eigenstates (all localized!)
W=0: Natural orbitals = plane waves (Bloch theorem!)

Two previous studies of OPDM eigenstates of HCBs with disorder (no connection to MBL transition):
Nessi, Iucci Phys. Rev. A 84, 063614 (2011); Gramsch, Rigol, Phys. Rev. A 86, 053615 (2012)

OPDM description of MBL

What we do, using exact diagonalization

→ **Fixed energy E**

→ **Obtain many-body eigenstates with E_n ~ E** $H|\psi_n\rangle = E_n|\psi_n\rangle; \quad E_n \approx E$

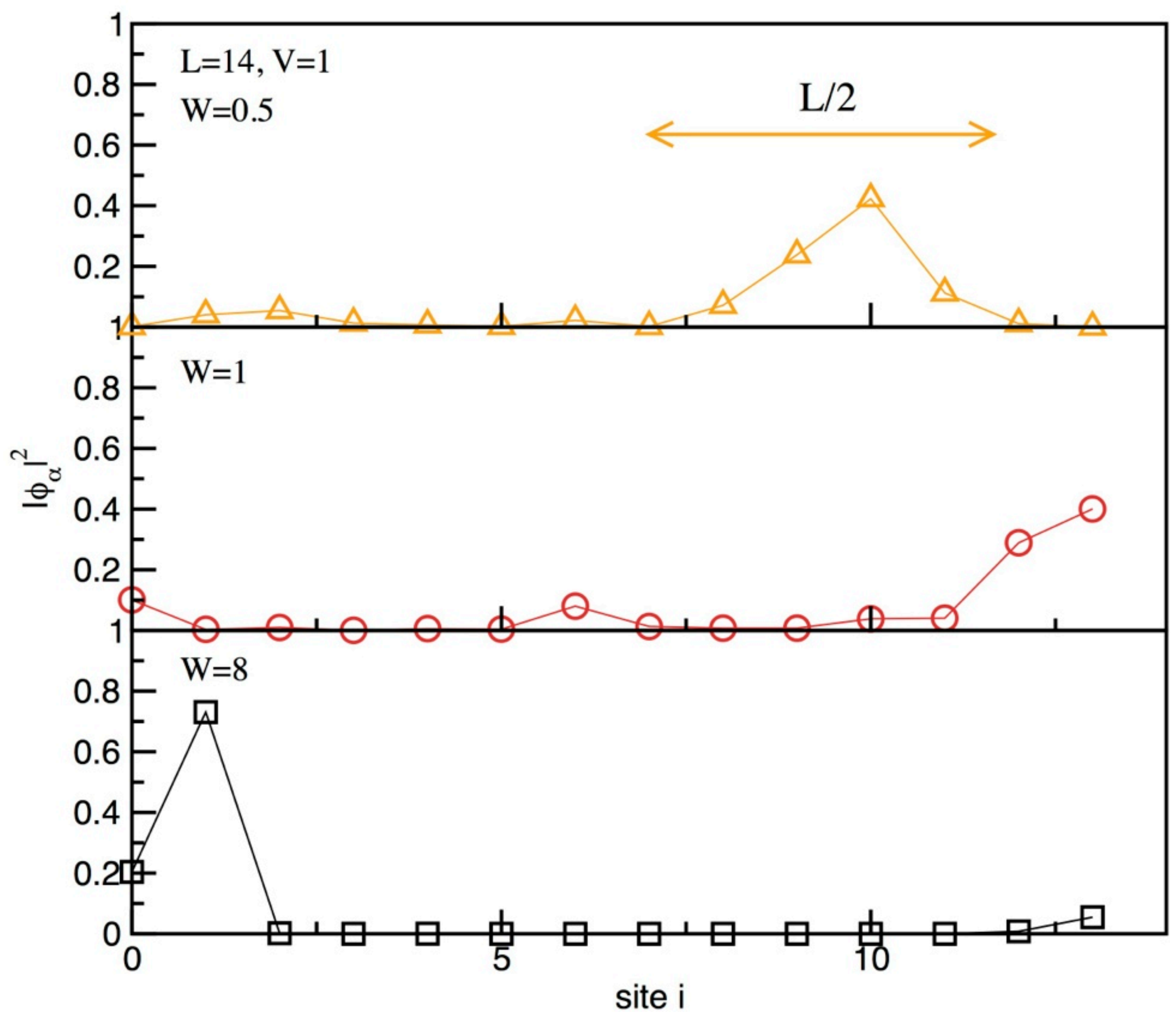
→ **Compute OPDM** $\rho^{(1)}_{ij} = \langle\psi_n|c^\dagger_i c_j|\psi_n\rangle$

→ **Diagonalize it !** $\rho^{(1)}|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle; \quad c^\dagger_\alpha = U_{\alpha j} c^\dagger_j$

→ **Average over impurity configurations**

Two previous studies of OPDM eigenstates of HCBs with disorder (no connection to MBL transition):
Nessi, Iucci Phys. Rev. A 84, 063614 (2011); Gramsch, Rigol, Phys. Rev. A 86, 053615 (2012)

Typical OPDM eigenstates



delocalized

localized

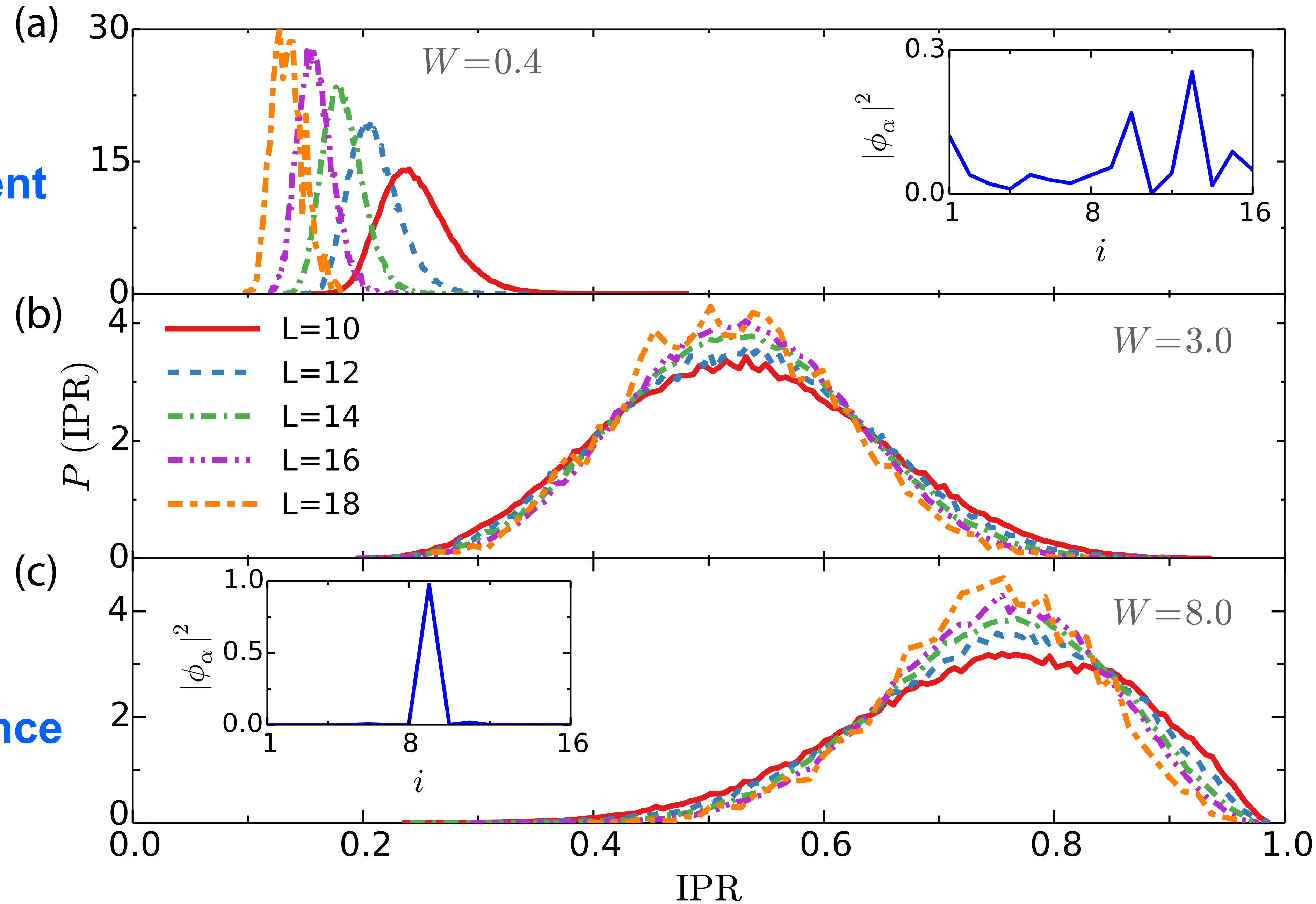
Inverse participation ratio

$$IPR = \frac{1}{N}\sum_{\alpha=1}^{L} n_\alpha \sum_{i=1}^{L} |\phi_\alpha(i)|^4$$

delocalized $\frac{1}{L} < IPR < 1$ **localized**

Clearly L-dependent

Weak L-dependence



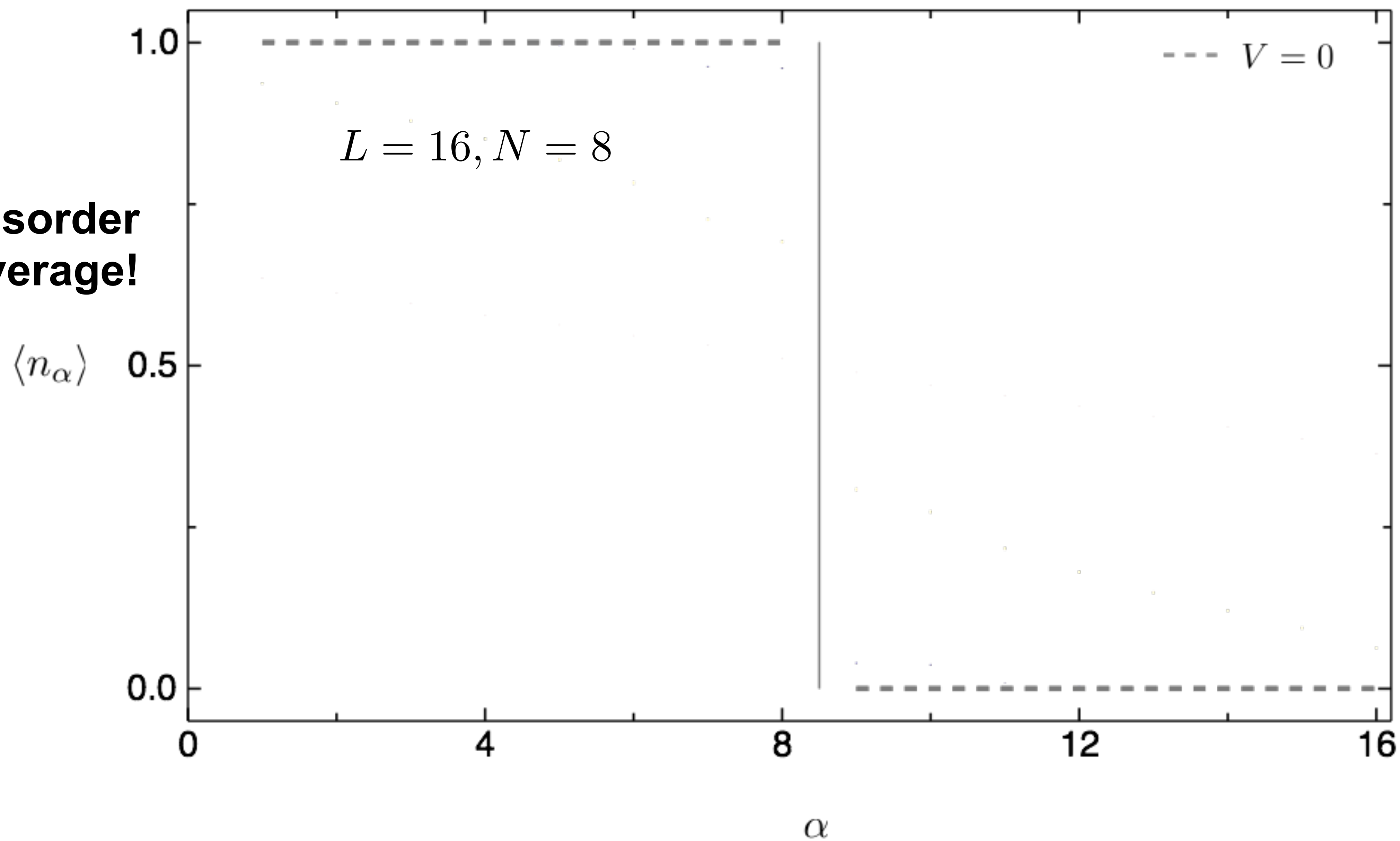
Tools used to analyze Anderson eigenstates can be applied to natural orbitals!

OPDM occupations: Non-interacting case

$\rho^{(1)}|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle$

fixed energy density

Disorder average!



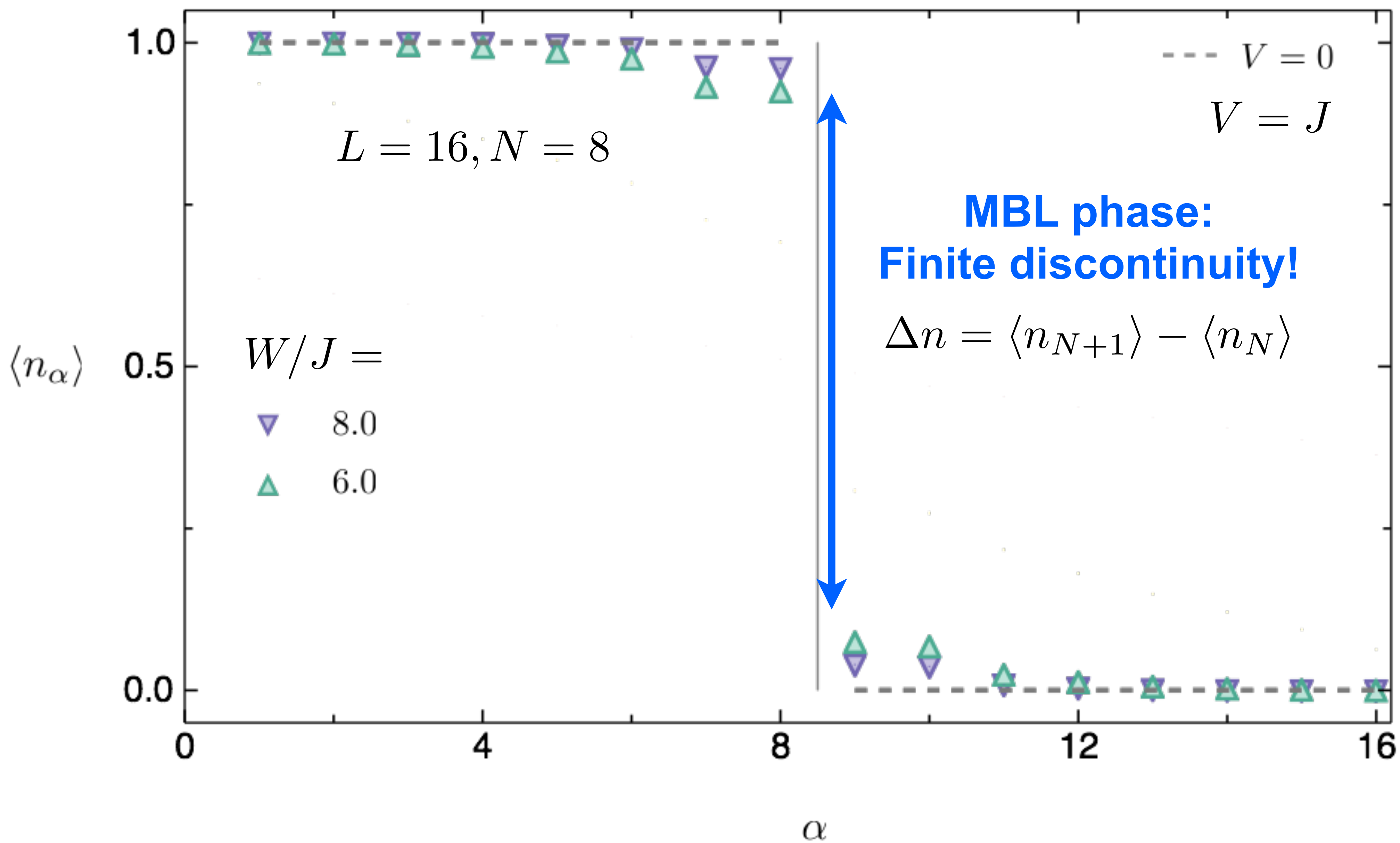
$$H = -J\sum_{j=1}^{L}(c_j^\dagger c_{j+1} + h.c.) - \sum_j \epsilon_j n_j$$

Anderson insulator

Many-body eigenstate: Slater determinant independent of disorder: step function

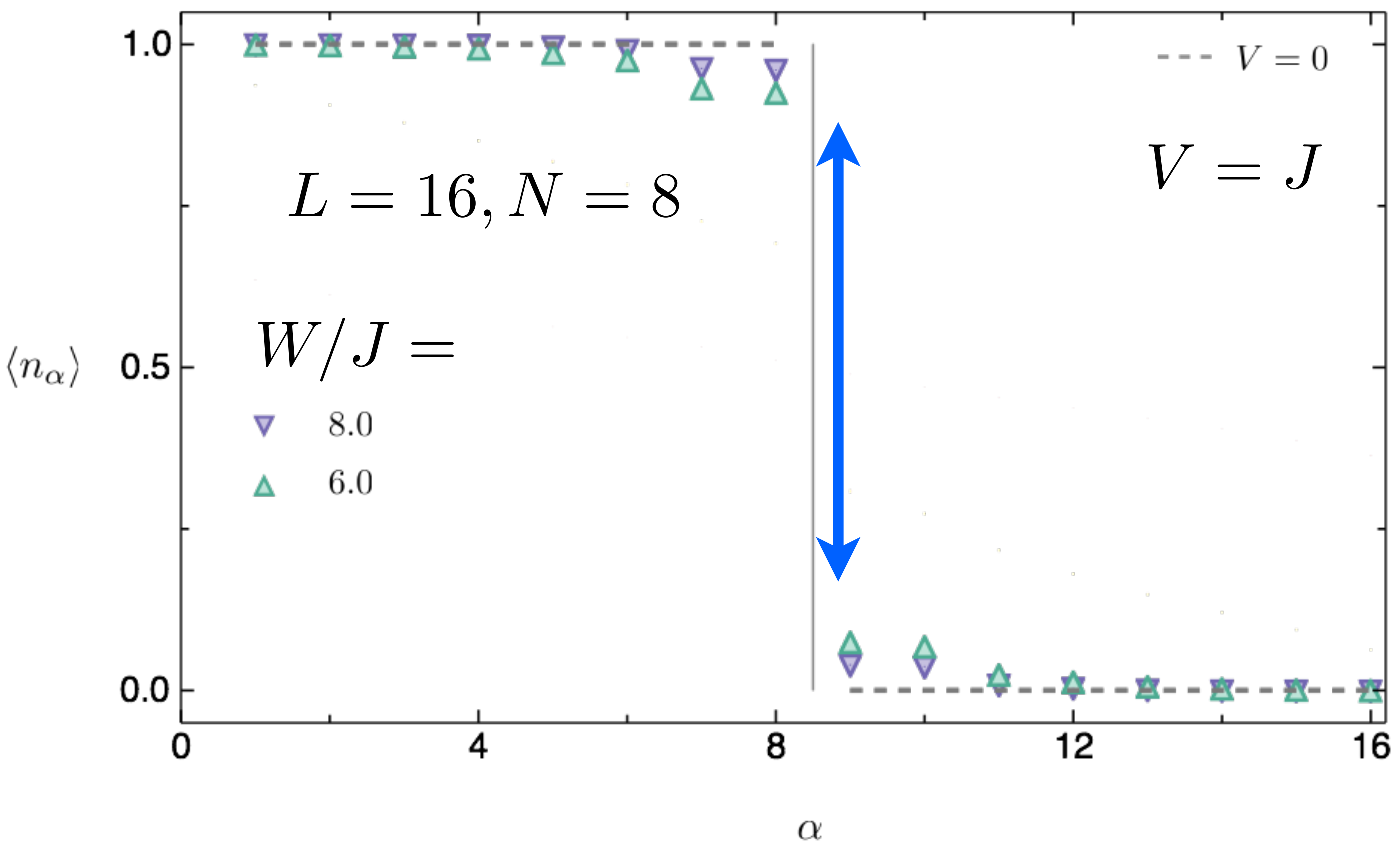
OPDM occupations in the MBL phase

fixed energy density

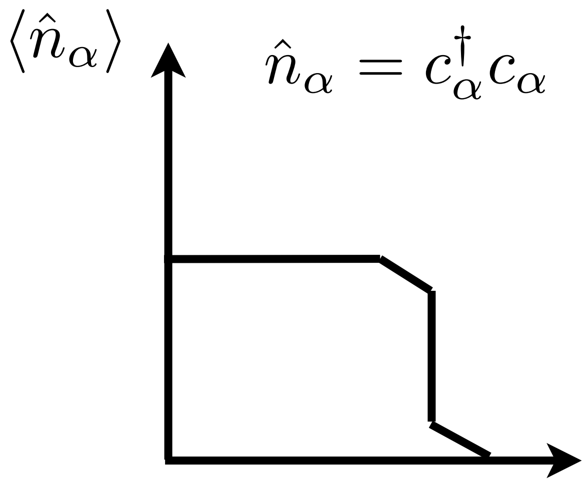


MBL phase: Finite discontinuity survives!
Fock-space localization

Quasi-particles: MBL phase vs Fermi-liquid



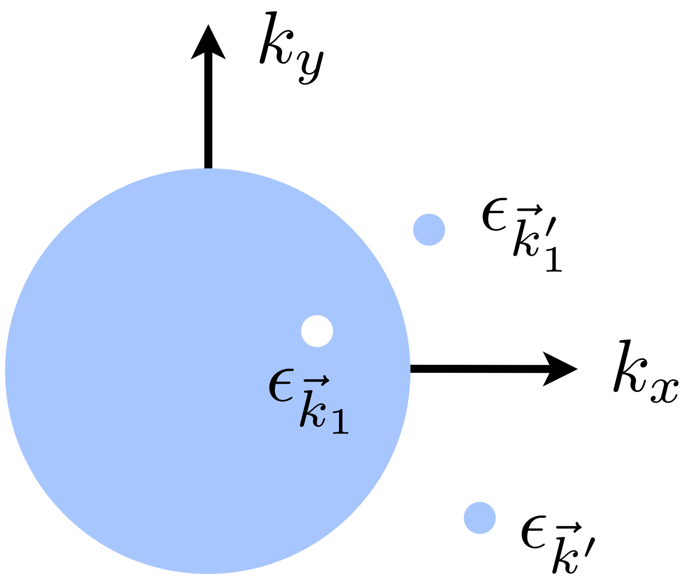
Analogy to a T=0 Fermi liquid!



$$\text{MBL}: \quad \alpha \rightarrow \text{"}j\text{"}$$

$$\text{FL}: \quad \alpha \rightarrow k$$

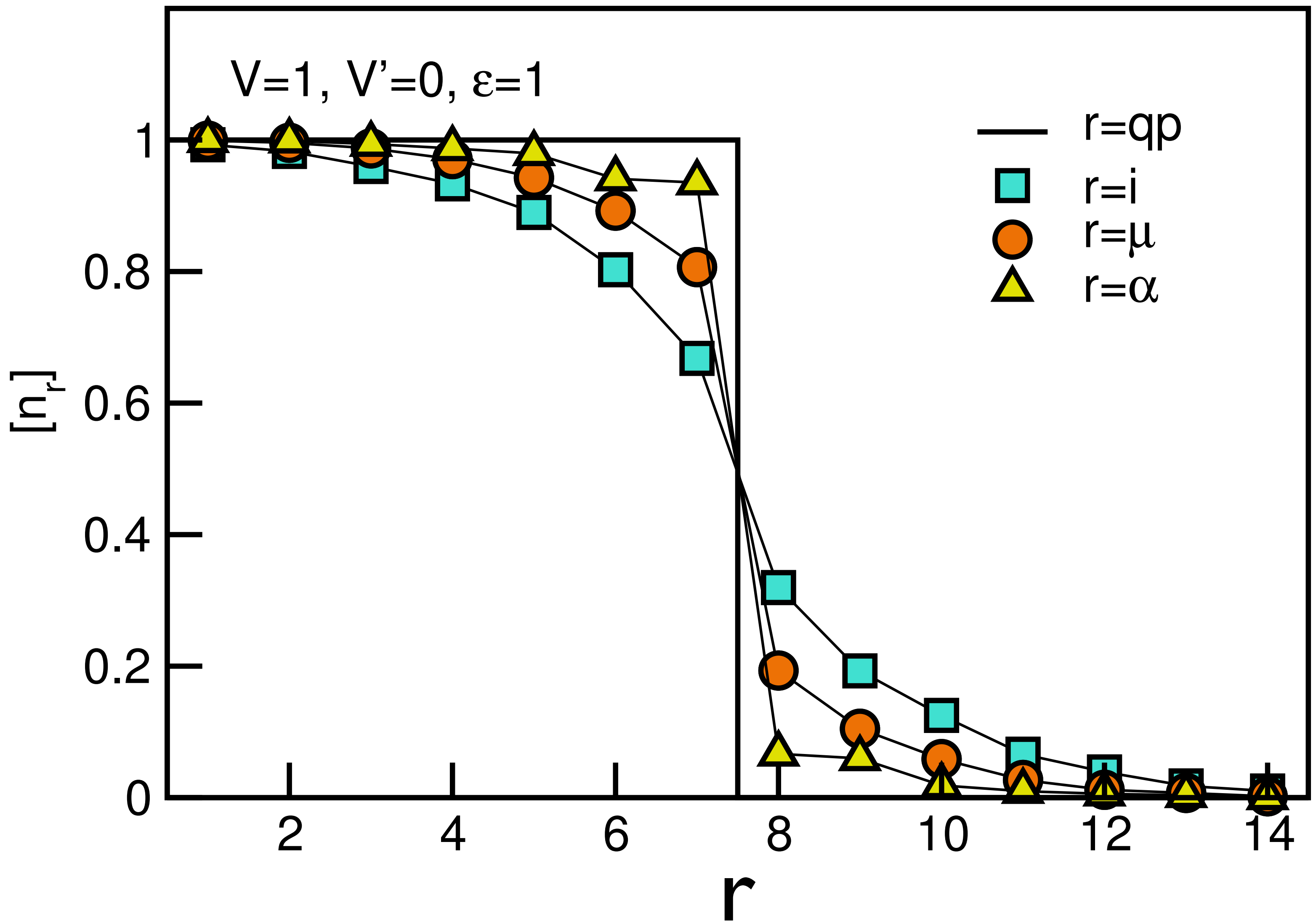
MBL to Anderson insulator
as
Fermi-liquid to free Fermi gas!
Localized Quasi-particles



Bera, Schomerus, FHM, Bardarson, Phys. Rev. Lett. 115, 046603 (2015)

Consistent with Basko, Aleiner, Altshuler Annals of Physics (2006),
quasi-local conserved charges:
Huse et al. PRB (2014), Serbyn et al. PRL (2013) Vosk, Altman PRL (2013)

Natural orbitals: Optimum approximation to l-bits

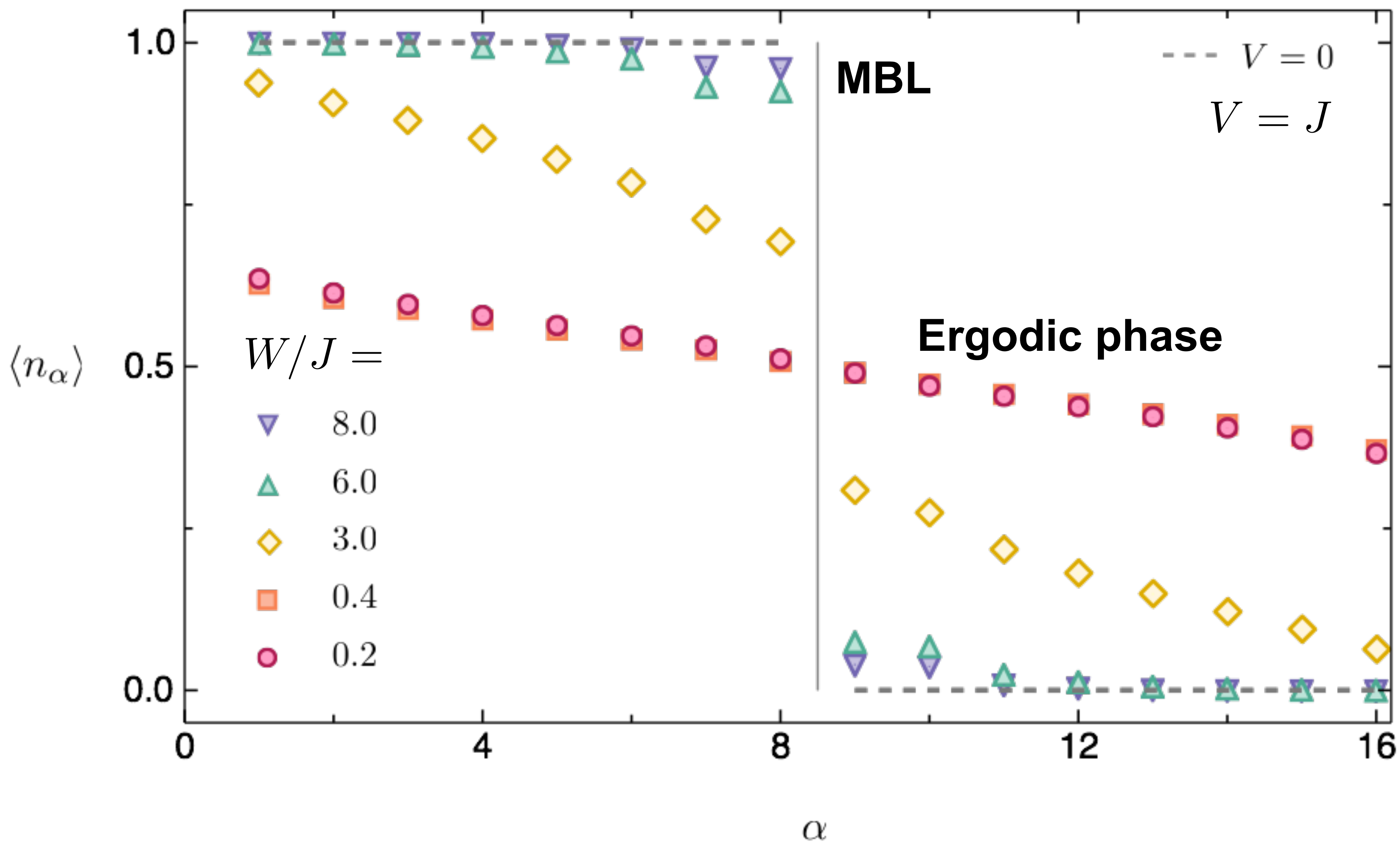


Quasi-particles
Physical density
Anderson orbitals
Natural orbitals

OPDM occupations closest to quasi-particle distribution:
Best single-particle approximation to l-bits

OPDM occupations in ergodic phase

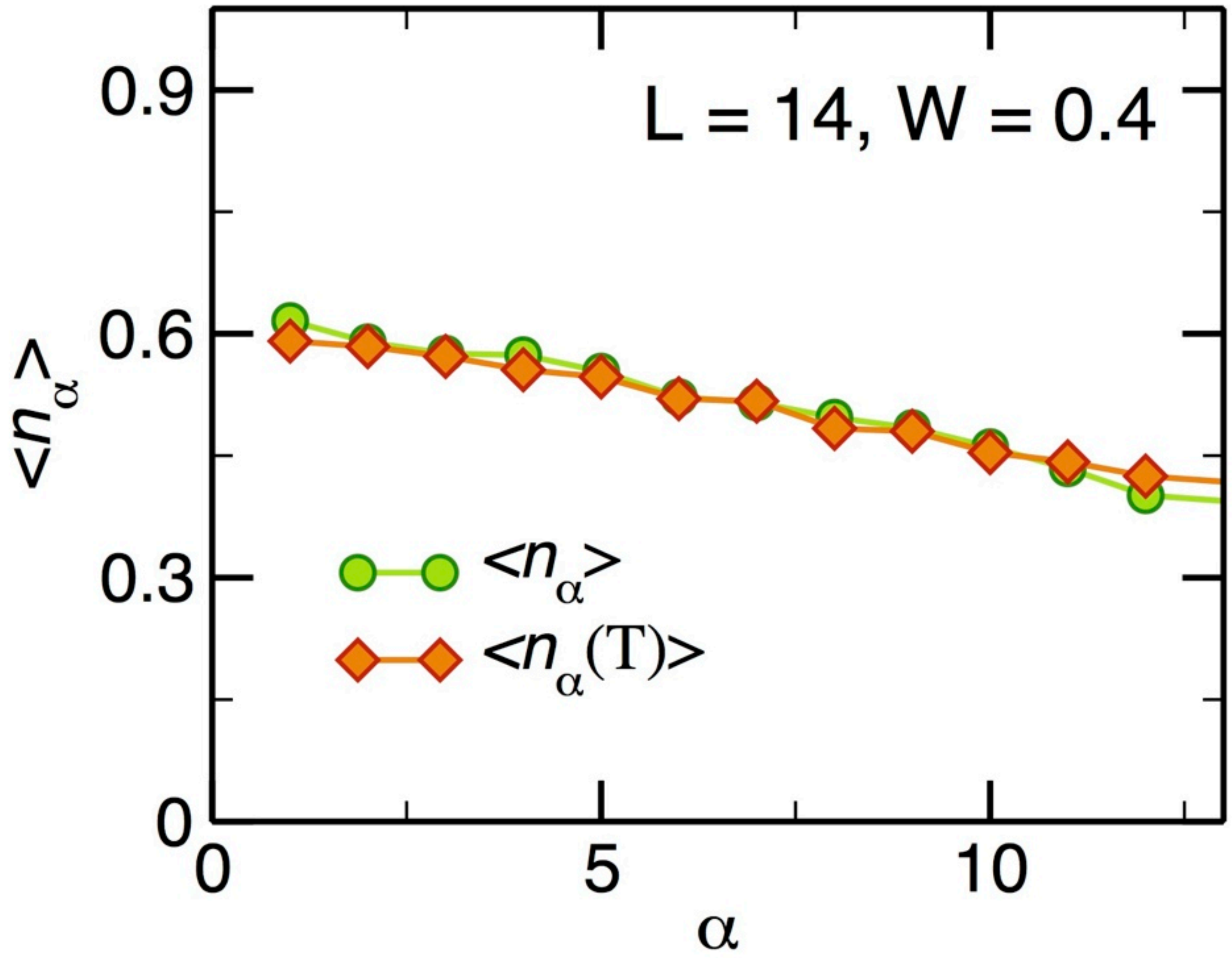
fixed energy density



thermal!

Discontinuity vanishes in ergodic phase - thermal distribution!
Phase diagram from calculating discontinuity!

OPDM occupations in ergodic phase

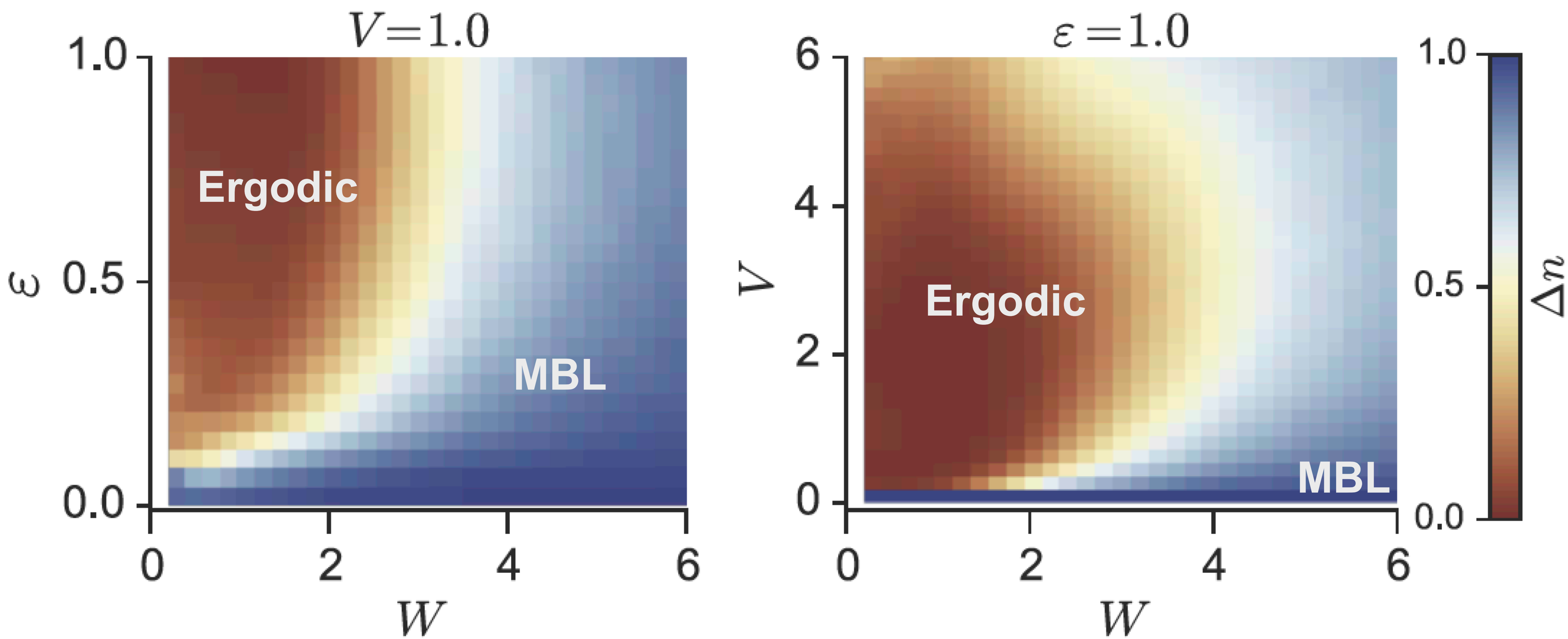


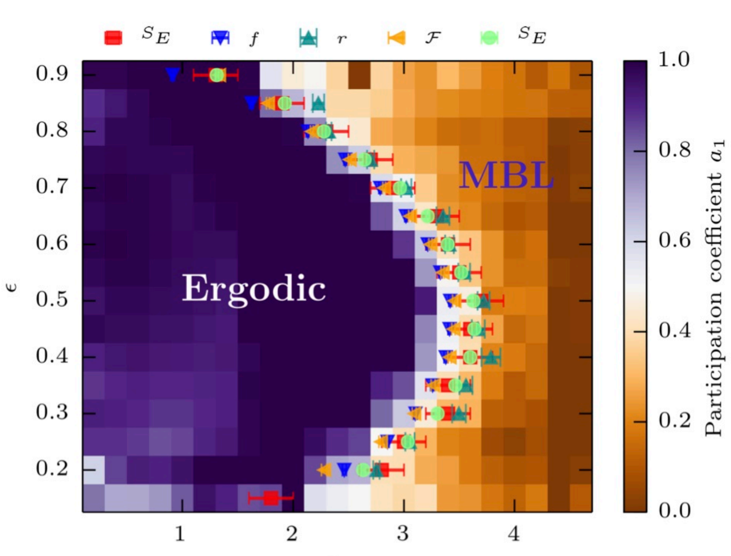
$$E \approx \langle \psi_n | H | \psi_n \rangle = \mathrm{tr}(\rho(T) H) \qquad \rho^{(1)}_{ij}(T) = \mathrm{tr}[\rho_{\mathrm{can}}(T) c_i^\dagger c_j] \to \langle n_\alpha(T) \rangle$$

Finite-size effects typical for non-integrable system, *Sorg, Vidmar, Pollet, FHM PRA 2014*

Discontinuity in occupations: “Phase” diagrams

$$\Delta n = n_{N+1} - n_N$$

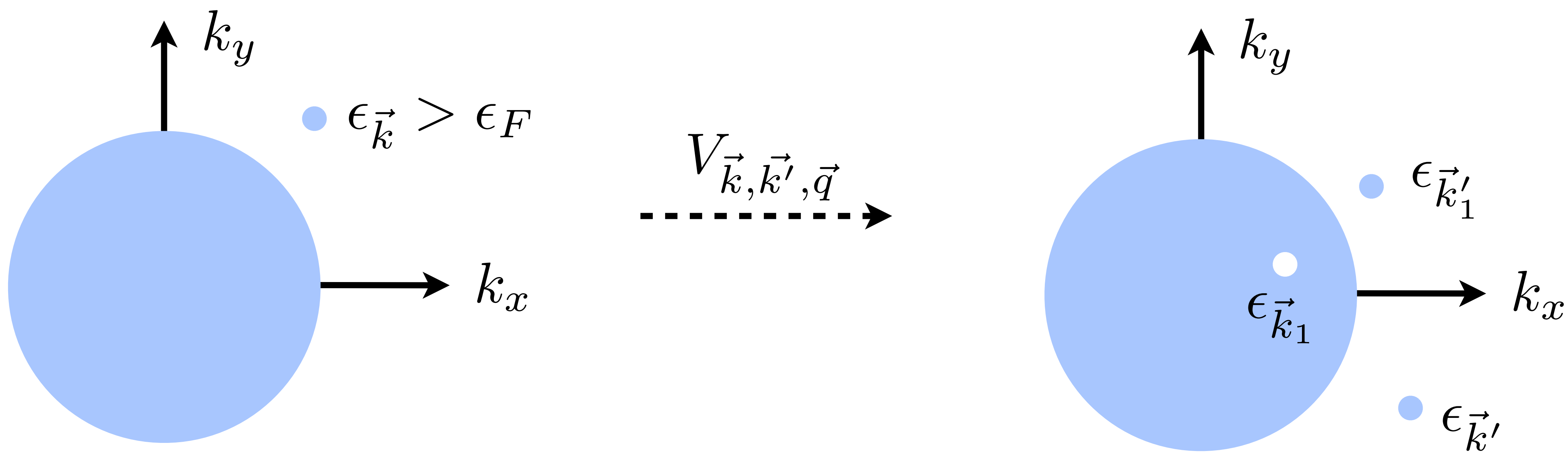




New measure to determine phases: Discontinuity Δn agrees with known phase diagram

Fermi-liquids

3D Fermi gas with short-range interactions



Low energies: $\frac{1}{\tau_{\vec{k}}} \propto (\epsilon_k - \epsilon_F)^2$

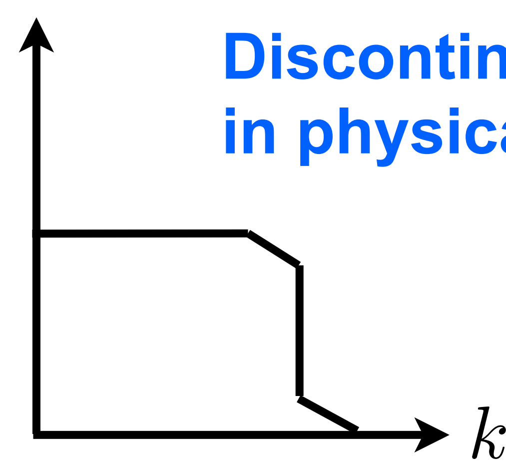
Long-lived quasi-particles: $\tilde{c}^\dagger_{\vec{k}} = Z_k c^\dagger_{\vec{k}} + \text{particle-hole pairs}$

Fermi-liquid Hamiltonian:

$$H_{\rm FL} = \sum_k \tilde{\epsilon}_k \tilde{n}_k + \sum_{k,k'} f_{k,k'} \tilde{n}_k \tilde{n}_{k'}$$

$$[H_{\rm FL}, \tilde{n}_k] = 0$$

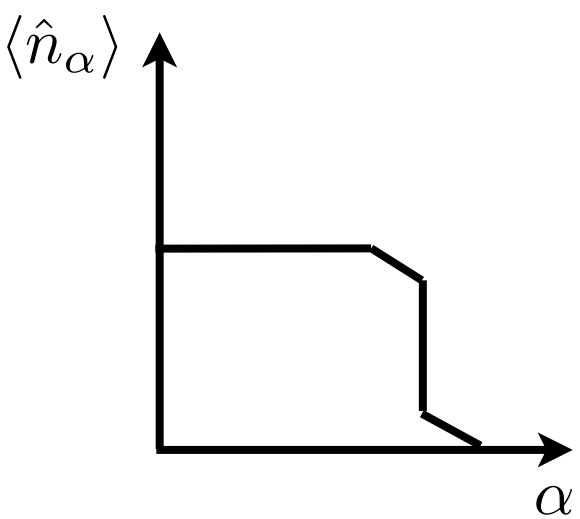
$n_k = \langle c^\dagger_k c_k \rangle$



Discontinuity in physical MDF!

MBL & Fermi-liquids

→ **Connection to conserved charges & Fermi-liquid interpretation**
(one realization!)

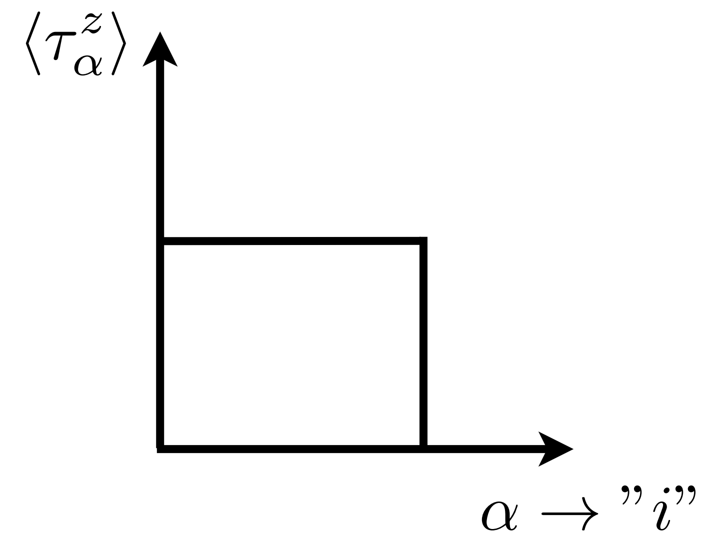


"Physical" particles in FL

OPDM eigenstates

$$\hat{n}_\alpha = c_\alpha^\dagger c_\alpha \leftrightarrow \tau_\alpha^z$$

l-bits



Quasi-particles in FL

l-bits

$$H = \sum_i \epsilon_i \tau_i^z + \sum_{i,j} J_{i,j} \tau_i^z \tau_j^z + \sum_{i,j,\{k\}} K_{i,\{k\},j}^n \tau_i^z \tau_{k_1}^z \cdots \tau_{k_n}^z \tau_j^z$$

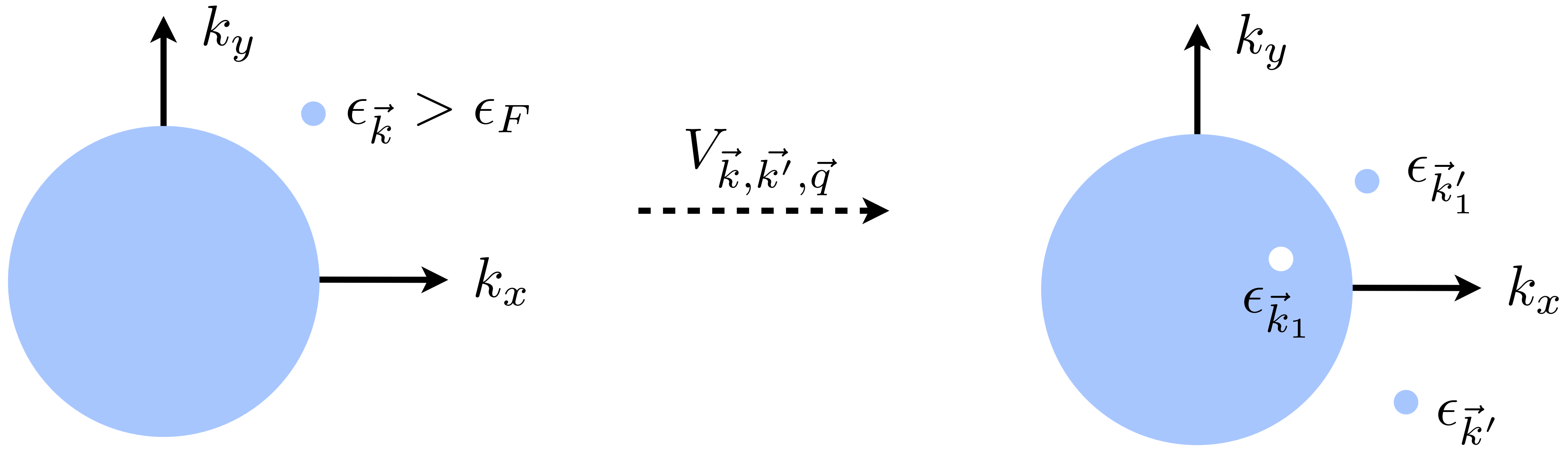
Fermi-liquid

$$H_{\mathrm{FL}} = \sum_k \tilde{\epsilon}_k \tilde{n}_k + \sum_{k,k'} f_{k,k'} \tilde{n}_k \tilde{n}_{k'}$$

MBL relates to Anderson insulator as FL relates to free Fermi gas

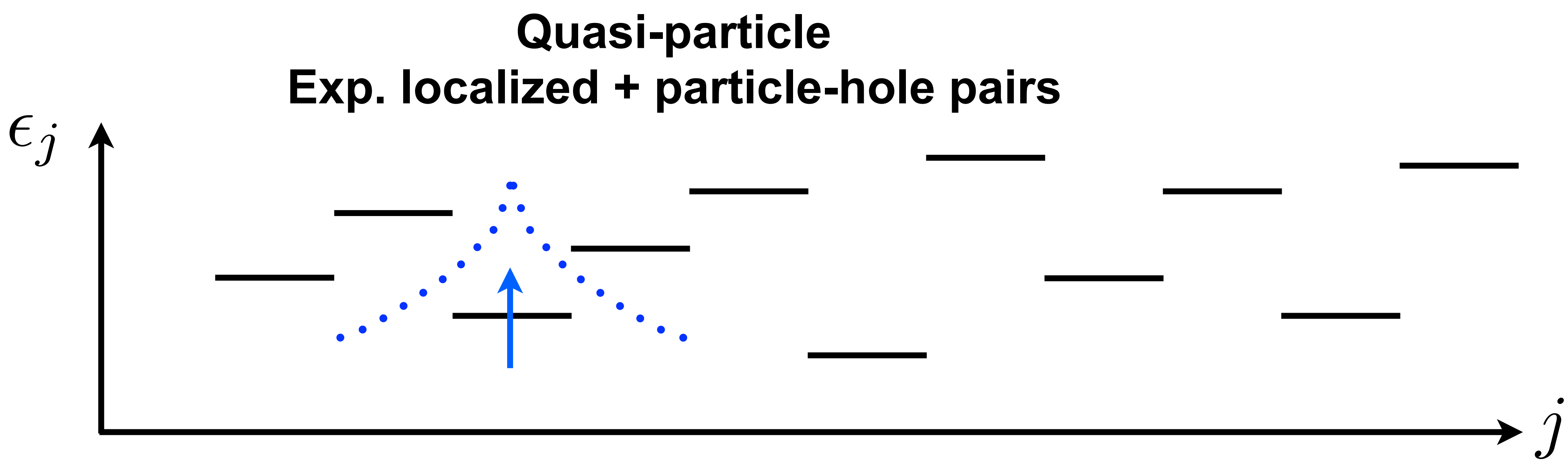
Fermi-liquids

3D Fermi gas with short-range interactions

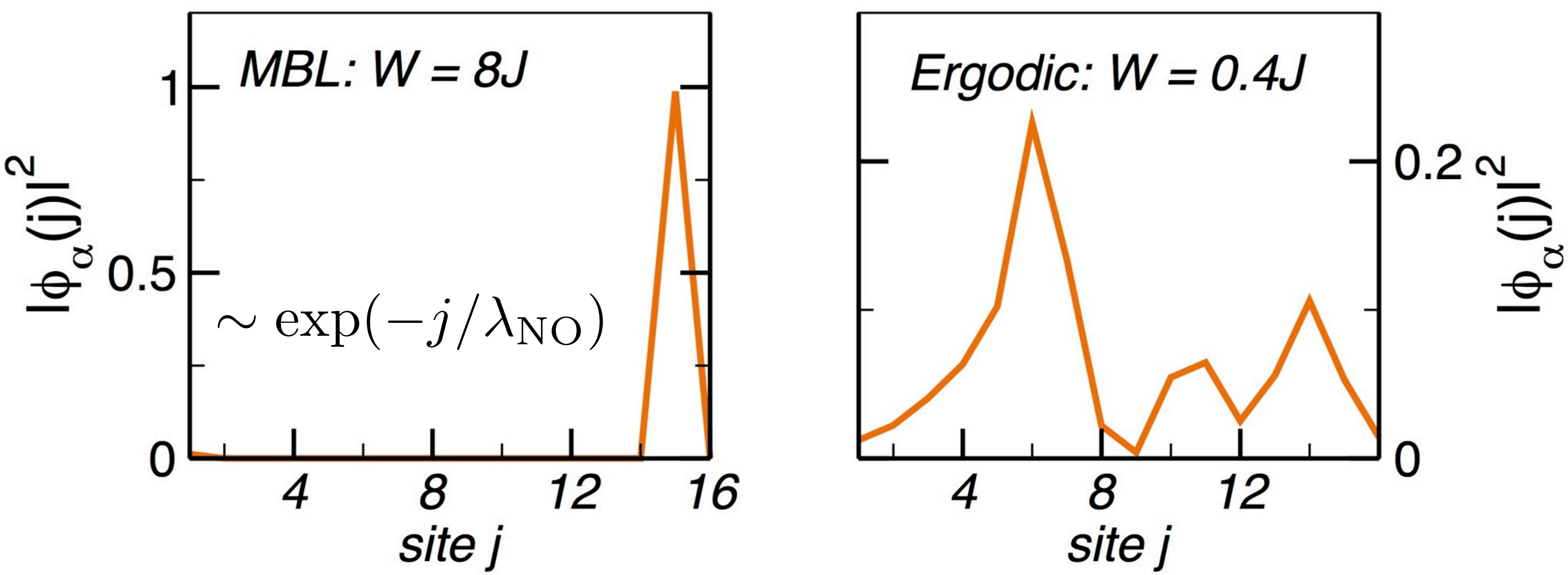


Low energies: $\frac{1}{\tau_{\vec{k}}} \propto (\epsilon_k - \epsilon_F)^2$

Localized quasi-particles & natural orbitals

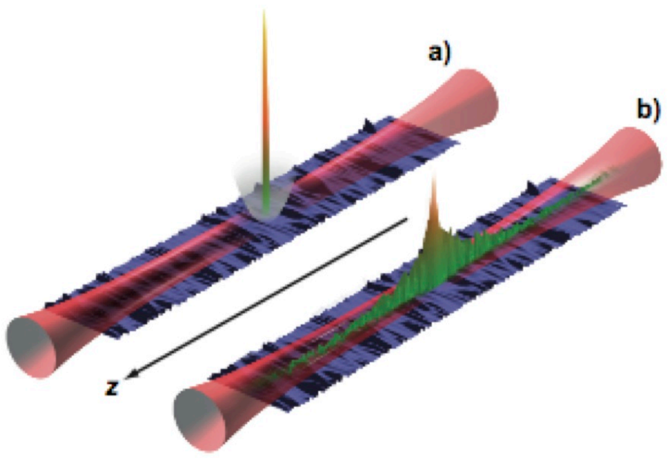


Natural orbitals: Localized/extended in MBL/ergodic phase



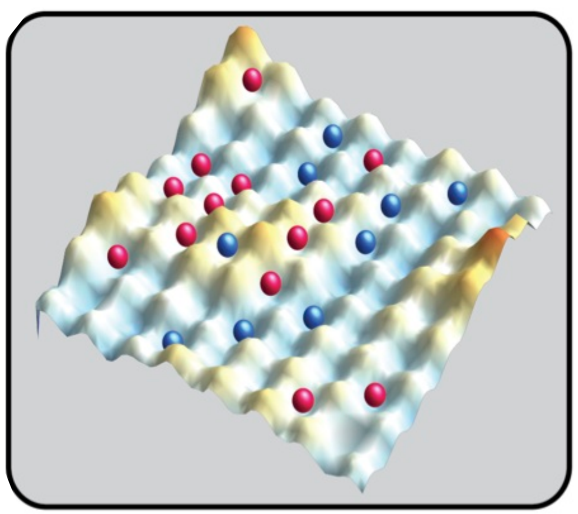
MBL experiments with quantum gases

Anderson localization



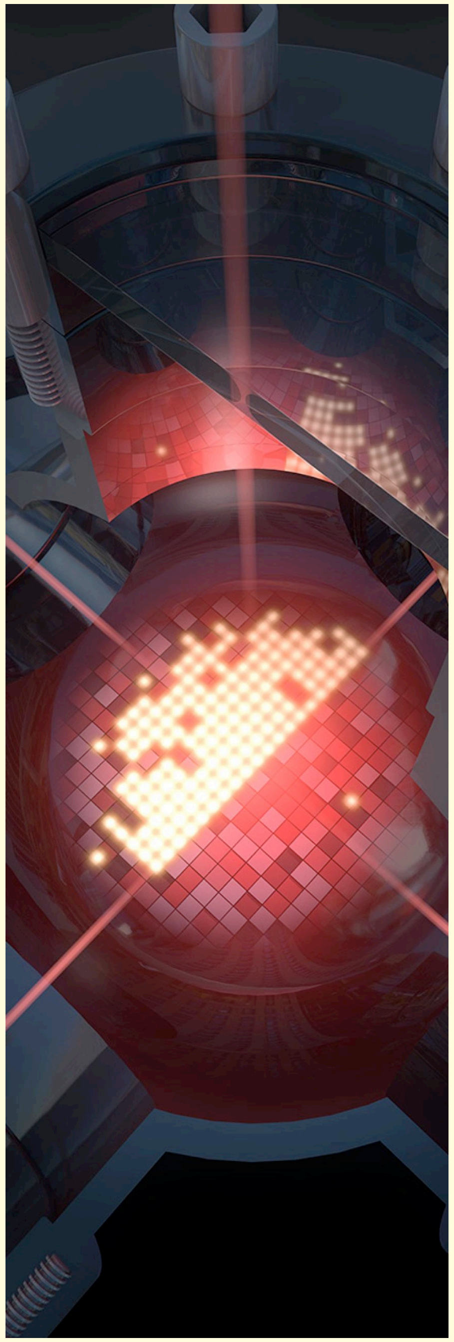
Billy, Aspect et al. Nature (2008)
Roati, Inguscio et al. Nature (2008)

3D Bose-Hubbard



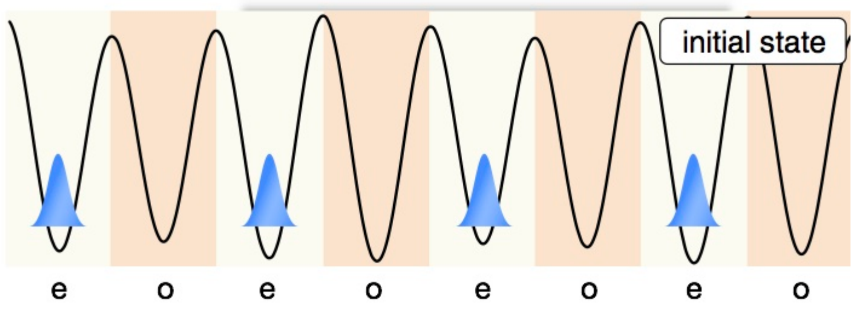
Kondov, de Marco et al. PRL (2015)

2D Bose-Hubbard



Choi, Bloch, Gross et al. Science (2016)

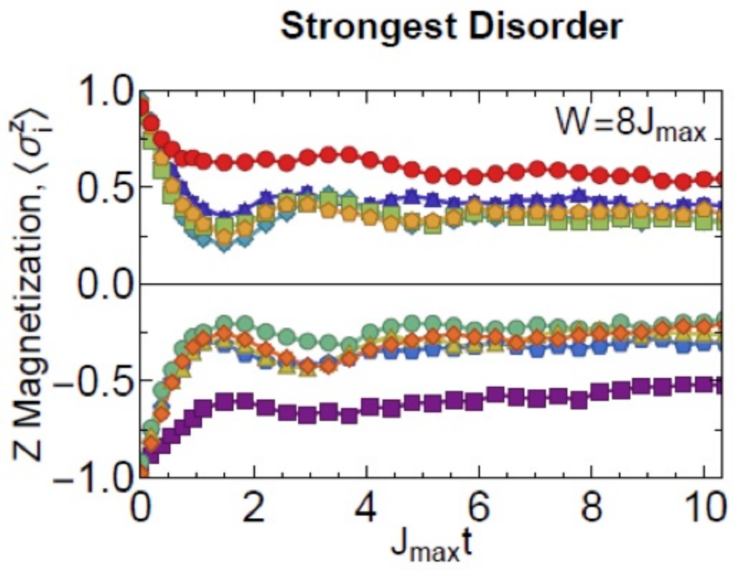
1D fermions, quasi-periodicity



MBL: No decay of CDW!

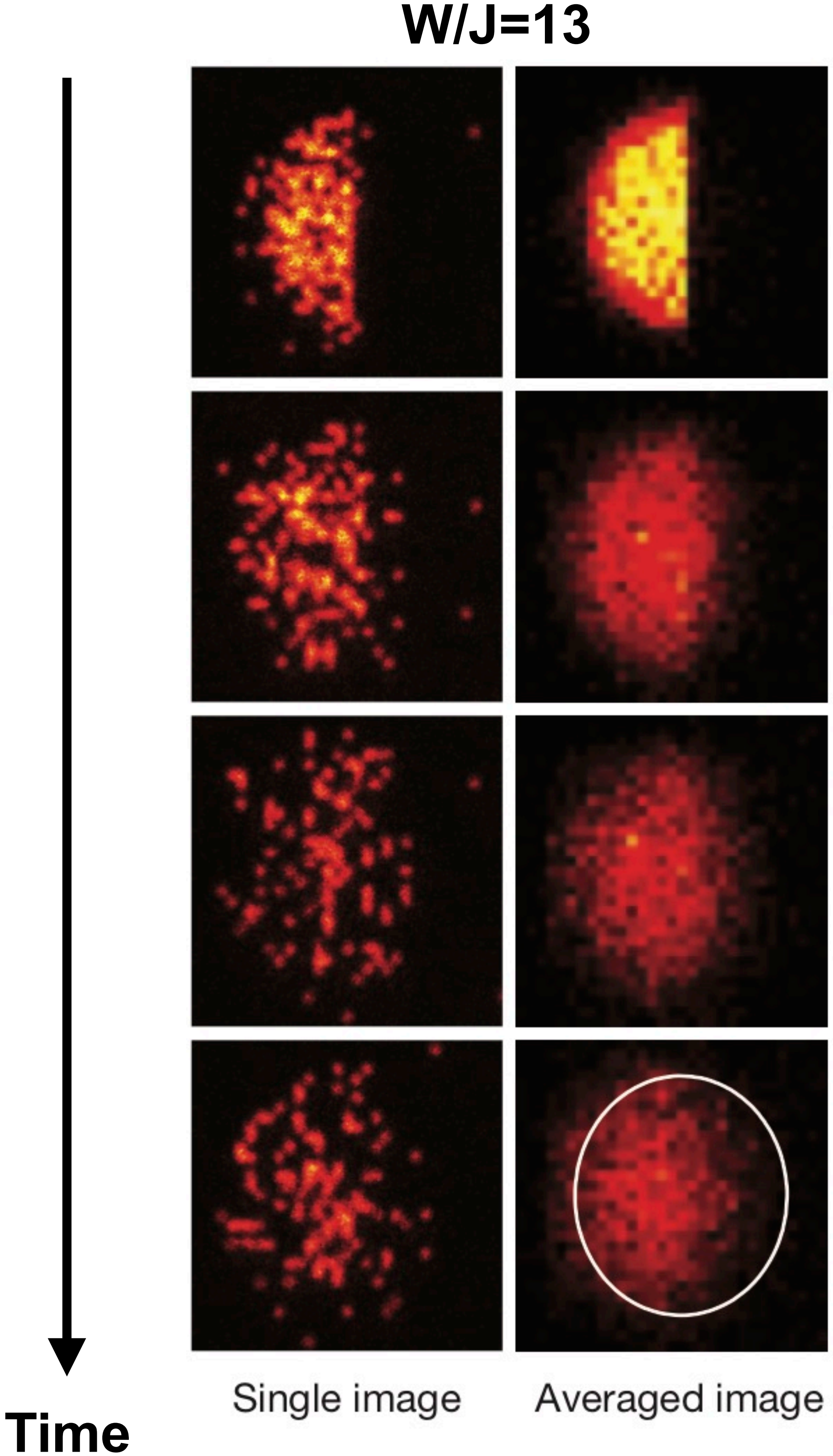
Schreiber, Bloch, Schneider et al. Science (2015)
Bordia et al. PRL (2016)

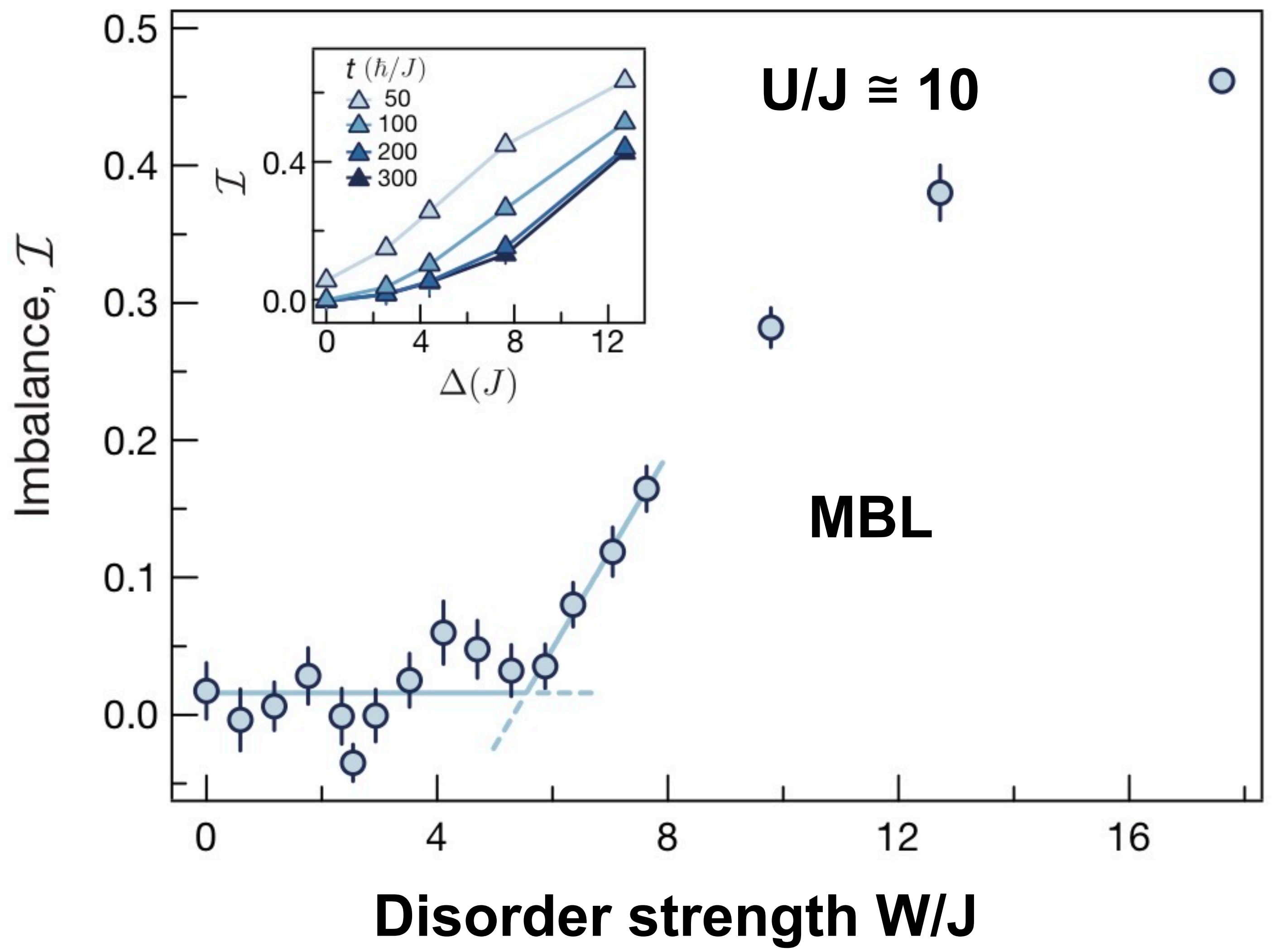
Ions, transverse Ising model



Smith, Monroe et al. Nature Phys. (2016)

2D Bose-Hubbard: Domain-wall melting



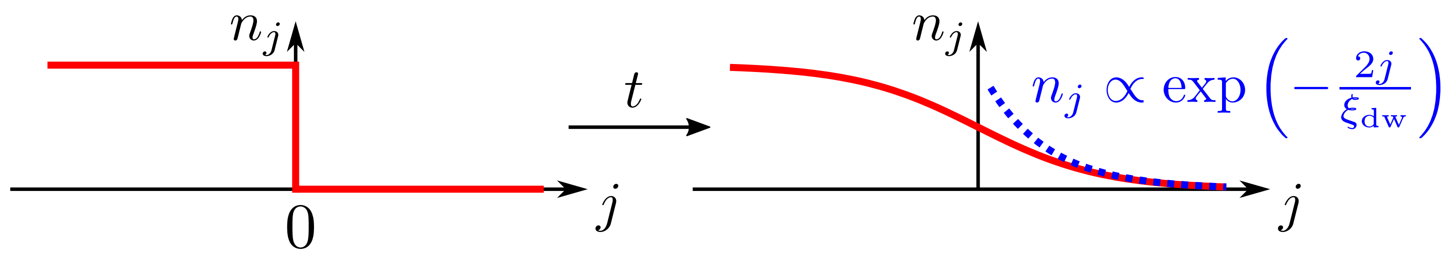


$$\mathcal{I} = \frac{N_{\mathrm{right}} - N_{\mathrm{left}}}{N_{\mathrm{right}} + N_{\mathrm{left}}}$$

Choi, Bloch, Gross et al. Science (2016)

Domain-wall melting: Absence of transport

Kubo: Gopalakrishnan et al. Phys. Rev. B 92, 104202 (2015); Barišić et al. Phys. Rev. B 94, 045126 (2016); Steinigeweg et al. Phys. Rev. B 94, 180401 (2016)

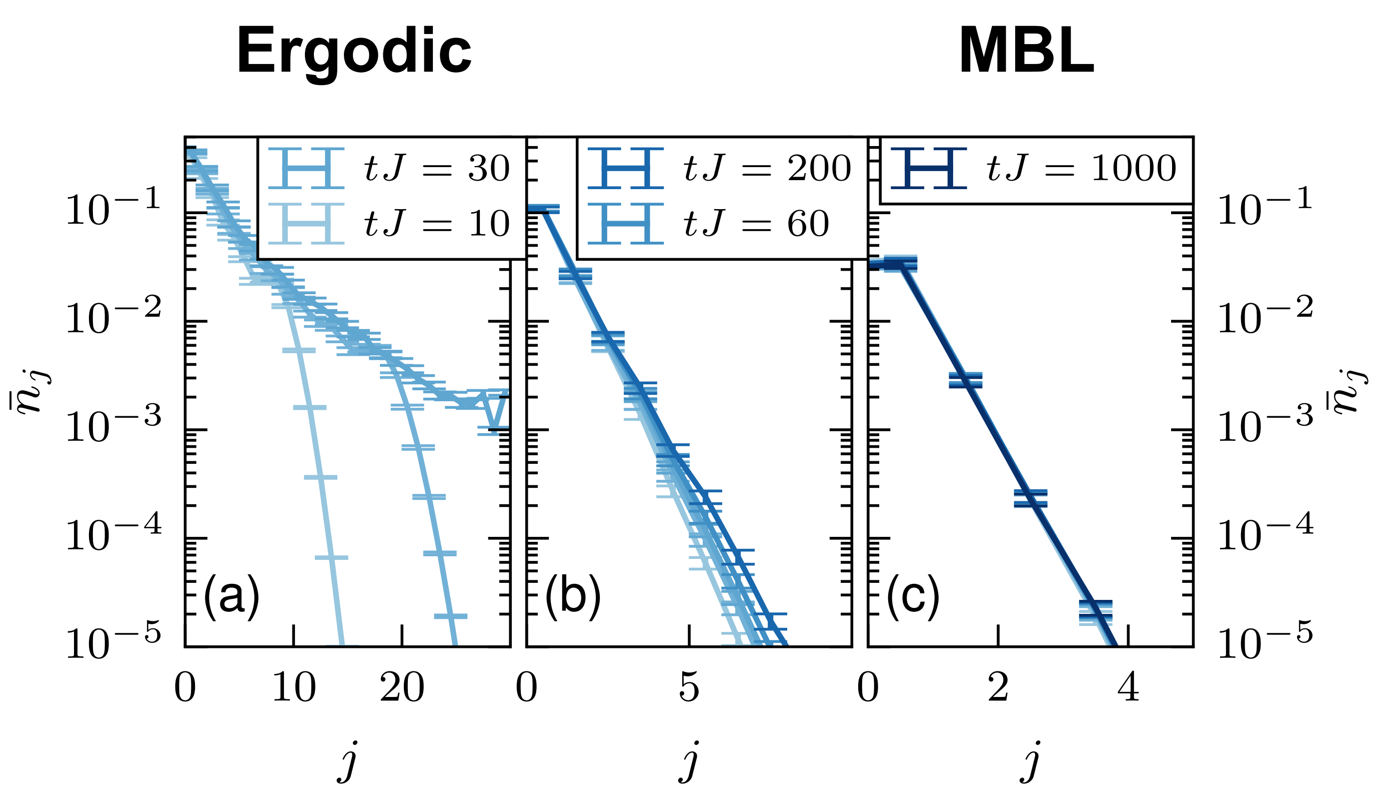


Time-dependent Density-Matrix Renormalization Group

Hauschild, FHM, Pollmann, Phys. Rev. B 94, 161109(R) (2016)

$V/J = 1$

no stationary profiles

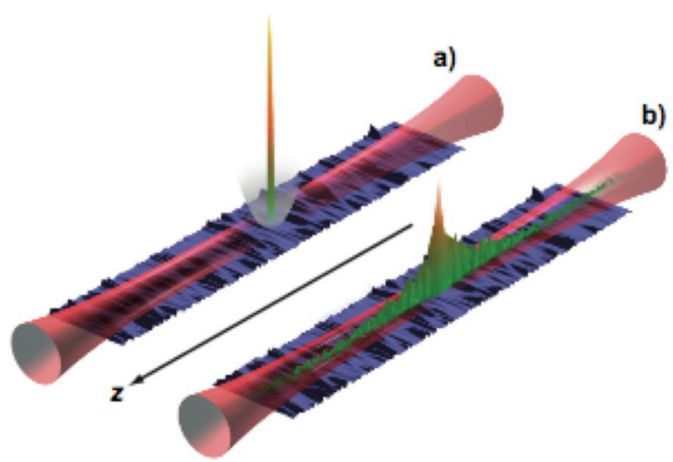


Stationary, exponential

MBL phase: No transport, exponential length scales!

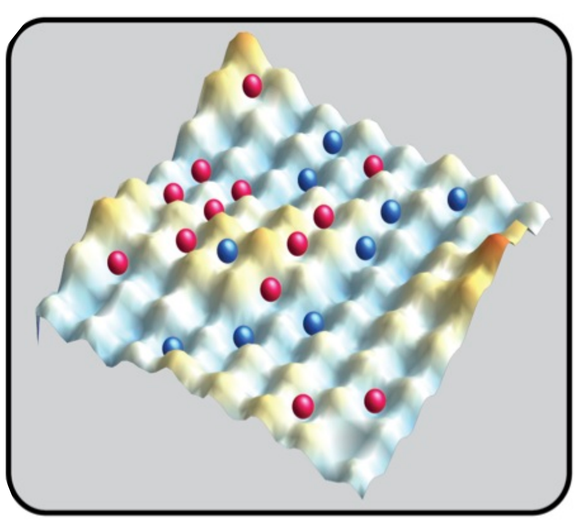
MBL experiments with quantum gases

Anderson localization



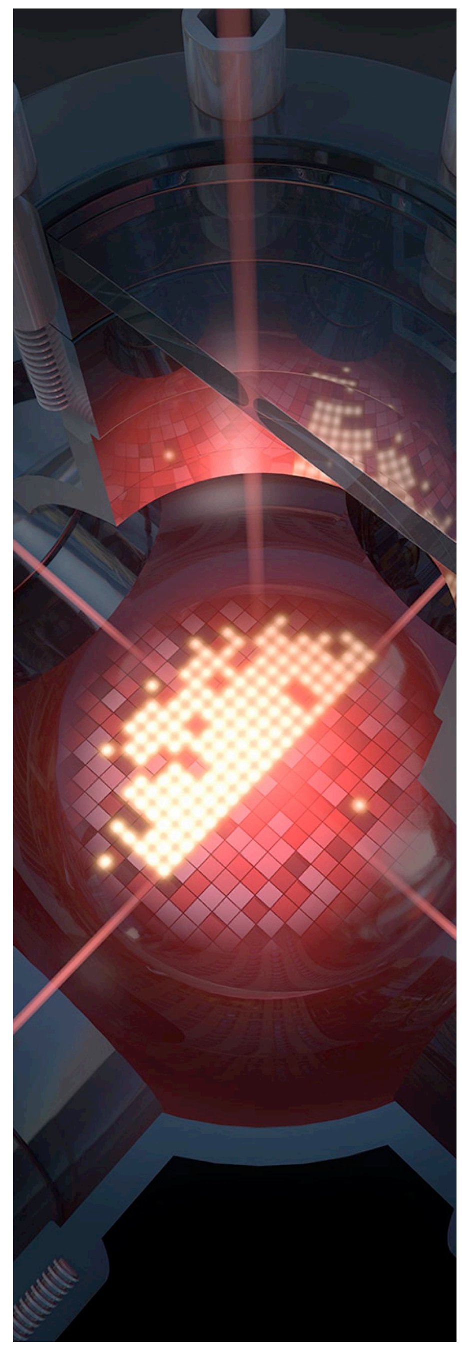
Billy, Aspect et al. Nature (2008)
Roati, Inguscio et al. Nature (2008)

3D Bose-Hubbard



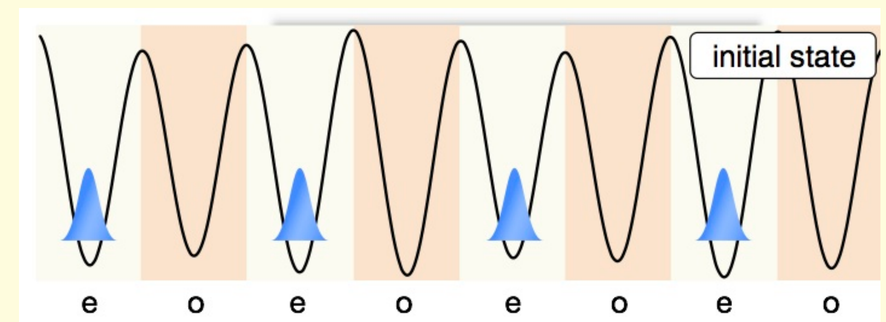
Kondov, de Marco et al. PRL (2015)

2D Bose-Hubbard



Choi, Bloch, Gross et al. Science (2016)

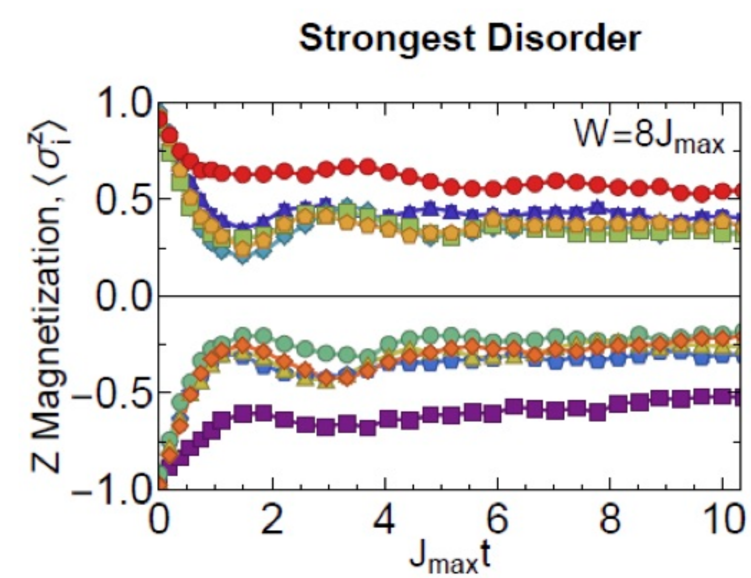
1D fermions, quasi-periodicity



MBL: No decay of CDW!

Schreiber, Bloch, Schneider et al. Science (2015)
Bordia et al. PRL (2016)

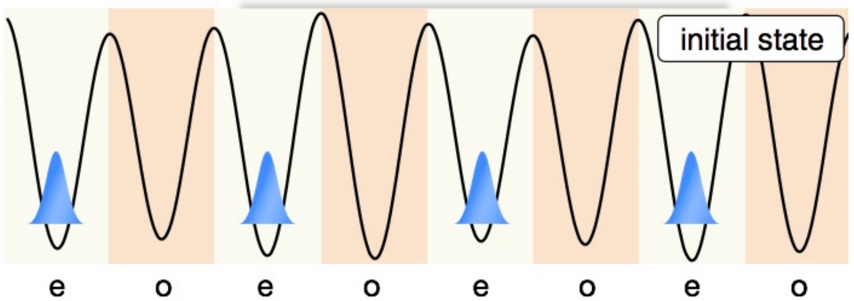
Ions, transverse Ising model



Smith, Monroe et al. Nature Phys. (2016)

Quantum quench dynamics

1D fermions, quasi-periodicity
onsite interactions,



Schreiber, Bloch, Schneider et al. Science (2015)
Bordia et al. PRL (2016)

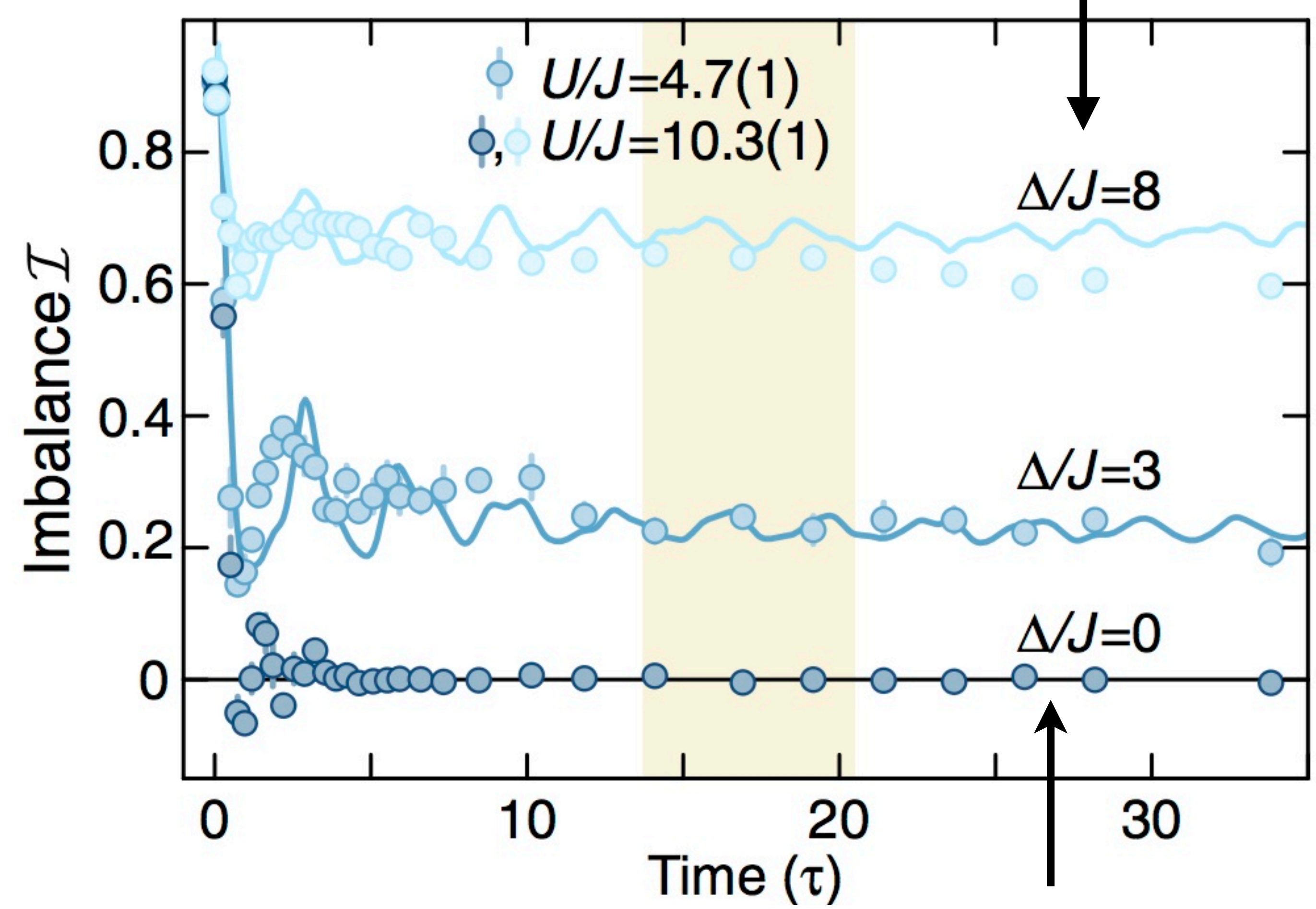
$$|\psi_0\rangle = |1,0,1,0,1,0,\dots\rangle$$

(spin randomized)

Density imbalance

$$\mathcal{I}_{\text{phys}} = \frac{N_{\text{odd}} - N_{\text{even}}}{N}$$

strong disorder



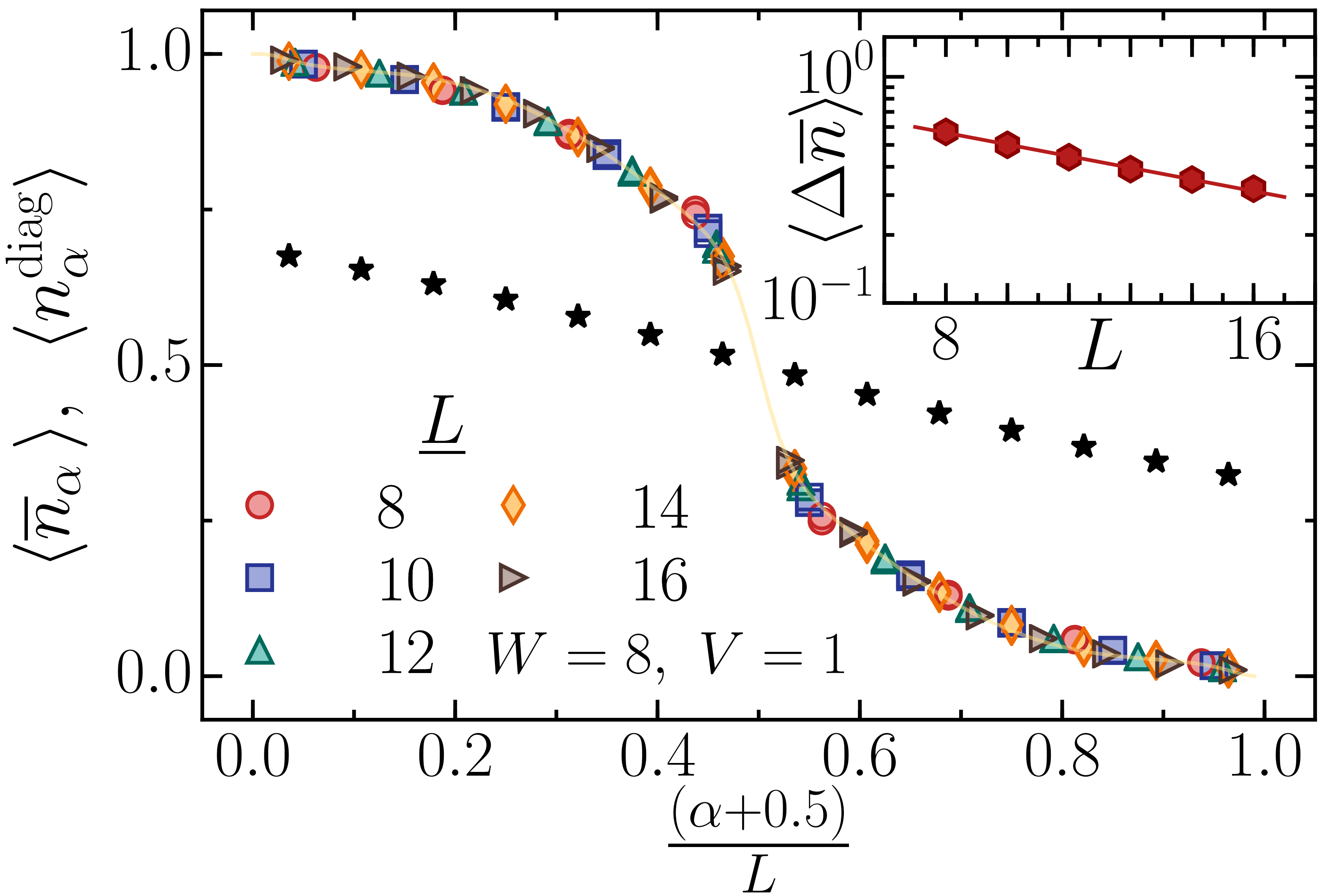
no disorder

MBL: No decay of density imbalance/density wave!

OPDM & quantum quench dynamics

$$H|n\rangle = E_n|n\rangle$$
$$|\psi_0\rangle = \sum_n a_n|n\rangle$$

$$\rho^{(1)}_{ij,\mathrm{diag}} = |a_0|^2\langle 0|c_i^\dagger c_j|0\rangle + \sum_{n>0}|a_n|^2\langle n|c_i^\dagger c_j|n\rangle$$



Finite-size discontinuity vanishes as:

$$\Delta n \approx |a_0|^2 \Delta n^{(0)} \approx \exp(-bL)$$

Partial quasi-particle occupations: Analog of finite-T Fermi liquid?

OPDM & quantum quench dynamics

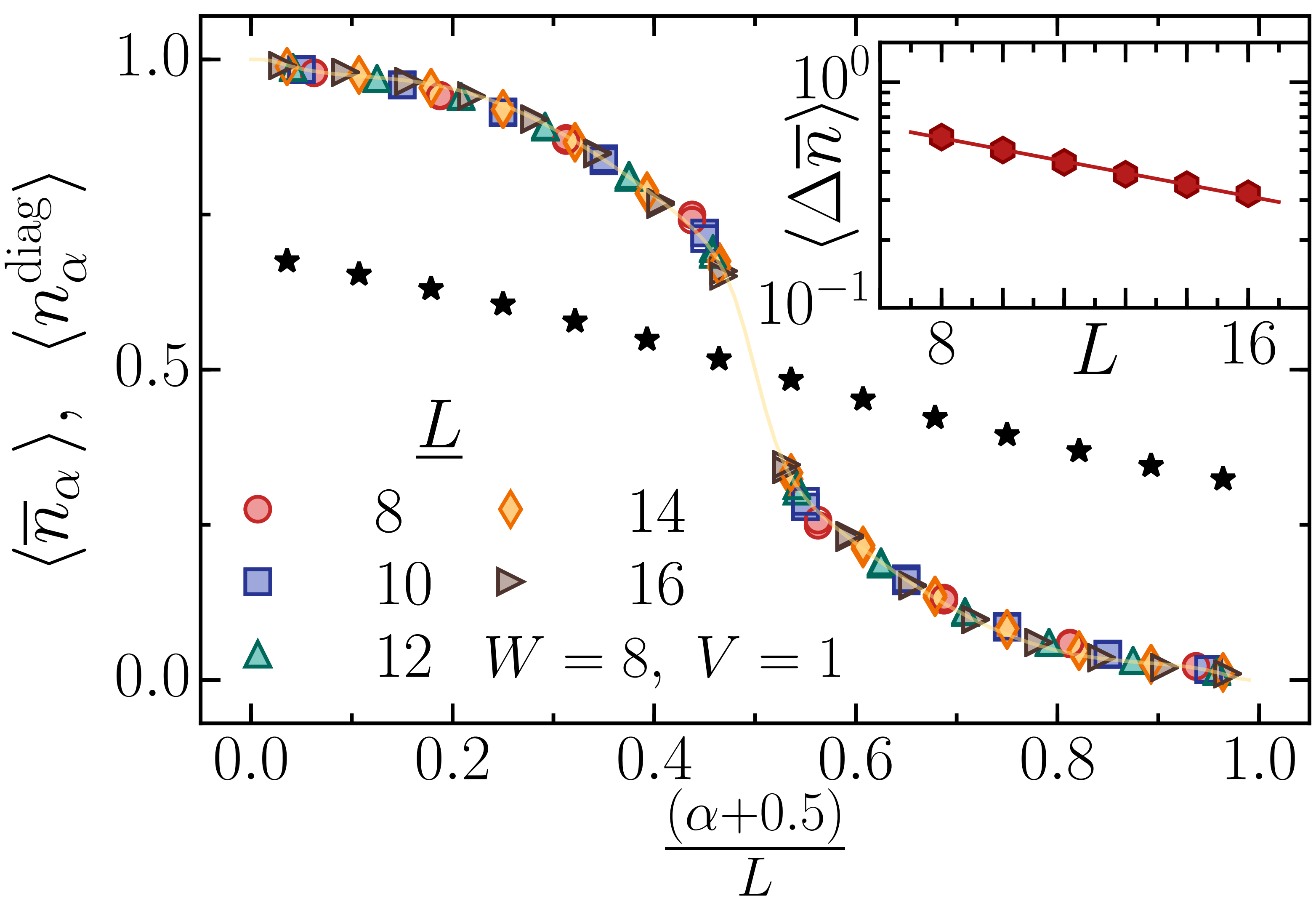
OPDM imbalance

$$\mathcal{I}_{\mathrm{OPDM}} = \frac{N_+ - N_-}{N}$$

Initial state:

$$|\psi_0\rangle = |1,0,1,0,1,0,\ldots\rangle$$

$$\mathcal{I}_{\mathrm{OPDM}}(t=0) = 1$$

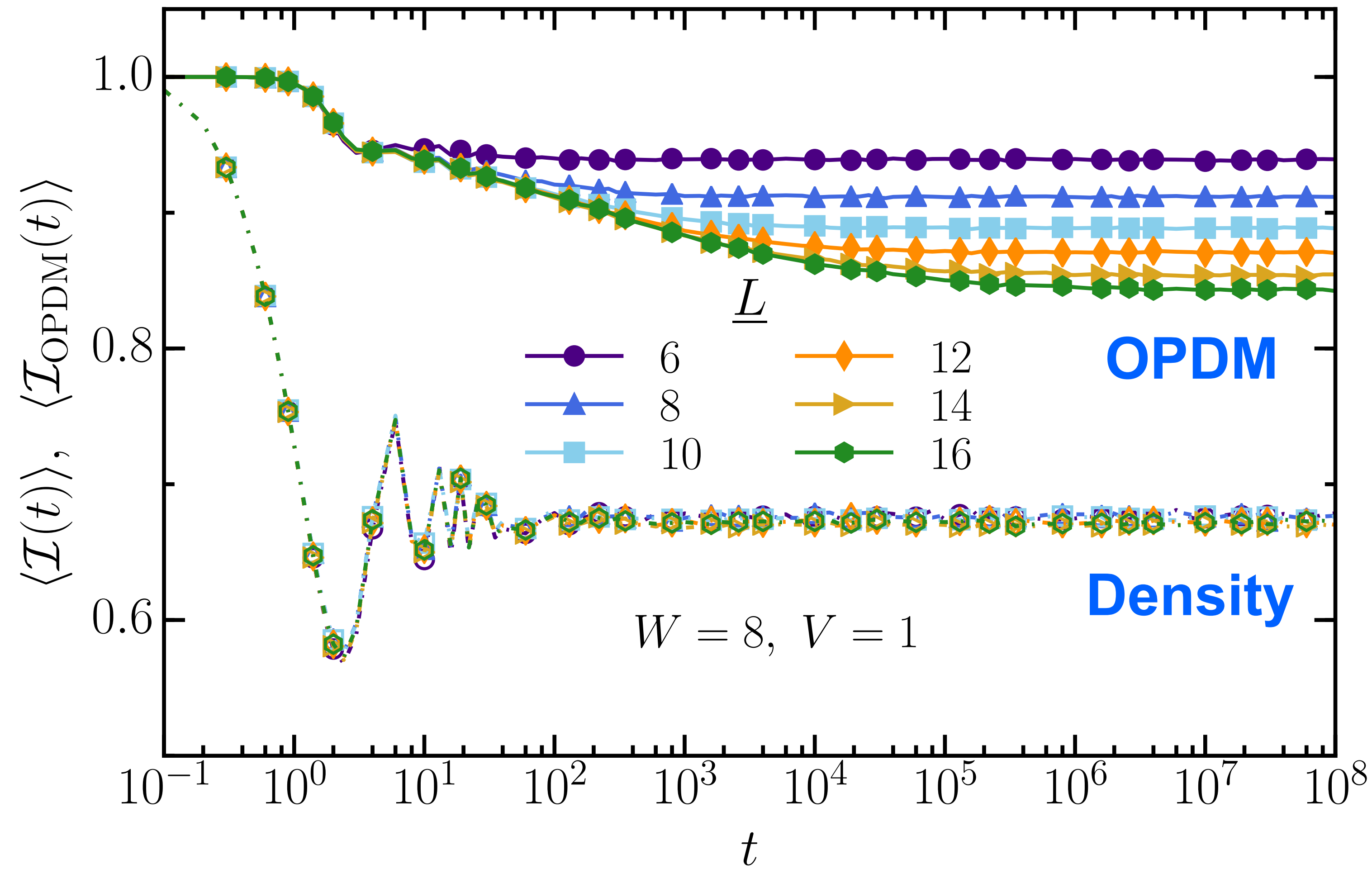


OPDM & quantum quench dynamics

OPDM imbalance

$$\mathcal{I}_{\rm OPDM} = \frac{N_+ - N_-}{N}$$

Remains nonzero as well!



Quench dynamics from product states:

Statonary regime: No discontinuity, but nonthermal occupation spectrum

OPDM and physical-density imbalance remain nonzero in MBL phase

Summary of the OPDM part

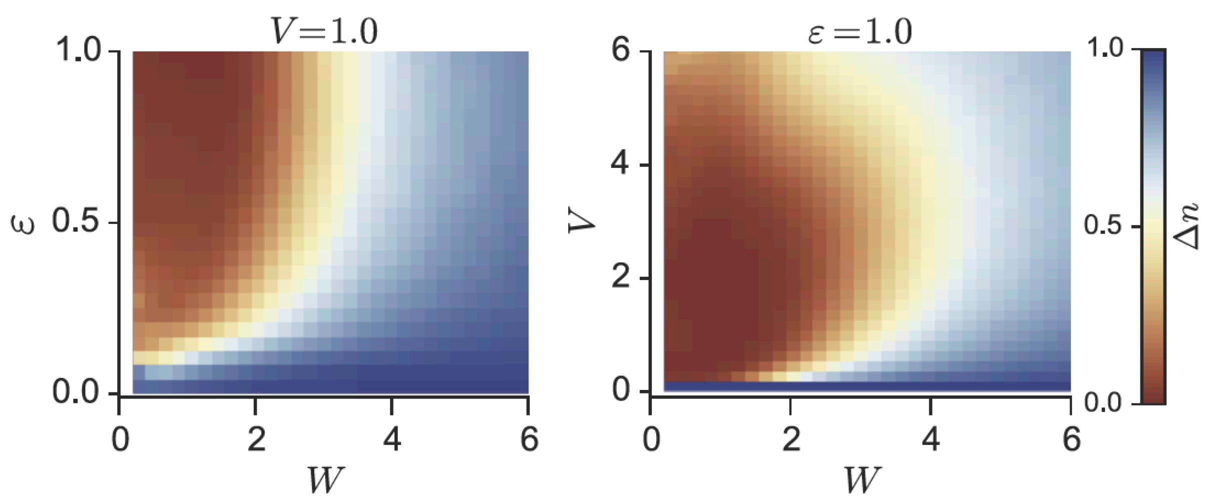
→ **Single-particle description based on one-particle density matrix: Additional tool for diagnostics of MBL phase**

→ **Natural orbitals (de)localized in (ergodic)MBL phase**

→ **Discontinuity in occupation spectrum: Similar to T=0 Fermi-liquid!**

→ **Localization in real space: Localization in Fock space**

→ **Survival of discontinuity Δn with bath coupling - prethermalization?**



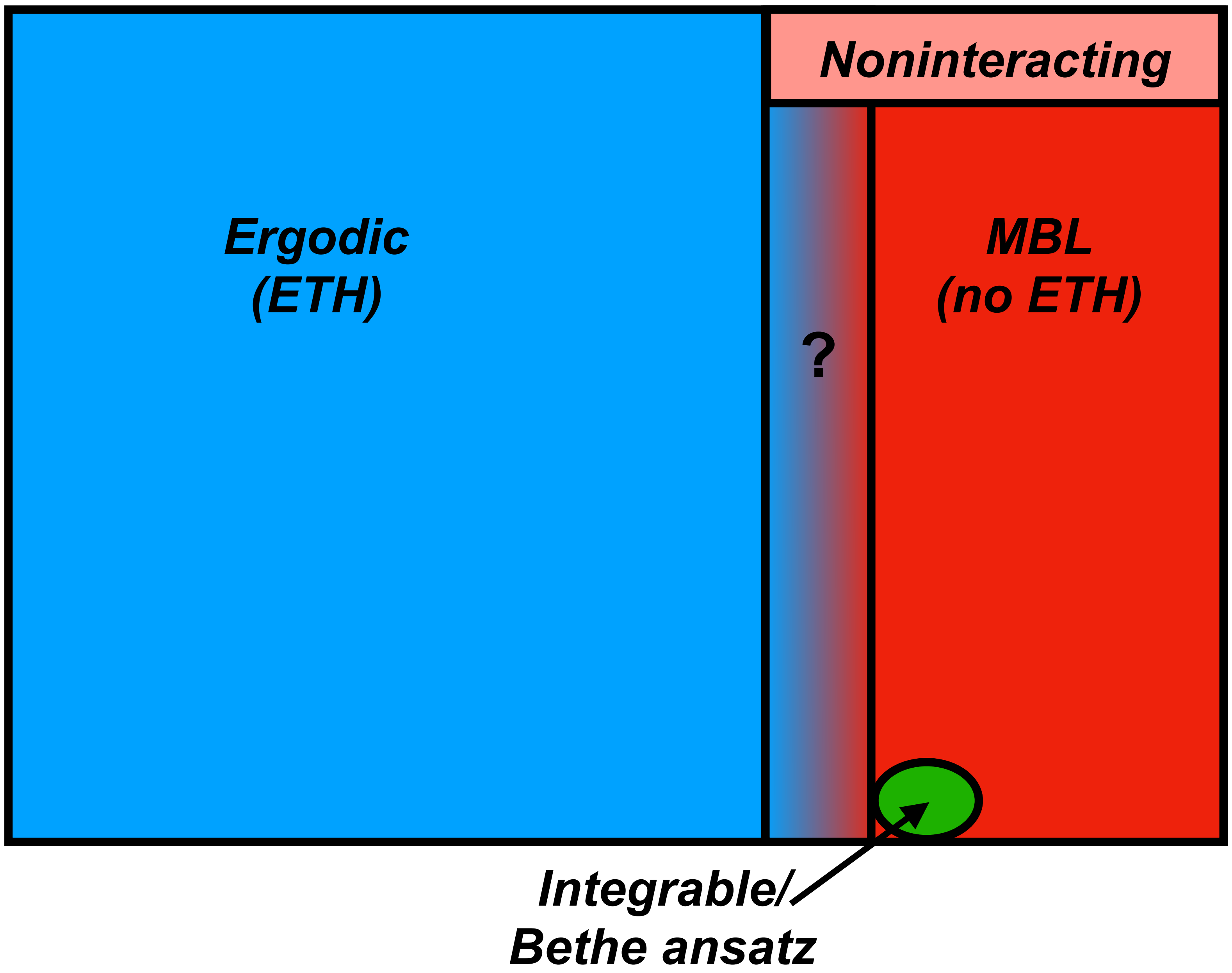
Bera, Schomerus, FHM, Bardarson,
Phys. Rev. Lett. 115, 046603 (2015)

Bera, Martynec, Schomerus, FHM, Bardarson,
Annalen der Physik (2017)

Lezama, Bera, Schomerus, FHM, Bardarson
Phys. Rev. B (R) (2017)

Thank you!

Summary (for Natan)



Thank you!