Beyond mean-field effects in a cold gas: multibody interactions and quantum droplets

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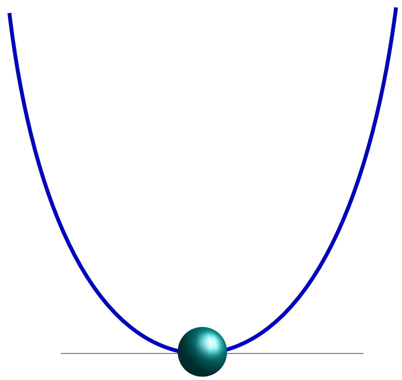
Quantum stabilization

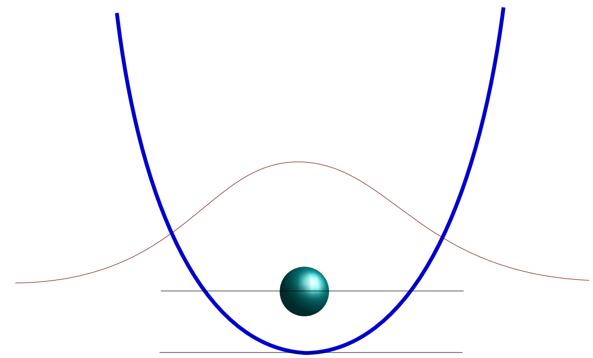
Classical



BEC analog:
Classical or mean-field limit =
Gross-Pitaevskii equation

Classical vacuum

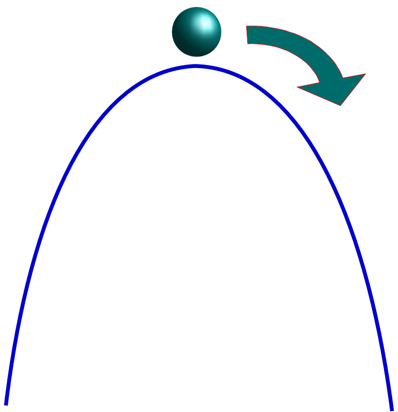
Quantum



BEC analog:
Mean field + Gaussian fluctuations
= GP+LHY

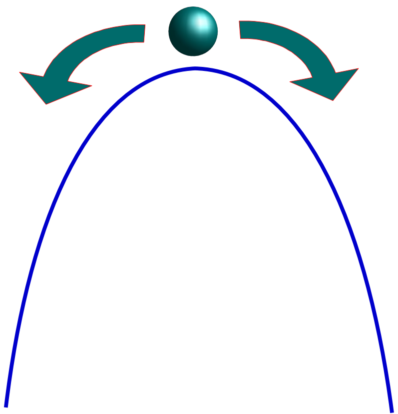
Bogoliubov vacuum

Classical



BEC analog:
collapse

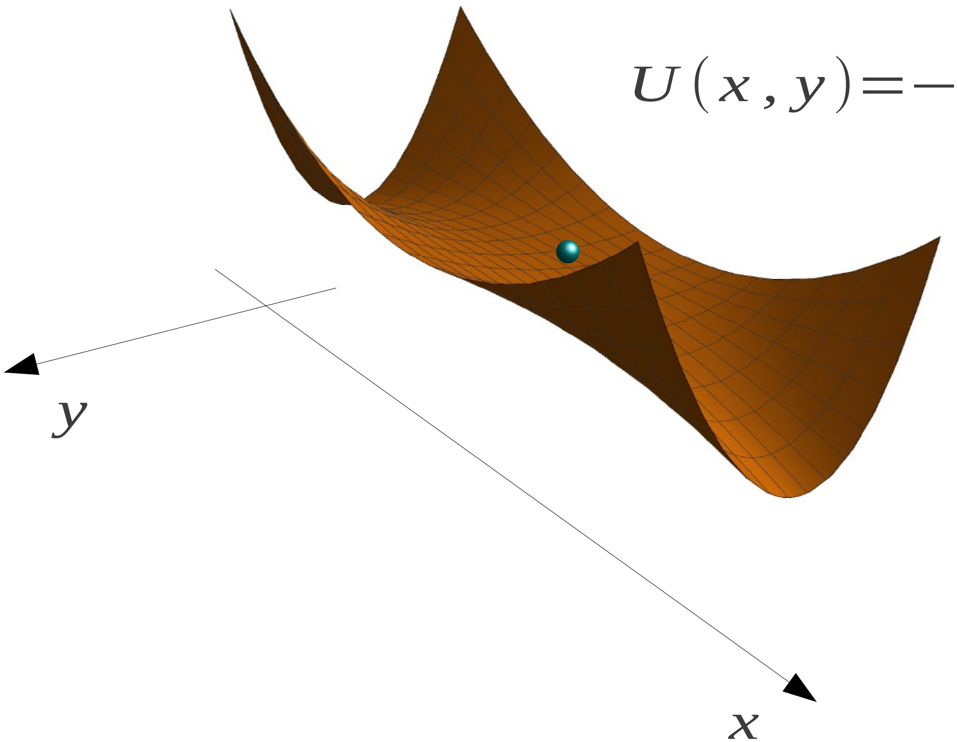
Quantum



BEC analog:
collapse :(

Can there be a classically unstable system,
yet stable when quantum mechanics is “switched on” ?

Quantum stabilization idea



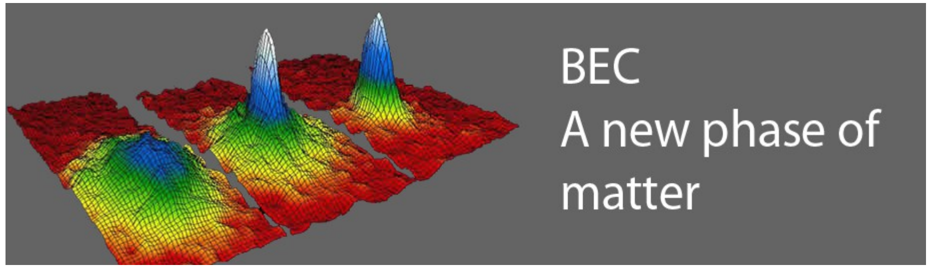
$$U(x,y) = -\alpha x^2 + \frac{\omega^2(x)}{2} y^2$$

Stable for sufficiently fast growing $\omega(x)$

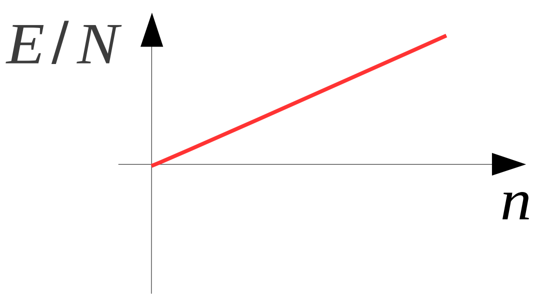
Classically unstable degree of freedom stabilized by quantum fluctuations in another degree of freedom!

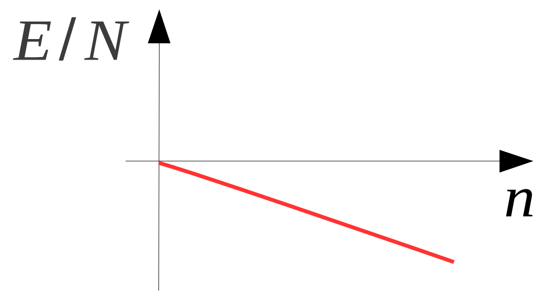
BEC analog:

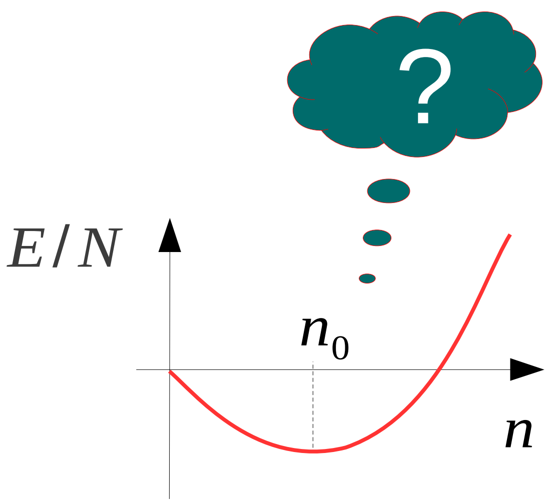
quantum droplet!



GAS requires trapping or collapses

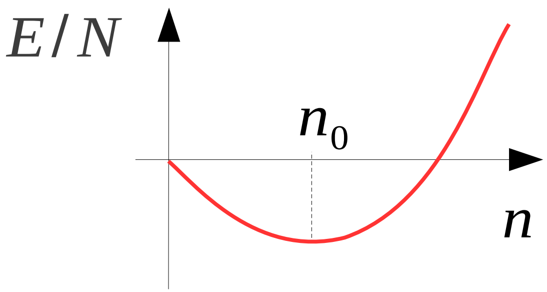






$$E/N \propto g_2 n + g_{\alpha+1} n^{\alpha}, \quad \alpha > 1$$

The gas should remain dilute, otherwise short lifetime!



$$E/N \propto a\,n + L^{3\alpha-2} n^{\alpha}, \quad \alpha > 1$$

$$\Downarrow$$

$$n_0 \sim \frac{1}{L^3}\left(\frac{a}{L}\right)^{\frac{1}{\alpha-1}}$$

Dilute = simultaneously small a and large L
and prefer small α

3-body $(\alpha = 2)$

Lee-Huang-Yang $(\alpha = 3/2)$

Resonant (Efimov)
3-body force
(Bulgac'02)

lossy :(

Non-resonant
3-body force
(DP'14)

Beyond-mean-field QUANTUM MECHANISM!

Realized with dipoles Dy (Stuttgart), Er (Innsbruck) and bosonic mixtures K (Barcelona, Florence)

Lots of ``beyond-mean-field prospects''!

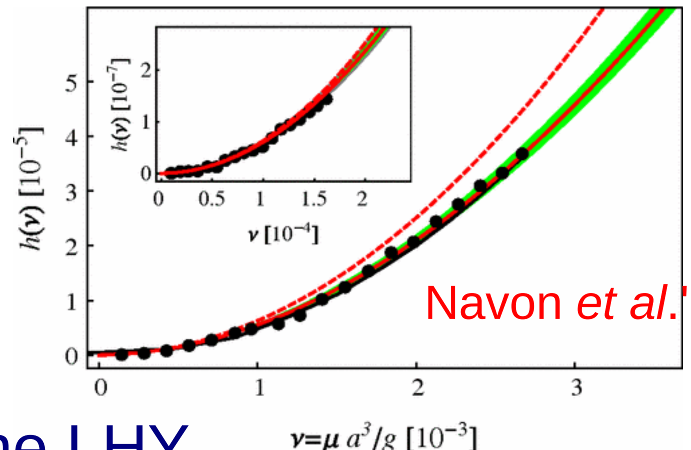
Quantum droplets

For spinless BEC: $$\frac{E}{\text{Volume}} = \frac{g_2 n^2}{2}\left(1 + \frac{128}{15}\sqrt{\frac{na^3}{\pi}} + \ldots\right)$$

Lee-Huang-Yang correction $\propto g_2^{5/2} n^{5/2}$

LHY correction is UNIVERSAL (depends only on the scattering length) and QUANTUM (zero-point energy of Bogoliubov phonons)!

Observed in ultracold gases where the scattering length is tunable by using Feshbach resonances (Innsbruck, MIT, ENS, JILA, Rice)



Navon *et al.* '11

Unfortunately, the effect is perturbative and the LHY term is smaller than the mean-field one!

Bose-Bose mixture, mean field

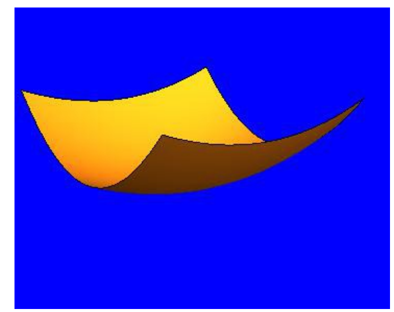
Mean-field energy density: $$\frac{E_{MF}}{\text{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2}$$

g_{12}

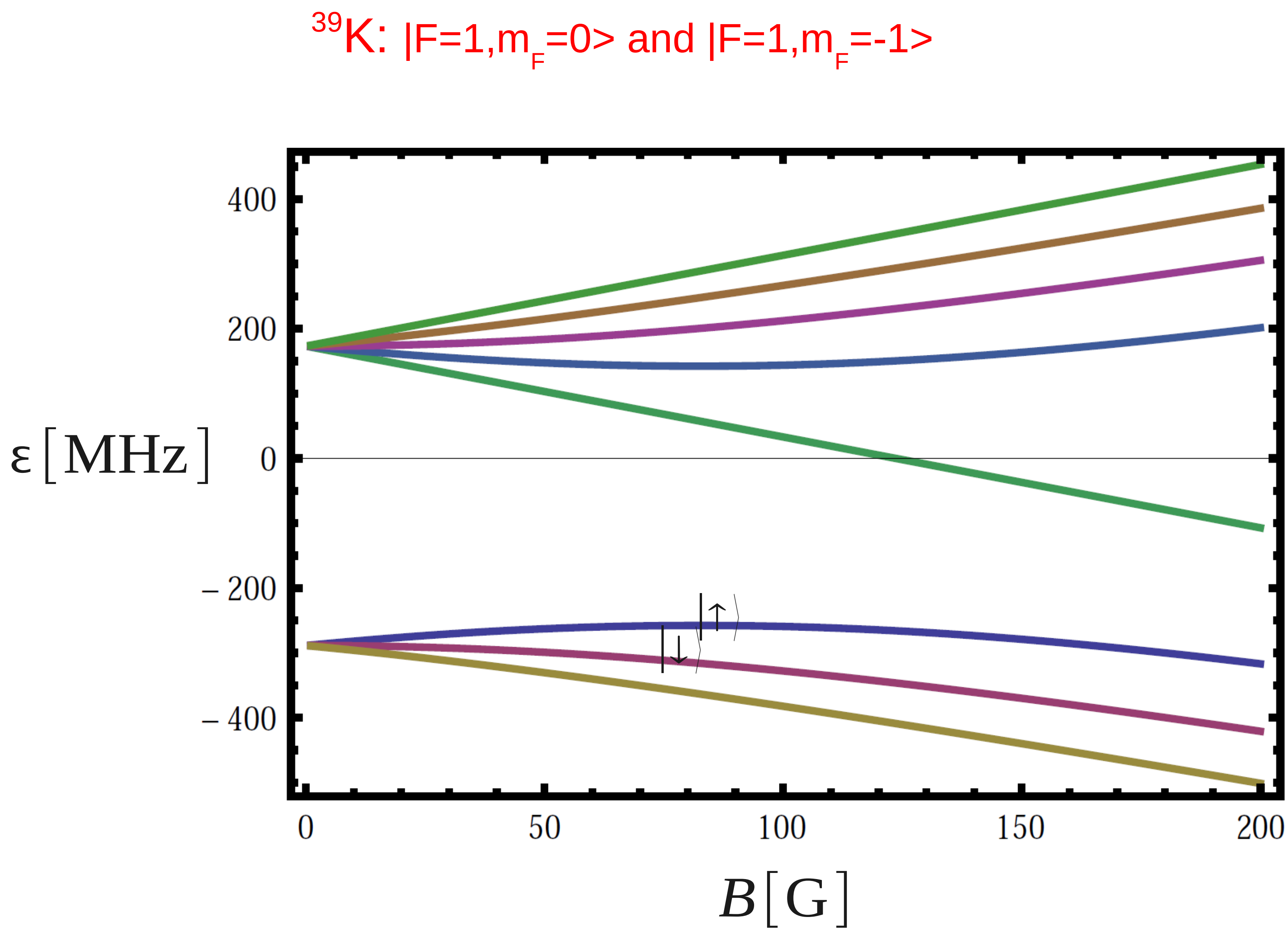
$g_{12} > \sqrt{g_{11} g_{22}}$ phase separation

mean-field stability

$g_{11} > 0, \quad g_{22} > 0, \quad \text{and} \quad g_{12}^2 < g_{11} g_{22}$

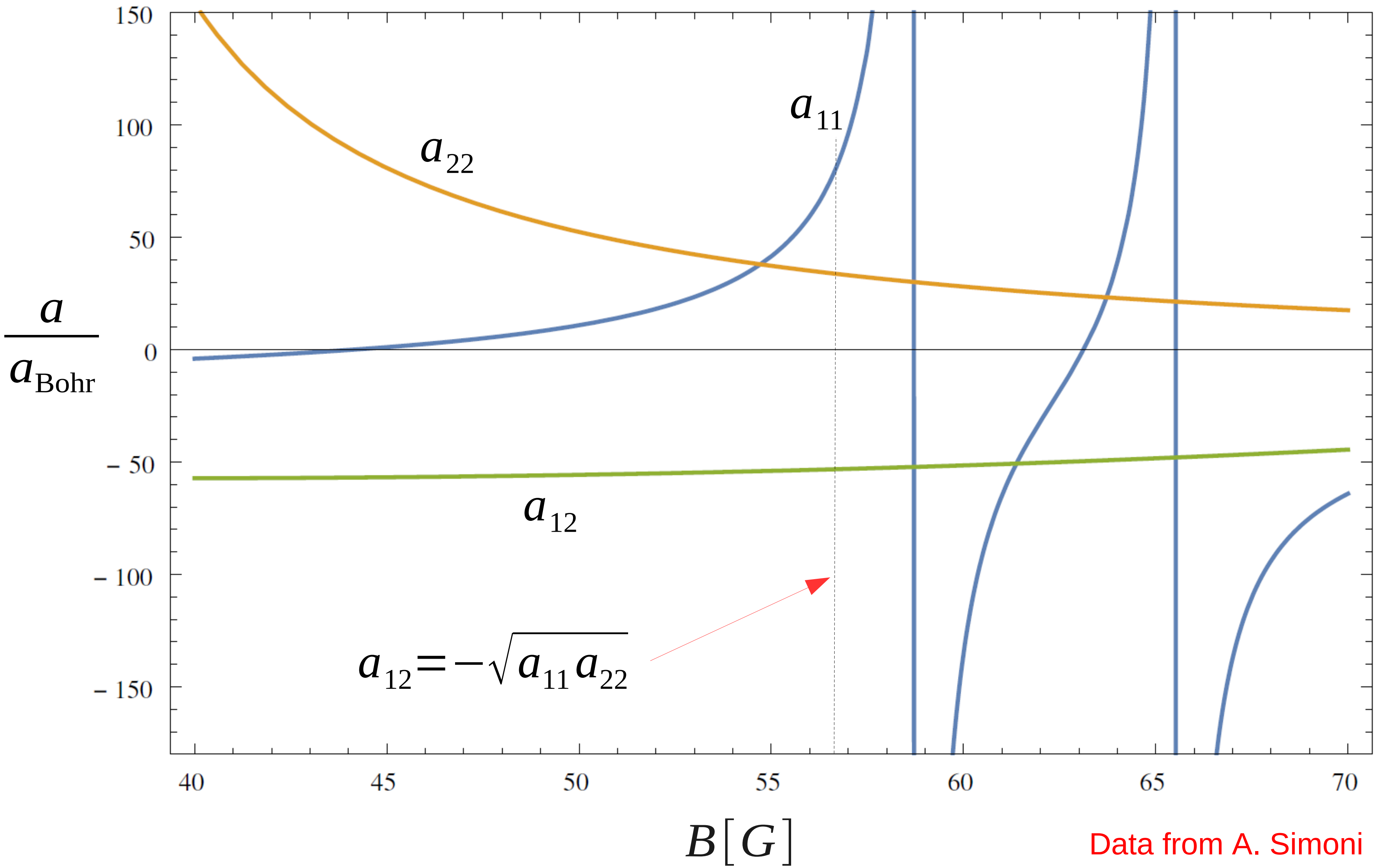


$g_{12} < -\sqrt{g_{11} g_{22}}$ collapse



39K: |F=1,mF=0> and |F=1,mF=-1>
ε [MHz]
400
200
0
− 200
− 400
0
50
100
150
200
B [G]
|↑⟩
|↓⟩

^{39}K: |F=1,m$_F$=0> and |F=1,m$_F$=-1>



Data from A. Simoni

LHY correction

Bogoliubov theory

$$E_\pm(k)=\sqrt{c_\pm^2 k^2+k^4/4}; \quad c_\pm^2=\frac{g_{11}n_1+g_{22}n_2\pm\sqrt{(g_{11}n_1-g_{22}n_2)^2+4g_{12}^2 n_1 n_2}}{2}$$

$$\frac{E}{\mathrm{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_\pm\sum_k\left[E_\pm(k)-k^2/2-c_\pm^2\right]=$$

$$=\underbrace{\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}}_{\mathrm{MF}\propto g n^2}+\underbrace{\frac{8}{15\pi^2}(c_+^5+c_-^5)}_{\mathrm{LHY}\propto (gn)^{5/2}}$$

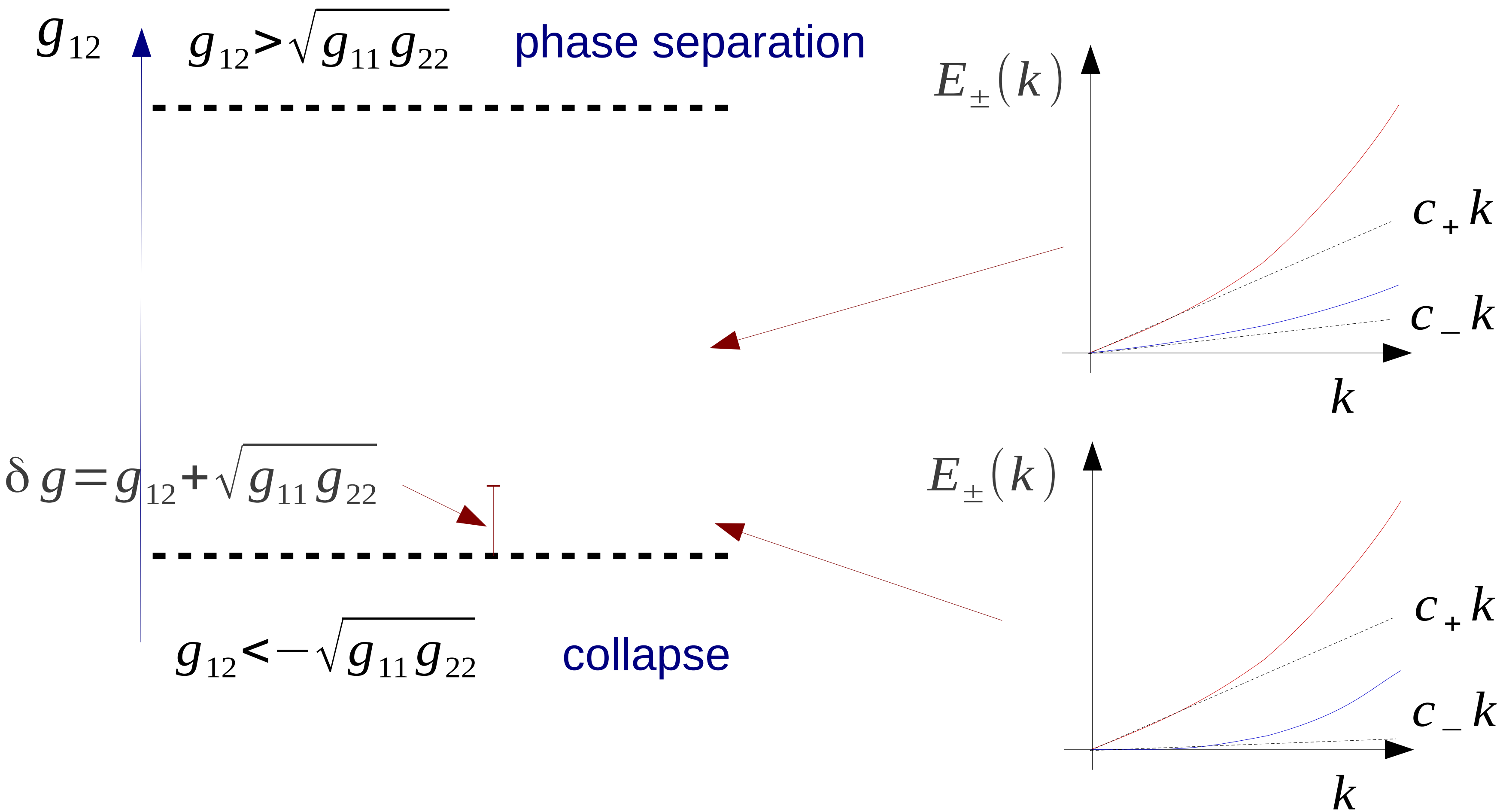
(Larsen'63)

In contrast to one-component case, MF and LHY depend on (different) combinations of $g_{\sigma\sigma'}n_\sigma$

$$\boxed{\text{...and, thus, can be independently controlled!}}$$

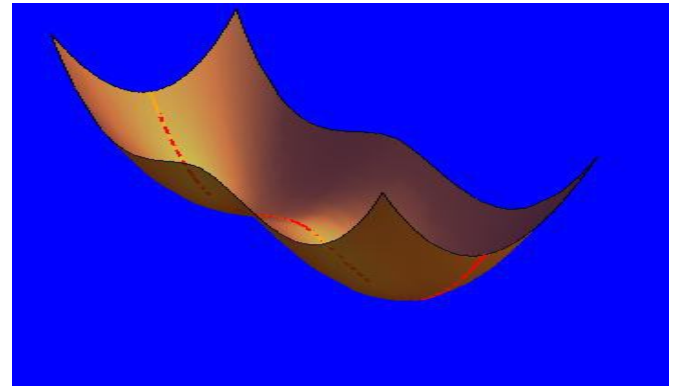
LHY correction

$$\frac{E}{\text{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2} + \frac{8}{15\pi^2} \frac{m^4}{\hbar^3} \left(c_+^5 + c_-^5 \right) + \dots$$



Quantum stabilization

$$\delta g = g_{12} + \sqrt{g_{11} g_{22}} \ll \sqrt{g_{11} g_{22}} = g$$



The mean-field term "locks" the ratio $\dfrac{n_2}{n_1} = \sqrt{\dfrac{g_{11}}{g_{22}}}$

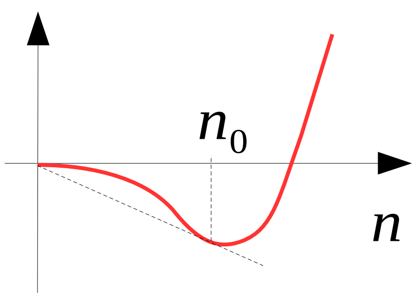
Softening of lower Bog. mode $c_- \ll c_+ \propto \sqrt{gn/m}$

only the ``hard'' + branch contributes to the LHY term

The structure of the energy-density functional:

$$\frac{E}{\text{Volume}} = A_1 \times \delta g \times n^2 + A_2 \times (m/\hbar^2)^{3/2} (g n)^{5/2}$$

$\delta g < 0$



Gas exists in equilibrium with vacuum. Saturation density $\boxed{n_0 \propto \dfrac{1}{a^3} \left(\dfrac{\delta g}{g} \right)^2}$

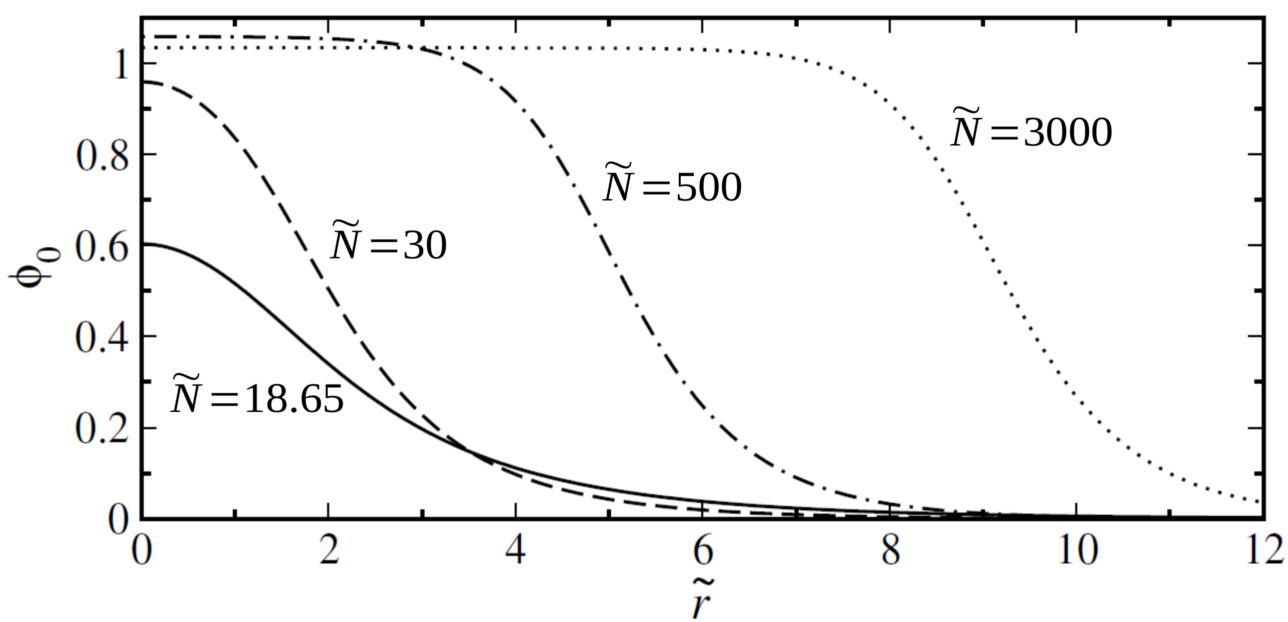
Density is tunable by modifying interaction parameters!

Gross-Pitaevskii eq., droplet shape

Rescaling $\vec{r}=\xi\,\vec{\tilde{r}}$, $t=\tau\,\tilde{t}$, $N=n\,\xi^3\,\tilde{N}$, where $\xi\propto 1/\sqrt{m|\delta g|n}$, $\tau\propto 1/|\delta g|n$

$$\boxed{\begin{gathered} i\,\partial_{\tilde{t}}\,\varphi=\left(-\nabla_{\tilde{\vec{r}}}^{\;2}/2-3|\varphi|^2+5|\varphi|^3/2-\tilde{\mu}\right)\varphi \\ \tilde{N}=\int|\varphi|^2\,d^3\tilde{r} \end{gathered}}$$

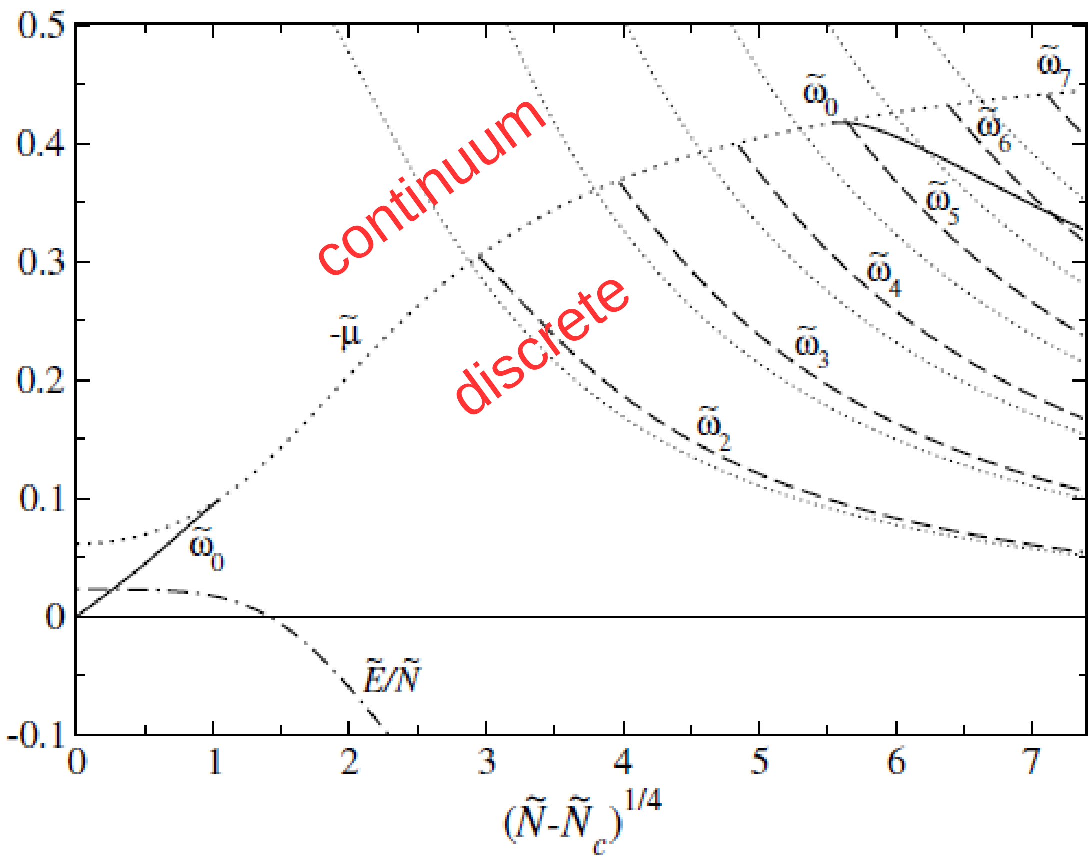
Modified Gross-Pitaevskii equation cubic-quartic nonlinearities



Bogoliubov-de Gennes eqs., excitations

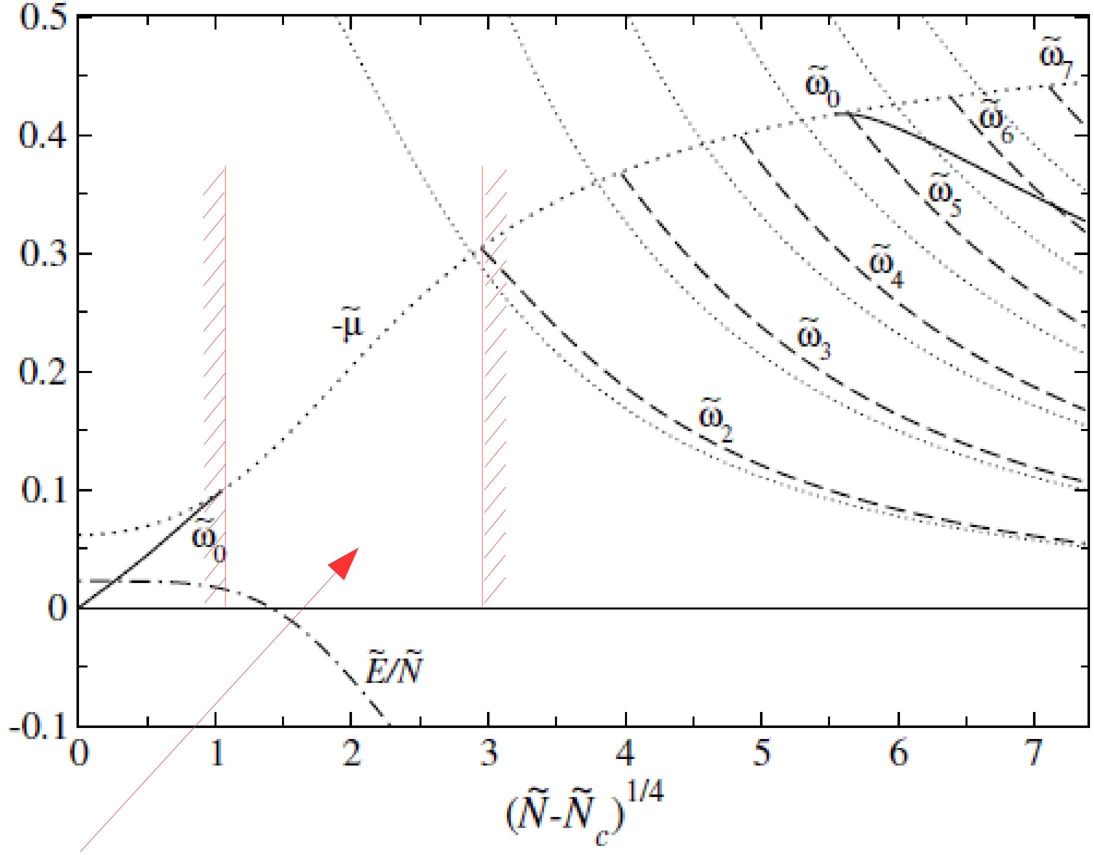
$$\varphi(\tilde t,\vec{\tilde r})=\varphi_0(\vec{\tilde r})+\delta\varphi(\tilde t,\vec{\tilde r})$$

linearize $i\partial_{\tilde t}\varphi=(-\nabla_{\vec{\tilde r}}^2/2-3|\varphi|^2+5|\varphi|^3/2-\tilde\mu)\varphi$ with respect to small $\delta\varphi(\tilde t,\vec{\tilde r})$



Surface modes

Zero-temperature object



No discrete modes → the droplet evaporates itself to zero T!

(by contrast, ^{4}He droplets always have discrete modes)

Macroscopic zero-temperature object:

- is interesting by itself
- can be used for sympathetic cooling of other systems

LHY depends on ...

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}[E_{\pm}(k)-k^2/2-c_{\pm}^2]=$$

number of components

density of states (dimension)

shape of the Bogoliubov spectrum
(anisotropy of the interaction,
driving the mixture, etc.)

...and life becomes ~~harder~~ more interesting in the inhomogeneous case
particularly if LDA is not valid

Low-dimensional case

Bogoliubov theory – ``Mean field + LHY'' (DP, Astrakharchik'16)

$$\frac{E}{\mathrm{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2} + \frac{1}{2} \sum_{\pm} \sum_{k} [E_{\pm}(k) - k^2/2 - c_{\pm}^2]$$

Dimension enters here!

Bogoliubov theory – ``Mean field + LHY'' (DP, Astrakharchik'16)

3D: $$\frac{E_{3D}}{\mathrm{Volume}}=\frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'}+\frac{8}{15\pi^2}\sum_{\pm} c_\pm^5 \sim \delta g n^2+(gn)^{5/2}$$

$$\sqrt{n g^3}\ll 1$$

2D: $$\frac{E_{2D}}{\mathrm{Surface}}=\frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'}+\frac{1}{8\pi}\sum_{\pm} c_\pm^4 \ln\frac{c_\pm^2\sqrt{e}}{\kappa^2} \sim g^2 n^2 \ln\frac{n}{n_0}$$

$$g_{\sigma\sigma'}=2\pi/\ln\left(2e^{-\gamma}/a_{\sigma\sigma'}\kappa\right)\ll 1$$

1D: $$\frac{E_{1D}}{\mathrm{Length}}=\frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'}-\frac{2}{3\pi}\sum_{\pm} c_\pm^3 \sim \delta g n^2-(gn)^{3/2}$$

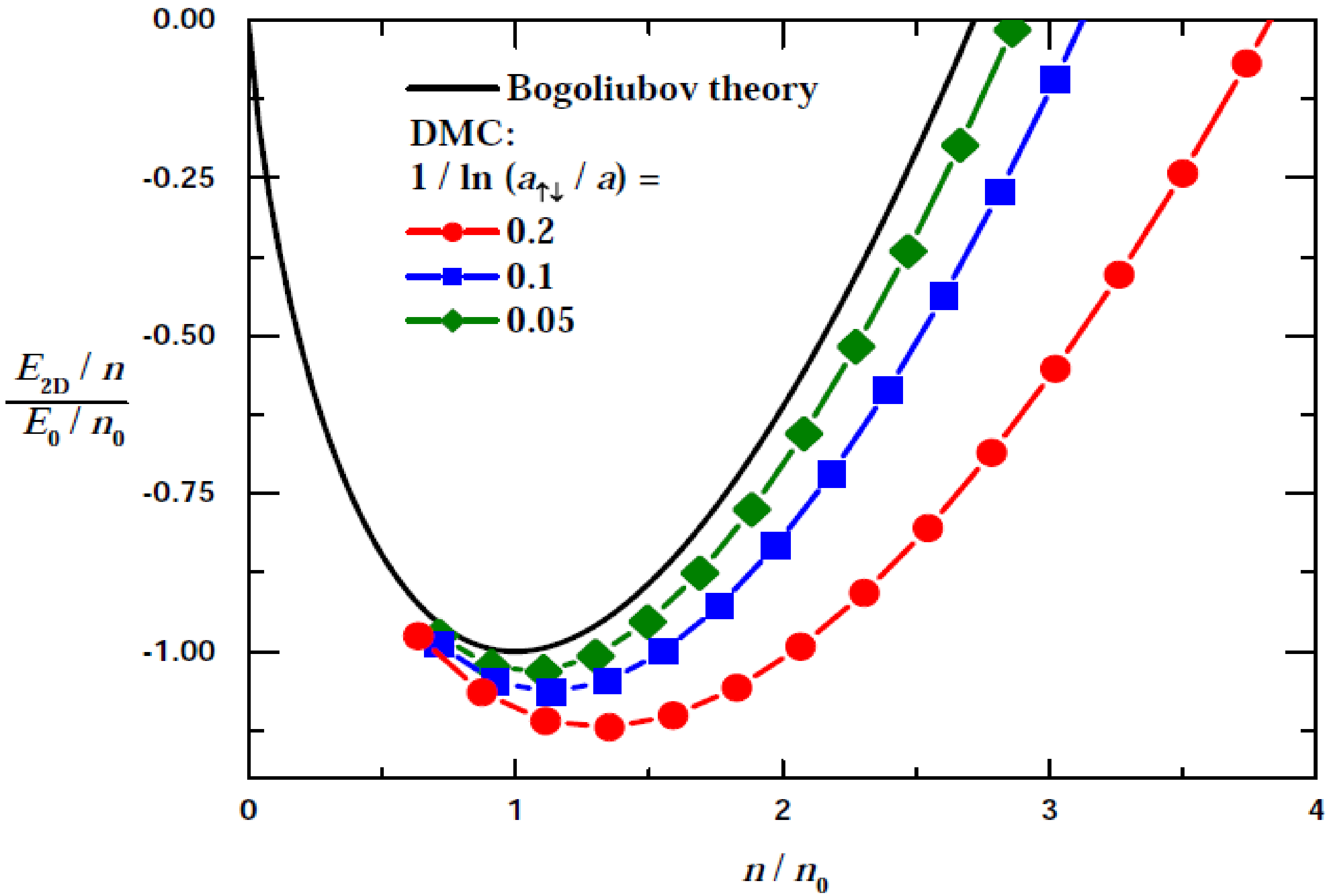
$$\sqrt{g/n}\ll 1$$

!

2D symmetric case

$$a_{\uparrow\uparrow}=a_{\downarrow\downarrow}=a \quad \Rightarrow \quad n_{\uparrow}=n_{\downarrow}=n \quad \Rightarrow \quad \frac{E_{2D}}{\text{Surface}}=\frac{8\pi n^2}{\ln^2(a_{\uparrow\downarrow}/a)}[\ln(n/n_0)-1]$$

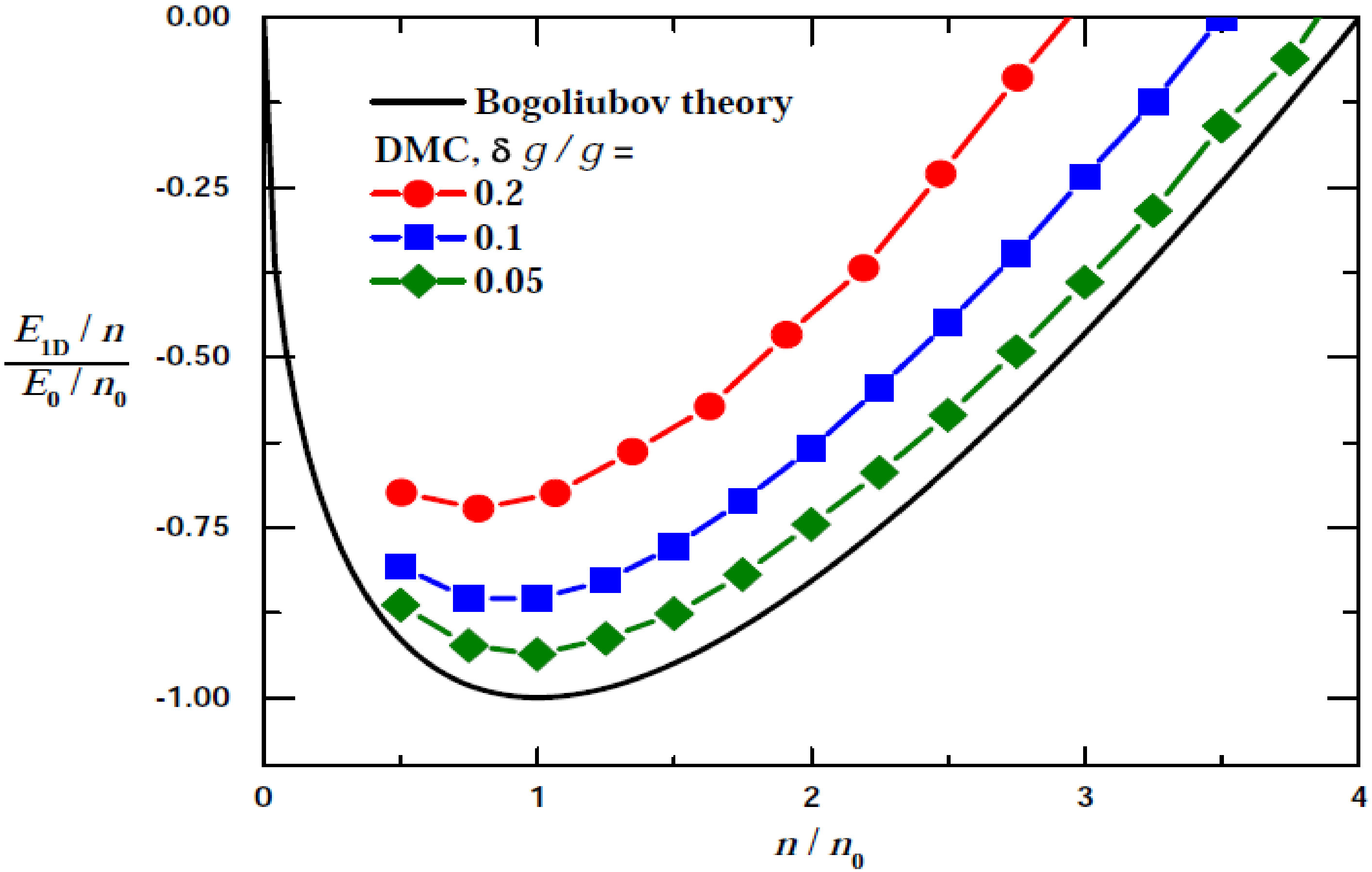
where $n_0=\frac{e^{-2\gamma-3/2}}{2\pi}\frac{\ln(a_{\uparrow\downarrow}/a)}{a a_{\uparrow\downarrow}}$



1D symmetric case

$$g_{\uparrow\uparrow}=g_{\downarrow\downarrow}=g \;\Longrightarrow\; n_{\uparrow}=n_{\downarrow}=n \;\Longrightarrow\; \frac{E_{1D}}{\text{Length}}=\delta g\, n^2-\frac{4\sqrt{2}}{3\pi}(gn)^{3/2}$$

Minimum for $\delta g>0$ at $n_0=\dfrac{8g^3}{9\pi^2\delta g^2}$



3D vs low-D

- 3D droplet disappears for $N<N_c$
- Low-D droplets are bound for any N

- 3D liquids are in the mean-field unstable regime ($\delta g<0$)
- 2D mixture liquefies for any weakly repulsive intra- and weakly attractive interspecies interaction
- 1D liquid is in the regime stable from the mean-field viewpoint ($\delta g>0$)

- The 1D modified stationary GP equation is solvable → full analytic solution for the shape of the droplet

$$\psi(x)=\frac{\sqrt{n_0}\,\mu/\mu_0}{1+\sqrt{1-\mu/\mu_0}\cosh\left(\sqrt{-2\mu}\,x\right)}$$

LHY depends on ...

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}\left[E_{\pm}(k)-k^2/2-c_{\pm}^2\right]=$$

number of components

density of states (dimension)

shape of the Bogoliubov spectrum
(anisotropy of the interaction,
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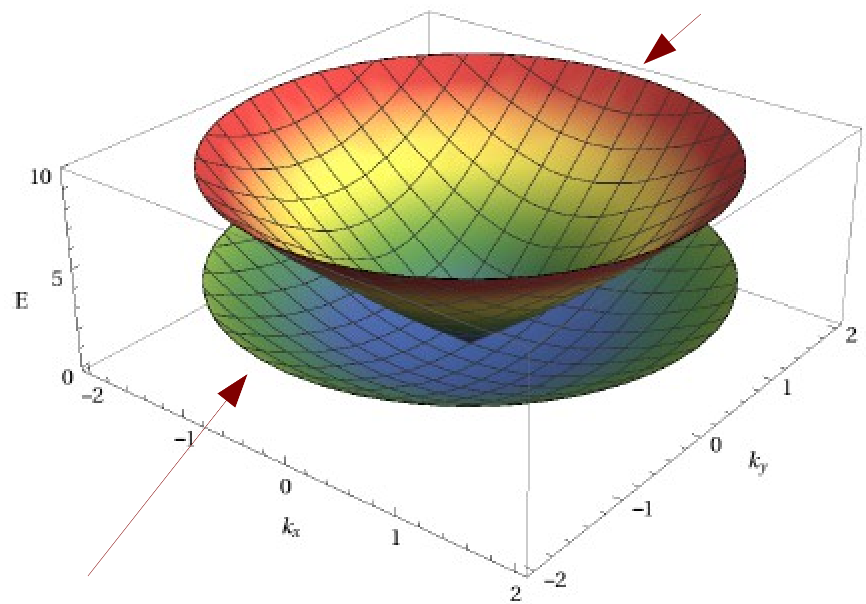
Bose-Bose mixture vs Dy/Er

Bose-Bose mixture

“stiff” mode



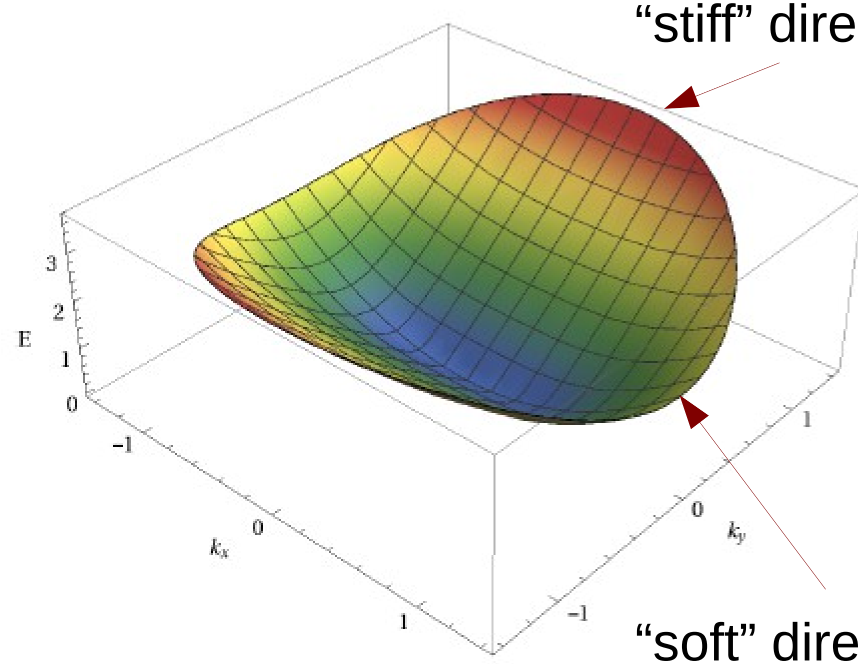
“soft” mode

$$\mathrm{LHY} = \frac{8}{15\pi^2}\frac{m^4}{\hbar^3}\left(c_+^5 + c_-^5\right)$$

(Larsen'63)

dipolar Bose gas

“stiff” direction



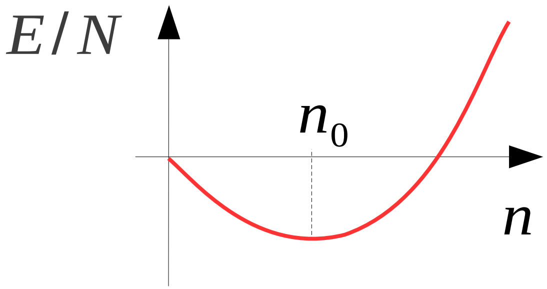
“soft” direction

$$\mathrm{LHY} = \frac{8}{15\pi^2}\frac{m^4}{\hbar^3}\left\langle c^5(\hat{k})\right\rangle_{\hat{k}}$$

(Lima&Pelster'11)

Multi-Body Interactions

The gas should remain dilute, otherwise short lifetime!



$$E/N \propto a\,n + L^{3\alpha-2} n^{\alpha}, \quad \alpha>1$$

$$\Downarrow$$

$$n_0 \sim \frac{1}{L^3}\left(\frac{a}{L}\right)^{\frac{1}{\alpha-1}}$$

Dilute = simultaneously small a and large L
and prefer small α

3-body $(\alpha=2)$

Lee-Huang-Yang $(\alpha=3/2)$

Resonant (Efimov)
3-body force
(Bulgac'02)

lossy :(

Non-resonant
3-body force
(DP'14)

Beyond-mean-field QUANTUM MECHANISM!

Realized with dipoles Dy (Stuttgart), Er (Innsbruck) and bosonic mixtures K (Barcelona, Florence)

Lots of ``beyond-mean-field prospects''!

Uniform dilute one-component gas

$$\frac{E}{\text{Volume}} = \frac{g_2 n^2}{2}\left(1 + \frac{128}{15}\sqrt{\frac{na^3}{\pi}}\right) + g_3 \frac{n^3}{3!} + \ldots$$

3-body interaction is noticeable if $g_3 \sim g_2 / n$, i.e., in the dilute limit we basically need $g_2 = 0$

… so, the aim is

$g_2 = 0$ AND $g_3 > 0$ AND STRONG!

Why interesting?

Bosons + $g_2<0$ → Collapse

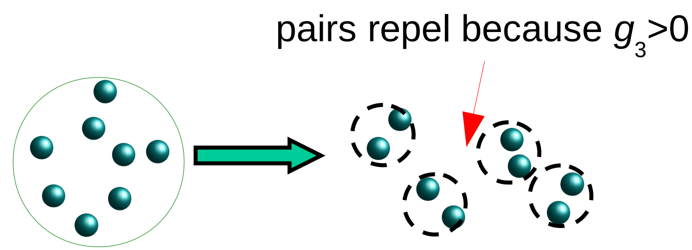
Bosons + $g_2<0$ + $g_3>0$ → Free space → self-trapped droplet state Bulgac'02:

- Neglecting surface tension, flat density profile $n=3|g_2|/2g_3$
- Including surface tension → surface modes

Increasing $g_2<0$ → bosonic pairing Nozieres&Saint James'82

<u>Topological transition</u>, not crossover!

Radzihovsky et al., Romans et al., Lee&Lee'04



Pairing on a lattice with three-body constraint:

Daley et al.'09-, Ng&Yang'11, Bonnes&Wessel'12,...

g_3 is necessary! = Pauli pressure in the BCS-BEC crossover!

Why interesting? (contd.)

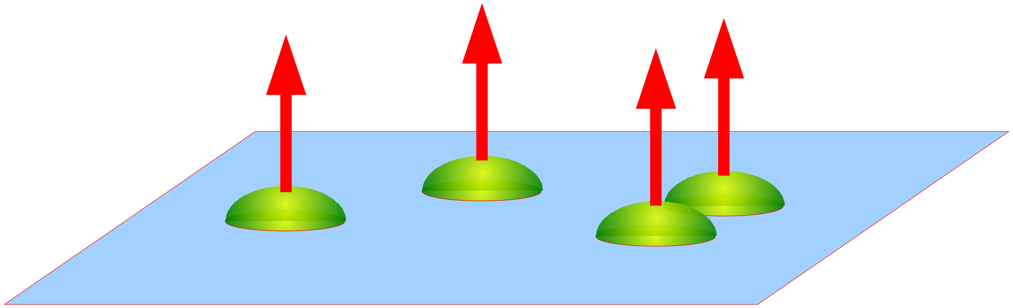
Local repulsive g_{k+1} is the ``parent'' Hamiltonian for the k-th state of the Read-Rezayi series of quantum Hall states, Nayak et al., Rev. Mod. Phys.'08

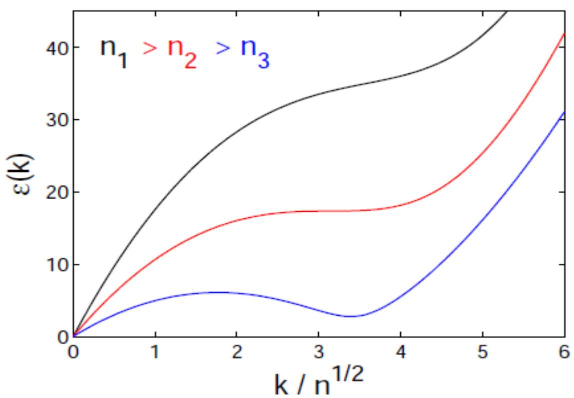
- k=1 (*2-body int.*) → Laughlin state (abelian anyons)
- k=2 (*3-body int.*) → Moore-Read state (non-abelian anyons, some topologically protected operations)
- k=3 (*4-body int.*) → Read-Rezayi state (non-abelian anyons, universal quantum computing)

Ground state degeneracy protected by gap ~ g_{k+1} Important to maximize !

Why interesting? (contd.#2)

2D dipoles





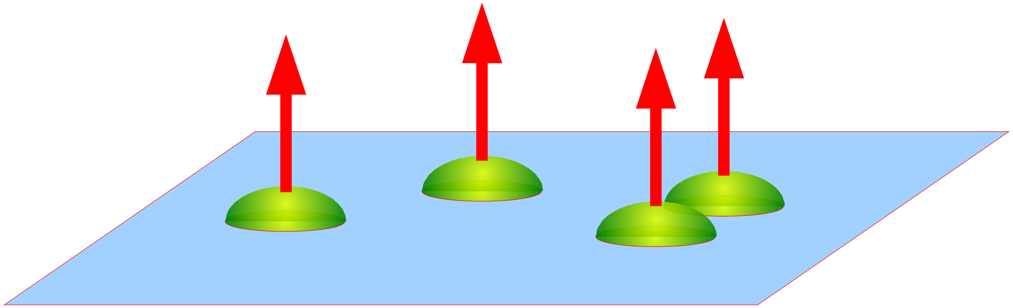
Rotonized superfluid & supersolid

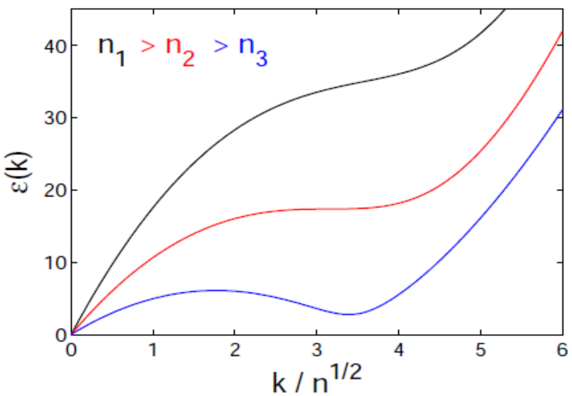
Mechanical stability for $g_3 > 0$

Lu et al'15

Why interesting? (contd.#2)

2D dipoles



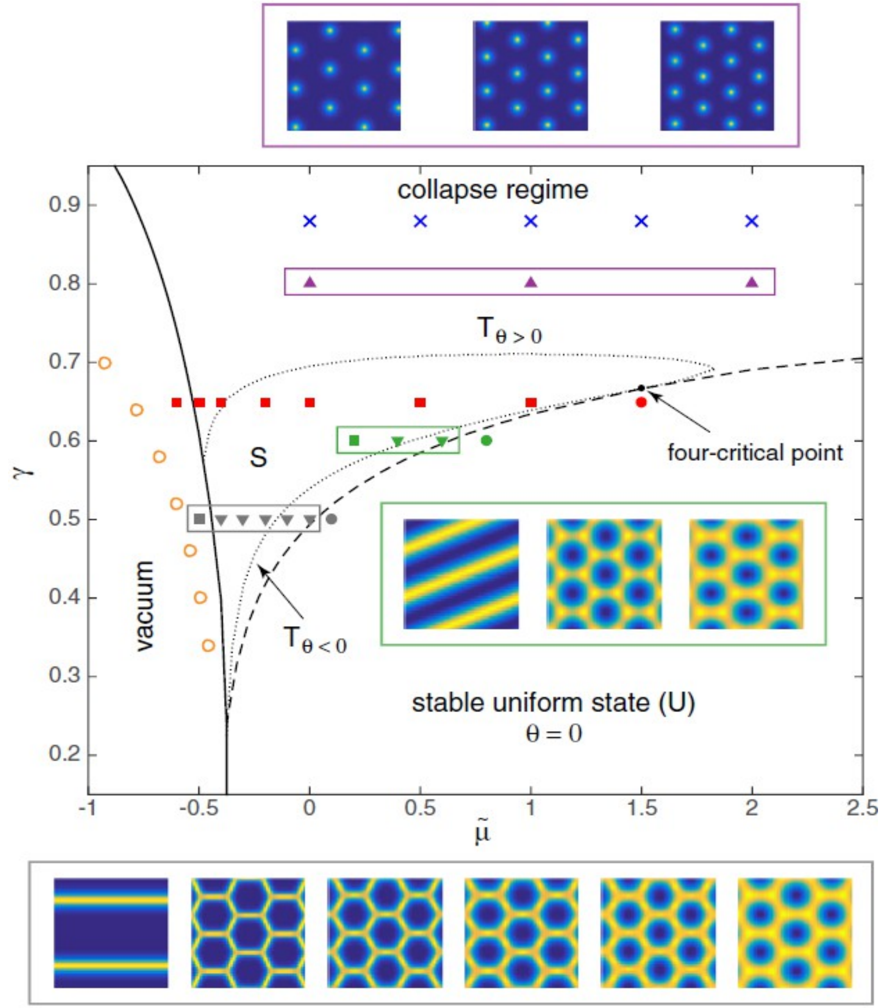


Rotonized superfluid & supersolid

Mechanical stability for $g_3 > 0$

Lu et al'15

Phase diagram



Dimensions of X-dimensional coupling constants

X-dimensional 2-body scattering $\longrightarrow$ $g_2 \propto \left(\hbar^2/m\right) \times length^{X-2}$

$\Downarrow$

$\boxed{\text{X-dimensional 3-body scattering = (2X)-dimensional 2-body scattering}}$

$\Downarrow$

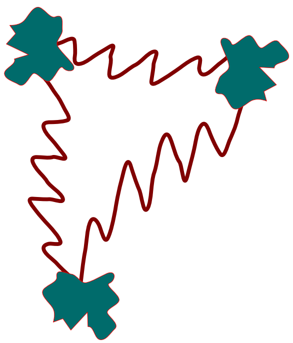
X-dimensional 3-body scattering $\longrightarrow$ $g_3 \propto \left(\hbar^2/m\right) \times length^{2X-2}$

In particular,

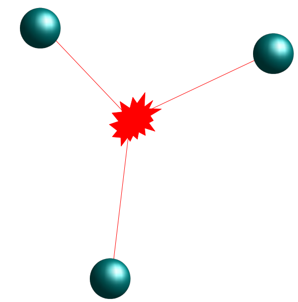
- 3D: $g_3 \propto \left(\hbar^2/m\right) \times length^4$
- 2D: $g_3 \propto \left(\hbar^2/m\right) \times length^2$
- 1D: $g_3 \propto \left(\hbar^2/m\right) / \ln\left(k \times length\right)$

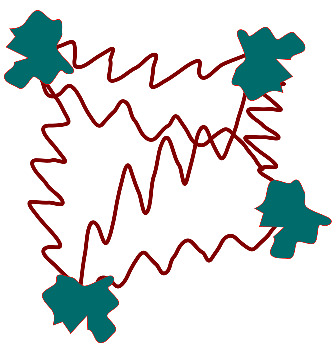
Effective multi-body interactions

Parasites which appear when we want to simplify our life:

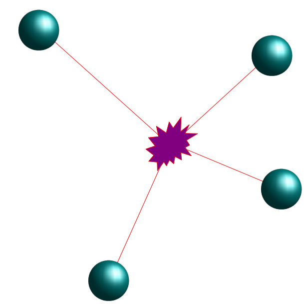


Simple 2-body pot. +





Simple 2-body pot. + 3-body +



⋮

Hammer et al., Rev. Mod. Phys. (2013)

Examples

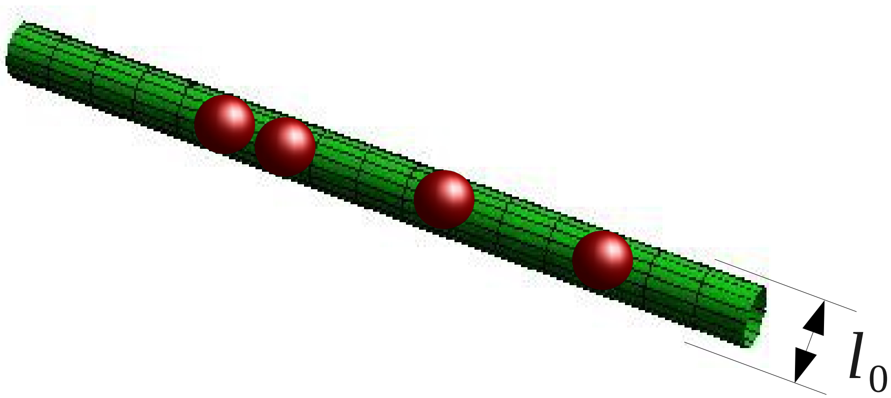
Lee, Huang, Yang'57, Wu'59 ...

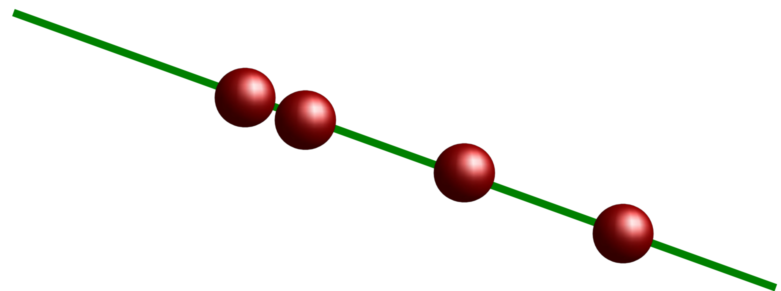
Hard sphere gas

Zero-range interacting gas
+ three-body interaction/
three-body parameter
(Zhu and Tan, 2017)

Examples

Quasi-1D bosons → 1D Lieb-Liniger model





2-body Olshanii'98 + 3-body Muryshev et al.'02

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \frac{2a}{l_0}\sum_{i<j}\delta(x_i - x_j) - 12\log\left(\frac{4}{3}\right)\frac{a^2}{l_0^2}\sum_{i<j<k}\delta(x_i - x_j)\delta(x_j - x_k)$$

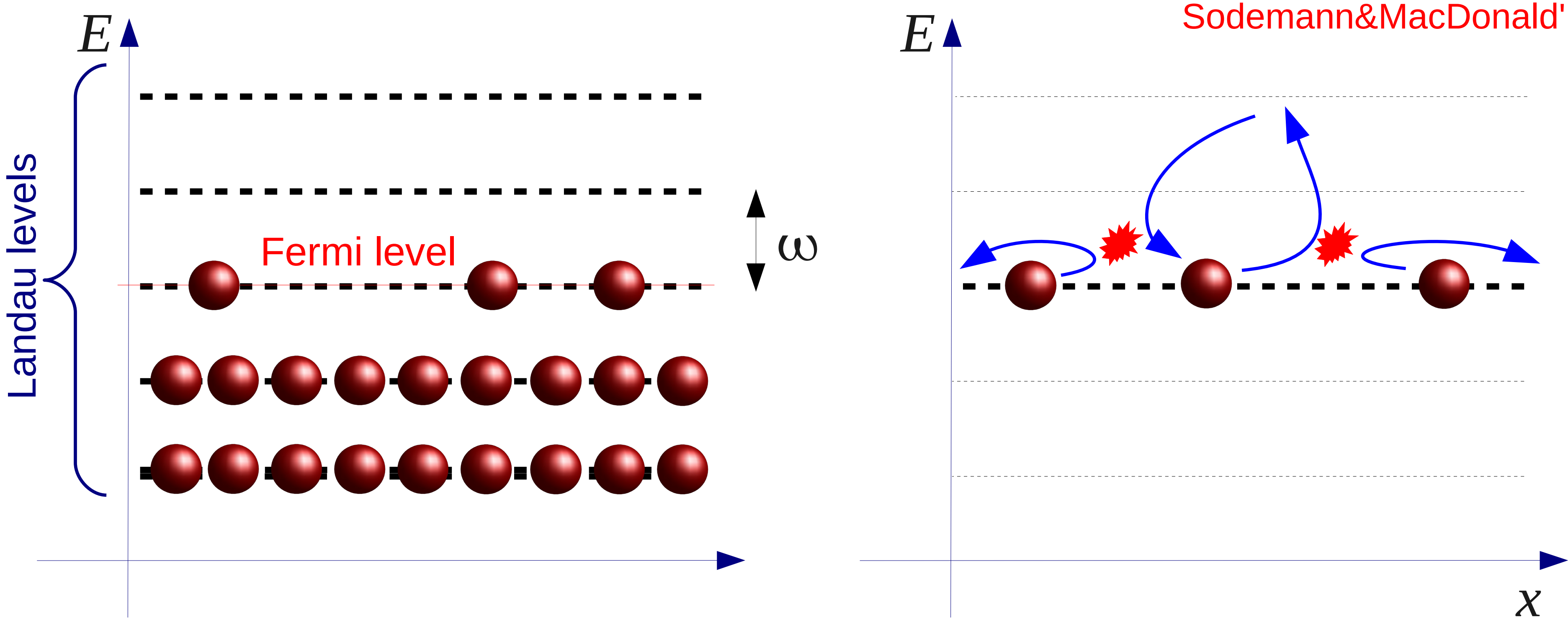
Perturbative, second order, weak,
attractive, but BREAKS INTEGRABILITY!

Examples

All quantum Hall numerics

Quantum Hall layer → Single Landau level model
+ higher order interactions

Nayak et al.'09,'13,
Sodemann&MacDonald'13



Weak, higher order in g/ω , but important due to high Landau level degeneracy!

Vol 465 | 13 May 2010 | doi:10.1038/nature09036

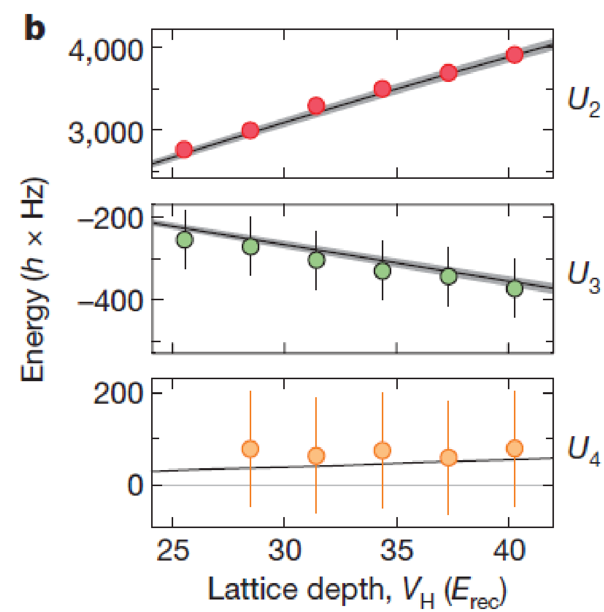
LETTERS

Time-resolved observation of coherent multi-body interactions in quantum phase revivals

Sebastian Will[1,2], Thorsten Best[1], Ulrich Schneider[1,2], Lucia Hackermüller[1], Dirk-Sören Lühmann[3] & Immanuel Bloch[1,2,4]

$$E(N)=U_2\frac{N(N-1)}{2!}+U_3\frac{N(N-1)(N-2)}{3!}+U_4\frac{N(N-1)(N-2)(N-3)}{4!}+\dots$$

$$g\sim a\,\omega^{3/2} \gg g^2/\omega \gg g^3/\omega^2$$



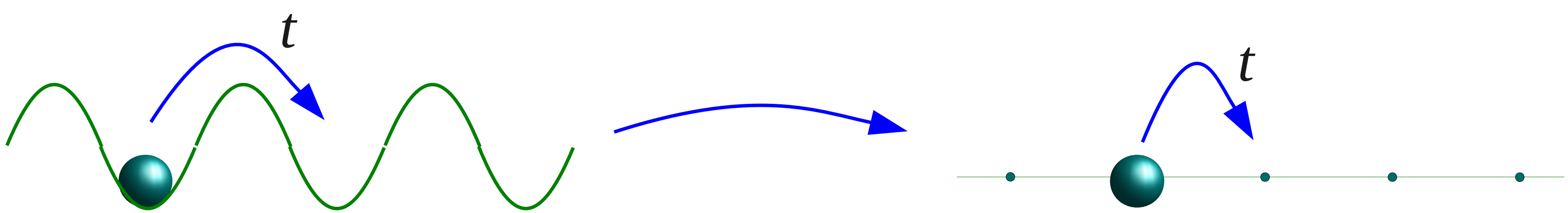
Optical lattice → Hubbard model

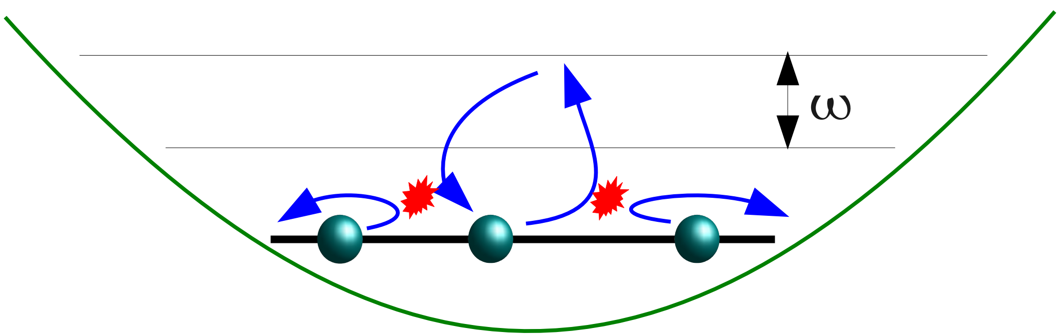
Jaksch et al.'98...

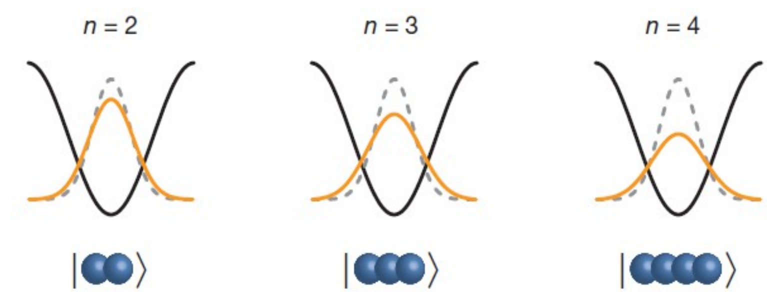
Bosons in an optical lattice

Bose-Hubbard model
+ higher order interactions

Niu et al.'06, Tiesinga et al.'09,11, Hazzard&Mueller'10







Weak, high order in g/ω, but measurable
Campbell et al.'06, Will et al.'10

3-body interacting case: perturb. prosp.

$$E(N)=U_2\frac{N(N-1)}{2!}+U_3\frac{N(N-1)(N-2)}{3!}+U_4\frac{N(N-1)(N-2)(N-3)}{4!}+\dots$$

Perturbative approach Johnson et al.'09

$$E(2)=U_2=\langle\psi_0|V|\psi_0\rangle+\underbrace{\sum_\nu\frac{|\langle\psi_\nu|V|\psi_0\rangle|^2}{\epsilon_0-\epsilon_\nu}+\dots}_{\text{Higher order terms}}$$

$$E(3)=3E(2)-\underbrace{\underbrace{O(V^2)+\sum_{\bar\nu}\frac{|\langle\bar\psi_{\bar\nu}|V|\bar\psi_0\rangle|^2}{\bar\epsilon_0-\bar\epsilon_{\bar\nu}}+\dots}_{\text{Additional non-additive higher order terms}}}_{U_3\sim V^2/|\epsilon_0-\epsilon_1|}$$

Double counting compensation

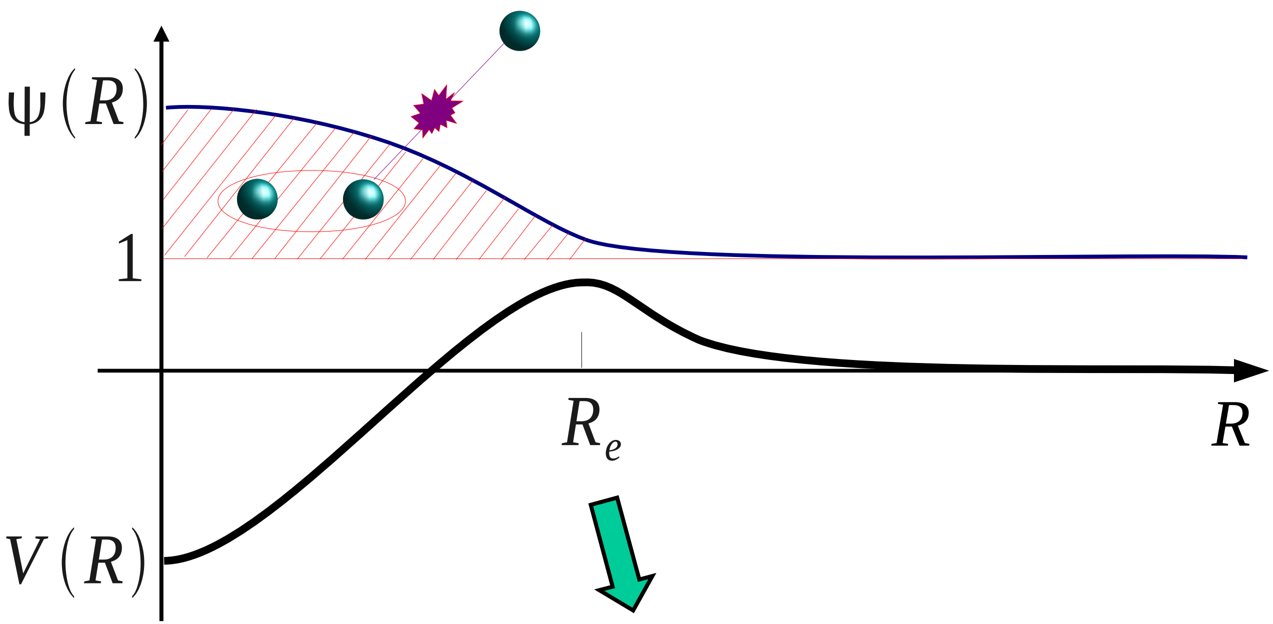
This talk → $\boxed{U_2=0 \quad \text{AND} \quad U_3>0 \quad \text{AND STRONG!}}$

Where to look?

$\langle \psi_0 | V | \psi_0 \rangle \approx 0$ → vanishing on-shell scattering... OK

$\langle \psi_\nu | V | \psi_0 \rangle \neq 0$ ↔ large off-shell contribution...

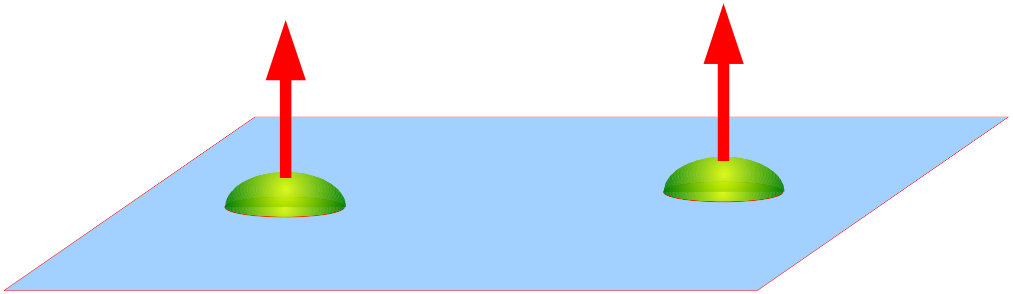
which should repel the third particle... ???

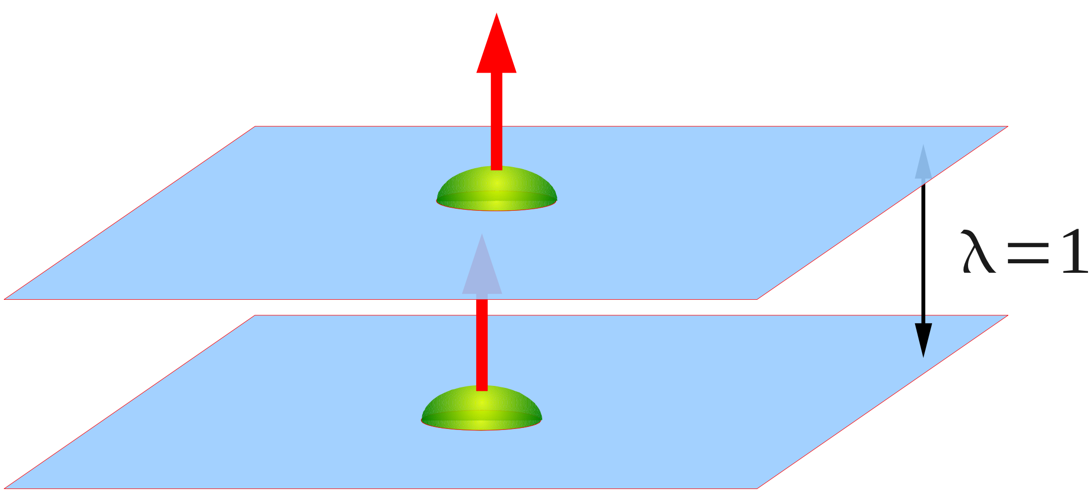


$g_3^{(3D)} \sim R_e^4$

Just two-body zero crossing is not enough !

Dipoles on layers

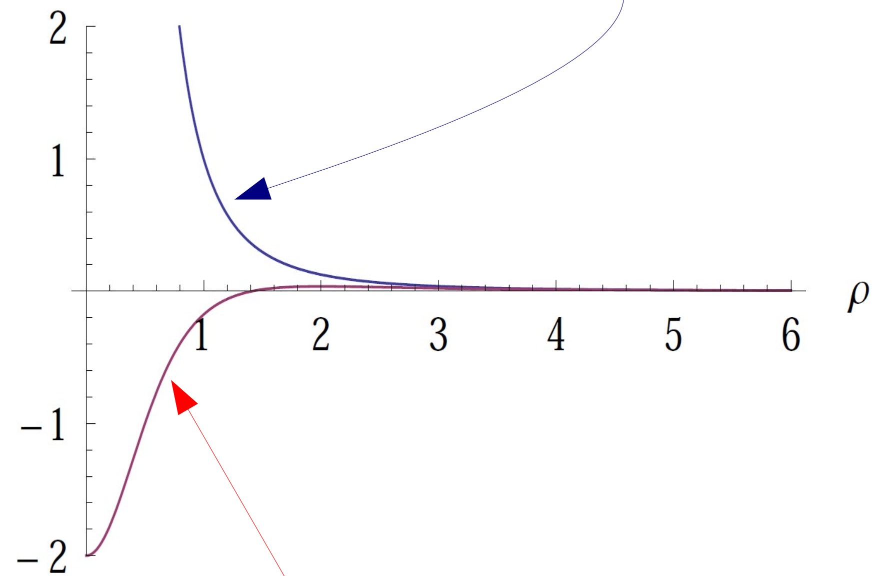




Repulsive intralayer potential

$$V_{\uparrow\uparrow}(\rho)=r_*/\rho^3$$

No bound state



Interlayer potential averages to zero

$$V_{\uparrow\downarrow}(\rho)=r_*(\rho^2-2)/(\rho^2+1)^{5/2}$$

At least one bound state

Vertex function and bound state

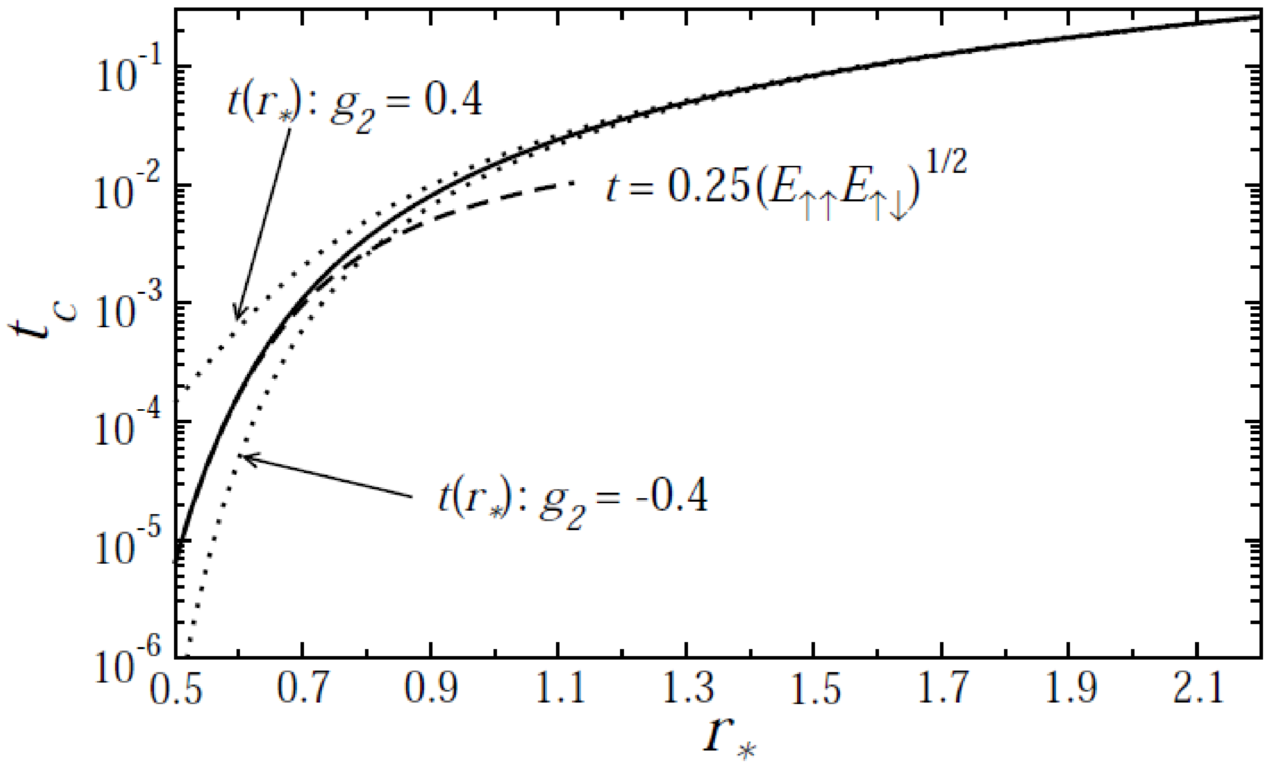
Vertex function for 2D scattering with weakly-bound state + dipolar tails Baranov et al.'11

$$\Gamma(E,\vec{k},\vec{k}\,')\approx\frac{4\pi}{\ln(4t/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}\,'|$$

$$\varepsilon_0=4t\exp(4\pi/g_2)$$

Exponentially weakly bound state for small negative g_2

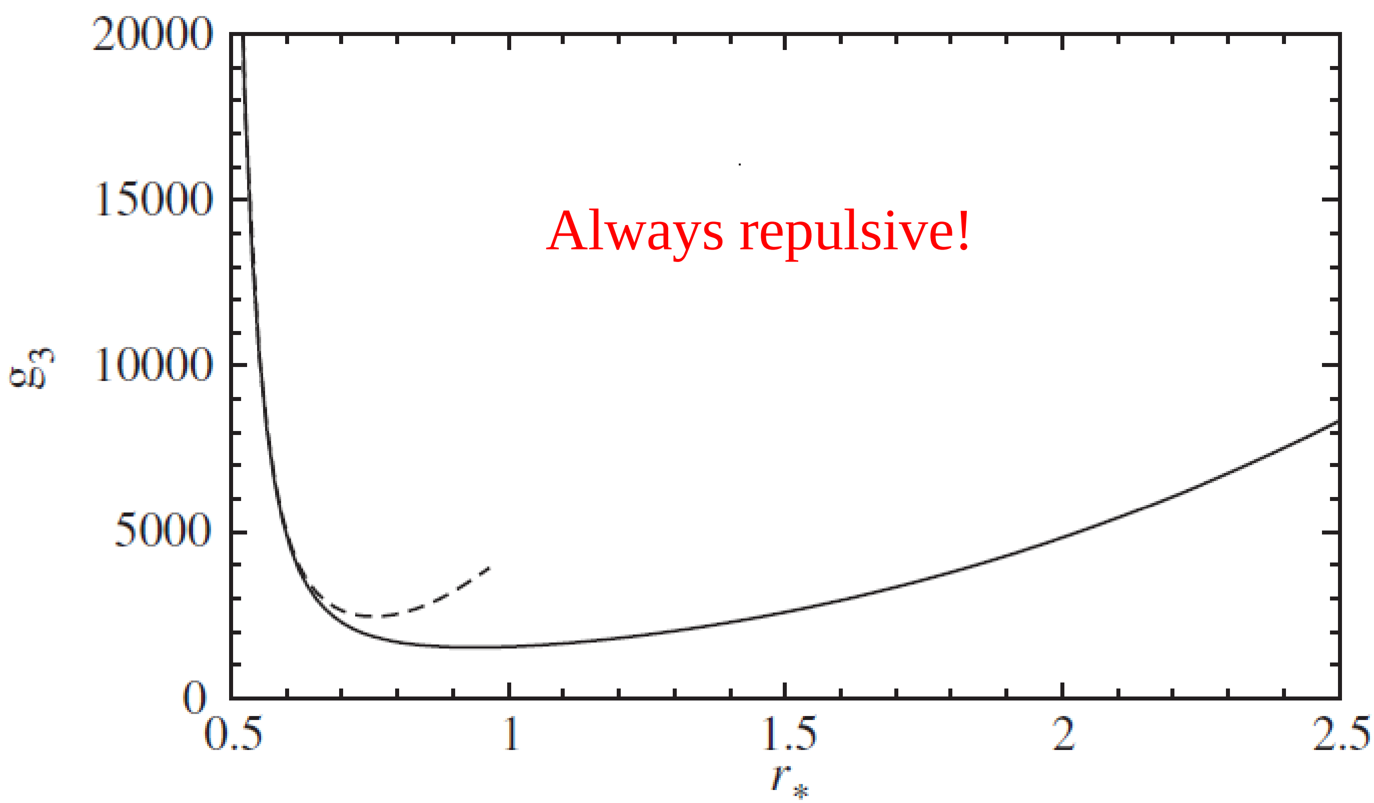
Critical *t*



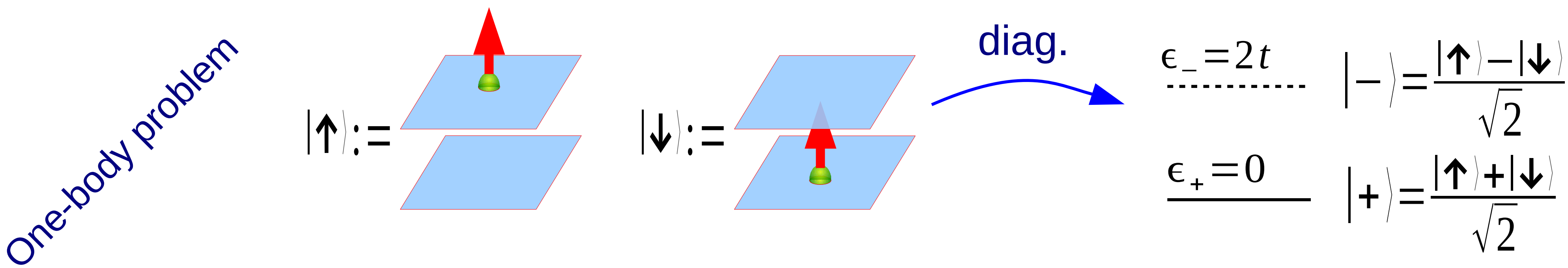
3-body coupling constant

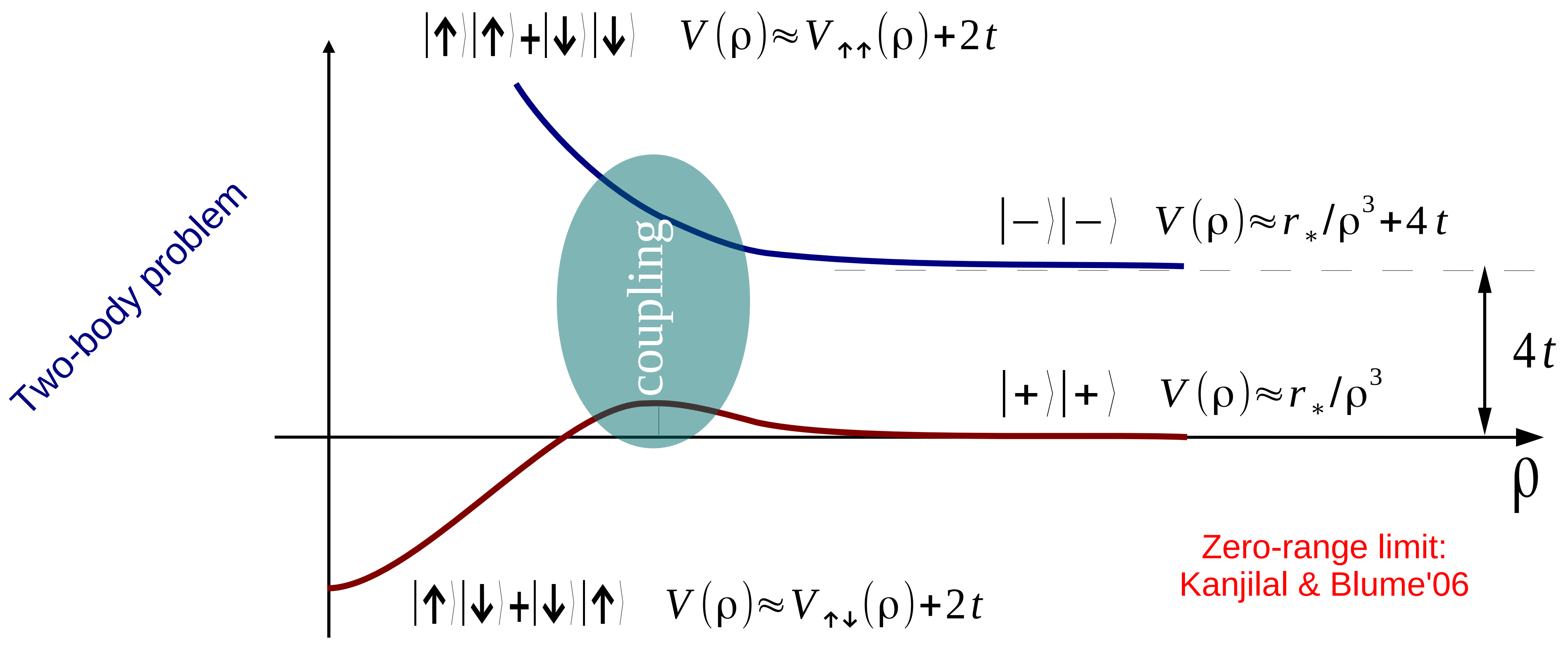
$$g_3 = \langle free_3 | \sum V | true_3 \rangle - 3 \langle free_2 | V | true_2 \rangle$$

Hyperspherical method
on the line $g_2=0$

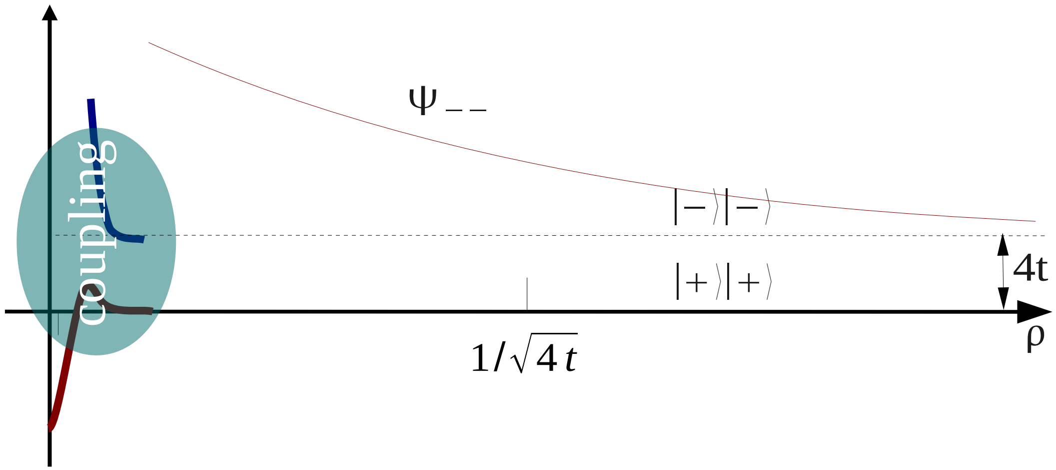


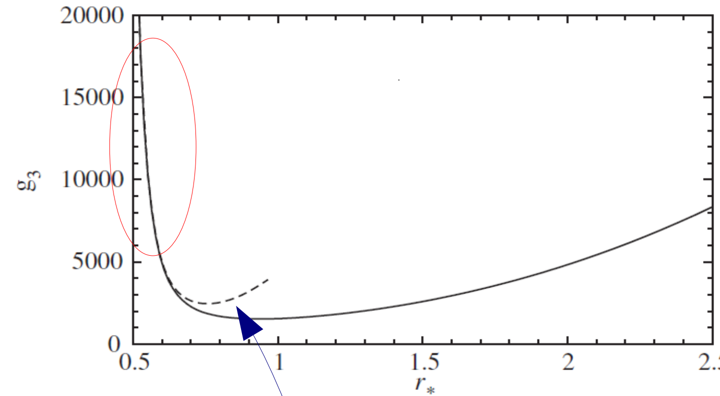
Bilayer with tunneling

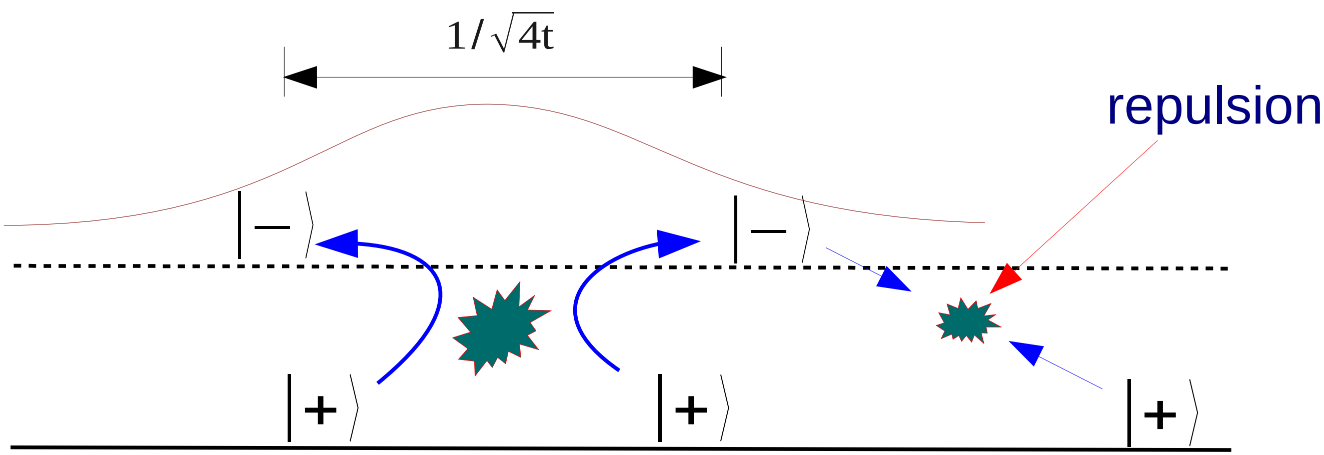




Small r_* – large off shell contribution



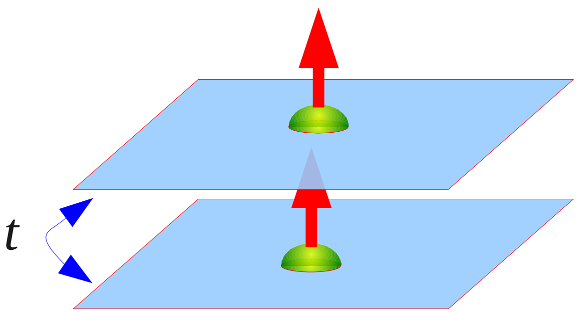




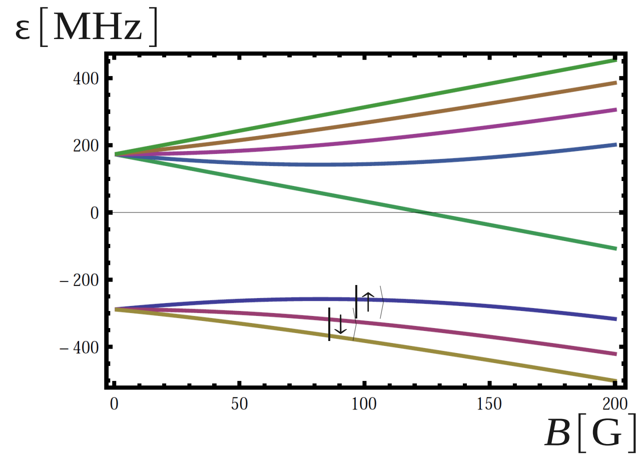
$1/\sqrt{4t} \gg r_*$ → zero-range model →

$$\boxed{g_3 = \frac{24\pi^2}{t_c}\left[\frac{1}{\ln^3\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} - \frac{3\ln(4/3)}{\ln^4\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} + \ldots\right]}$$

Long-range is not essential → hyperfine states of an atom



^{39}K: |F=1,m$_F$=0> and |F=1,m$_F$=-1>



Couple them with RF (~50MHz)

Analog of the interlayer tunneling *t* → Ω = Rabi frequency (~kHz)

Δ = Detuning (~kHz)

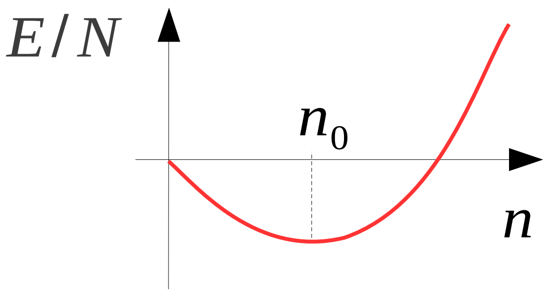
Works in any dimension: 3D, 2D, 1D, and lattice!

X-dimensional 3-body scattering = (2X)-dimensional 2-body scattering on a potential characterized by the range $R_3 \propto 1/\sqrt{\Omega}$

Three-body interaction grows with decreasing Ω What happens when $\Omega = 0$?

Connection between LHY and three-body force

The gas should remain dilute, otherwise short lifetime!



$$E/N \propto a\,n + L^{3\alpha-2} n^{\alpha}, \quad \alpha > 1$$

$$n_0 \sim \frac{1}{L^3}\left(\frac{a}{L}\right)^{\frac{1}{\alpha-1}}$$

Dilute = simultaneously small a and large L
and prefer small α

3-body $(\alpha = 2)$

Lee-Huang-Yang $(\alpha = 3/2)$

Resonant (Efimov)
3-body force
(Bulgac'02)

lossy :(

Non-resonant
3-body force
(DP'14)

Beyond-mean-field QUANTUM MECHANISM!

Realized with dipoles Dy (Stuttgart), Er (Innsbruck) and bosonic mixtures K (Barcelona, Florence)

LHY depends on ...

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}\left[E_{\pm}(k)-k^2/2-c_{\pm}^2\right]=$$

number of components

density of states (dimension)

shape of the Bogoliubov spectrum
(anisotropy of the interaction,
driving the mixture, etc.)

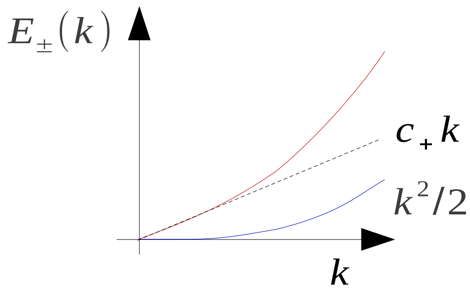
...and life becomes ~~harder~~ more interesting in the inhomogeneous case
particularly if LDA is not valid

Symmetric case close to collapse $\quad g_{11}=g_{22}=-g_{12}=g \quad$ and $\quad n_1=n_2=n/2$

without coupling

$$E_+(k)=\sqrt{(k^2/2)(k^2/2+2\mathrm{gn})}$$

$$E_-(k)=k^2/2$$

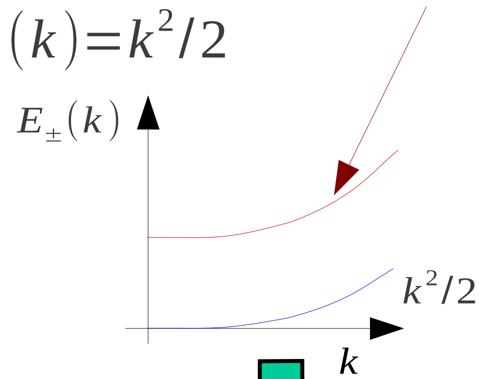


$$\mathrm{LHY}=\frac{8}{15\pi^2}(g\,n)^{5/2}$$

with coupling (Goldstein&Meystre'97)

$$E_+(k)=\sqrt{(k^2/2+\Omega)(k^2/2+\Omega+2\mathrm{gn})}$$

$$E_-(k)=k^2/2$$



$$\mathrm{LHY}=\frac{2}{\pi^2}(g\,n)^{5/2}\int_0^1\sqrt{x(1-x)\left(x+\frac{\Omega}{2\mathrm{gn}}\right)}\,dx$$

$gn \ll \Omega$

$$=\frac{\sqrt{\Omega}\,g^2}{2\sqrt{2}\pi}\frac{n^2}{2}+\frac{3\,g^3}{4\sqrt{2}\pi\sqrt{\Omega}}\frac{n^3}{3!}+\dots$$

Renormalization of two-body interaction

Effective three-body force

DSP and Recati, to be published... since a long time...

Summary

Beyond-mean-field effects

3-body repulsion

Lee-Huang-Yang

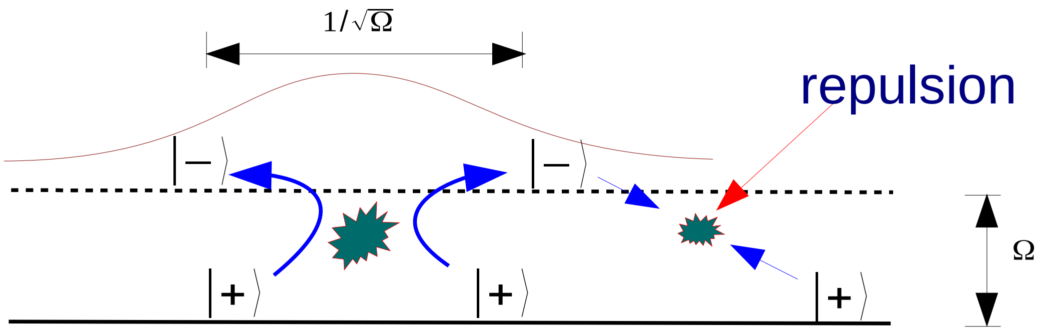
``Easily'' observable in 1D because of the logarithmic scaling

Needed to stabilize supersolid of 2D dipoles, topological paired superfluid, and possibly other exotic phases

Quantum droplets in mixtures and in scalar dipolar gases

谢谢大家的关注。

Summary



→ Effective three-body interaction $\propto n^3$ with the range

$R_3 \propto 1/\sqrt{\Omega}$ → $\boxed{g_3 \propto 1/\Omega^{X-1}}$

$\Omega = 0$ → 3D: LHY correction = 2.5-body repulsion $\propto n^{5/2}$

2D: MF+ beyond MF $\propto n^2 \ln n$

1D: LHY correction = 1.5-body attraction $\propto -n^{3/2}$

Useful conclusions

Gapped stiff mode → LHY correction transforms into effective 3-body force

Same in low dimensions (coincides with direct three-body calculations)

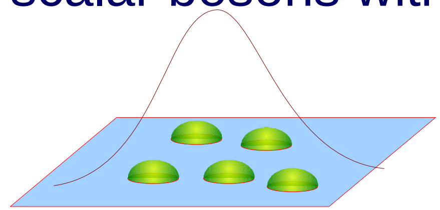
Also checked the nonsymmetric case.

Bogoliubov theory – powerful tool for calculating three-body force. More efficient than direct solution of the three-body problem.

Prospects

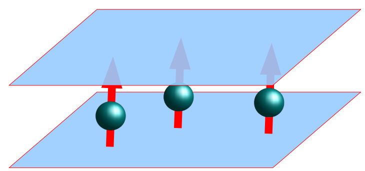
Theory beyond local density approximation (LDA)

- droplet of two-dimensional scalar bosons with zero-range interactions. Theory beyond Hammer&Son'04

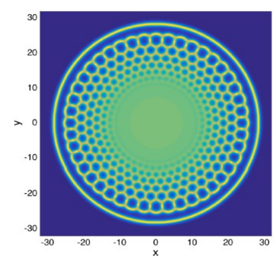


$$B_N / B_{N-1} \rightarrow 8.567$$

- quasi-low-dimensional problems with short- and long-range interactions when the healing length of the mode responsible for the LHY correction is comparable to the size of the condensate



- LHY correction to the supersolid phase?

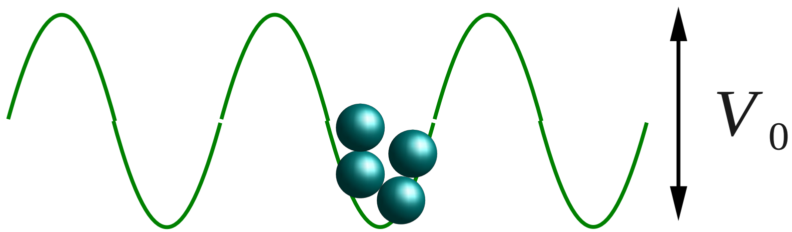


Get out of the weakly-interacting regime while staying dilute?

- use sign-problem-free bosonic Monte Carlo

Fermions?

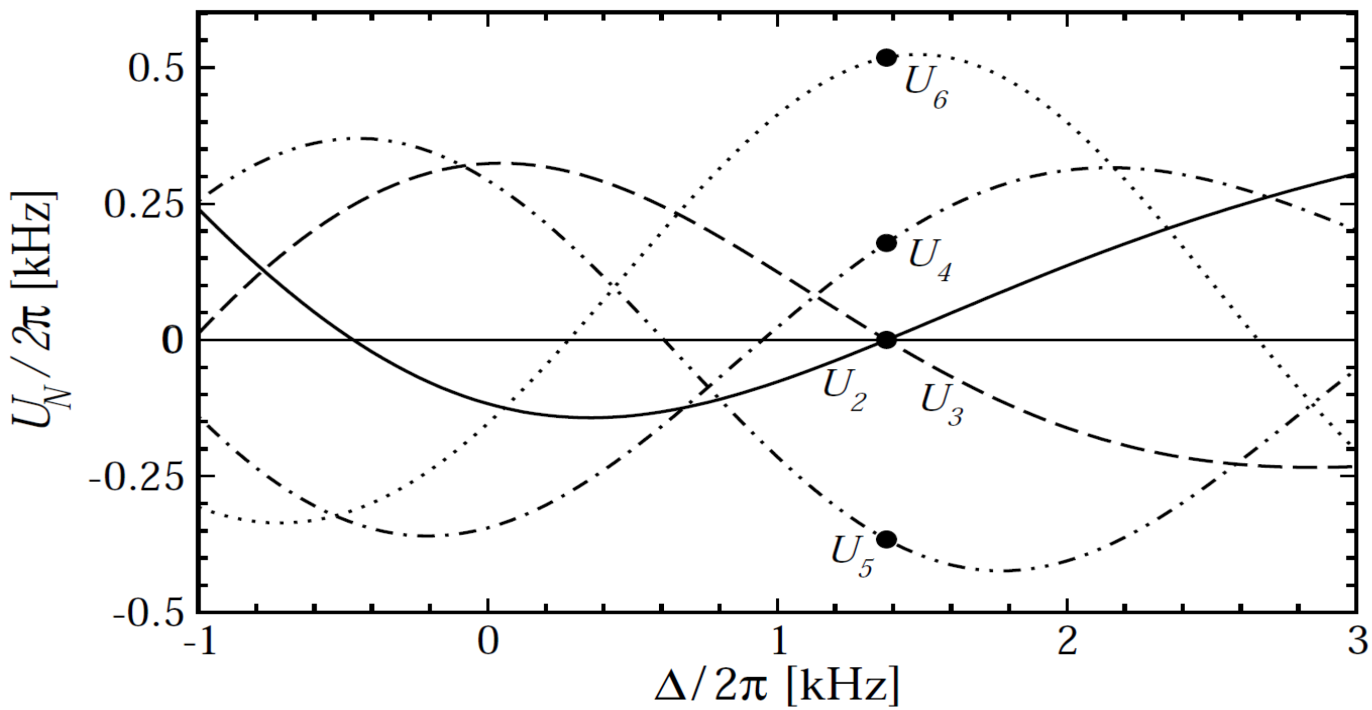
BTW, 4-body interacting lattice case



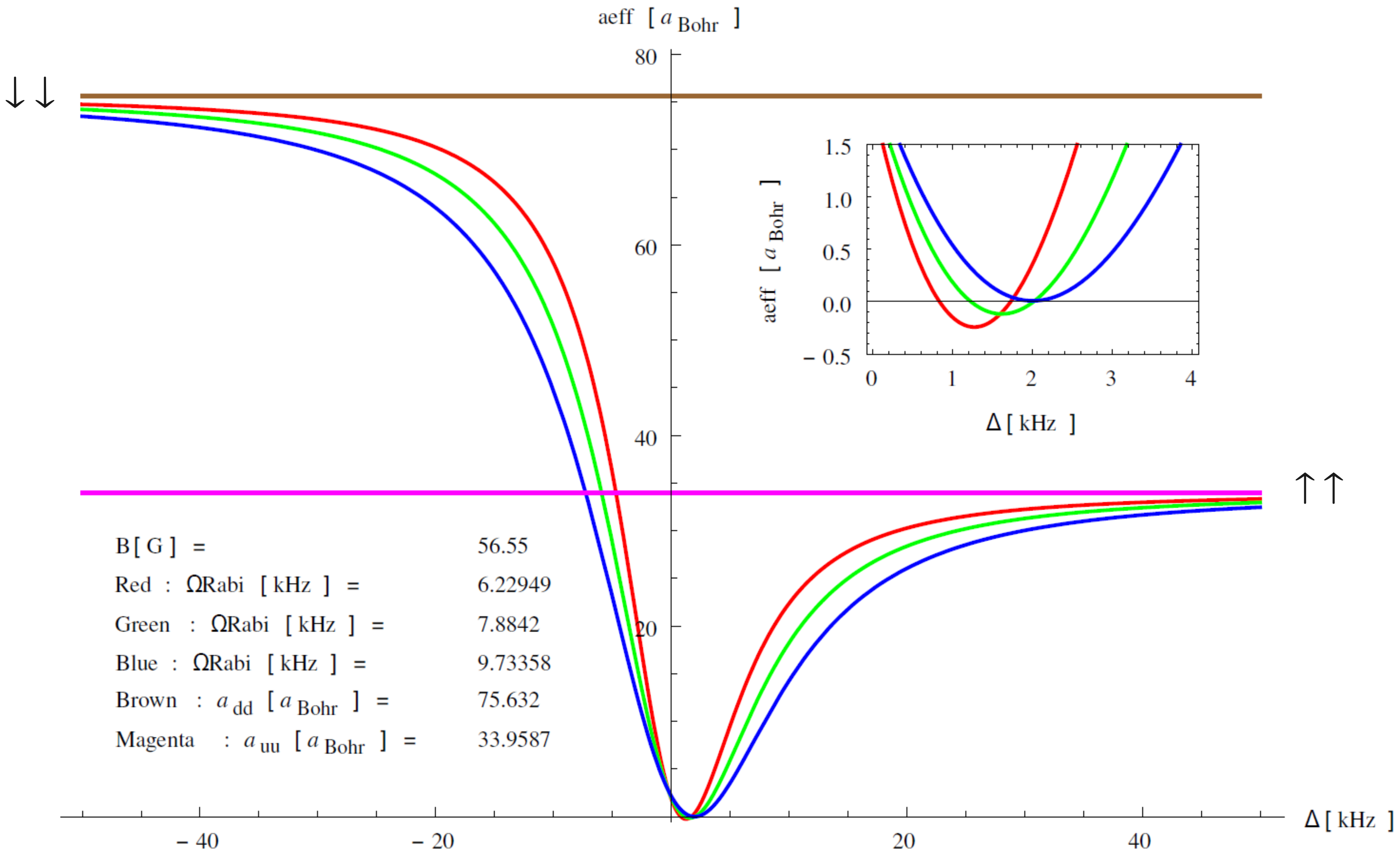
^{39}K: $|F=1,m_F=0\rangle$ and $|F=1,m_F=-1\rangle$

$\boxed{\Omega=2\pi\times 1.7\,\mathrm{kHz}}$

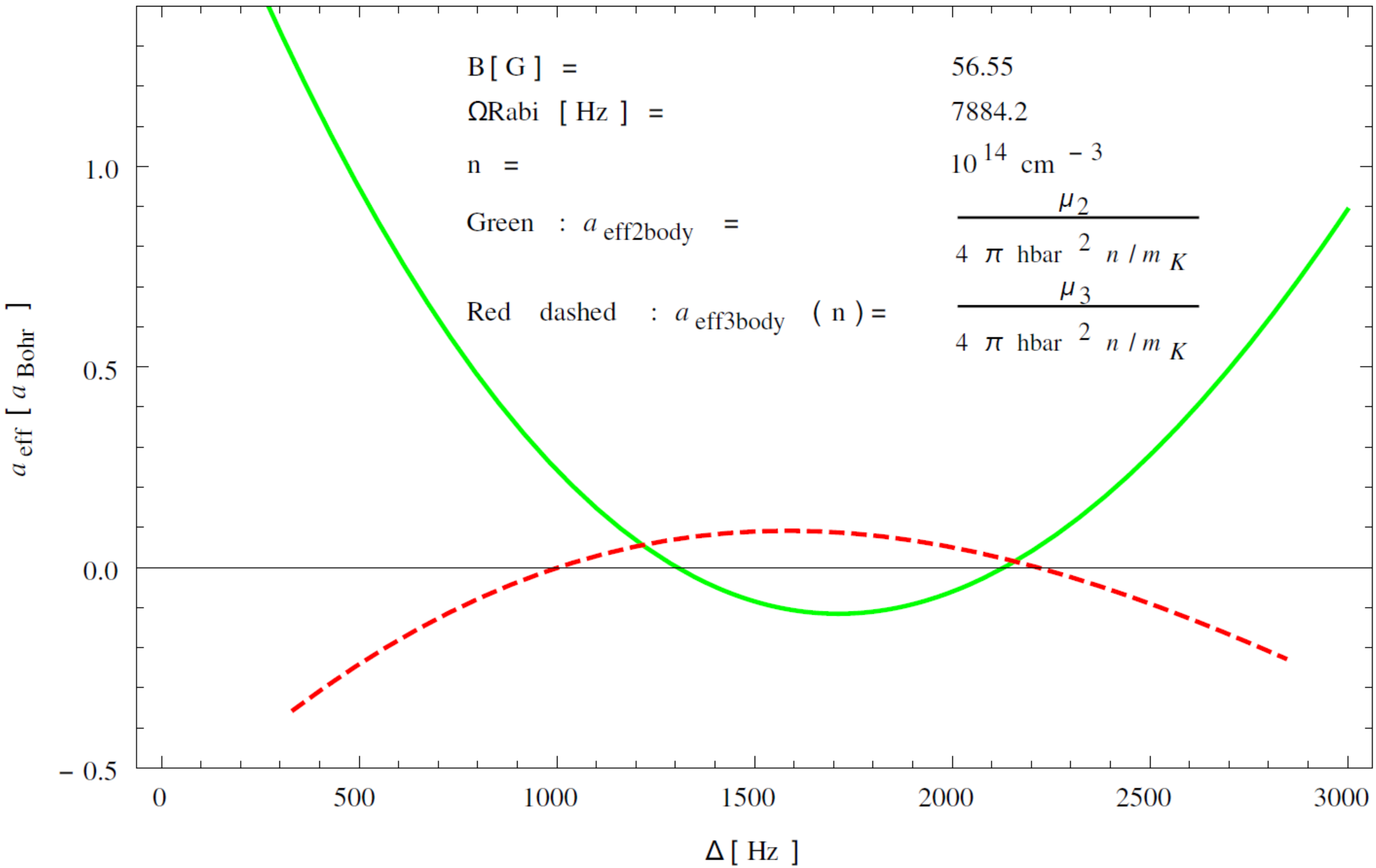
lattice constant $=532\,\mathrm{nm}$
$V_0=15\,E_R$
on-site osc. freq. $=2\pi\times 35\,\mathrm{kHz}$
$l_x=l_y=l_z=86\,\mathrm{nm}$
tunneling amp. $=2\pi\times 30\,\mathrm{Hz}$
$a_{\downarrow\downarrow}=9.4\,\mathrm{nm}\rightarrow g_{\downarrow\downarrow}=2\pi\times 3.05\,\mathrm{kHz}$
$a_{\uparrow\uparrow}=1.7\,\mathrm{nm}\rightarrow g_{\uparrow\uparrow}=2\pi\times 0.55\,\mathrm{kHz}$
$a_{\uparrow\downarrow}=-2.8\,\mathrm{nm}\rightarrow g_{\uparrow\downarrow}=-2\pi\times 0.91\,\mathrm{kHz}$

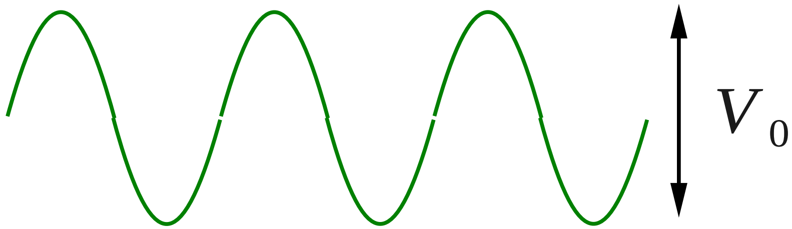


Scattering length (free space)



Three-body interaction (free space)





lattice constant $= 532$ nm

$V_0 = 15\, E_R$

on-site osc. freq. $= 2\pi \times 35$ kHz

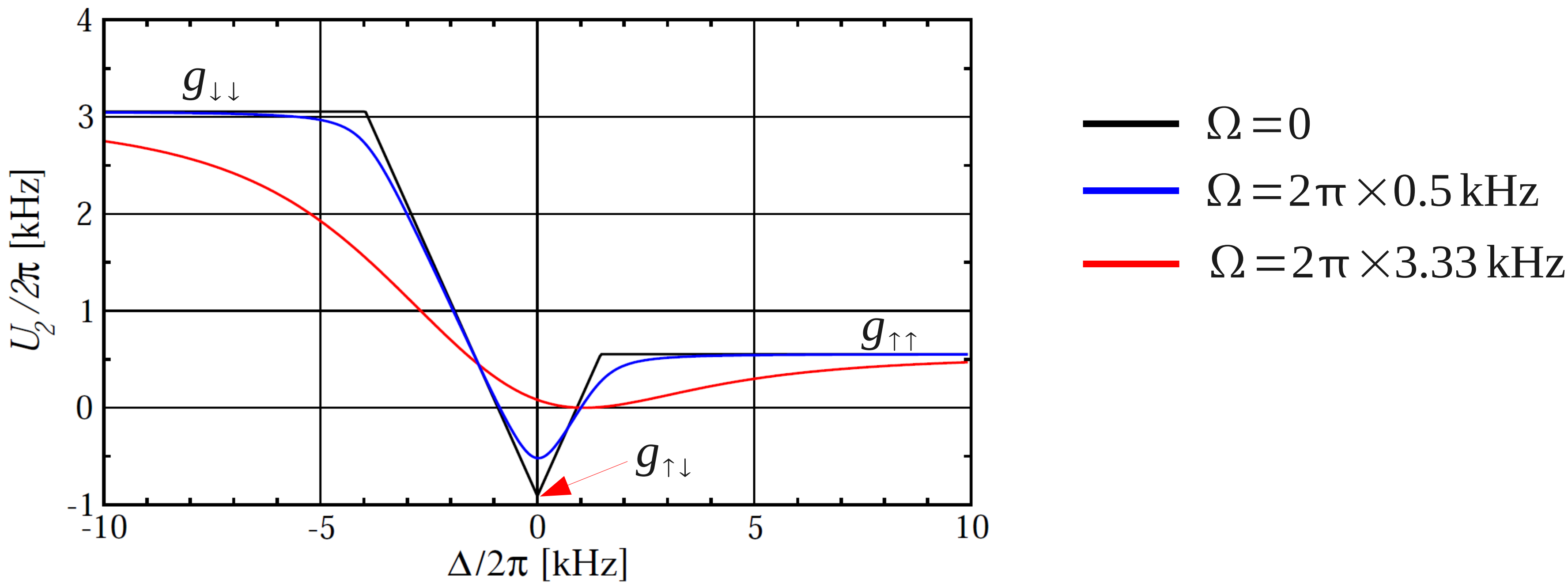
$l_x = l_y = l_z = 86$ nm

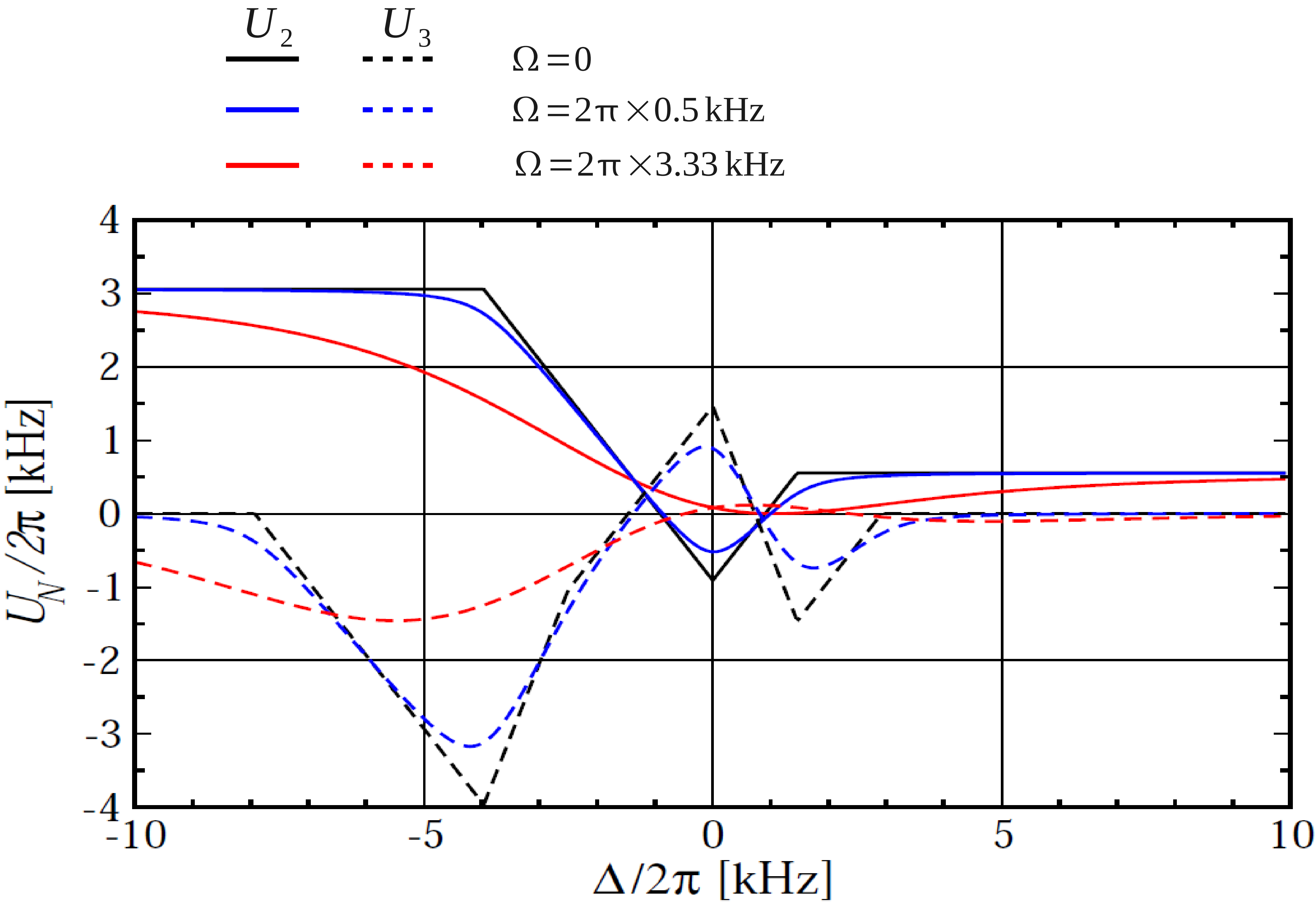
tunneling amp. $= 2\pi \times 30$ Hz

$a_{\downarrow\downarrow} = 9.4\,\text{nm} \rightarrow g_{\downarrow\downarrow} = 2\pi \times 3.05\,\text{kHz}$

$a_{\uparrow\uparrow} = 1.7\,\text{nm} \rightarrow g_{\uparrow\uparrow} = 2\pi \times 0.55\,\text{kHz}$

$a_{\uparrow\downarrow} = -2.8\,\text{nm} \rightarrow g_{\uparrow\downarrow} = -2\pi \times 0.91\,\text{kHz}$

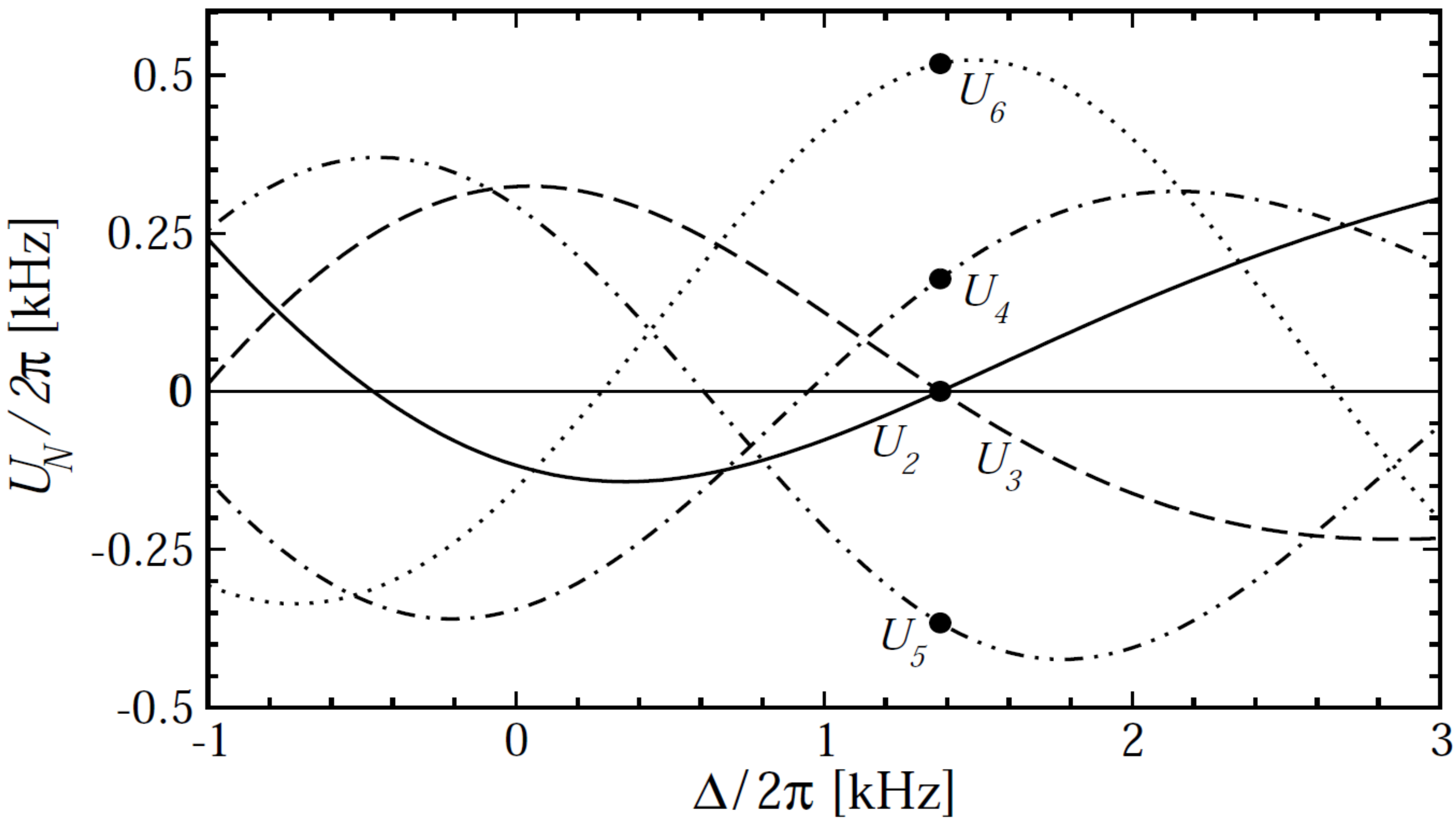




U_2
U_3
$\Omega = 0$
$\Omega = 2\pi \times 0.5\,\mathrm{kHz}$
$\Omega = 2\pi \times 3.33\,\mathrm{kHz}$
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10
$U_N/2\pi$ [kHz]
$\Delta/2\pi$ [kHz]

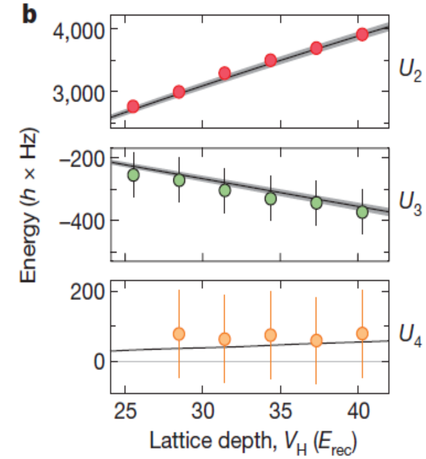
4-body interacting case

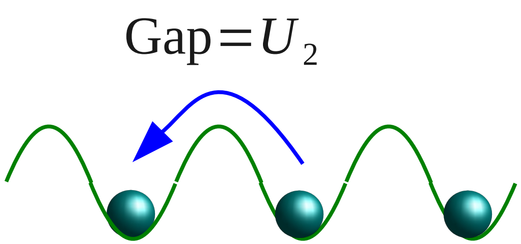
$\boxed{\Omega = 2\pi \times 1.7\,\mathrm{kHz}}$

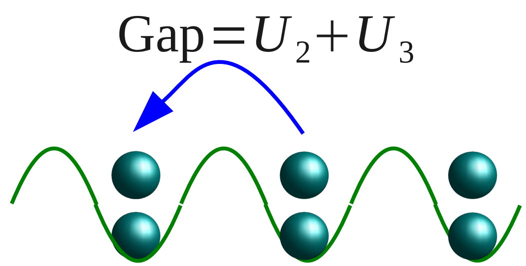


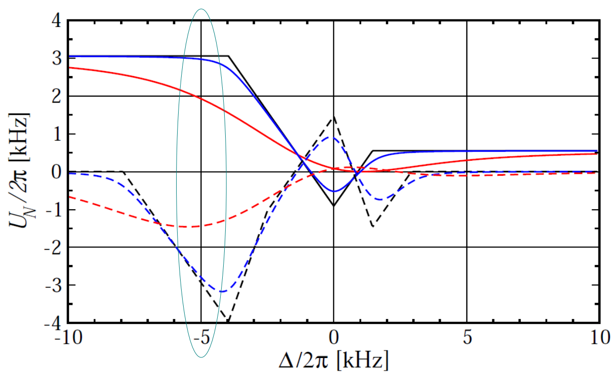
Perspectives

- Bosonic dipoles... Have to wait a bit...
- ^{39}K on a lattice
  - collapse and revivals
  - solitonic-like self-trapping
  - Mott lobes Chen et al.'08







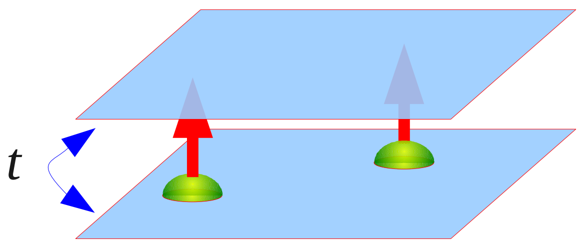


- Strong three-body interaction = large-size off-shell effects!

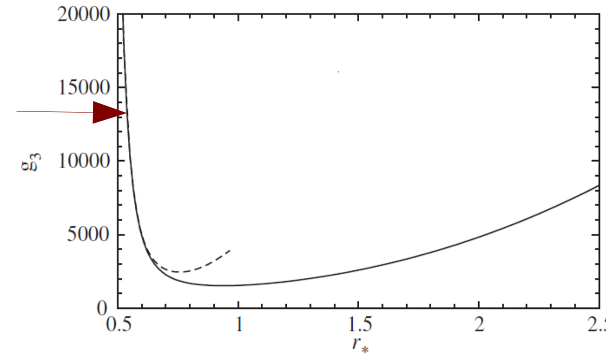
  Need small t (bilayers) or small $\sqrt{\Omega^2+\Delta^2}$ (RF coupling)

Implementation

Dipoles in the bilayer geometry with interlayer tunneling



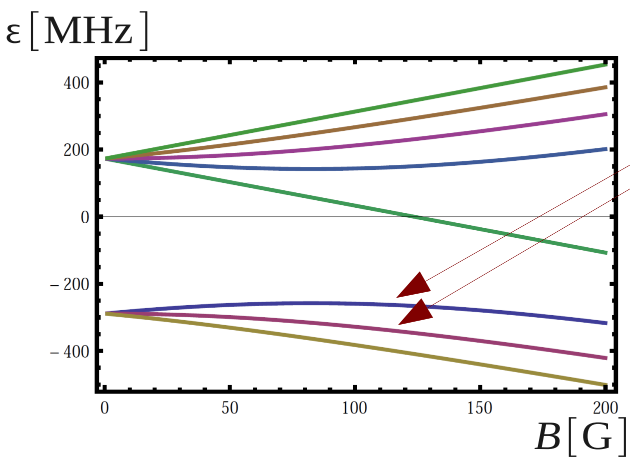
$\delta E \propto t$ ⇒ $g_3 \propto (1/\sqrt{t})^2 = 1/t$



Integrating out the interlayer degree of freedom we obtain 2D dipoles with 3-body repulsion!

Same ideas applied to hyperfine states of an atom

^{39}K: $|F=1,m_F=0\rangle$ and $|F=1,m_F=-1\rangle$ (suitable inter- and intra-species interactions)



Couple them with RF (~50MHz)

$\Omega = \text{Rabi frequency (\textasciitilde kHz)}$

$\Delta = \text{Detuning (\textasciitilde kHz)}$

$\sqrt{\Omega^2 + \Delta^2} = \text{spin splitting} \gg T$

Should work in any dimension: 3D, 2D, 1D, 0D (lattice)!

Vertex function and bound state

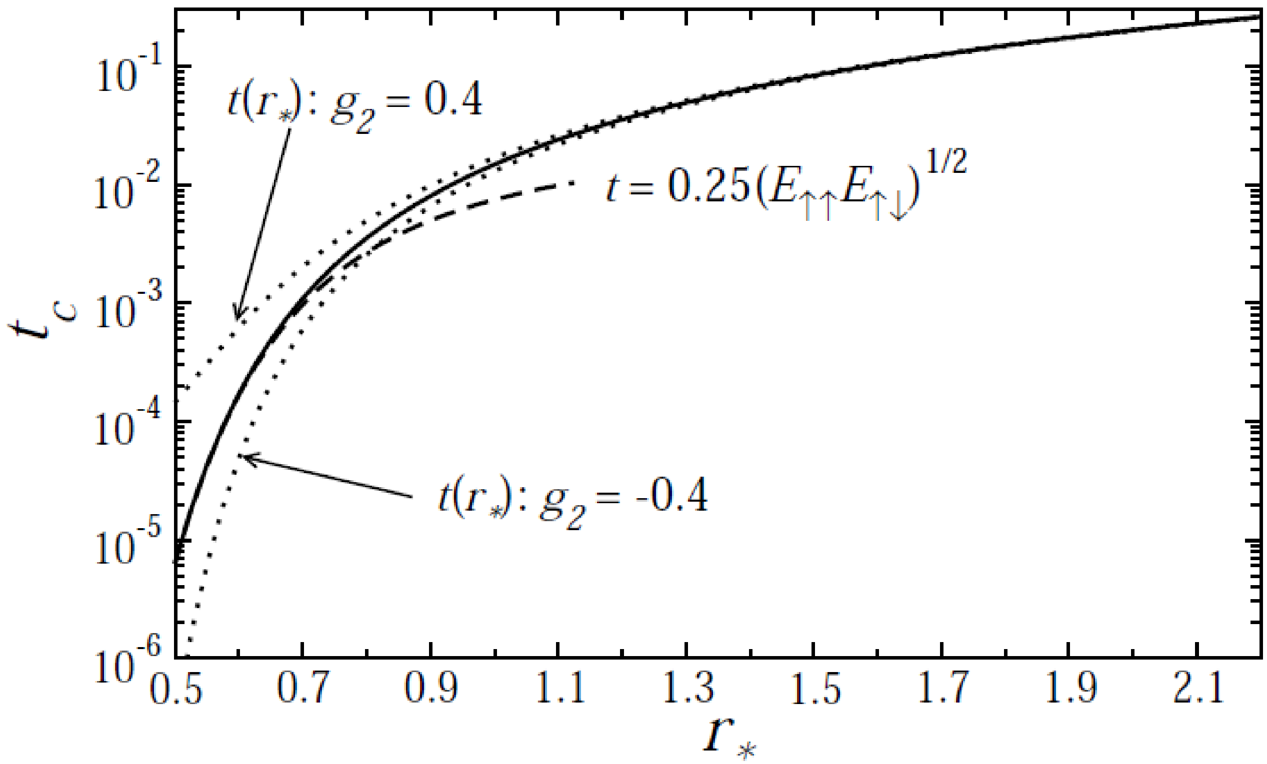
Vertex function for 2D scattering with weakly-bound state + dipolar tails:

$$\Gamma(E, \vec{k}, \vec{k}') \approx \frac{4\pi}{\ln(4t/E) + 4\pi/g_2 + i\pi} - 2\pi r_* |\vec{k} - \vec{k}'|$$

$$\varepsilon_0 = 4t \exp(4\pi/g_2)$$

Exponentially weakly bound state for small negative g_2

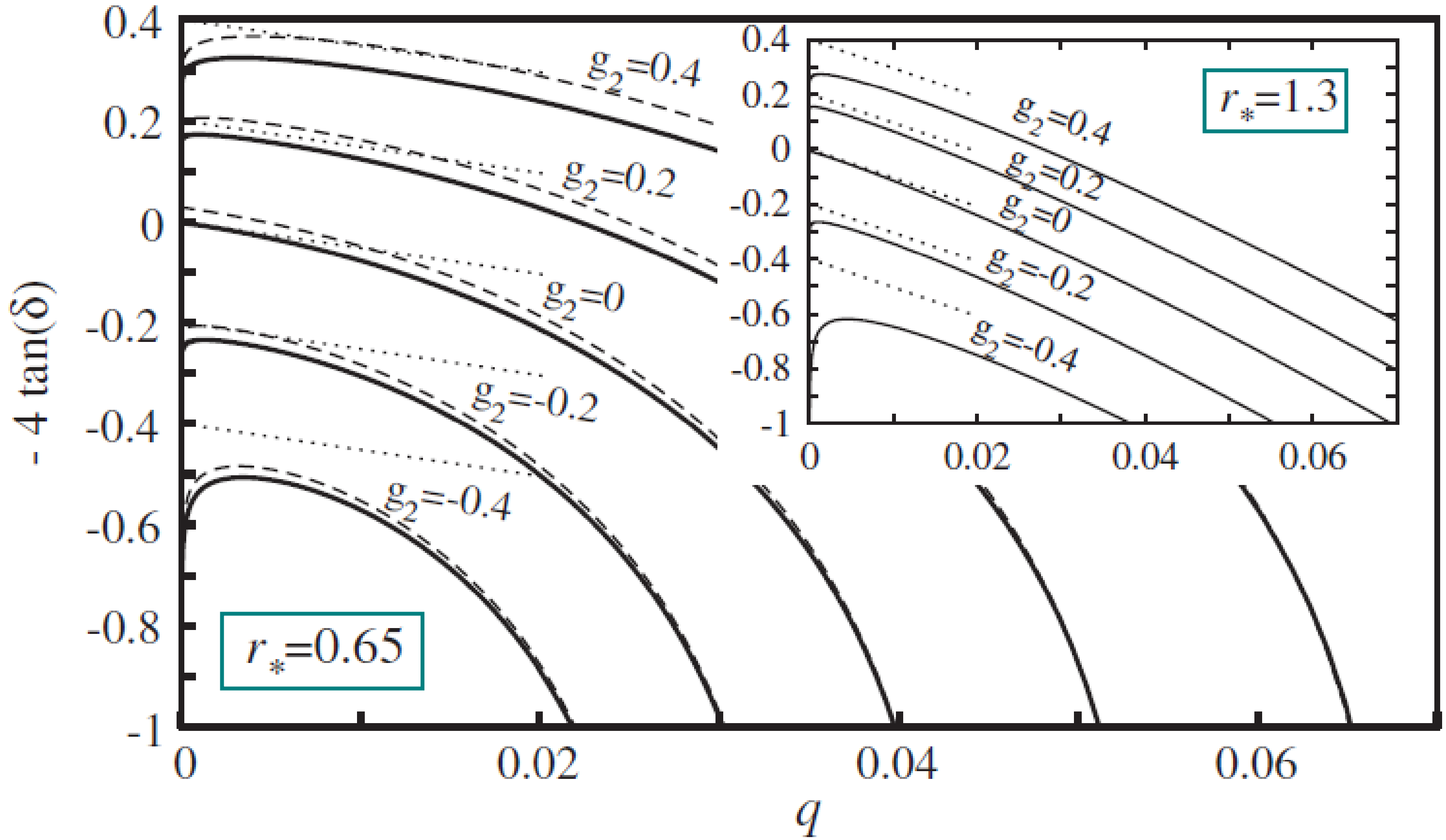
Critical *t*



S-wave scattering at finite collision energy

$$\Gamma(E,\vec{k},\vec{k}')\approx\frac{4\pi}{\ln(4t/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}'| \longrightarrow -4\tan\delta_s(q)\approx g_2-8r_*q$$

s-wave and on shell

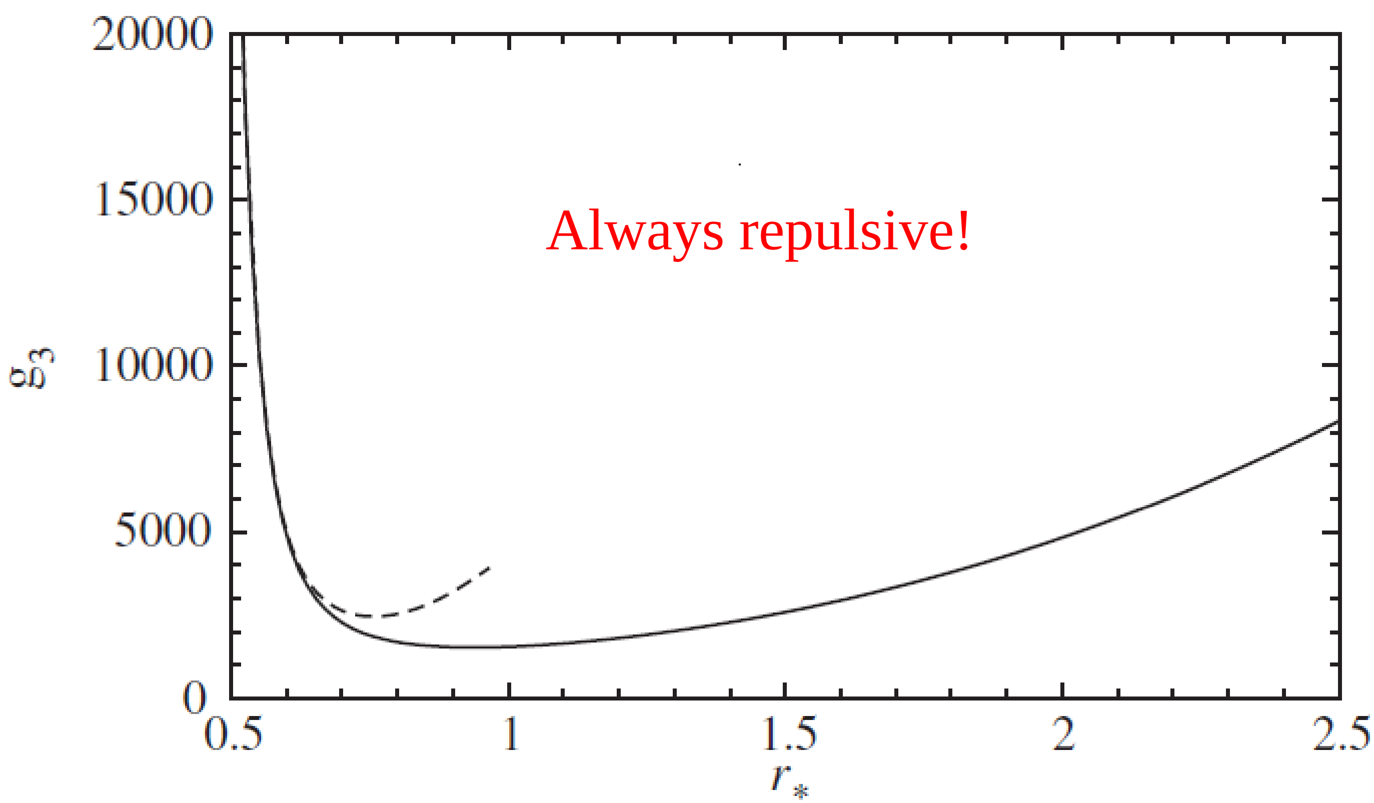


Two-body zero crossing + dipolar tail → rotonization, density wave, etc

3-body coupling constant

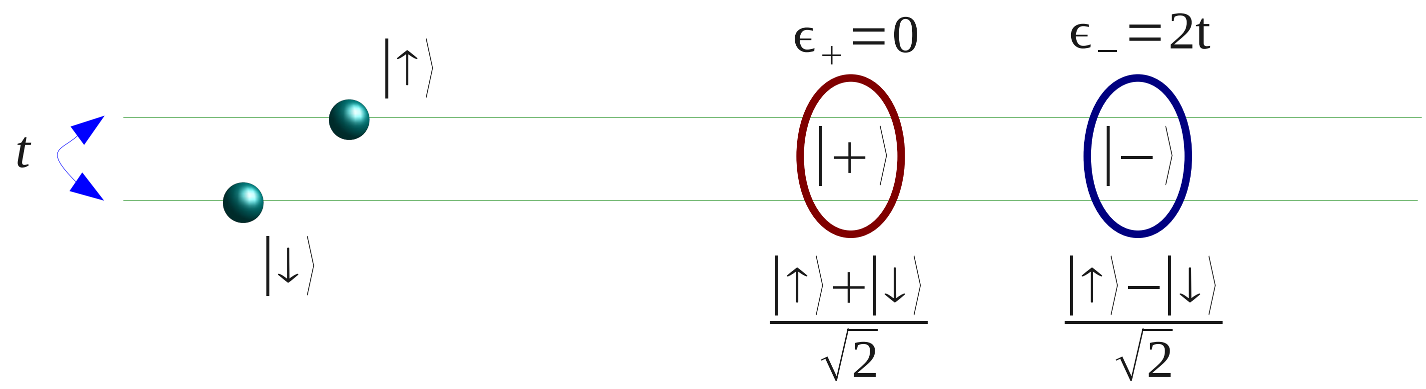
$$g_3 = \langle free_3 | \sum V | true_3 \rangle - \langle free_2 | V | true_2 \rangle$$

Hyperspherical method
on the line $g_2=0$

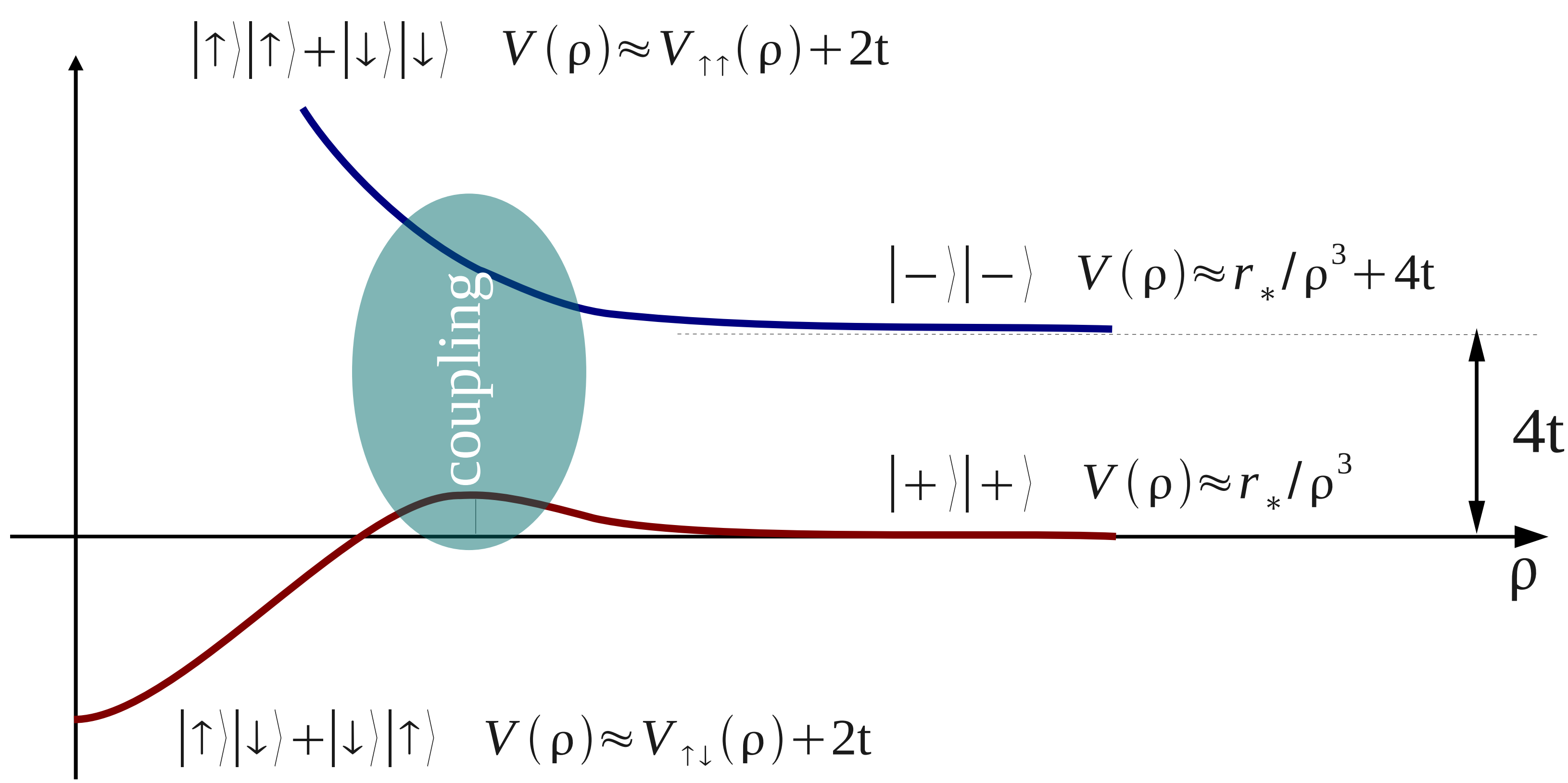


Bilayer with tunneling

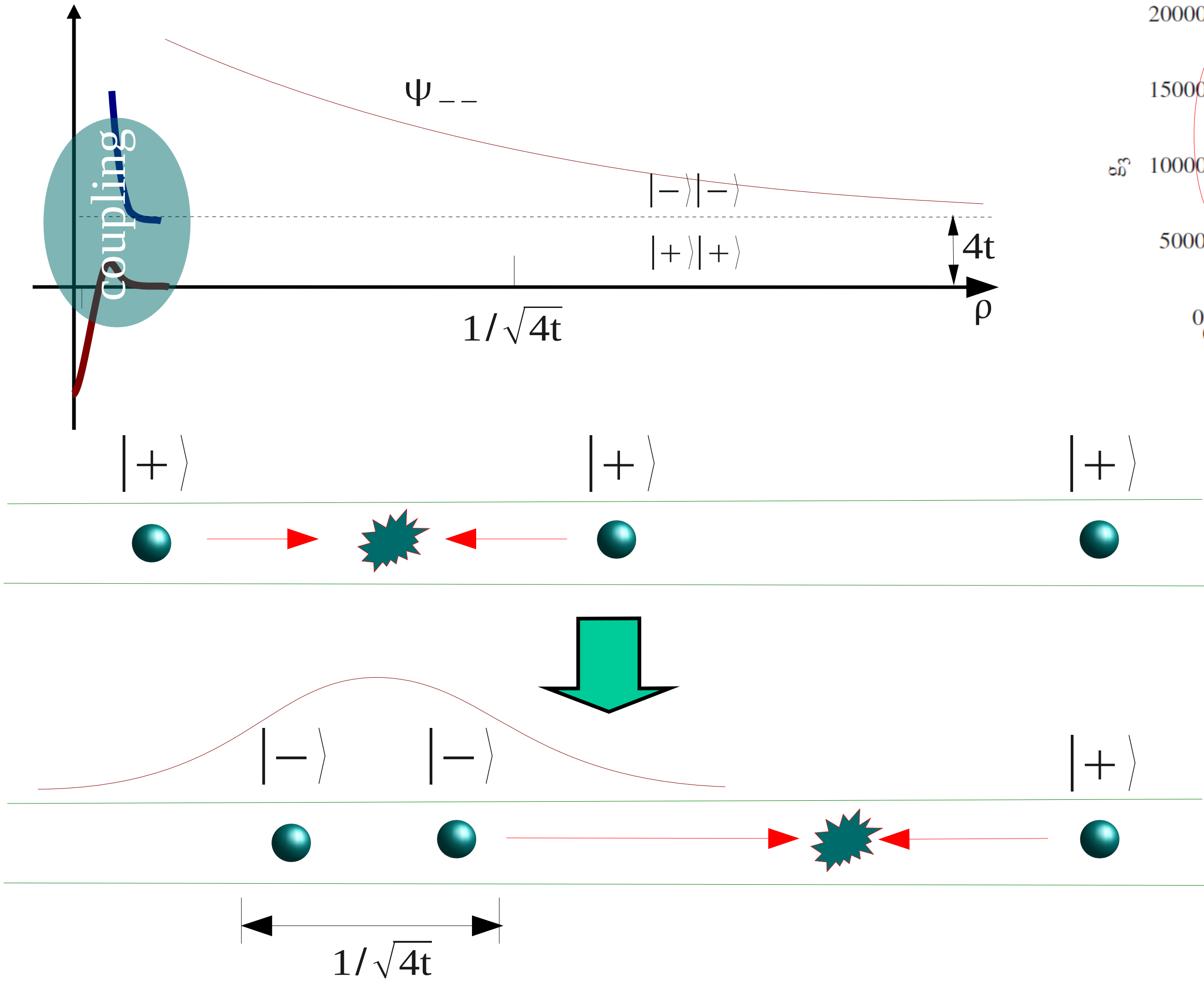
One-body problem

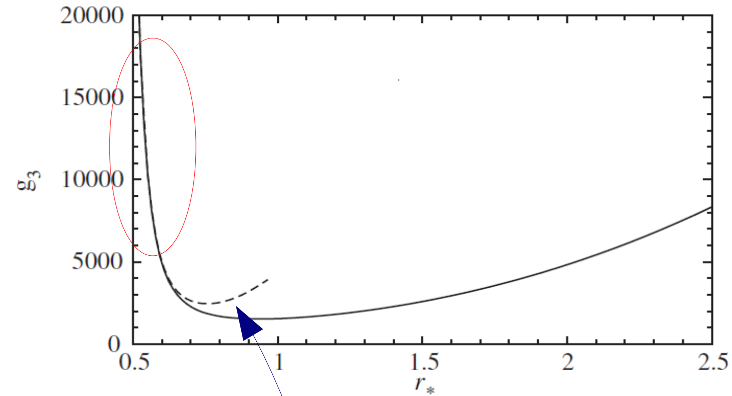


Two-body problem



Small r_* – large off shell contribution

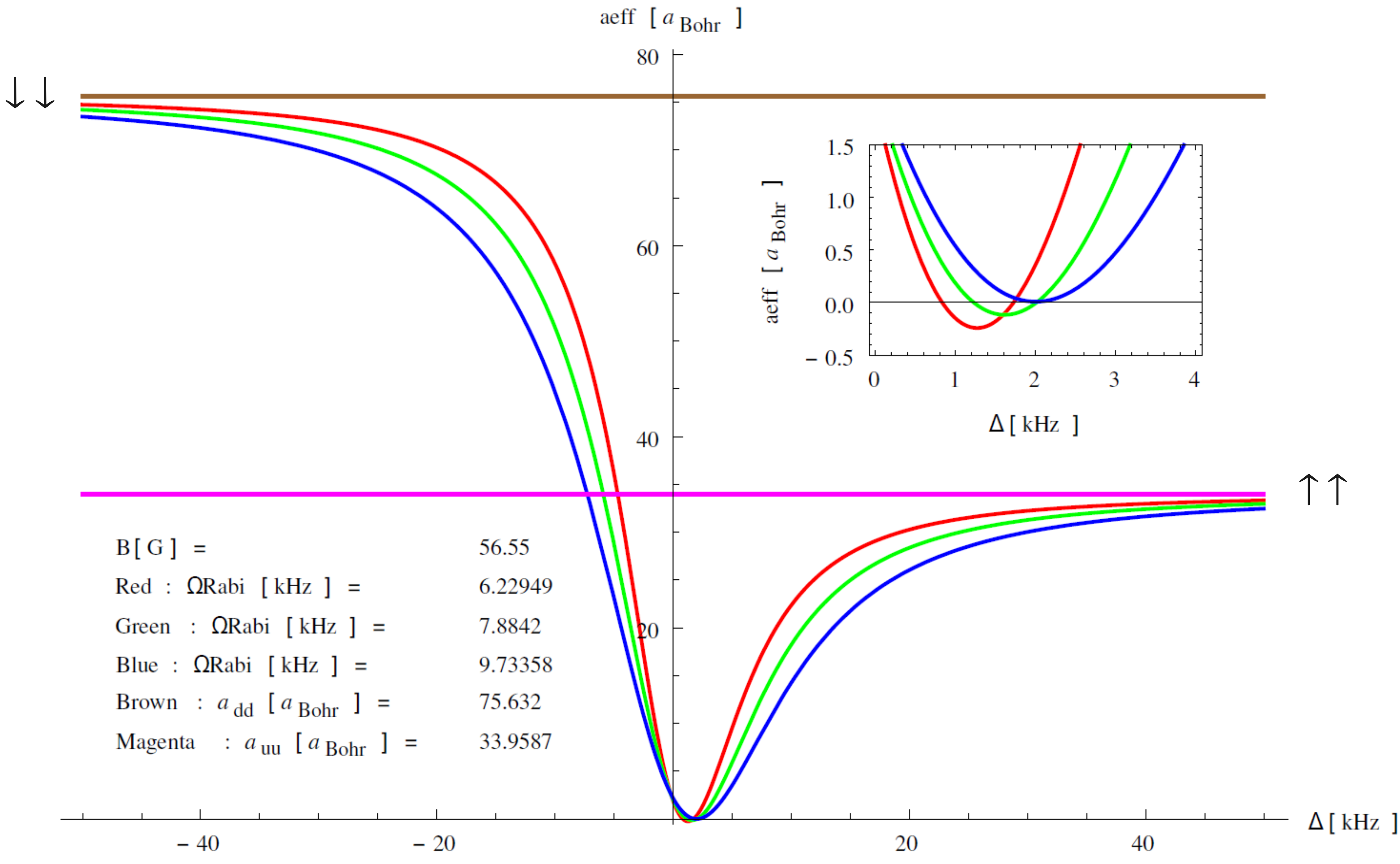




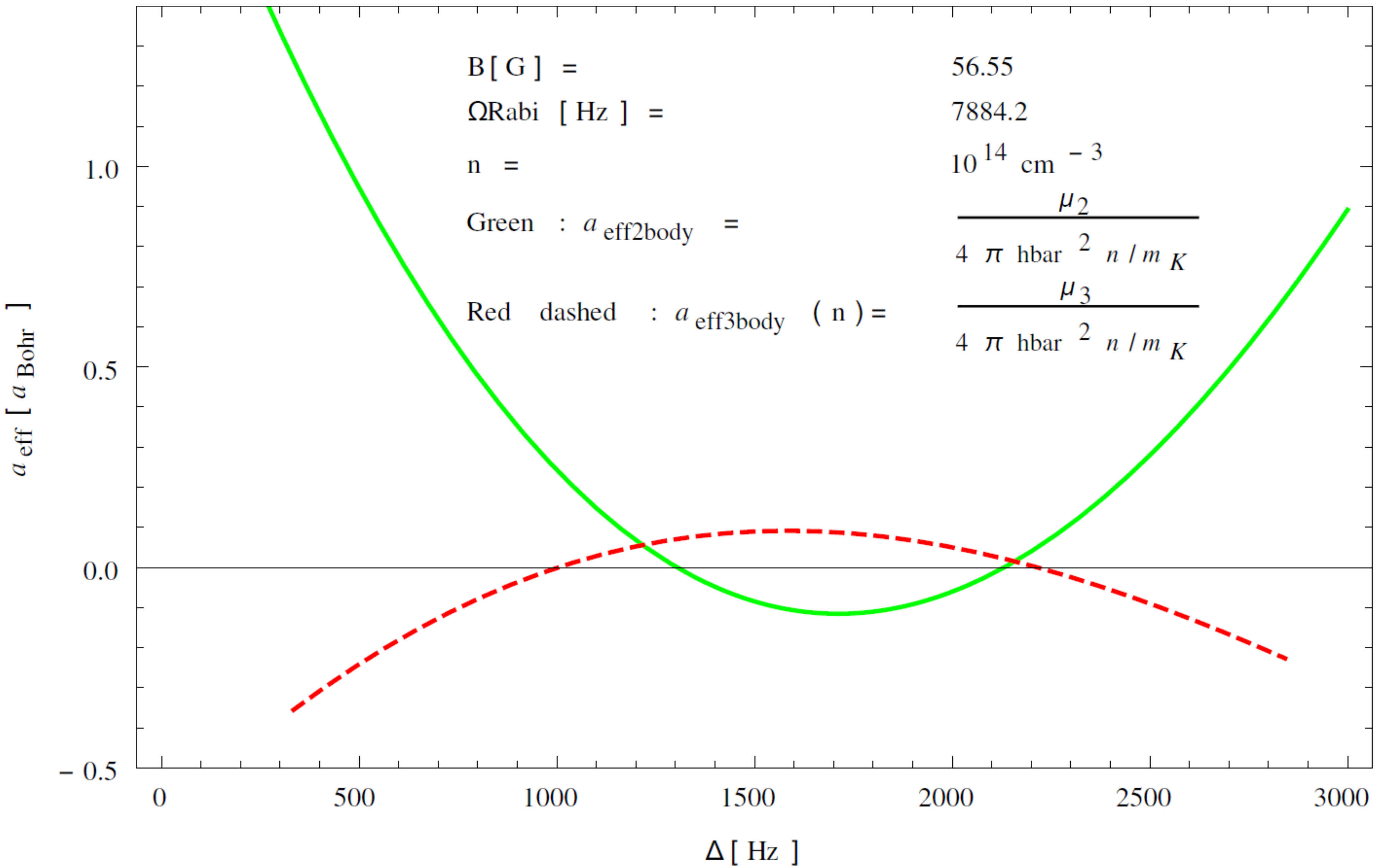
Zero-range model →

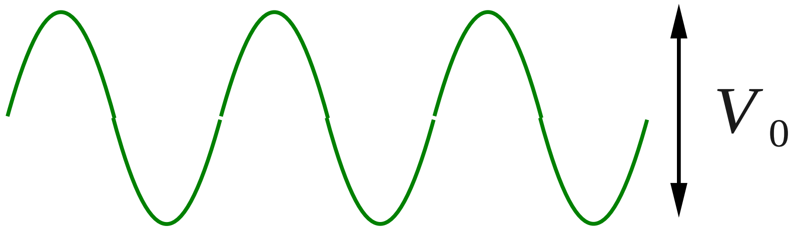
$$g_3 = \frac{24\pi^2}{t_c}\left[\frac{1}{\ln^3\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} - \frac{3\ln(4/3)}{\ln^4\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} + \ldots\right]$$

Scattering length (free space)



Three-body interaction (free space)





lattice constant $= 532$ nm

$V_0 = 15\,E_R$

on-site osc. freq. $= 2\pi \times 35$ kHz

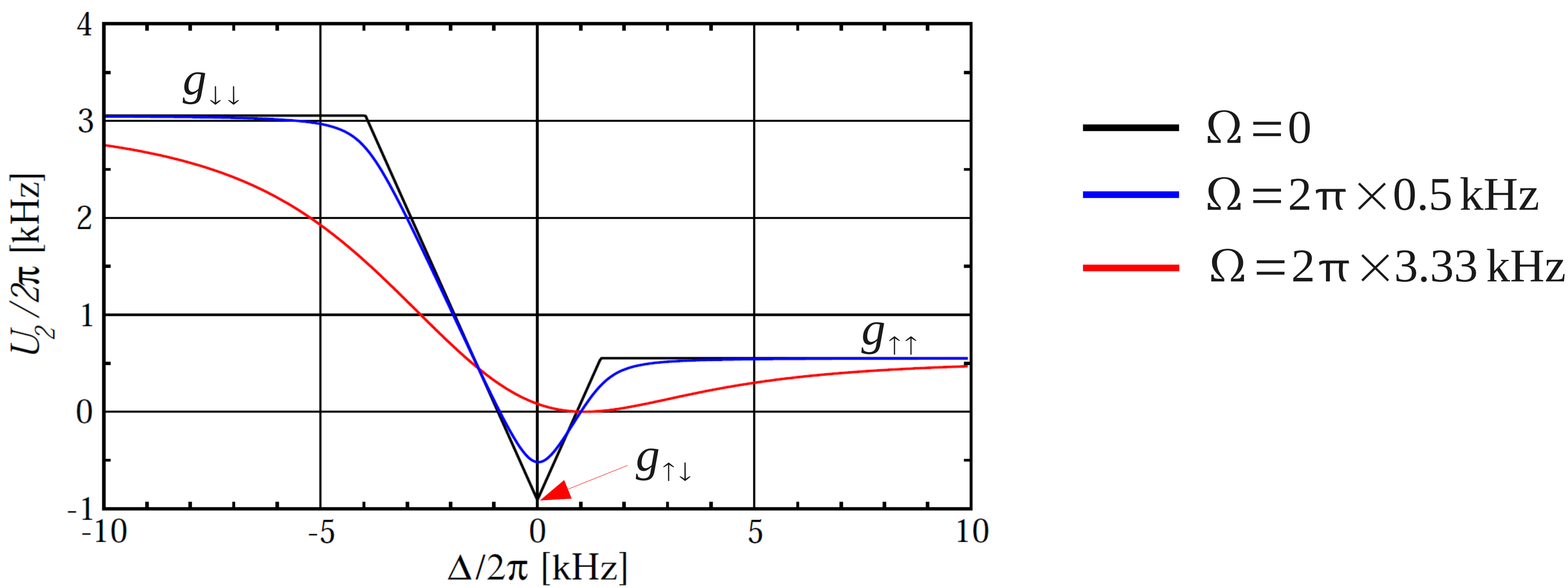
$l_x = l_y = l_z = 86$ nm

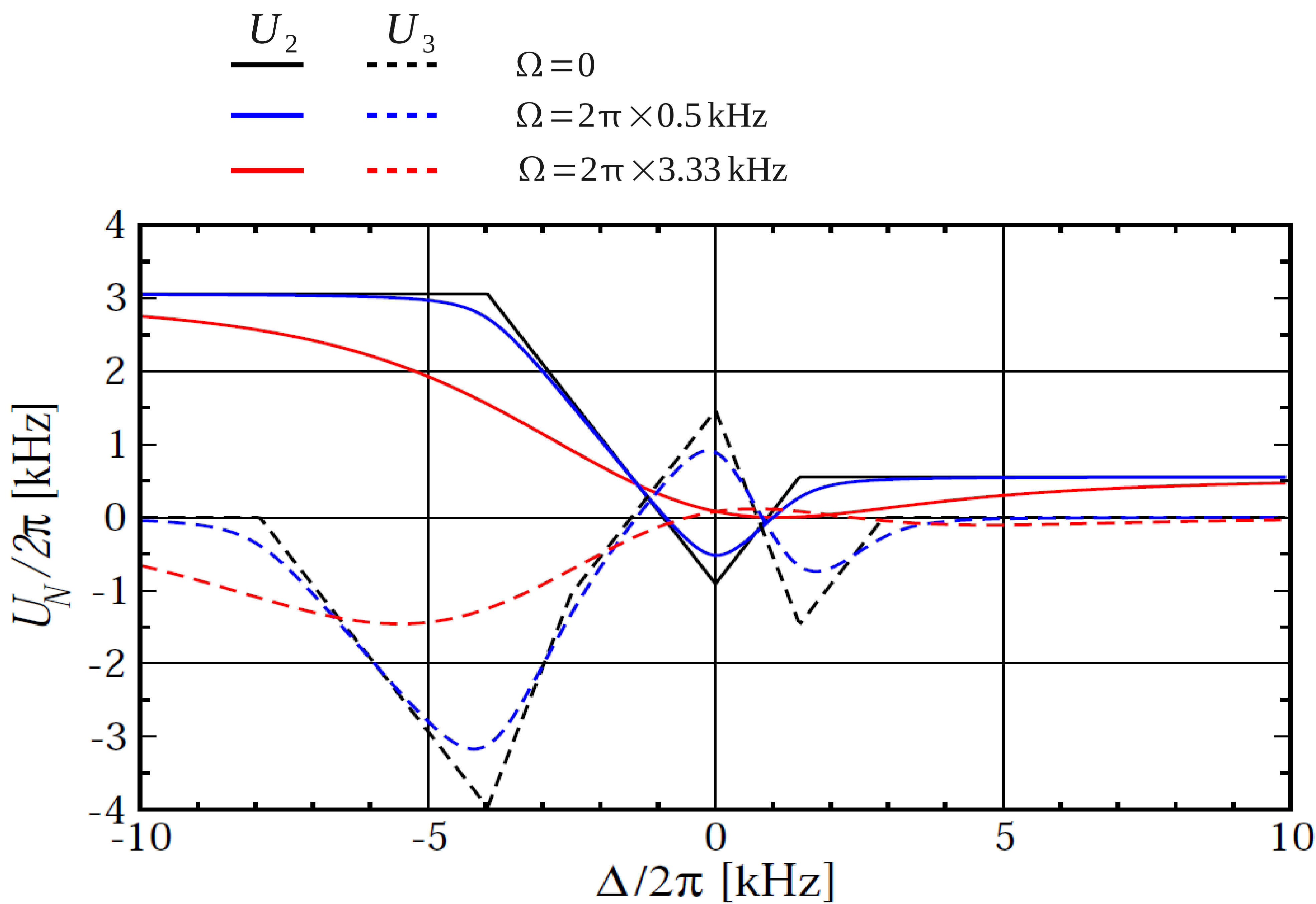
tunneling amp. $= 2\pi \times 30$ Hz

$a_{\downarrow\downarrow} = 9.4\text{ nm} \rightarrow g_{\downarrow\downarrow} = 2\pi \times 3.05$ kHz

$a_{\uparrow\uparrow} = 1.7\text{ nm} \rightarrow g_{\uparrow\uparrow} = 2\pi \times 0.55$ kHz

$a_{\uparrow\downarrow} = -2.8\text{ nm} \rightarrow g_{\uparrow\downarrow} = -2\pi \times 0.91$ kHz



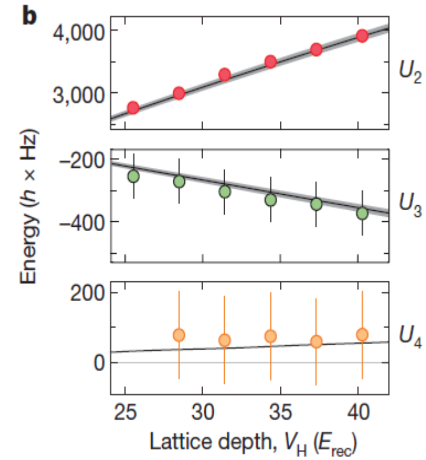


U_2
U_3
$\Omega = 0$
$\Omega = 2\pi \times 0.5$ kHz
$\Omega = 2\pi \times 3.33$ kHz
$U_N/2\pi$ [kHz]
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10
$\Delta/2\pi$ [kHz]

Perspectives

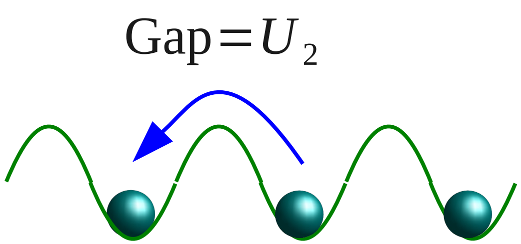
- Bosonic dipoles... Have to wait a bit...
- ^{39}K on a lattice

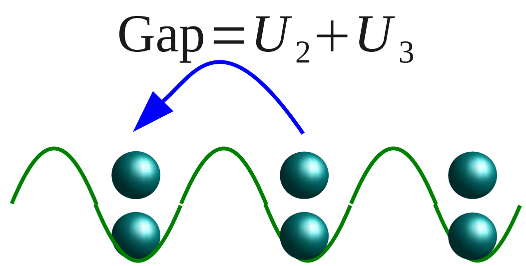
 - collapse and revivals

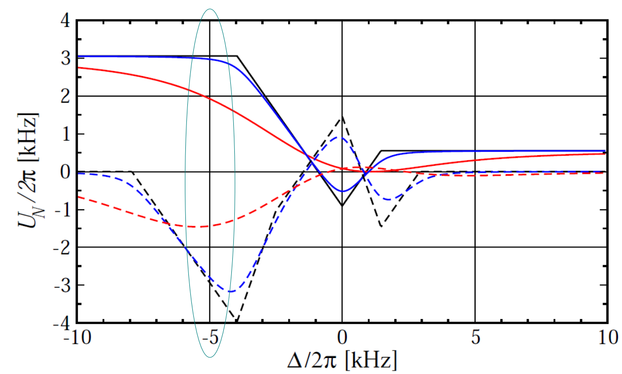


 - solitonic-like self-trapping

 - Mott lobes Chen et al.'08







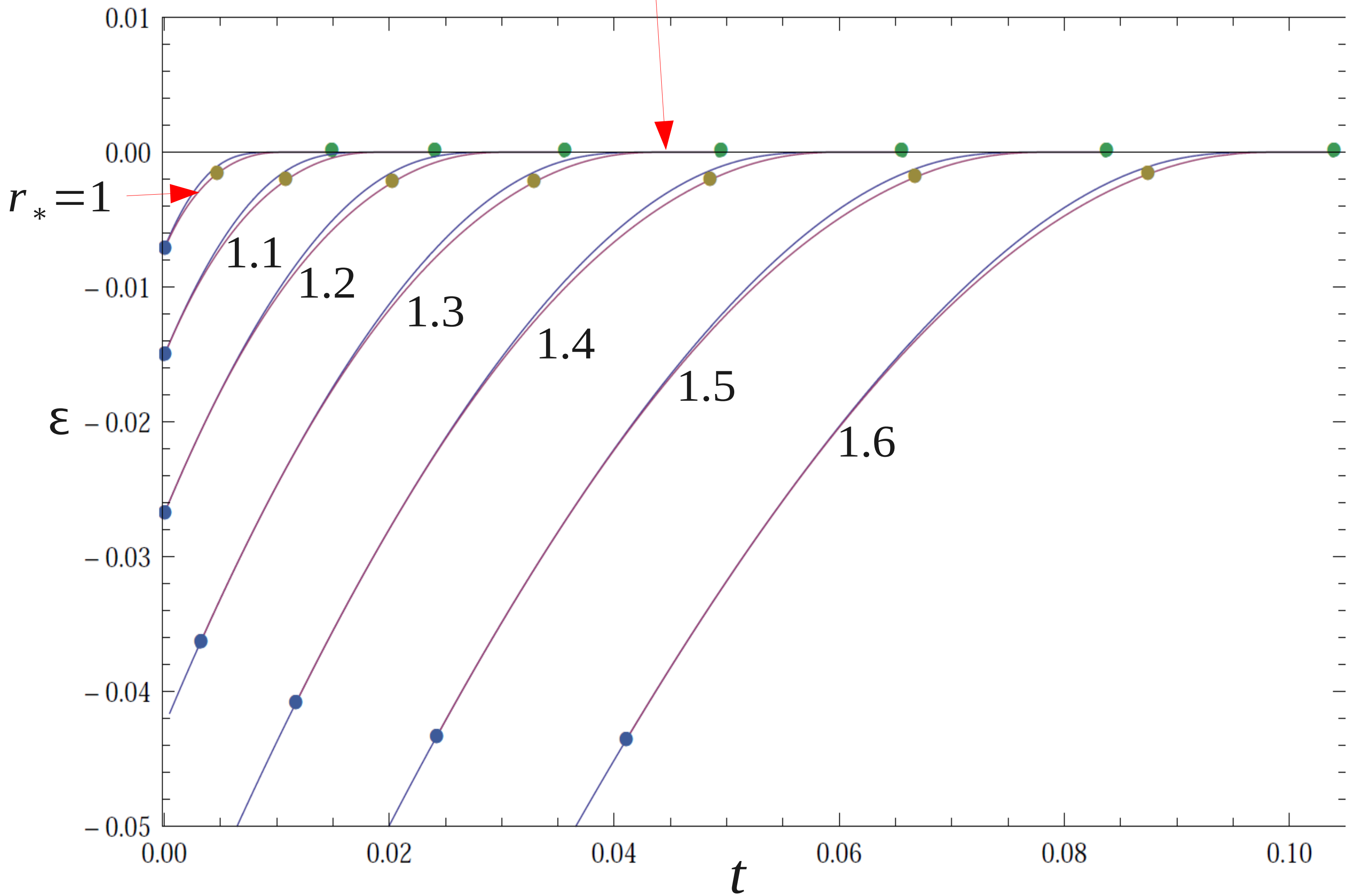
- Strong three-body interaction = large-size off-shell effects!

 Need small t (bilayers) or small $\sqrt{\Omega^2+\Delta^2}$ (RF coupling)

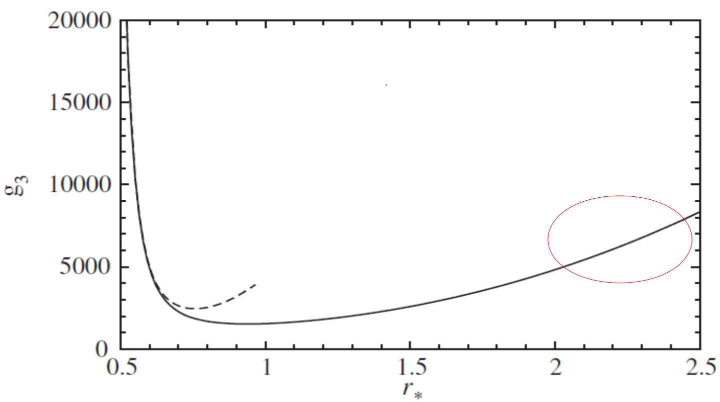
Three-body repulsion and trimers

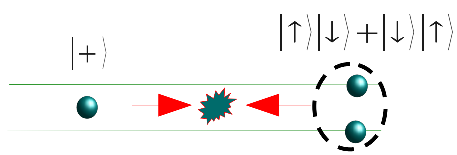
$$\boxed{B_3^{(0)} = 16.522\,688\,(1)\,B_2}$$

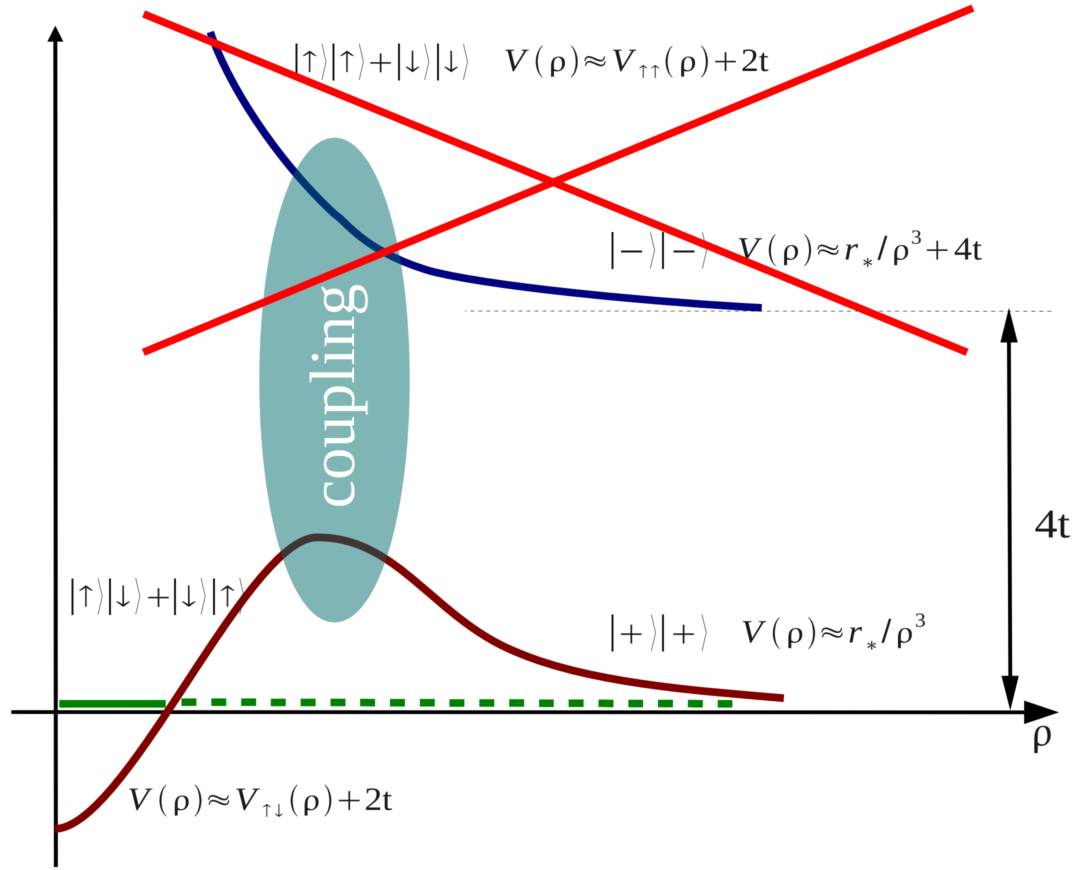
Bruch&Tjon'79,Nielsen et al.'99, Hammer&Son'04



Large r_* – dipolar frustration







Frustration is good!

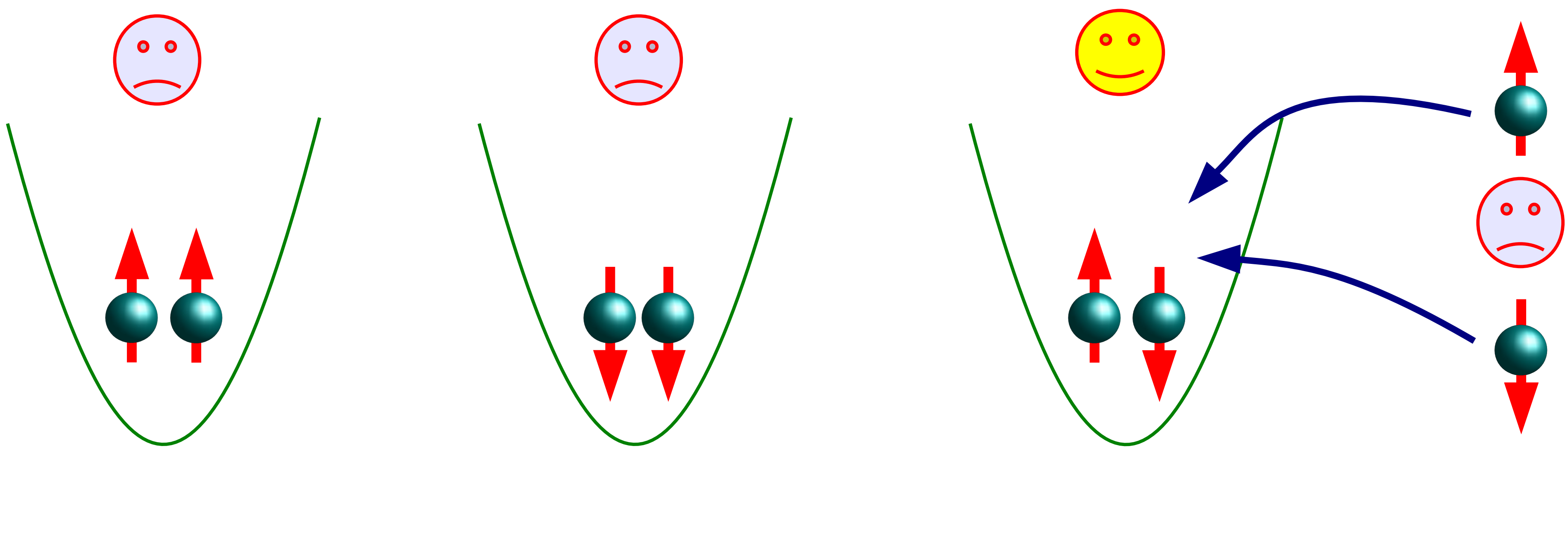
Lattice bosons with on-site Hamiltonian

$$H=\frac{\Delta}{2}\left(b_\downarrow^+ b_\downarrow - b_\uparrow^+ b_\uparrow\right) - \frac{\Omega}{2}\left(b_\uparrow^+ b_\downarrow + b_\downarrow^+ b_\uparrow\right) + \sum_{\sigma,\sigma'} \frac{g_{\sigma\sigma'}}{2} b_\sigma^+ b_{\sigma'}^+ b_\sigma b_{\sigma'}$$

Bilayer dipoles $\rightarrow$ $\Delta=0,\ \Omega=2t,\ g_{\uparrow\uparrow}=g_{\downarrow\downarrow}>0,$ and $g_{\uparrow\downarrow}<0$

Simple example

$$\Delta=\Omega=g_{\uparrow\downarrow}=0,\quad g_{\uparrow\uparrow}=g_{\downarrow\downarrow}>0$$



Optimization problem

$$\Downarrow$$

$$U_2 \text{ and } U_3$$

$$\Downarrow$$

Find parameters for which $\boxed{U_2 = 0 \text{ and } U_3 \rightarrow max}$

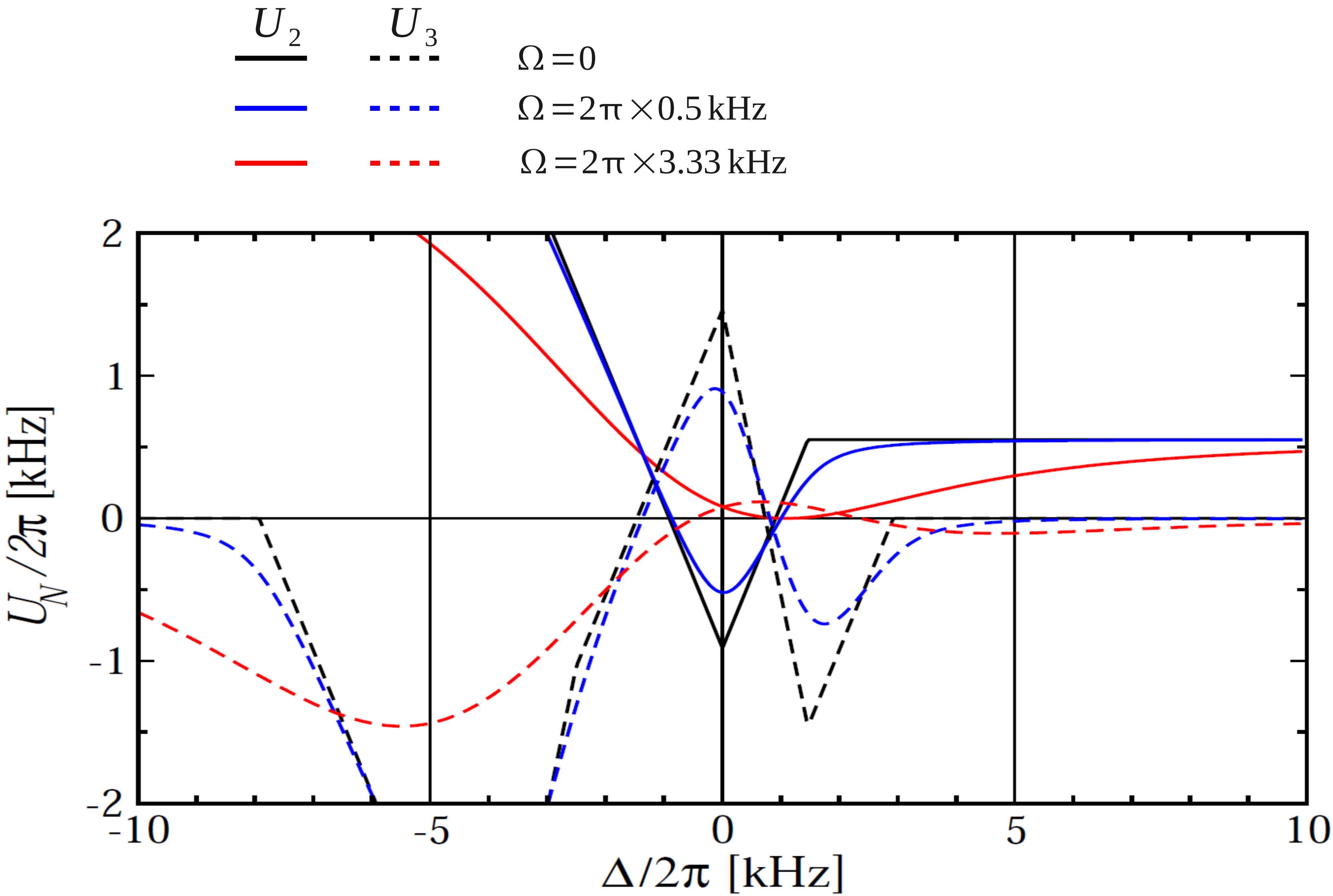
The result is:

$$U_{3,max} = \begin{cases} min(g_{\downarrow\downarrow}, g_{\uparrow\uparrow}), & |g_{\downarrow\downarrow} - g_{\uparrow\uparrow}| > -g_{\uparrow\downarrow} \\ max(g_{\downarrow\downarrow}, g_{\uparrow\uparrow}) + g_{\uparrow\downarrow}, & |g_{\downarrow,\downarrow} - g_{\uparrow,\uparrow}| < -g_{\uparrow\downarrow} \end{cases}$$

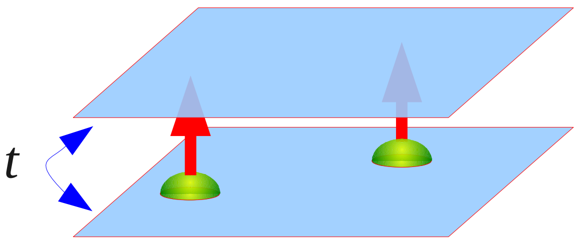
reached for

$$\Omega = 0 \;\; \&\& \;\; \Delta = g_{\uparrow\downarrow}\, sign(g_{\downarrow\downarrow} - g_{\uparrow\uparrow})$$

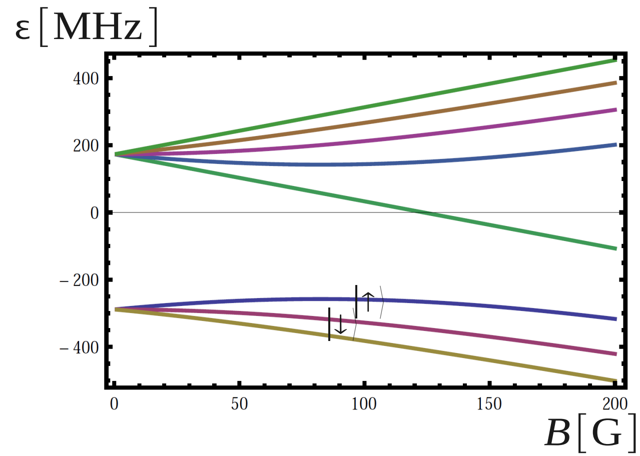
$\boxed{U_{3,max} > 0 \text{ requires } min(g_{\downarrow\downarrow}, g_{\uparrow\uparrow}) > 0 \;\; \&\& \;\; -max(g_{\downarrow\downarrow}, g_{\uparrow\uparrow}) < g_{\uparrow\downarrow} < 0}$


U_2
U_3
$\Omega = 0$
$\Omega = 2\pi \times 0.5\,\mathrm{kHz}$
$\Omega = 2\pi \times 3.33\,\mathrm{kHz}$
$U_N/2\pi$ [kHz]
$\Delta/2\pi$ [kHz]
2
1
0
-1
-2
-10
-5
0
5
10

Same ideas applied to hyperfine states of an atom



^{39}K: |F=1,m_F=0> and |F=1,m_F=-1>



Couple them with RF (~50MHz)

Analog of the interlayer tunneling *t*

Ω = Rabi frequency (~kHz)

Δ = Detuning (~kHz)

$\sqrt{\Omega^2+\Delta^2}$ = spin splitting $\gg T$

Should work in any dimension: 3D, 2D, 1D, 0D (lattice)!