The few-atom problem

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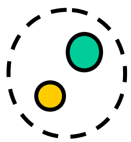
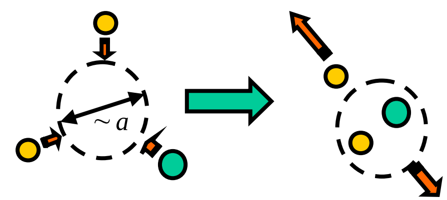
Few-body problem in ultracold gases?

Microscopic level

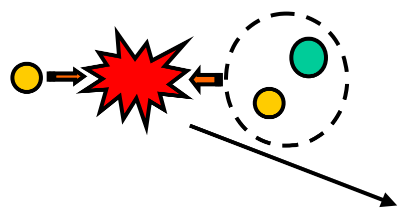
Structure of molecules



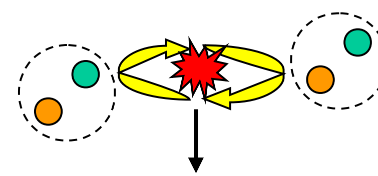
Three-body recombination



Atom-dimer scattering



Dimer-dimer scattering

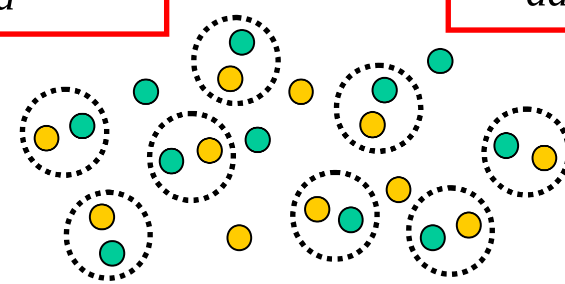


$$a_{ad} = ?$$

$$a_{dd} = ?$$

Macroscopic level

- compressibility, analysis of collapse or phase separation?
- equation of state and test for Monte-Carlo calculations



Path towards many-body case

Exact account of few-body correlations, cluster expansion, virial coefficients…

Size of configuration space

N-body problem in 3D ⇒ 3N (degrees of freedom)

- 3 (translational invariance)

- 3(2) (rotational invariance)

= 3N-6(5)

2-body ⇒ Single coordinate. Radial Schrödinger equation. Clculations possible for realistic potentials

3-body ⇒ 3 coordinates. Involved numerics with simple model potentials.

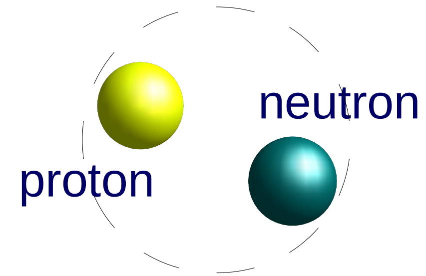
4-body ⇒ 6 coordinates. Reliable solution very difficult even for simple model potentials

Fortunately, many interesting few-body problems can be solved in an elegant way by using the short-range character of interparticle forces

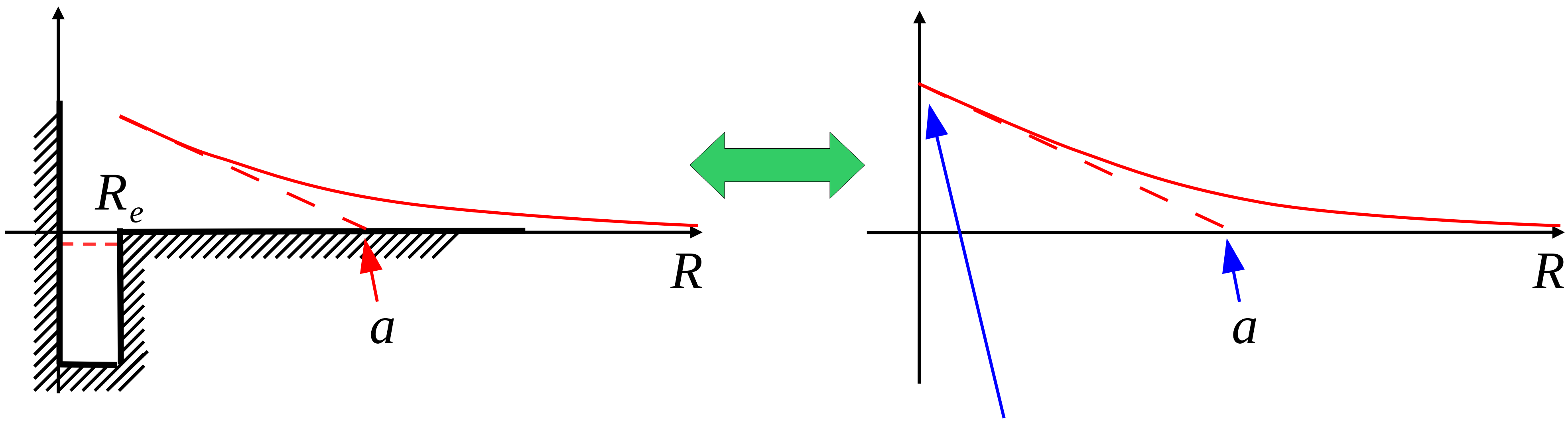
Some history
(of the zero-range approach)

Bethe – Peierls boundary conditions

Bethe and Peierls (1934), “Quantum theory of the diplon”



Diplon (deutron) is a weakly bound dimer with $R_e \approx 10^{-13}\,\mathrm{cm}$, $a \approx 4.5 \cdot 10^{-13}\,\mathrm{cm}$

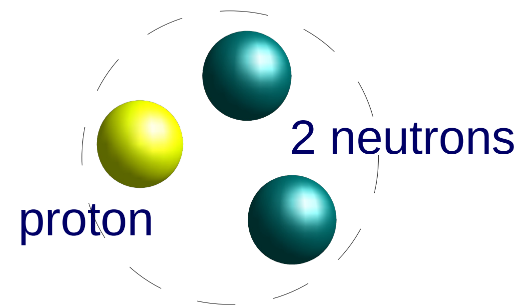


Bethe-Peierls boundary condition

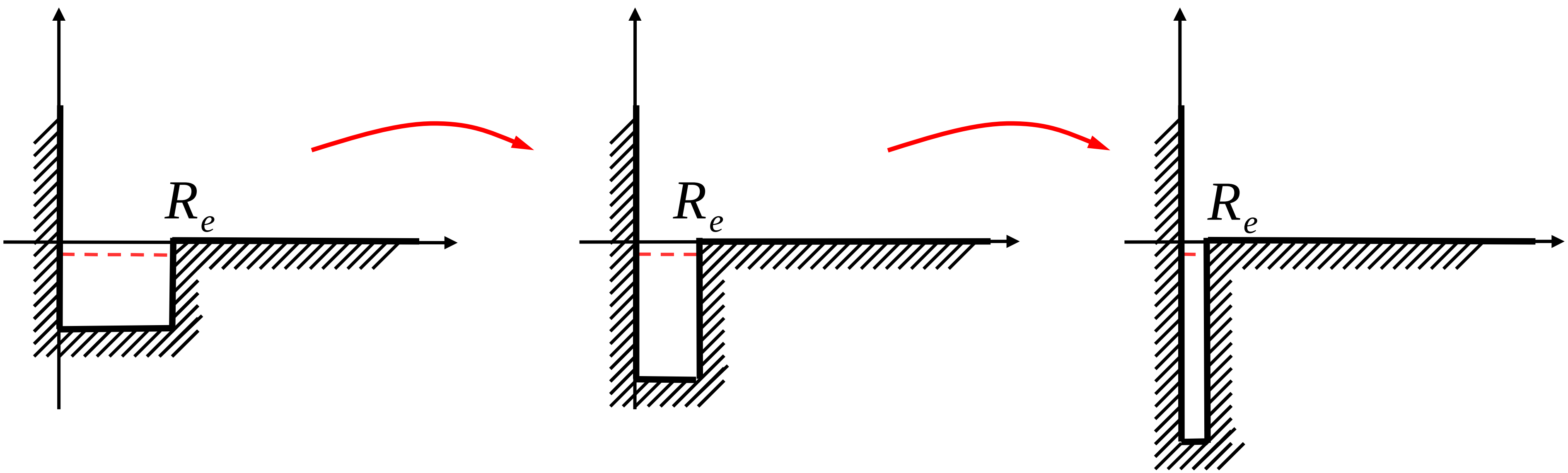
$$\frac{(R\psi(R))'}{R\psi(R)} = -\frac{1}{a} \quad \Longleftrightarrow \quad \psi(R) \underset{R\to 0}{\to} \frac{1}{R} - \frac{1}{a}$$

Thomas effect

Thomas (1935), “The interaction between a neutron and a proton and the structure of ^3H”



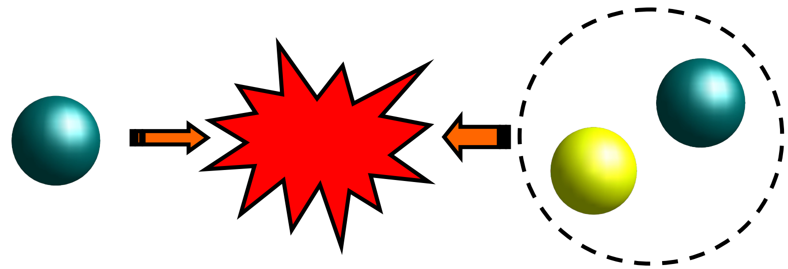
Decrease the range of the proton-neutron potential keeping their binding energy constant



The trimer binding energy tends to infinity! ⇒ Thomas effect or Thomas collapse

Example: three ^{4}He atoms form much deeper bound molecule than two ^{4}He atoms

Neutron-deuteron scattering



Skorniakov and Ter-Martirosian (1957) derived an integral equation for the neutron-neutron-proton 3-body problem in the zero-range approximation. They calculated the neutron - deuteron scattering length.

Exact in the limit $R_e \ll a$

This results are more applicable for the field of ultracold gases because

Nuclear matter:

$R_e \approx 10^{-13}\,\mathrm{cm}\,,\ a \approx 4.5 \cdot 10^{-13}\,\mathrm{cm}$

Ultracold atoms:

$R_e \sim 0.5 \cdot 10^{-6}\,\mathrm{cm}\,,\ a > 10^{-5}\,\mathrm{cm}$

Three Body Problem for Short Range Forces. I. Scattering of Low Energy Neutrons by Deuterons

G. V. SKORNIAKOV AND K. A. TER-MARTIROSIAN

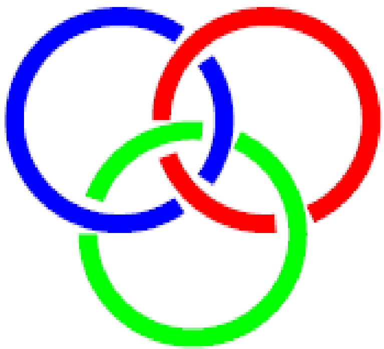
(Submitted to JETP editor July 23, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 775-790 (November, 1956)

$$(\alpha-\gamma_k)\chi(\mathbf{k}) + 8\pi\int\frac{d\mathbf{k}'}{(2\pi)^3}\frac{\chi(\mathbf{k}')}{k^2+k'^2+\mathbf{k}\mathbf{k}'-(ME/\hbar^2)-i\tau}=0. \tag{12}$$

The solution of this equation determines the wave function of the system, in accord with Eq. (11). For states with a definite quantity of momentum, Eq. (12) reduces to an equation for a function which depends on one independent variable, which can be solved numerically.

The idea for this consideration of the three body problem was supplied by L. D. Landau.

Borromean binding

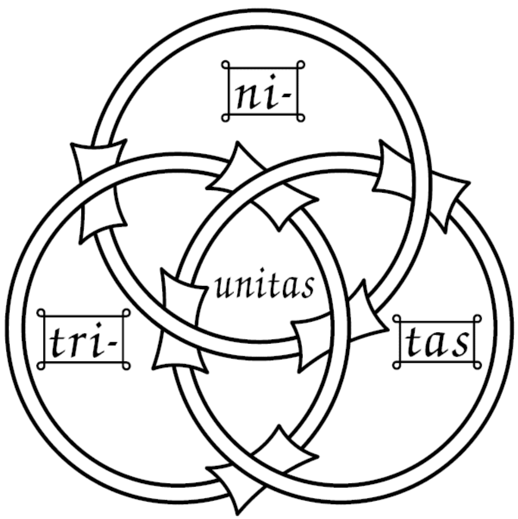


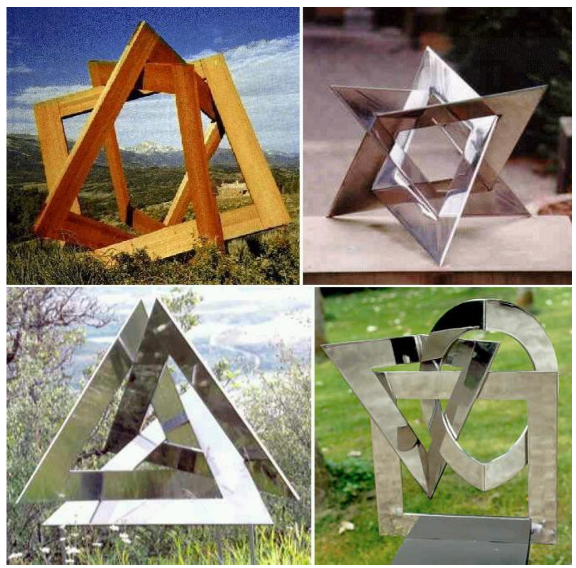
Borromean rings – symbol of strength in unity.
Remove one ring and the other two fall apart

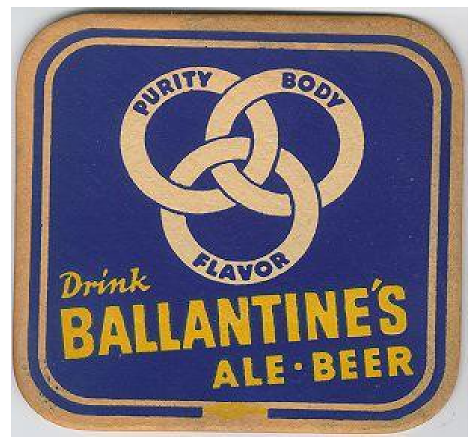
The symbol is used in a number of other applications

Borromean sculptures (John Robinson)

Christian Trinity

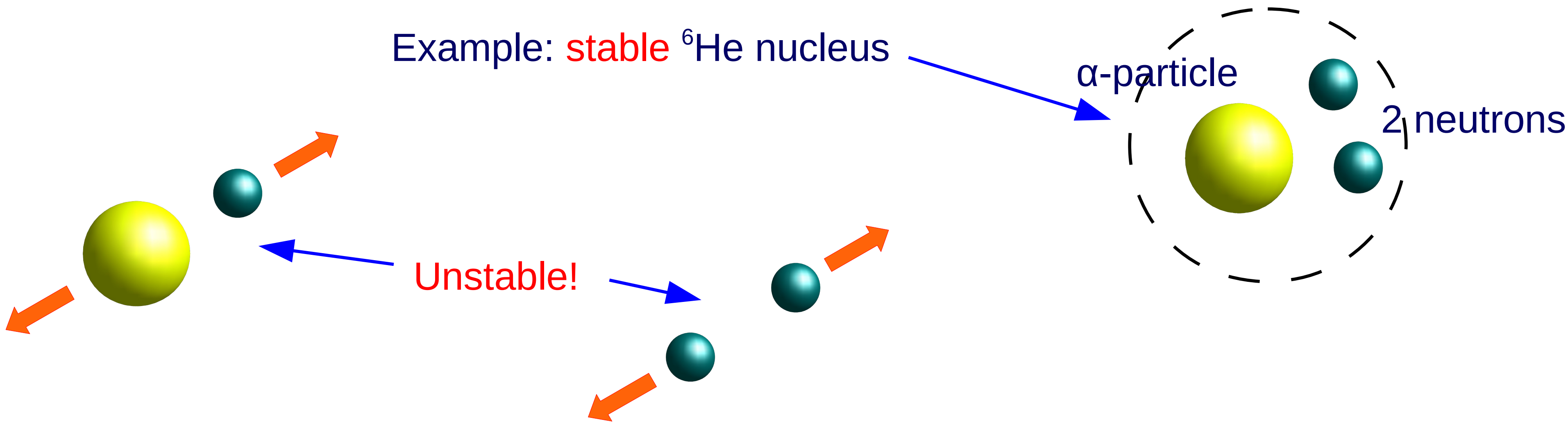




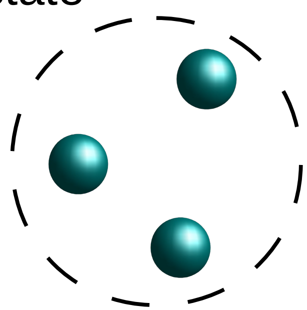


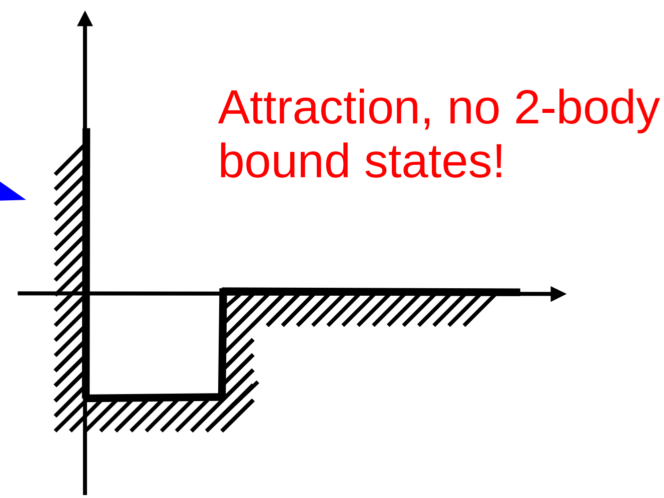
Borromean binding

... in nuclear physics to represent halo nuclei

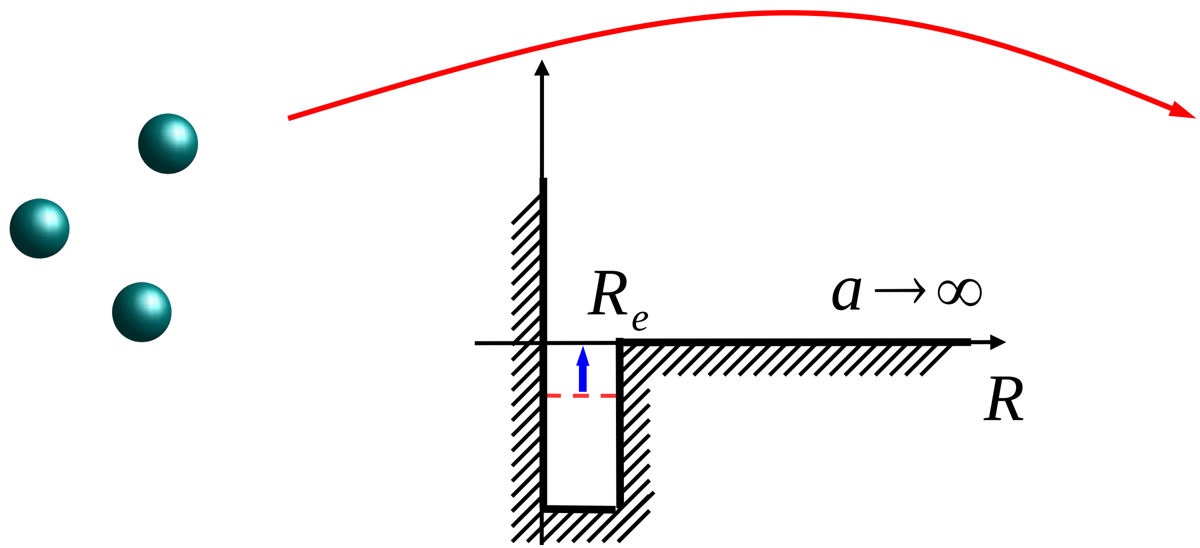


Not a big surprise. Three bosons attracting each other via this potential could form a trimer state





Efimov effect



Efimov (1970) found that the number of trimer states

$$N_{tr} \sim \frac{s_0}{\pi} \log(|a|/R_e) \to \infty$$

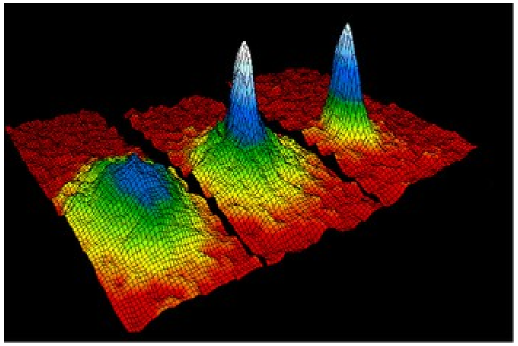
with the accumulation point at $E=0$

Efimov state – weakly bound trimer state

Discrete scaling symmetry: three-body observables depend on R_e , but if $R'_e = R_e \exp(\pm\pi/s_0)$, they do not change

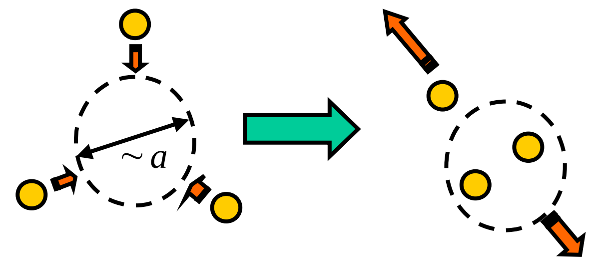
For three identical bosons $s_0 \approx 1.00624$. This number depends on the symmetry (Fermi, Bose) and on the masses of particles

Few-ultracold-atom physics after 1995



BEC of atoms is metastable. Increasing density leads to enhanced 3-body recombination

3-body recombination to a weakly bound state
Theoretical papers:
Fedichev, Reynolds, and Shlyapnikov (1996)
Nielsen and Macek (1999)
Esry, Greene, and Burke (1999)
Bedaque, Braaten, and Hammer (2000)



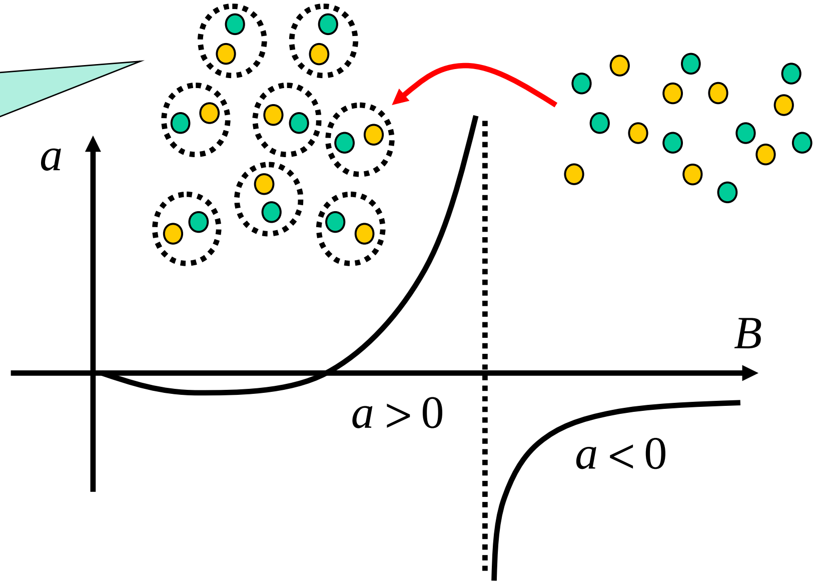
$$\alpha_{rec} \sim \hbar a^4 / m$$

MIT (1999) BEC + Feshbach resonance ⇨ Strong losses

Difficult to reach strongly interacting regime with bosons! Efimov states are not stable because of the relaxation to deep molecular states

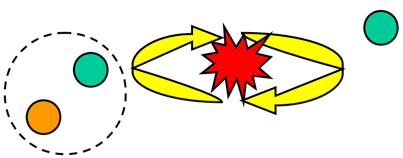
2003: BCS-BEC crossover, molecules

Bose gas of dimers ("BEC side" of the resonance)



Two-component Fermi gas ("BCS side" of the resonance)

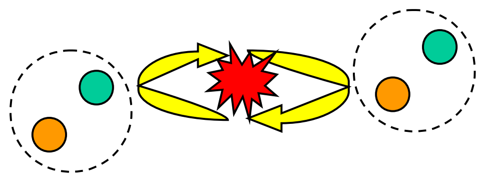
Few-body problem in this case is non-efimovian



$$a_{ad} = 1.2\, a$$

Skorniakov,

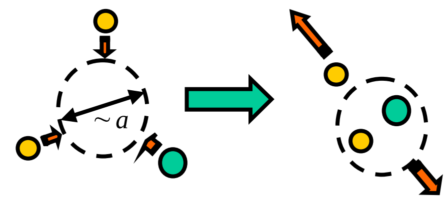
Ter-Martirosian (1957)



$$a_{dd} = 0.6\, a$$

DSP, Salomon,

Shlyapnikov (2003)



$$\alpha_{rec} = 148 \frac{\hbar a^4}{m} \cdot \frac{\bar{\epsilon}}{\epsilon_0}$$

DSP (2003)

Modern history

Efimovian theme:

- Homonuclear bosons (Innsbruck, Florence, Rice, Ramat Gan, Paris)
- Heteronuclear mixtures (Florence, JILA, Heidelberg, Chicago...)
- Three-component Li (Heidelberg, PennState)
- Effective-range effects; wide and narrow resonances
- Van der Waals universality of the three-body parameters

Non-efimovian theme:

- Fermi-Fermi Li-K mixture (Innsbruck, Singapour), Li-Cr (Florence), K-Dy (Innsbruck)...

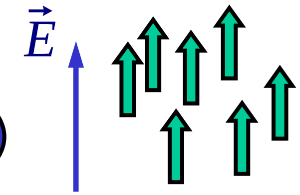
Theory (mostly):

- Mixed-dimensional
- Rabi- and spin-orbit-coupled mixtures

Heavy-heavy-light problem

Heteronuclear mixtures

Dipolar gases

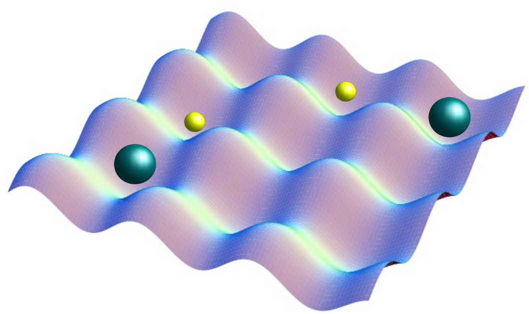


- Lots of interesting stuff

Mass imbalance

Controlled by optical lattices

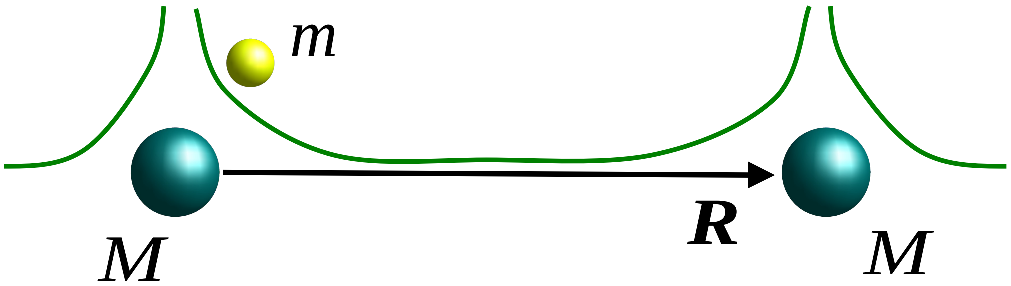
- Strong exchange interaction between molecules
- Crystallization



Few-body problem with different masses and statistics

- Efimov physics
- discrete scaling invariance
- weakly bound dimers
- atom-dimer scattering
- trimers
- tetramers
- pentamers...

3-body problem. Born-Oppenheimer approximation



Effective interaction between heavy atoms is provided by exchange of the fast light particle.
Born-Oppenheimer approximation.
Fonseca, Redish, Shanley (1979)

Light atom wavefunction: $$\psi(\boldsymbol{r}) = \frac{\exp(-\kappa|\boldsymbol{r}-\boldsymbol{R}/2|)}{|\boldsymbol{r}-\boldsymbol{R}/2|} + \frac{\exp(-\kappa|\boldsymbol{r}+\boldsymbol{R}/2|)}{|\boldsymbol{r}+\boldsymbol{R}/2|}$$

Bethe-Peierls boundary condition gives: $$\frac{1}{a} - \kappa + \frac{\exp(-\kappa R)}{R} = 0$$

$$R \ll a \;\Rightarrow\; \kappa \approx \frac{0.567}{R}$$

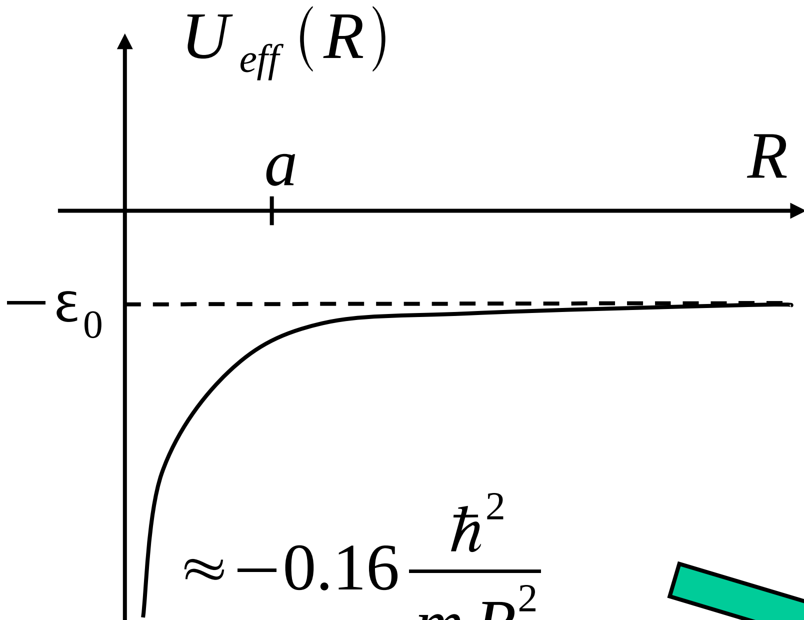
$$U_{eff}(R) = -\frac{\hbar^2 \kappa^2}{2m} \approx -0.16\frac{\hbar^2}{mR^2}$$

$$R \gg a \;\Rightarrow\; \kappa \approx \frac{1}{a}$$

$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$

Solving the Schrödinger equation for the heavy atoms we take into account their statistics:



$$\approx -0.16\frac{\hbar^2}{mR^2}$$

$$\tilde{U}_{eff}(R)\approx U_{eff}+\frac{\hbar^2 l(l+1)}{MR^2}$$

Heavy fermions ⇒ $l=1,3,5\ldots$

Heavy bosons ⇒ $l=0,2,4\ldots$

$$\tilde{U}_{eff}(R)\approx\frac{\hbar^2}{MR^2}\underbrace{\left(l(l+1)-0.16\frac{M}{m}\right)}_{\beta}$$

$R\ll a,\ E=0$ ⇒ $\left(-\partial^2/\partial R^2+\beta/R^2\right)\chi(R)=0$

$\chi(R)=R^{\nu}$ $\qquad \nu_{\pm}=1/2\pm\sqrt{\beta+1/4}$

Efimov effect. Discrete scaling symmetry

$$R \ll a\,, E=0 \;\Rightarrow\; \left(-\partial^2/\partial R^2 + \beta/R^2\right)\chi(R)=0$$

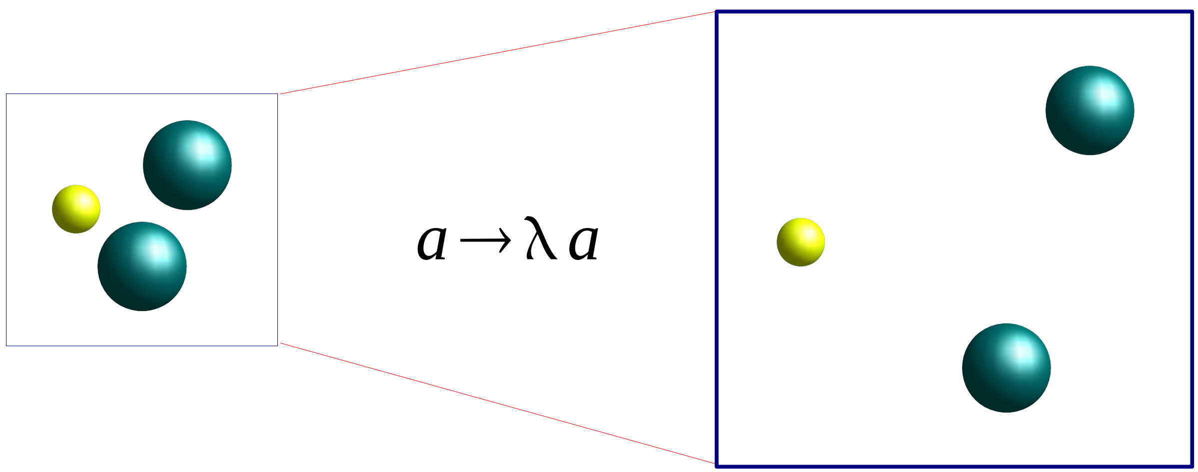
$$\chi(R) \propto R^{\nu} \qquad \nu_{\pm} = 1/2 \pm \sqrt{\beta + 1/4}$$

$$\beta = \left| l(l+1) - 0.16\frac{M}{m} \right| < -1/4 \;\Rightarrow\; \nu_{\pm} = 1/2 \pm i s_0 \;\Rightarrow\; \chi(R) \propto \sqrt{R}\sin(s_0 \log R/r_0)$$

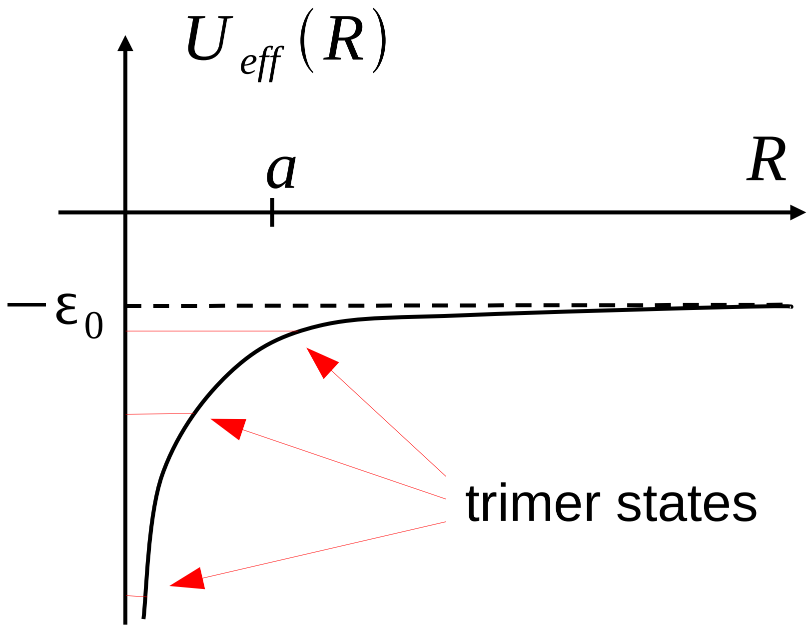
"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wavefunction. Infinite number of trimer states. Efimov effect

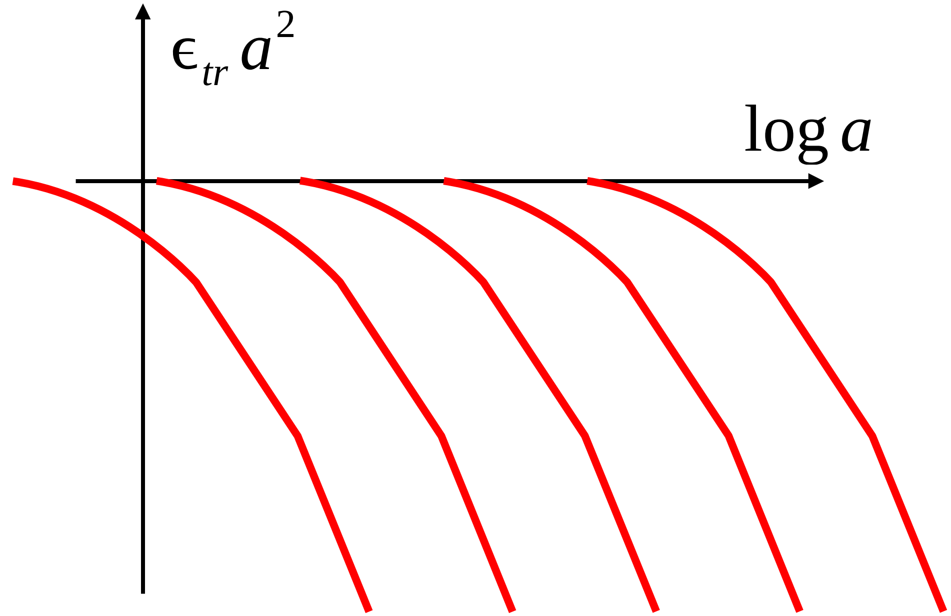
Discrete scaling symmetry. Multiplicative factor $\lambda = \exp(\pi/s_0)$

$$\chi(\lambda R) \propto \chi(R)$$

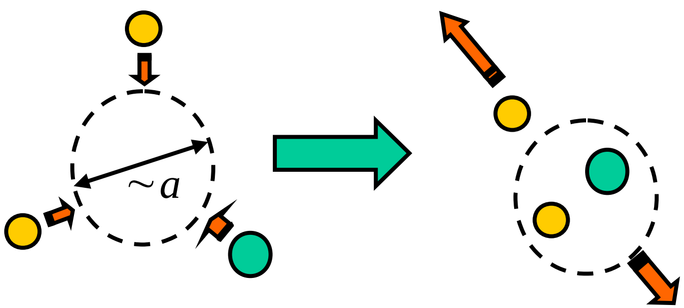


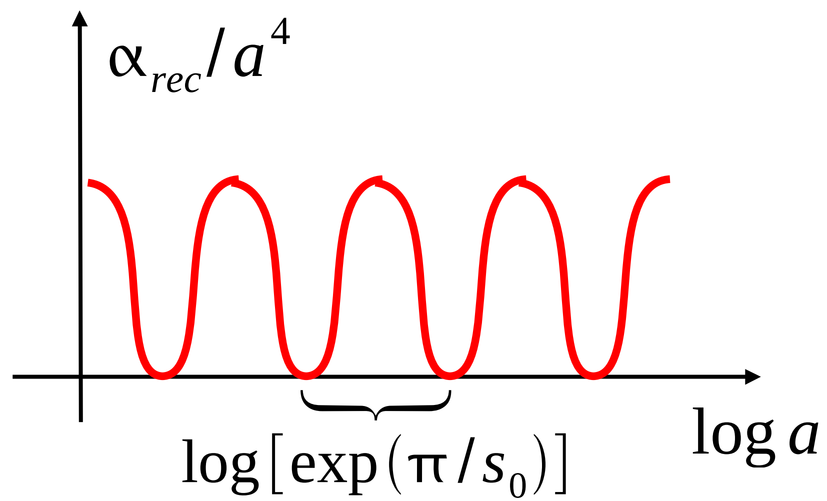
Manifestation of discrete scaling symmetry



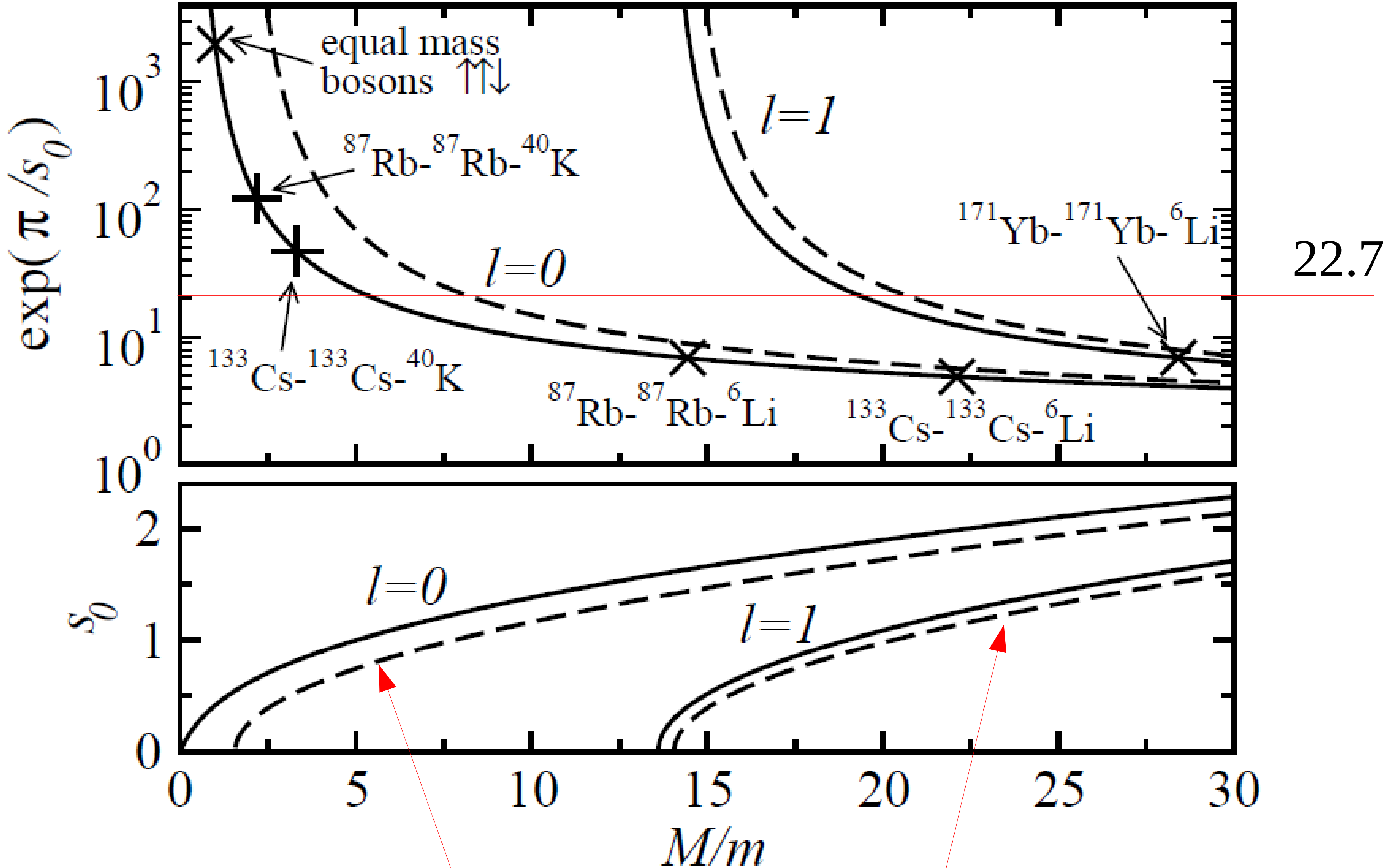


3-body recombination



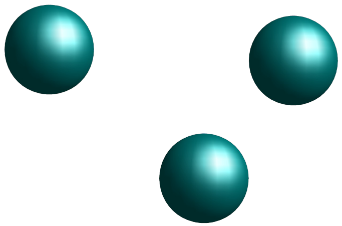


Values of s_0 and the scaling parameter $\lambda = \exp(\pi / s_0)$



$$s_0(BO) = \sqrt{-\beta - 1/4} = \sqrt{-l(l+1) + 0.16\, M/m - 1/4}$$

Observation of discrete scaling symmetry



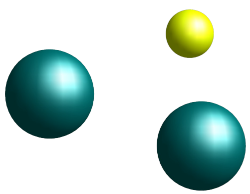
Cs-Cs-Cs, K-K-K, Li-Li-Li → $\lambda \approx 23$

Innsbruck, Florence, Heidelberg, PennState, Rice, Ramat Gan

Two Efimov features observed in Innsbruck (Huang et al (2014))

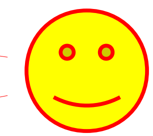
Rb-Rb-K → $\lambda \approx 120$

Florence, JILA



Rb-Rb-Li and Yb-Yb-Li → $\lambda \approx 7$

Cs-Cs-Li → $\lambda \approx 5$

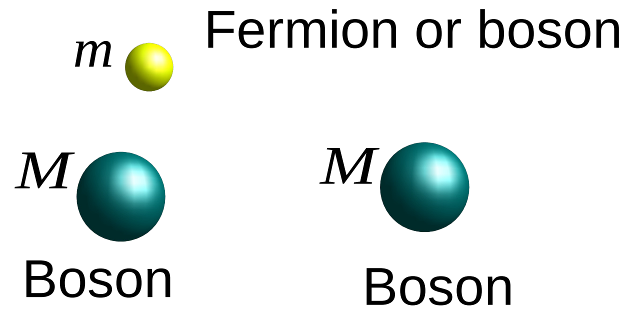


(Heidelberg, Chicago)

~3 Efimov features observed in 2014

Recombination loss rates in a cold mixture:

Helfrich, Hammer, Petrov (2010)



Rate constant for three-body recombination to a weakly bound molecular level →

$$\alpha_s = C(M/m)\frac{\sin^2[s_0 \log(a/a_{0*})] + \sinh^2\eta_*}{\sinh^2(\pi s_0 + \eta_*) + \cos^2[s_0 \log(a/a_{0*})]}\frac{\hbar a^4}{m}$$

… and to deeply bound states →

$$\alpha_d(a>0) = C(M/m)\frac{\coth(\pi s_0)\sinh(\eta_*)\cosh(\eta_*)}{\sinh^2(\pi s_0 + \eta_*) + \cos^2[s_0 \log(a/a_{0*})]}\frac{\hbar a^4}{m}$$

$$\alpha_d(a<0) = C(M/m)\frac{\coth(\pi s_0)\sinh(\eta_*)\cosh(\eta_*)}{\sinh^2\eta_* + \cos^2[s_0 \log(|a|/a_{0*})]}\frac{\hbar a^4}{m}$$

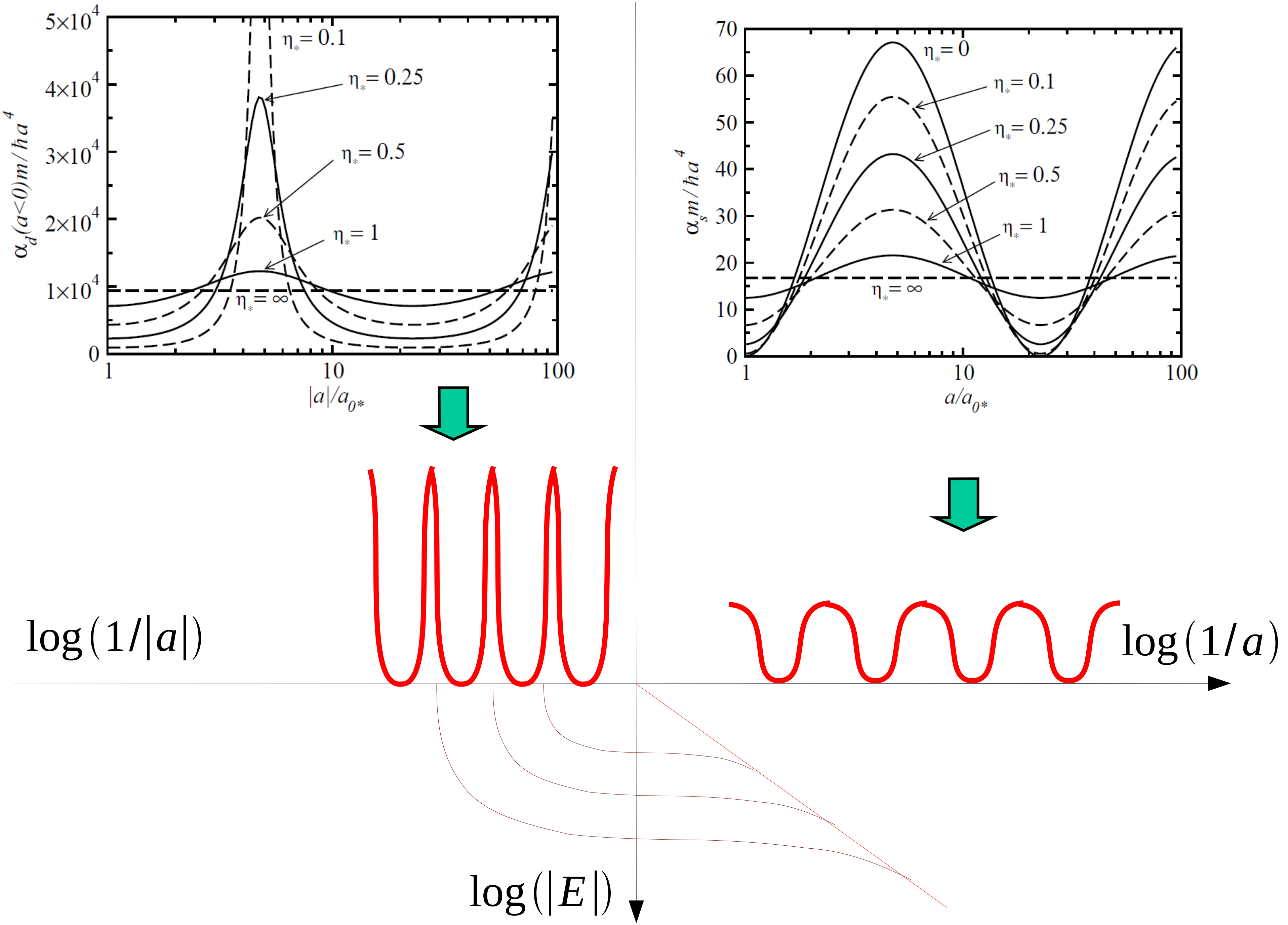
where

$$C(M/m) = 64\left[\left(\frac{M+m}{M}\right)^2 \arcsin\left(\frac{M}{M+m}\right) - \frac{\sqrt{m(2M+m)}}{M}\right]$$

η_* - elasticity parameter

a_{0*} - the value of a where $\alpha_s(a)$ reaches its minimum

Recombination maxima symmetric with respect to the resonance center!



5×10^4
4×10^4
3×10^4
2×10^4
1×10^4
0
η* = 0.1
η* = 0.25
η* = 0.5
η* = 1
η* = ∞
α_d(a<0)m/ħa^4
1
10
100
|a|/a_0*
70
60
50
40
30
20
10
0
η* = 0
η* = 0.1
η* = 0.25
η* = 0.5
η* = 1
η* = ∞
α_s m/ħa^4
1
10
100
a/a_0*
log(1/|a|)
log(1/a)
log(|E|)

Skorniakov and Ter-Martirosian zero-range approach

2D scattering by a curve

$$(-\nabla_{\rho}^{2}-E)\psi(\boldsymbol{\rho})=0$$

Homogeneous Helmholtz
equation with boundary condition

$$[\partial\psi/\partial\boldsymbol{n}]=g(x)\psi$$

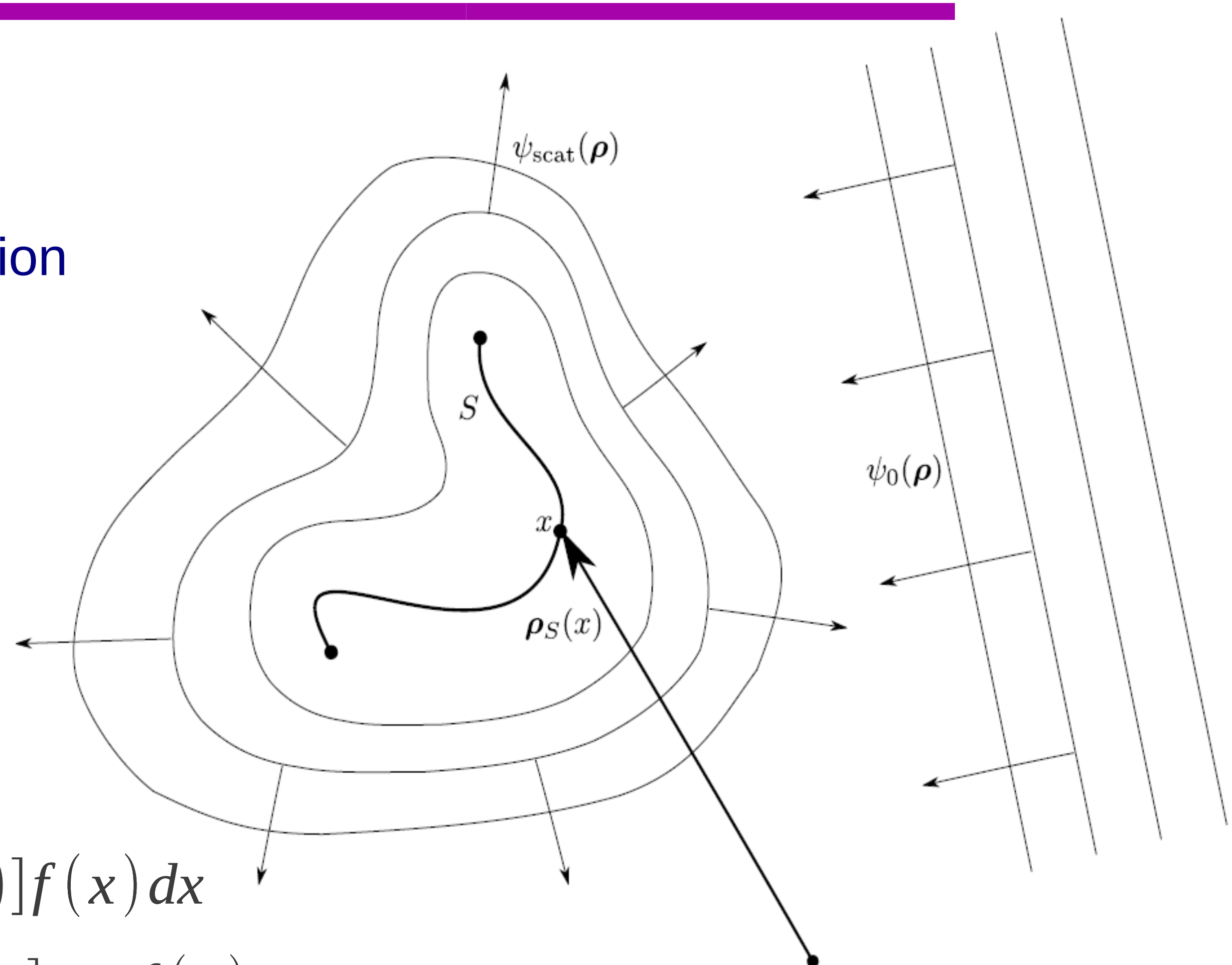
Introduce an auxiliary
function $f(x)$ (source or
distribution of charges)
defined on the boundary

$$\psi(\boldsymbol{\rho})=\psi_0(\boldsymbol{\rho})+\int_S G_E[\boldsymbol{\rho}-\boldsymbol{\rho}_S(x)]f(x)\,dx$$

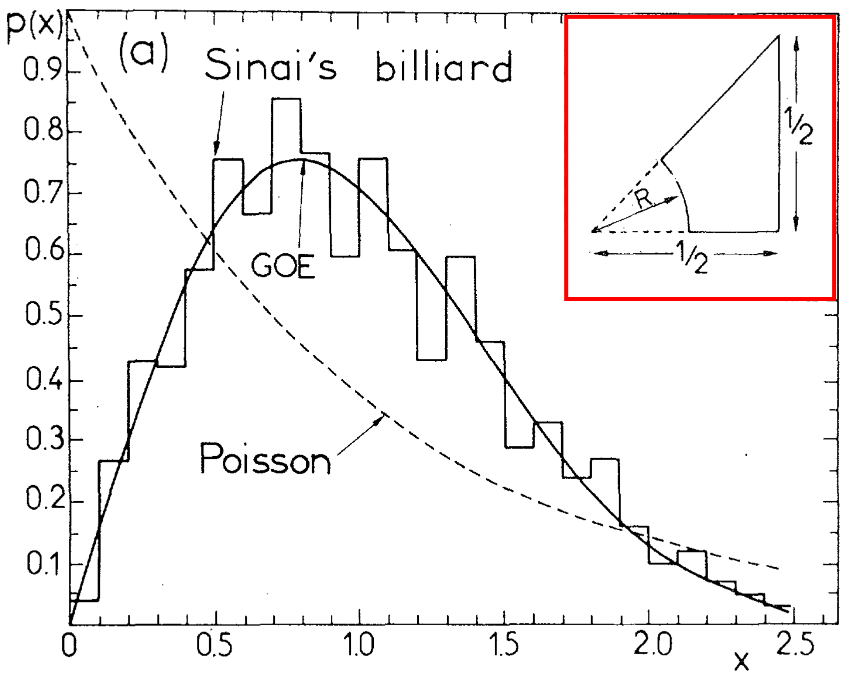
Gauss-Ostrogradsky: $[\partial\psi/\partial\boldsymbol{n}]=-f(x)$

$$\int_S G_E[\boldsymbol{\rho}_S(x)-\boldsymbol{\rho}_S(x')]f(x')\,dx'+f(x)/g(x)=-\psi_0[\boldsymbol{\rho}(x)]$$

Fredholm int. eq.



Example: quantum billiard



Calculation of ~1000 energy levels of Sinai billiard
Bohigas, Giannoni, Schmit (1984)

They call the method Korringa-Kohn-Rostoker and cite Berry...

$$-\nabla_\rho^2 \psi(\boldsymbol{\rho}) = E\,\psi(\boldsymbol{\rho})$$

Helmholtz vs Fredholm ;)

$$\int_S K_E(x, x') f(x')\,dx' = \lambda f(x)$$

- huge 2D configurational space

+ sparse matrix

- difficult to implement boundary conditions

+ more general; works with any potential

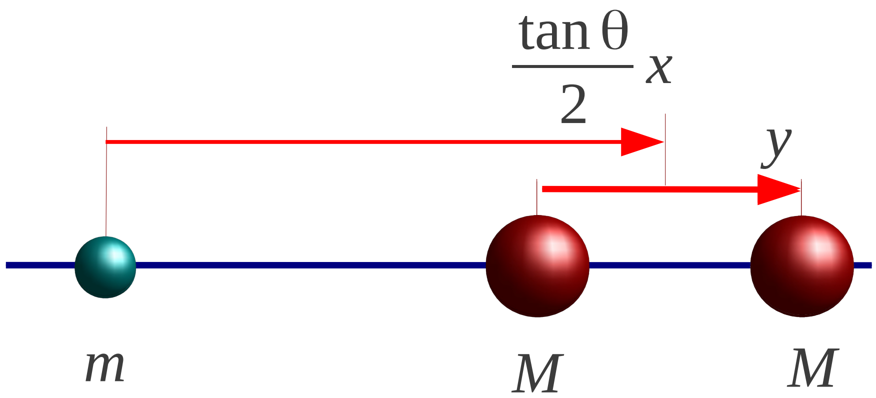
+ 1D configurational space

- full matrix

+ boundary conditions are taken into account naturally

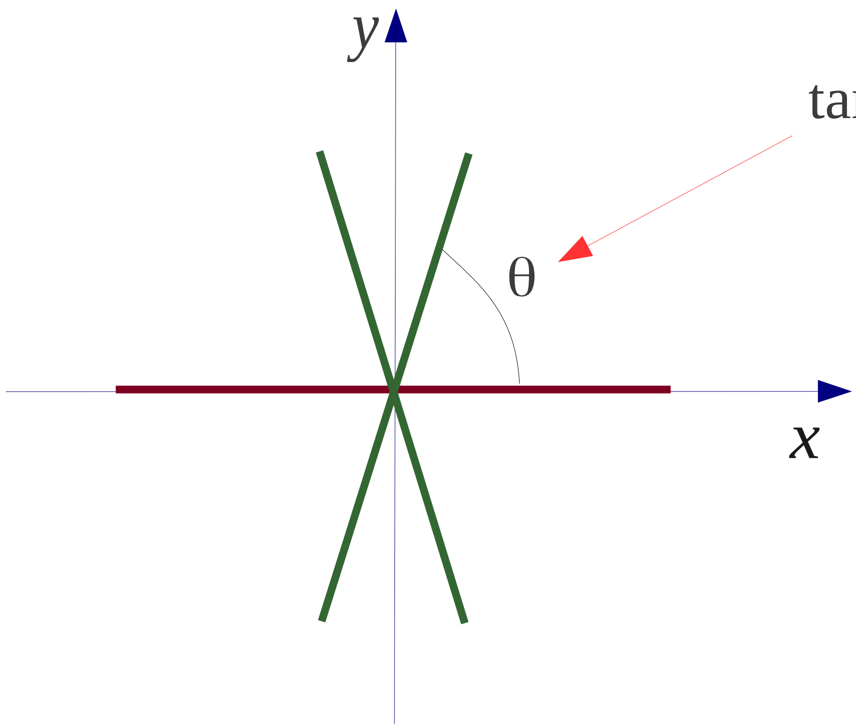
- only billiard

1D three-body = 2D one-body

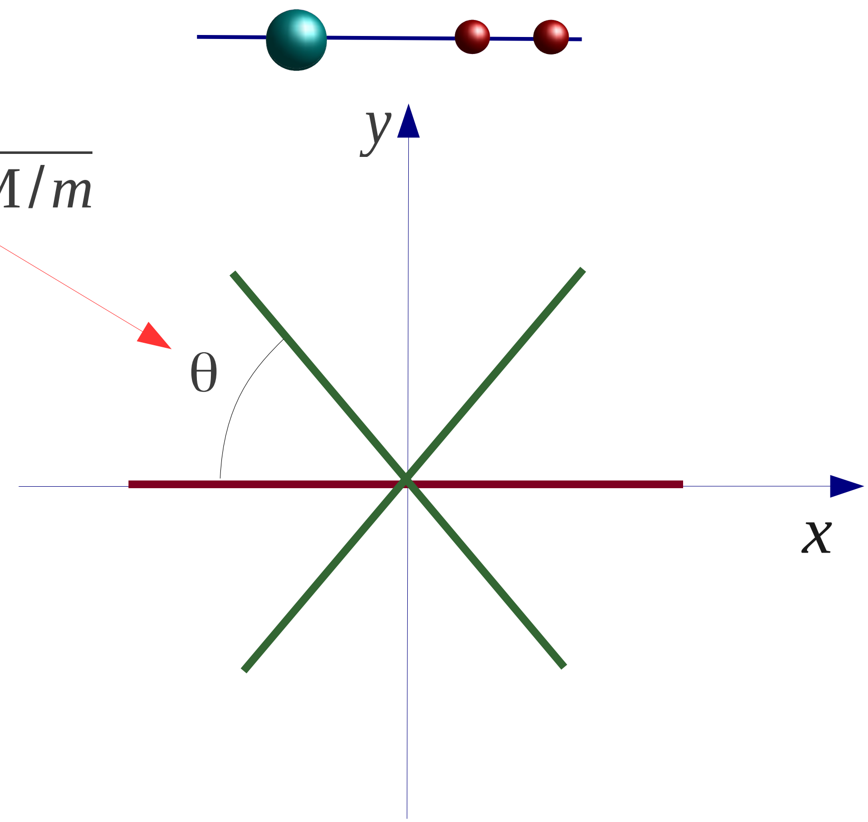


$$\left(-\partial^2/\partial x^2 - \partial^2/\partial y^2 - ME\right)\psi(\boldsymbol{\rho}) = 0$$

$$[\partial\psi/\partial\boldsymbol{n}] = -2\psi/a$$

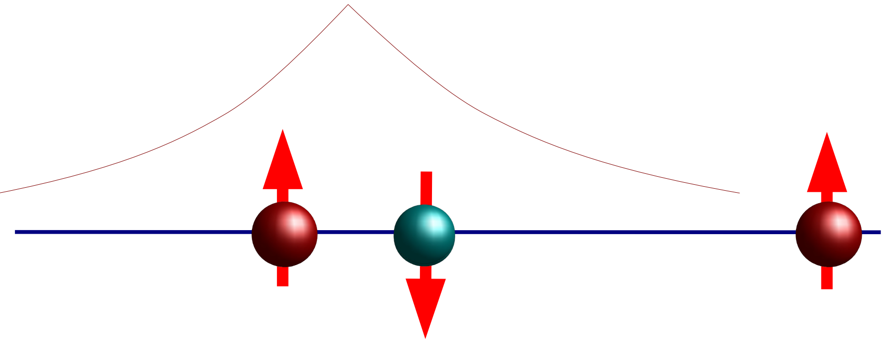


$$\tan\theta = \sqrt{1 + 2\mathrm{M}/m}$$



Some simple applications

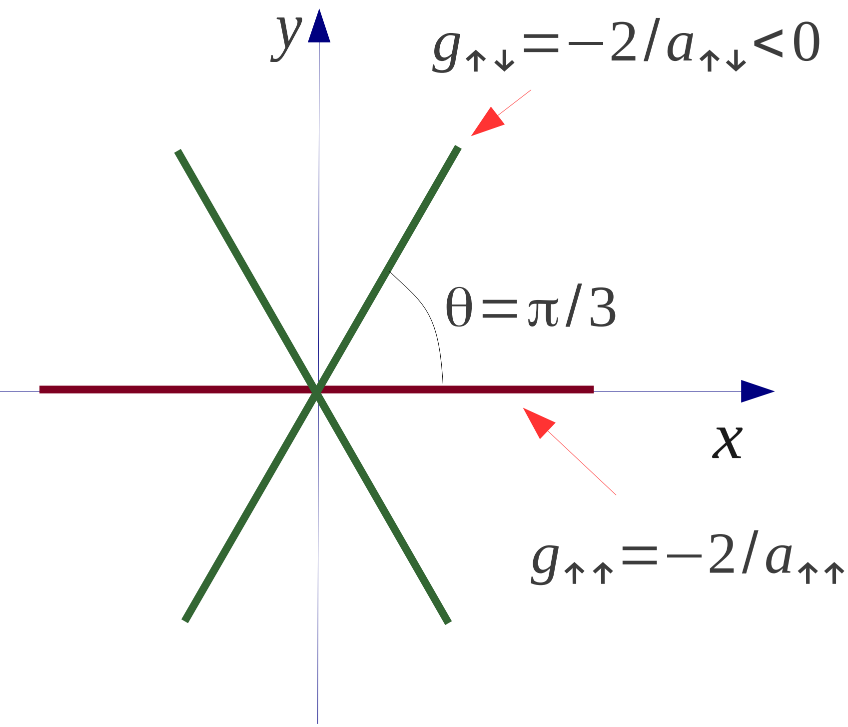
Trimer in a Bose-Bose mixture

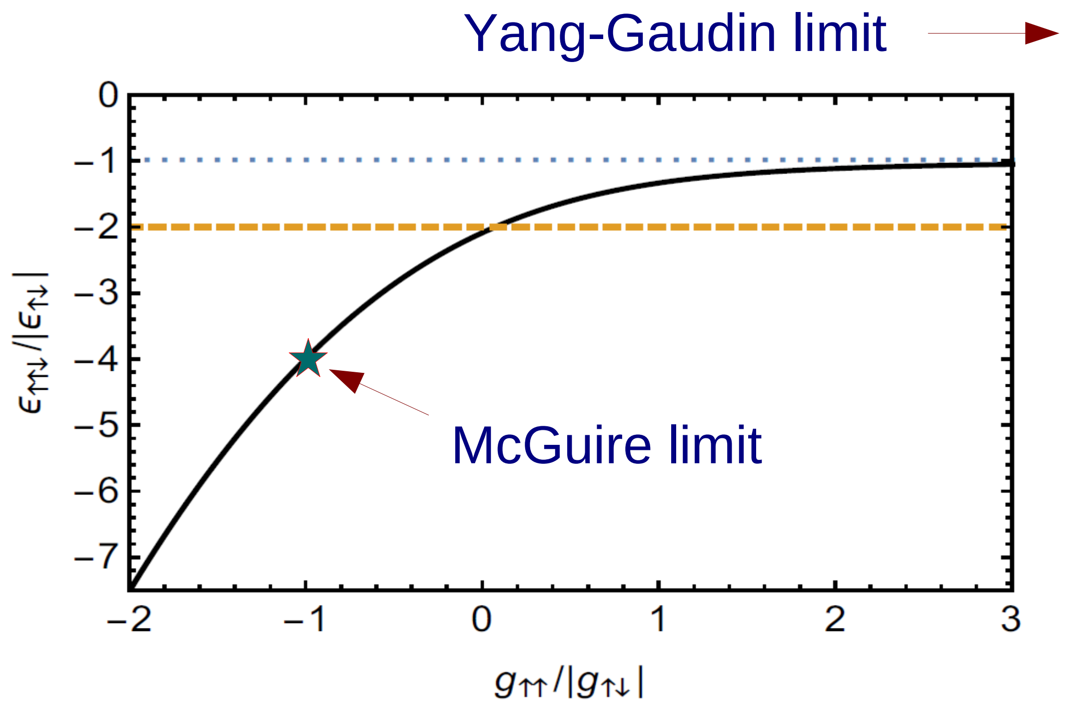


STM equation(s) in momentum space $\left(f = gF\right)$

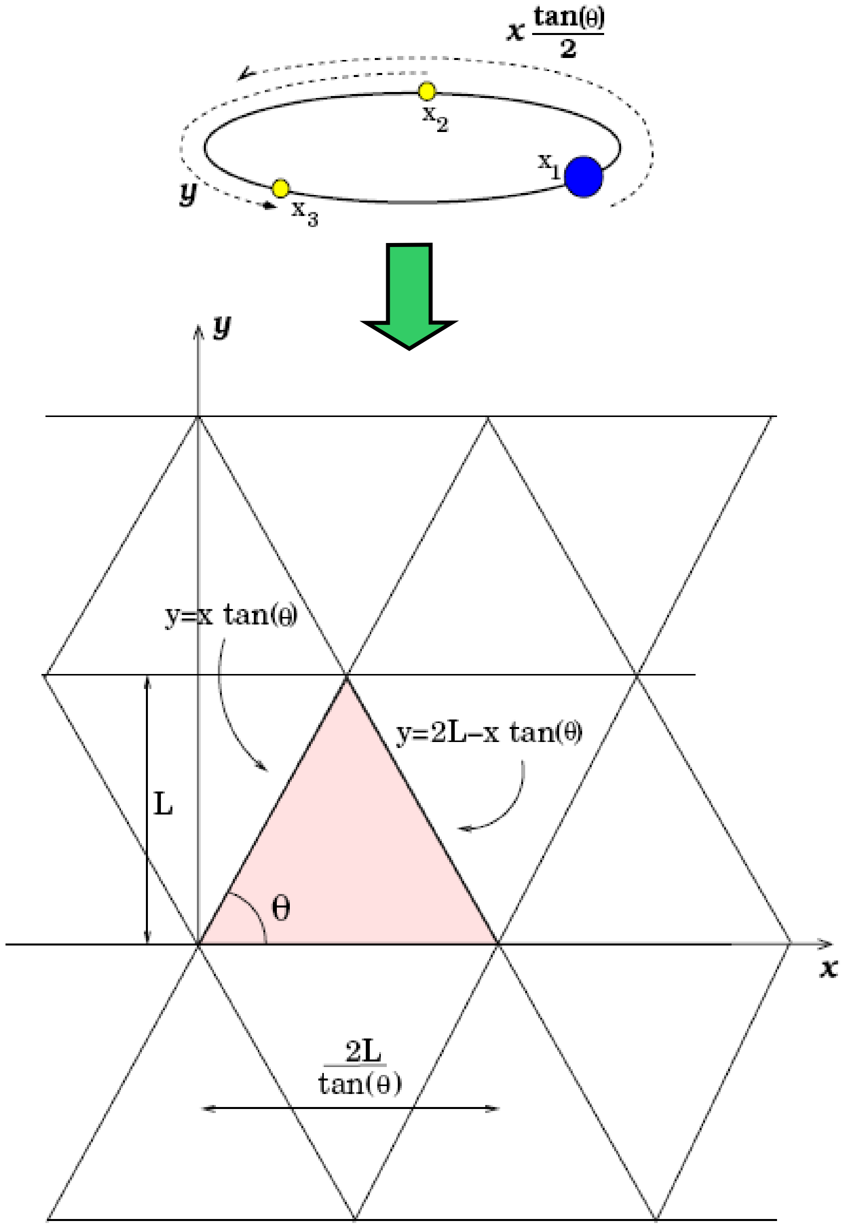
$$\left(1+\frac{g_{\uparrow\uparrow}}{\sqrt{3p^2-4E}}\right)F_{\uparrow\uparrow}(p) = \int \frac{2g_{\uparrow\downarrow}F_{\uparrow\downarrow}(q)}{E-p^2-pq-q^2}\frac{dq}{2\pi},$$

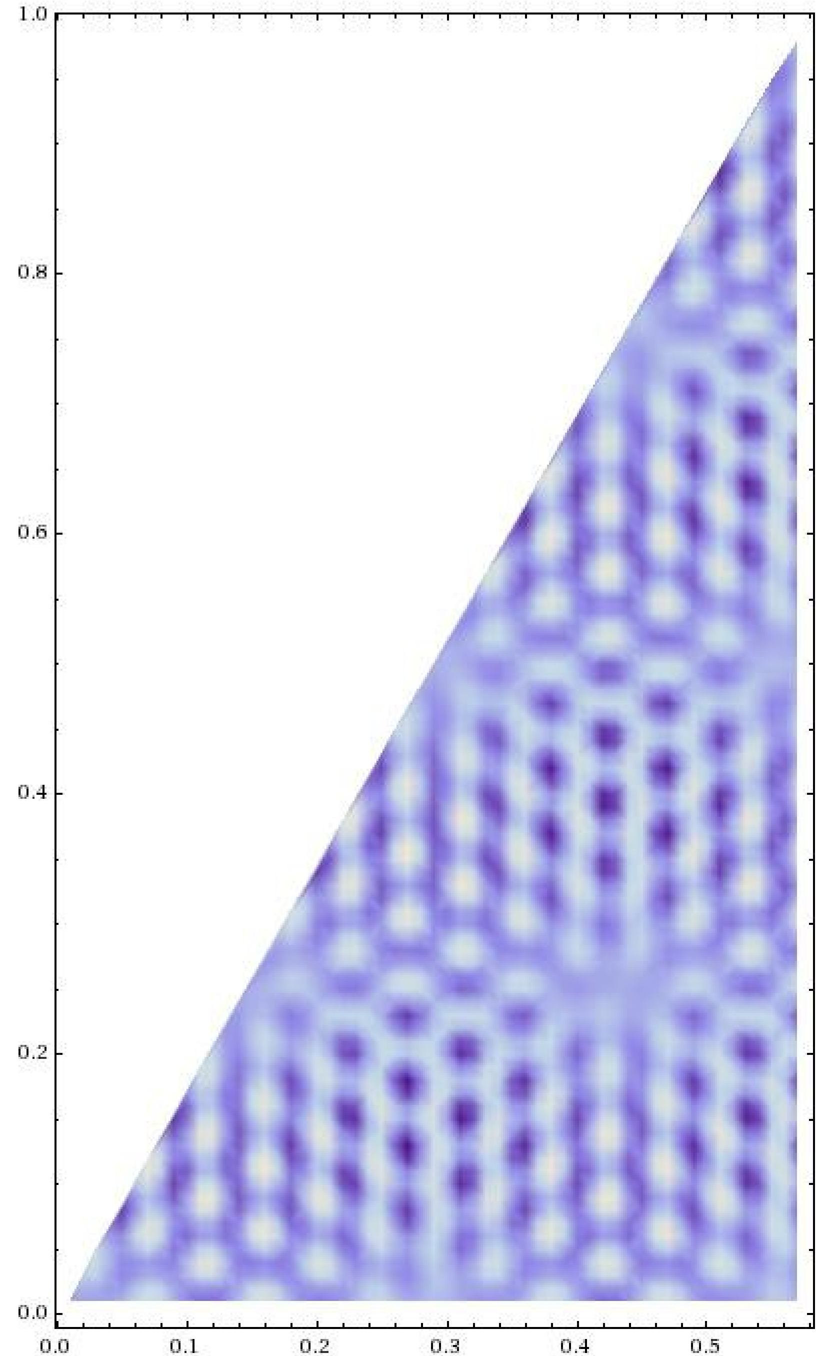
$$\left(1+\frac{g_{\uparrow\downarrow}}{\sqrt{3p^2-4E}}\right)F_{\uparrow\downarrow}(p) = \int \frac{g_{\uparrow\downarrow}F_{\uparrow\downarrow}(q)+g_{\uparrow\uparrow}F_{\uparrow\uparrow}(q)}{E-p^2-pq-q^2}\frac{dq}{2\pi}$$

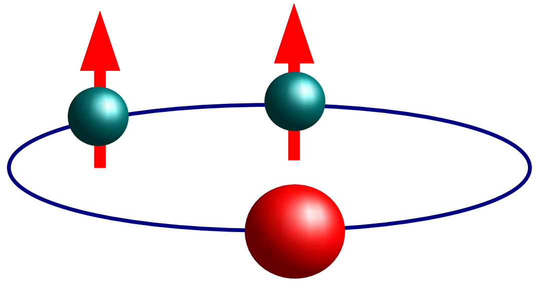




Equal-mass *up-up-down* fermions (Yang-Gaudin model)



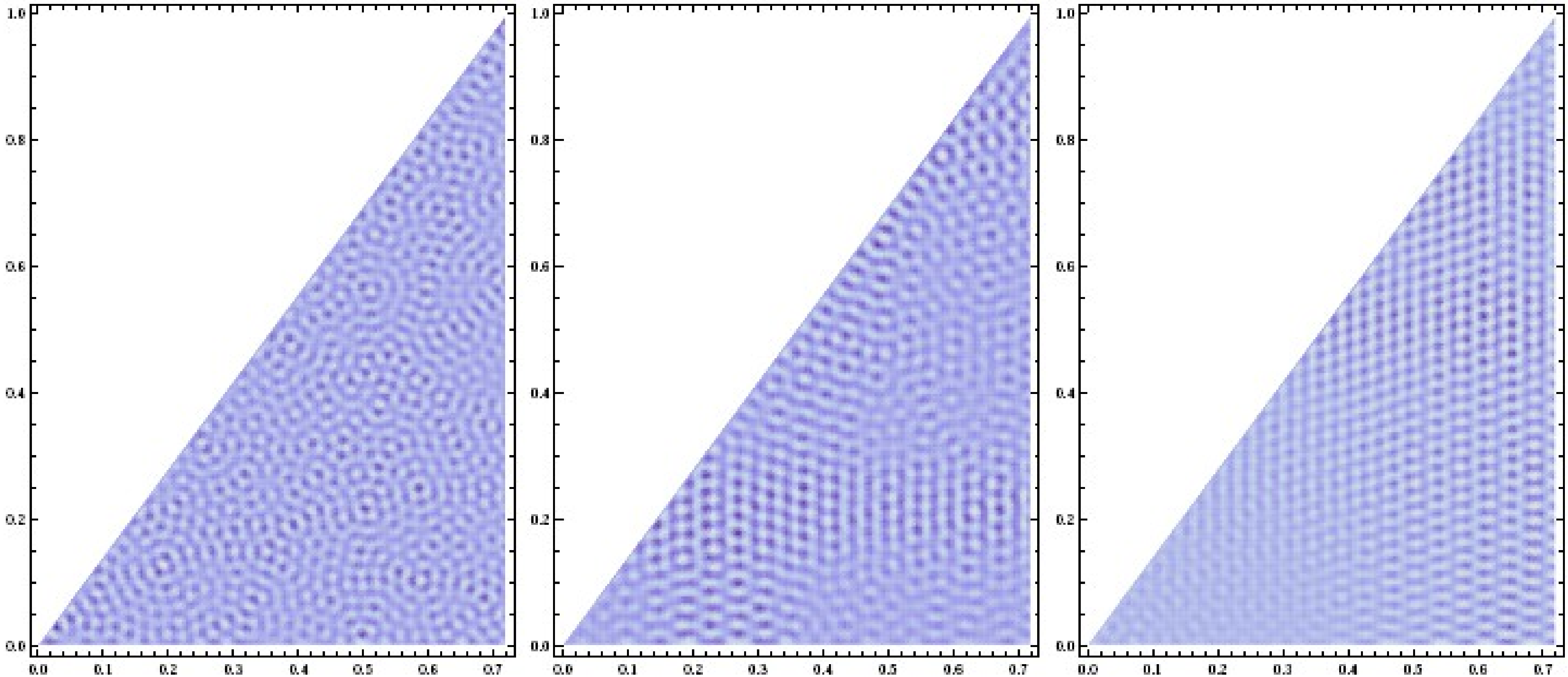




Rb-K-K Bose-Fermi-Fermi, non integrable

M. Colome-Tatche and DSP (2011)

State number 1835 1296 1676



Comparison with Born-Oppenheimer

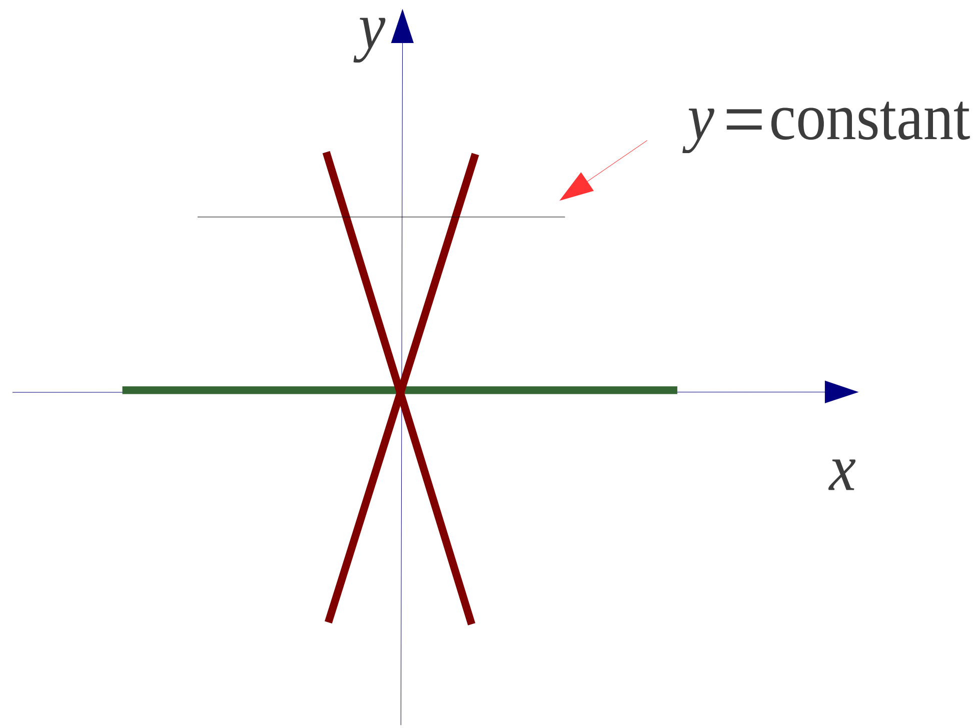
$$(-\partial^2/\partial x^2 - \partial^2/\partial y^2 - ME)\psi(\boldsymbol{\rho}) = 0$$

$$[\partial\psi/\partial \boldsymbol{n}] = -2\psi/a$$

$$-\partial^2\psi_y(x)/\partial x^2 = \lambda(y)\psi_y(x)$$

$$[\partial\psi_y/\partial x] = -2\psi_y/a$$

$$[-\partial^2/\partial y^2 + \lambda(y)]\chi(y) = ME\chi(y)$$



Comparison with hyperspherical approach

$$\left(-\partial^2/\partial x^2 - \partial^2/\partial y^2 - ME\right)\psi(\boldsymbol{\rho}) = 0$$

$$[\partial\psi/\partial\boldsymbol{n}] = -2\psi/a$$

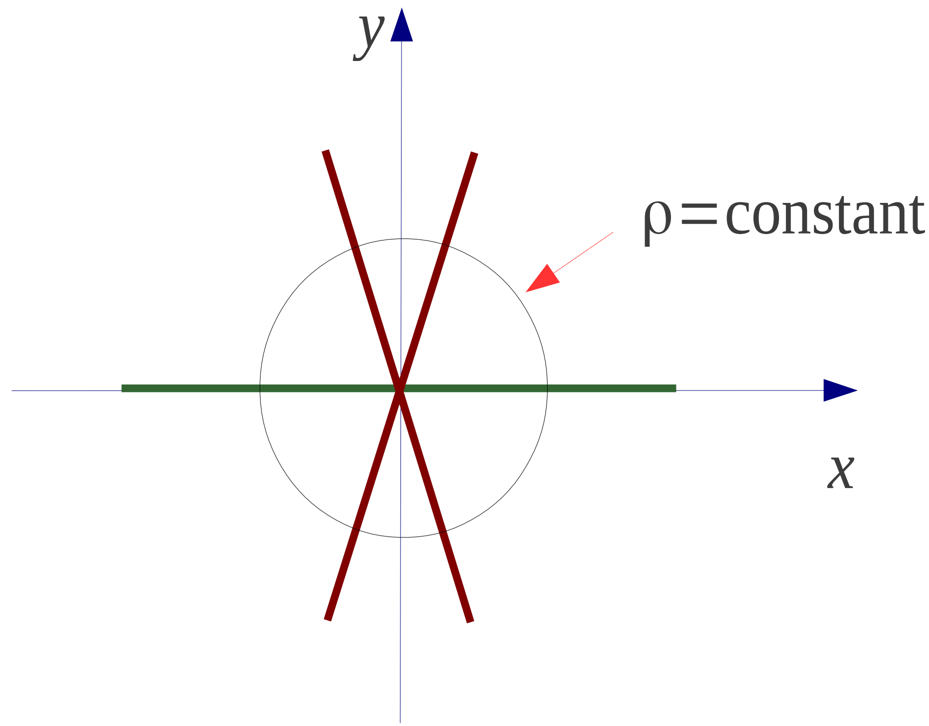
$$\Downarrow$$

$$-\partial^2\psi_{\nu,\rho}(\varphi)/\partial\varphi^2 = \lambda_\nu(\rho)\psi_{\nu,\rho}(\varphi)$$

$$\psi(\boldsymbol{\rho}) = \sum_\nu \chi_\nu(\rho)\psi_{\nu,\rho}(\varphi)$$

$$\Downarrow$$

$$\left[-\nabla_\rho^2 + \frac{\lambda_\nu(\rho)}{\rho^2} - ME\right]\chi_\nu(\rho) + \sum_\mu \hat{K}_{\nu\mu}\chi_\mu(\rho) = 0$$



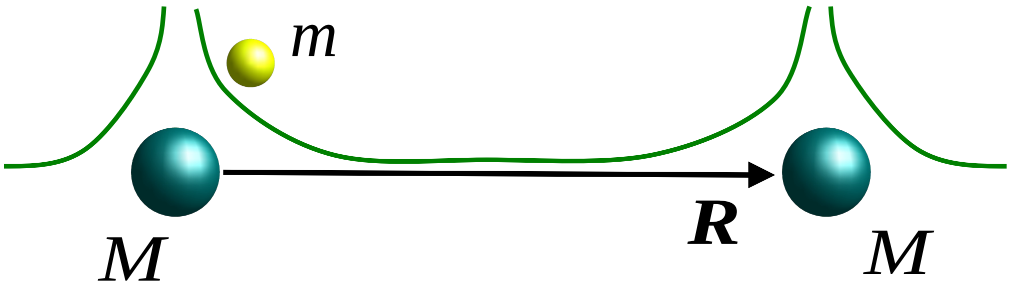
$$(\alpha - \gamma_k)\chi(\mathbf{k}) + 8\pi \int \frac{d\mathbf{k}'}{(2\pi)^3} \frac{\chi(\mathbf{k}')}{k^2 + k'^2 + \mathbf{k}\mathbf{k}' - (ME/\hbar^2) - i\tau} = 0. \tag{12}$$

The solution of this equation determines the wave function of the system, in accord with Eq. (11). For states with a definite quantity of momentum, Eq. (12) reduces to an equation for a function which depends on one independent variable, which can be solved numerically.

The idea for this consideration of the three body problem was supplied by L. D. Landau.

Heteronuclear Fermi-Fermi mixture
in the non-efimovian regime

3-body problem. Born-Oppenheimer approximation



Effective interaction between heavy atoms is provided by exchange of the fast light particle. Born-Oppenheimer approximation.

Light atom wavefunction: $$\psi(\boldsymbol{r}) = \frac{\exp(-\kappa|\boldsymbol{r}-\boldsymbol{R}/2|)}{|\boldsymbol{r}-\boldsymbol{R}/2|} + \frac{\exp(-\kappa|\boldsymbol{r}+\boldsymbol{R}/2|)}{|\boldsymbol{r}+\boldsymbol{R}/2|}$$

Bethe-Peierls boundary condition gives: $$\frac{1}{a} - \kappa + \frac{\exp(-\kappa R)}{R} = 0$$

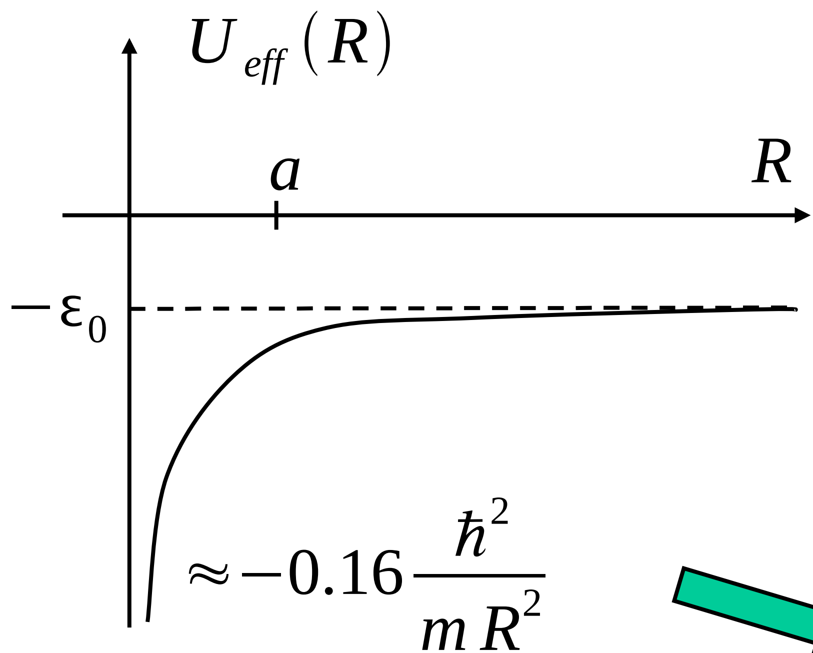
$R \ll a \Rightarrow \kappa \approx \frac{0.567}{R}$

$$U_{eff}(R) = -\frac{\hbar^2\kappa^2}{2m} \approx -0.16\frac{\hbar^2}{mR^2}$$

$R \gg a \Rightarrow \kappa \approx \frac{1}{a}$

$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$



Solving the Schrödinger equation for the heavy atoms we take into account their statistics:

$$\tilde{U}_{eff}(R)\approx U_{eff}+\frac{\hbar^2 l(l+1)}{MR^2}$$

Heavy fermions ⇨ $l=1,3,5\ldots$

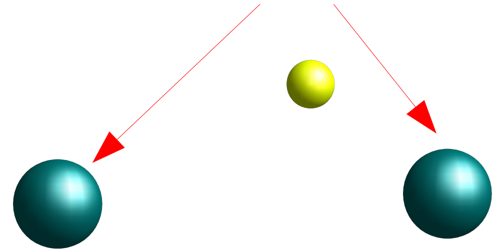
Heavy bosons ⇨ $l=0,2,4\ldots$

$$\tilde{U}_{eff}(R)\approx\frac{\hbar^2}{MR^2}\underbrace{\left(l(l+1)-0.16\frac{M}{m}\right)}_{\beta}$$

$R\ll a,\ E=0$ ⇨ $\left(-\partial^2/\partial R^2+\beta/R^2\right)\chi(R)=0$

$\chi(R)=R^{\nu}$ $\qquad \nu_{\pm}=1/2\pm\sqrt{\beta+1/4}$

Identical fermions



$\Rightarrow \; l=1$

$$R \ll a \;\Rightarrow\; \tilde{U}_{eff}(R) \approx \frac{\hbar^2}{MR^2}\underbrace{\left(2 - 0.16\frac{M}{m}\right)}_{\beta}$$

$\beta < -1/4, \quad M/m > 13.6$

$$\chi(R) \propto \sqrt{R}\sin\left(\sqrt{-1/4-\beta}\;\log R/r_3\right)$$

"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wavefunction. Infinite number of trimer states. Efimov effect

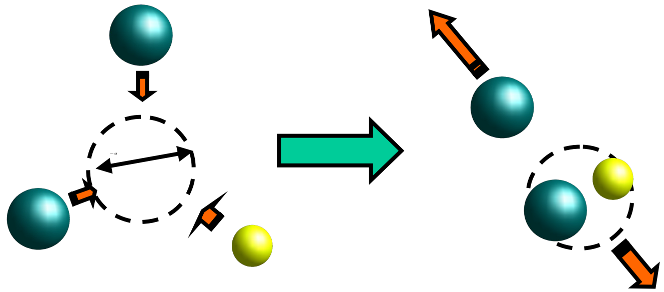
$\beta > -1/4, \quad M/m < 13.6$

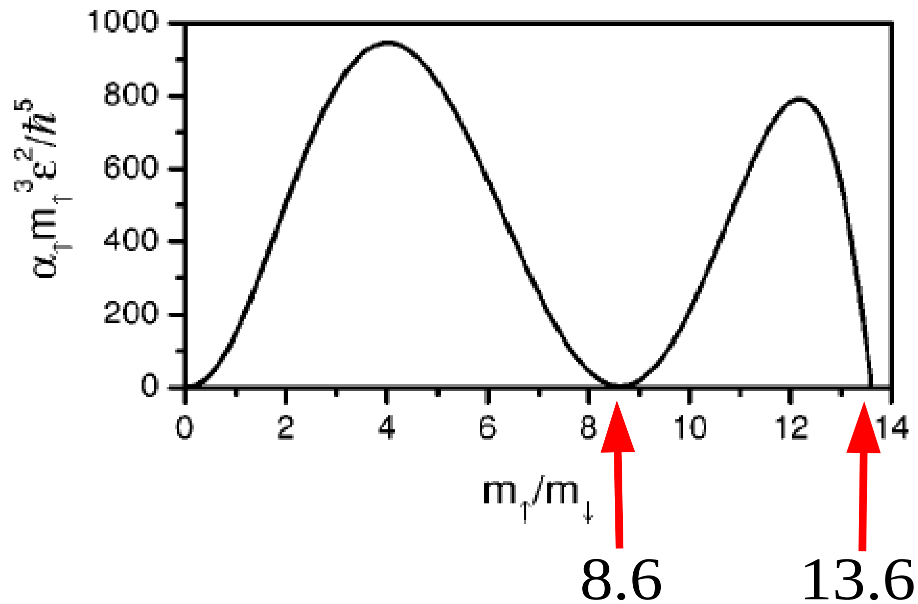
$$\chi(R) \propto R^{1/2+\sqrt{\beta+1/4}}$$

"Universal" regime in the sense that one needs no three-body parameter. Fermi statistics wins over the induced attraction

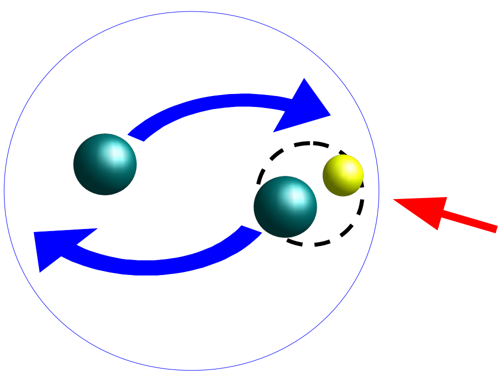
Heavy-heavy-light problem, magic mass ratios

3-body recombination to a weakly bound level DSP'03

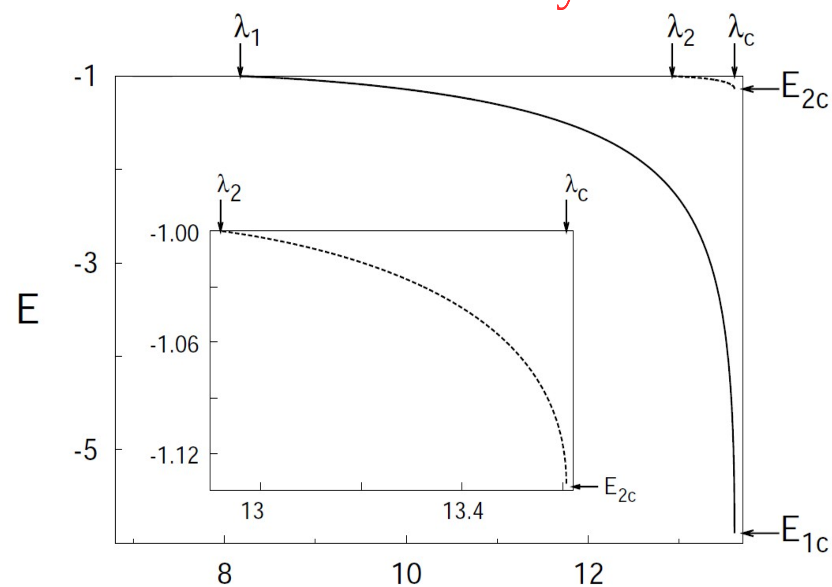




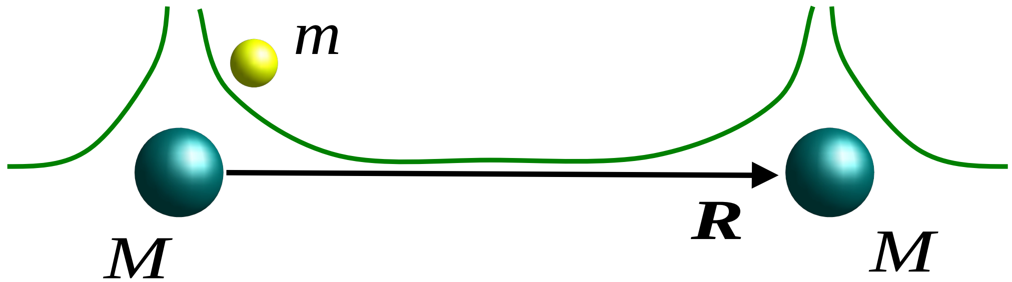
Emergence of a non-Efimovian trimer state for M/m>8.2 Kartavtsev&Malykh'06

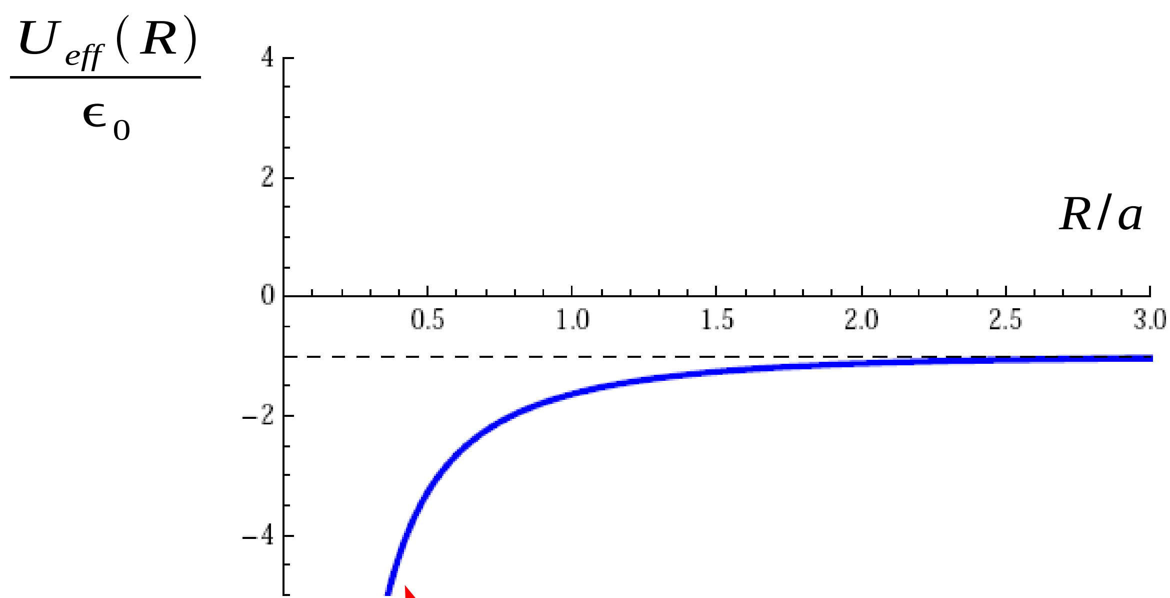


- M/m<8.2 *p*-wave atom-dimer scattering resonance
- M/m>8.2 trimer state with *l*=1



Born-Oppenheimer approximation

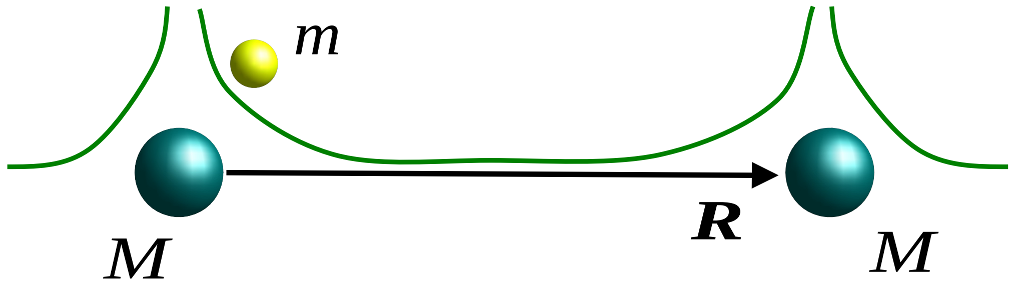


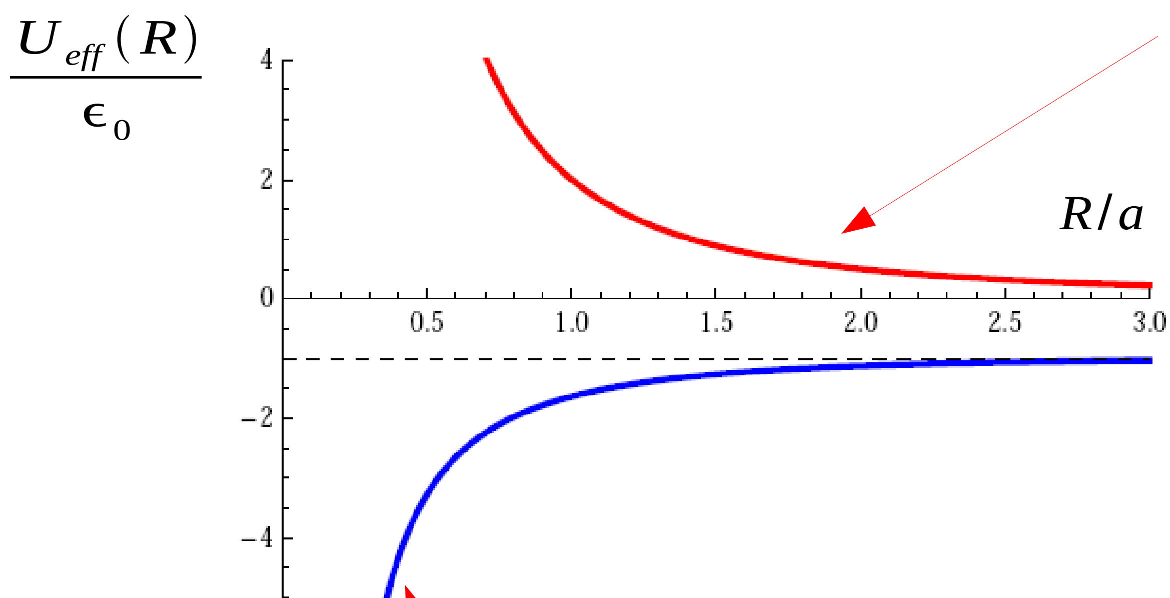


$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$U_{eff}(R) \approx -0.16\frac{\hbar^2}{mR^2}$$

Born-Oppenheimer approximation





$$U_{centrifugal}(R)=\frac{\hbar^2 l(l+1)}{MR^2}$$

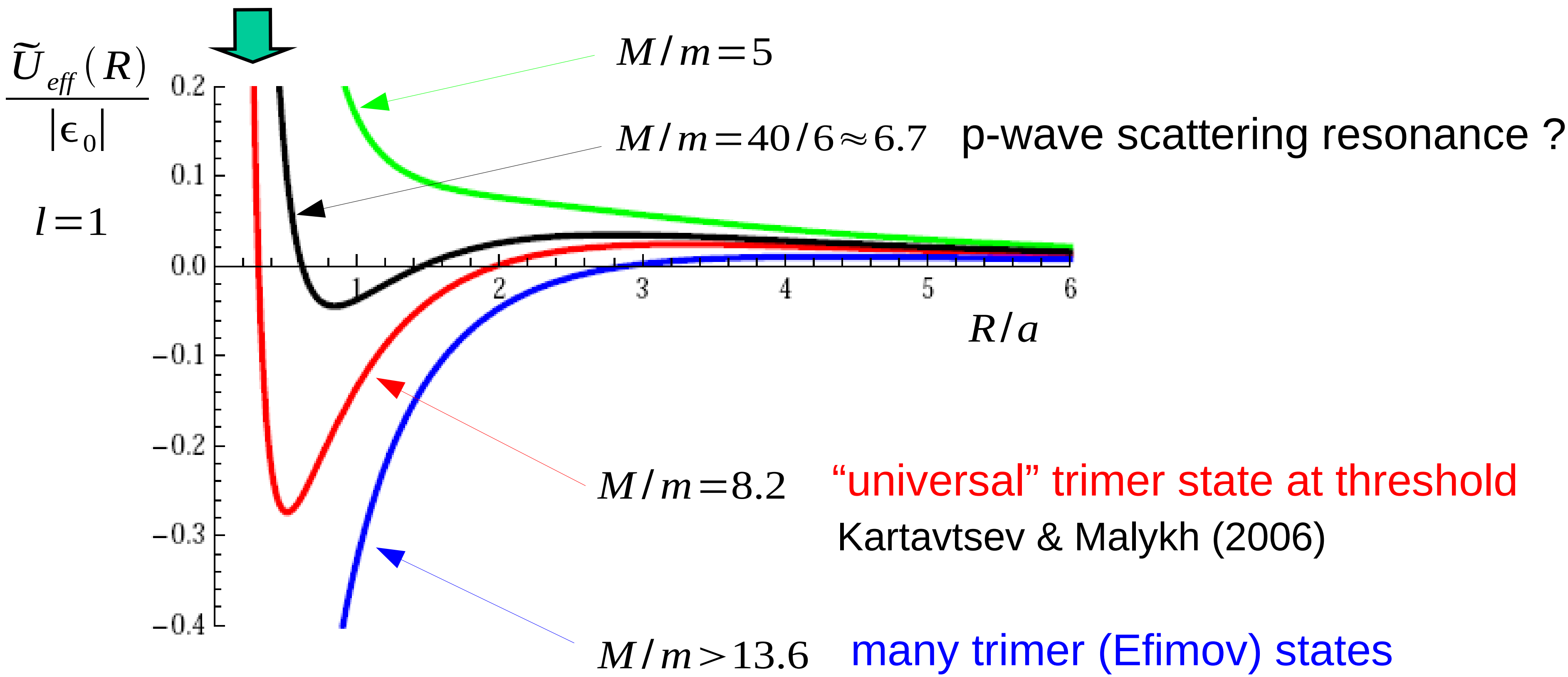
$$U_{eff}(R)\approx-\frac{\hbar^2}{2ma^2}=-|\epsilon_0|$$

$$U_{eff}(R)\approx-0.16\frac{\hbar^2}{mR^2}$$

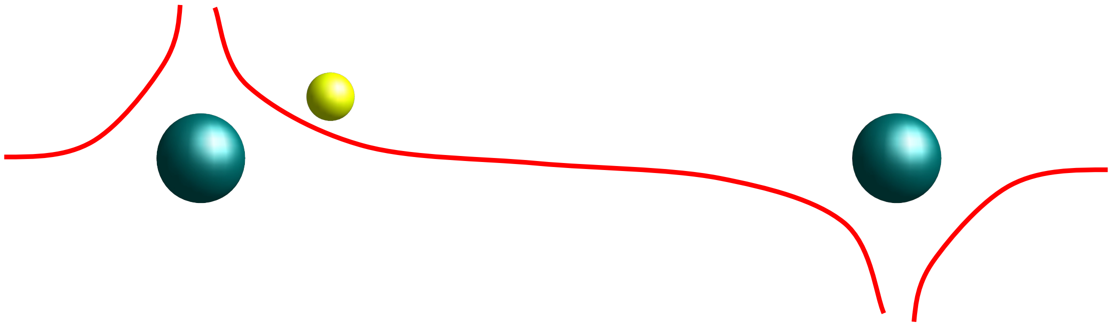
$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$

$$\frac{\hbar^2}{MR^2}\left(l(l+1)-0.16\frac{M}{m}\right)$$

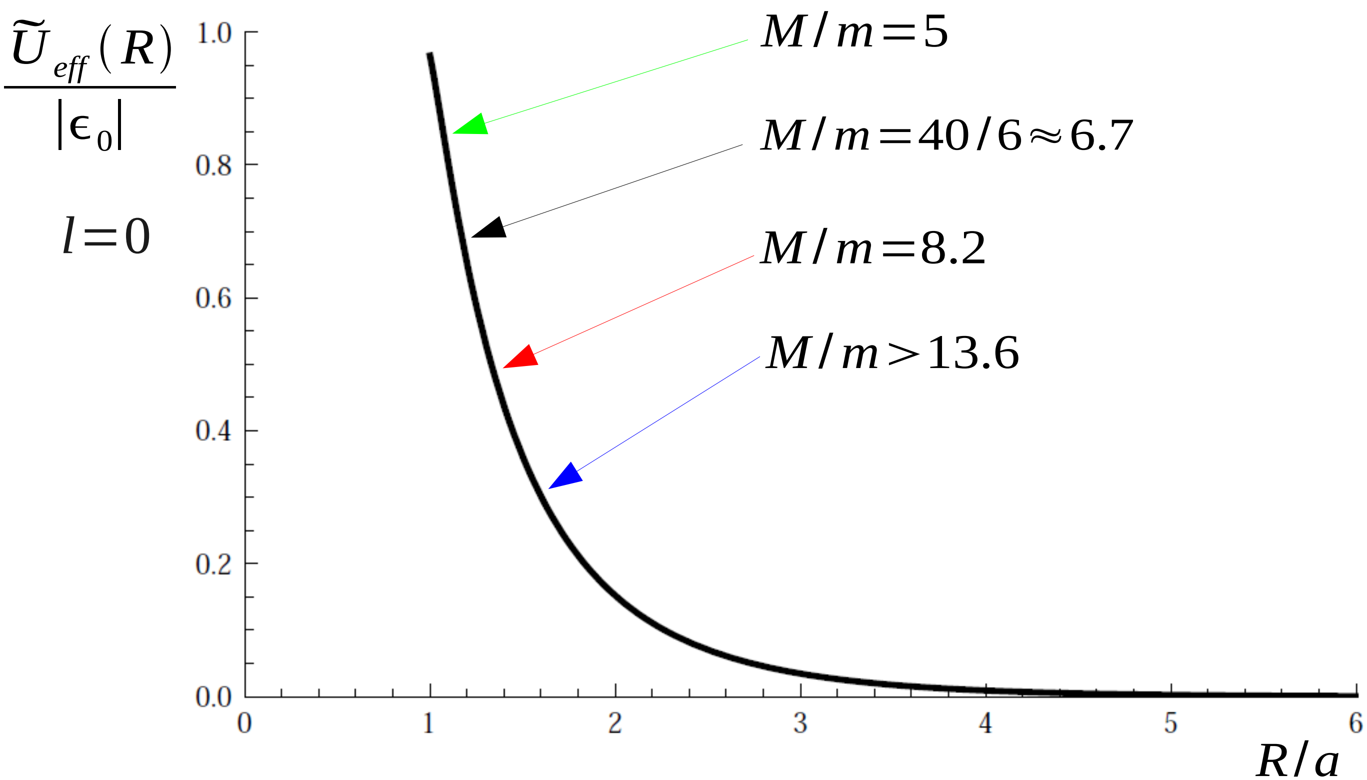
$$\tilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$$

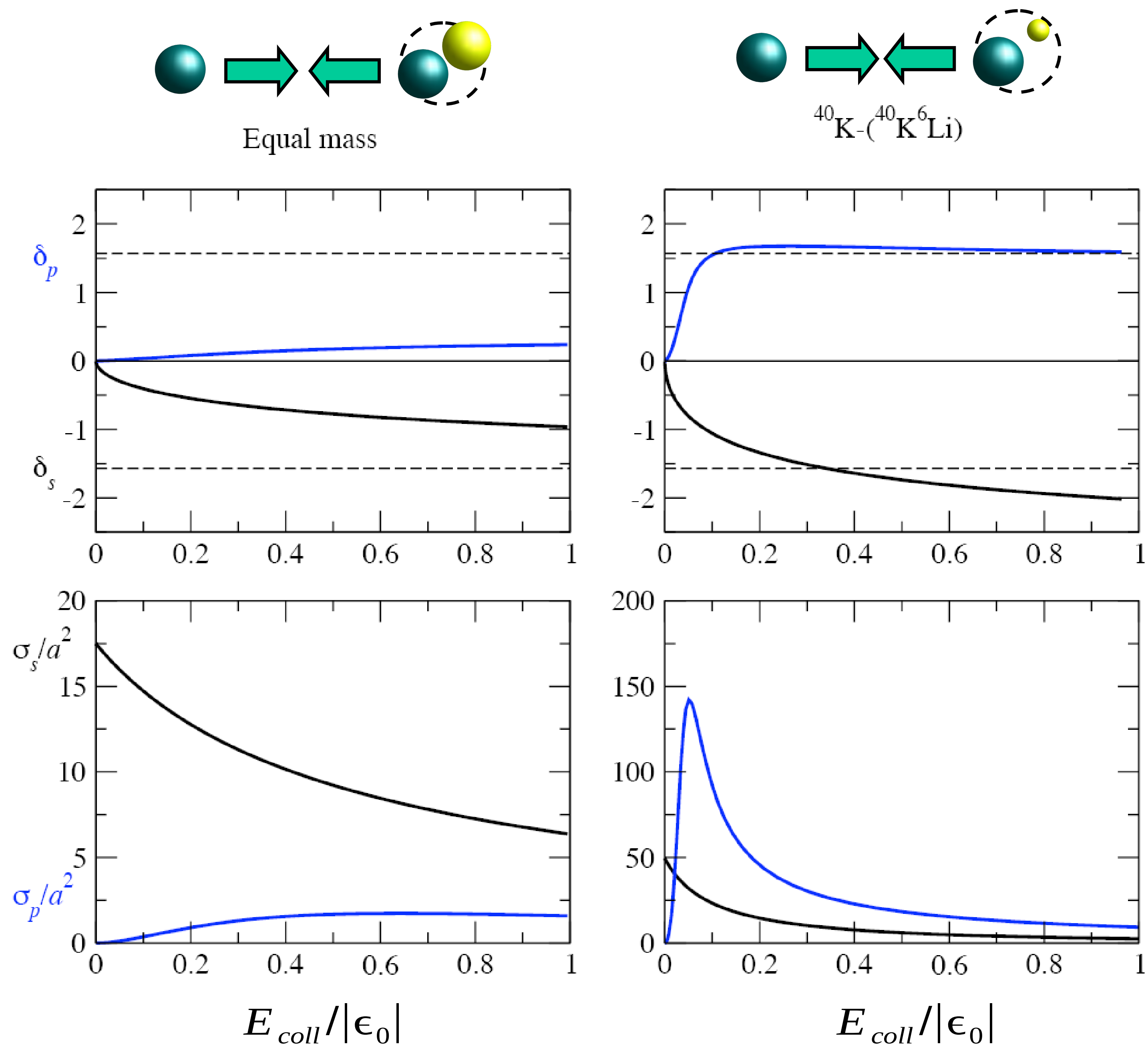


S-wave atom-dimer repulsion



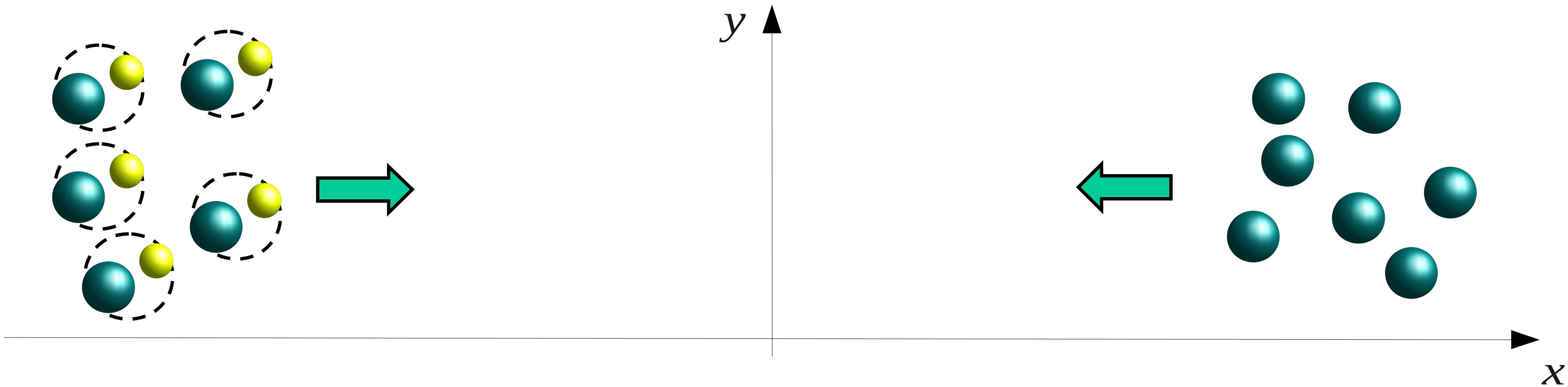
Light fermion in the antisymmetric “ungerade” state

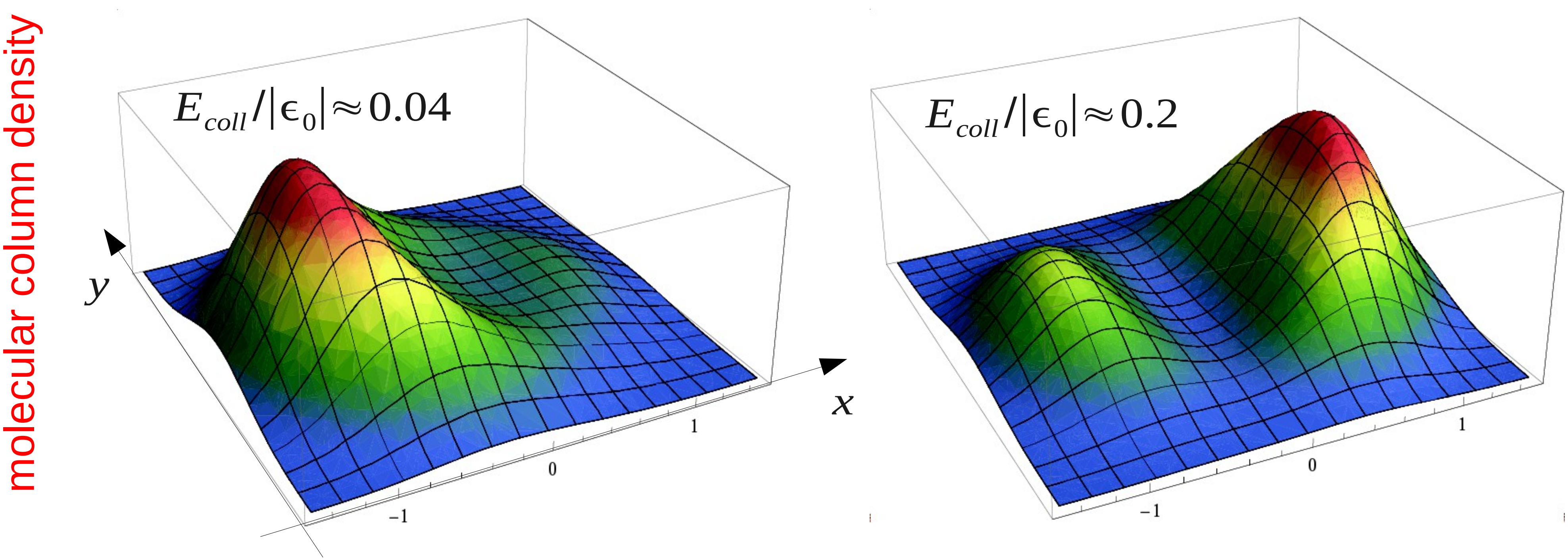




Equal mass
^{40}K-(^{40}K^{6}Li)
δ_p
δ_s
σ_s/a^2
σ_p/a^2
$E_{coll}/|\epsilon_0|$
$E_{coll}/|\epsilon_0|$

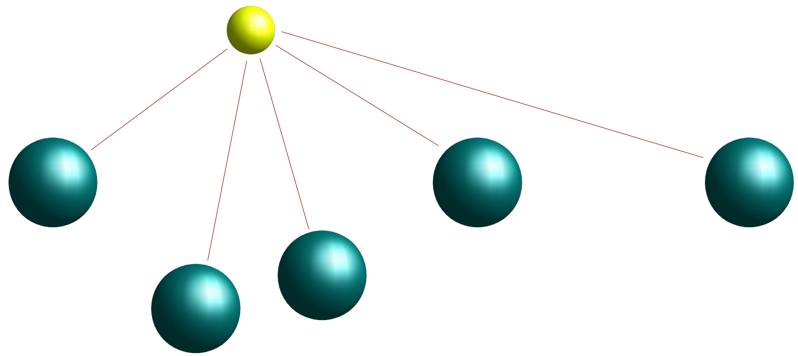
Collision of atomic and molecular thermal clouds

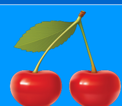
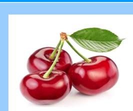
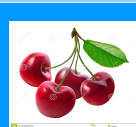




(N+1)-body problem

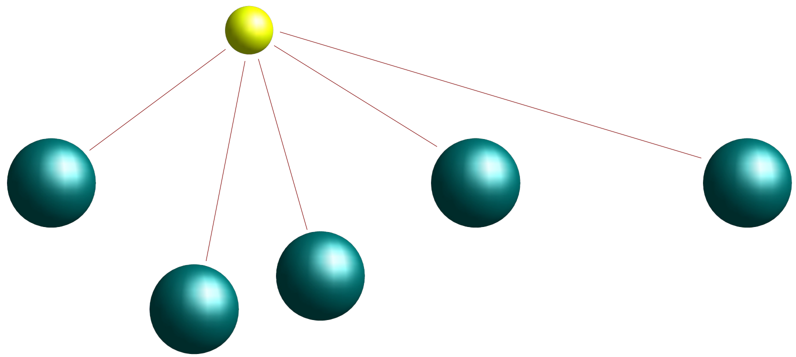
How many heavy fermions can be bound by a single light atom?



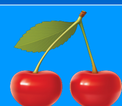
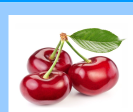
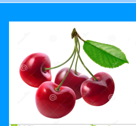
	Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer 	1^{-}	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer 	1^{+}	~9.5 Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer 	$?^{?}$	?	?
:	?	?	?
N+1-mer 	?	?	?

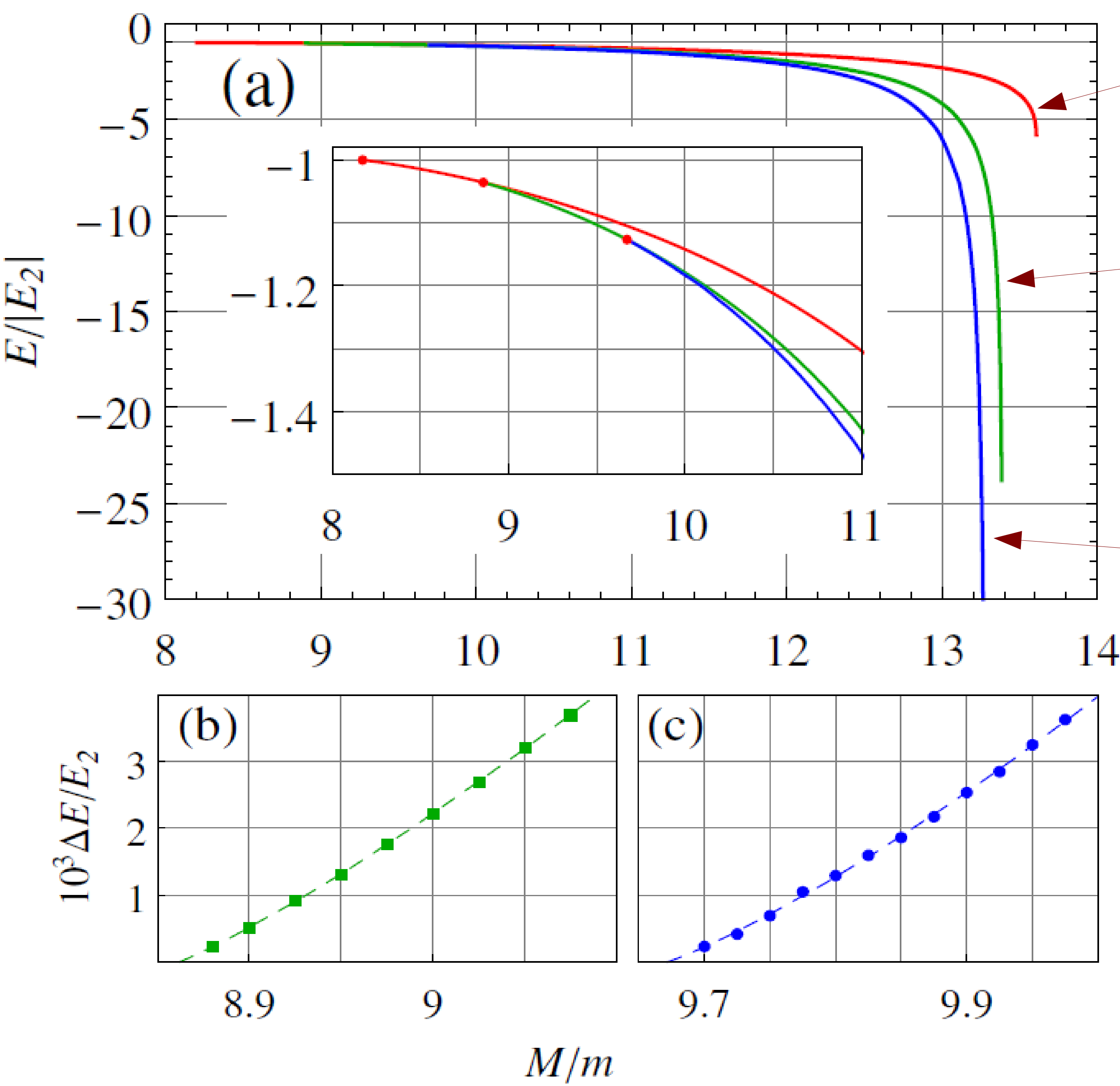
(N+1)-body problem

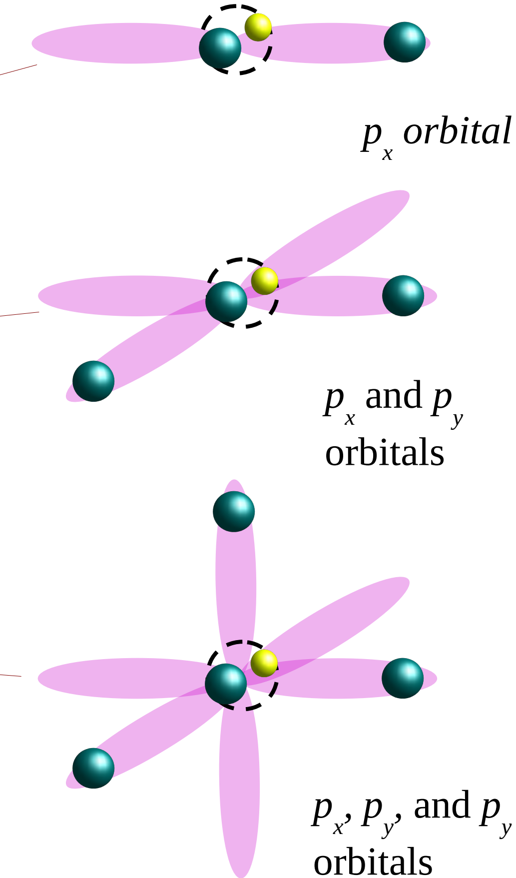
How many heavy fermions can be bound by a single light atom?



B. Bazak and DSP (2017)

	Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer 	1^{-}	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer 	1^{+}	~9.5 → 8.862(1) Blume'12	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer 	0^{-}	9.672(6)	13.279(2)
:	?	?	?
N+1-mer 	?	?	?





pentamer = closed p-shell

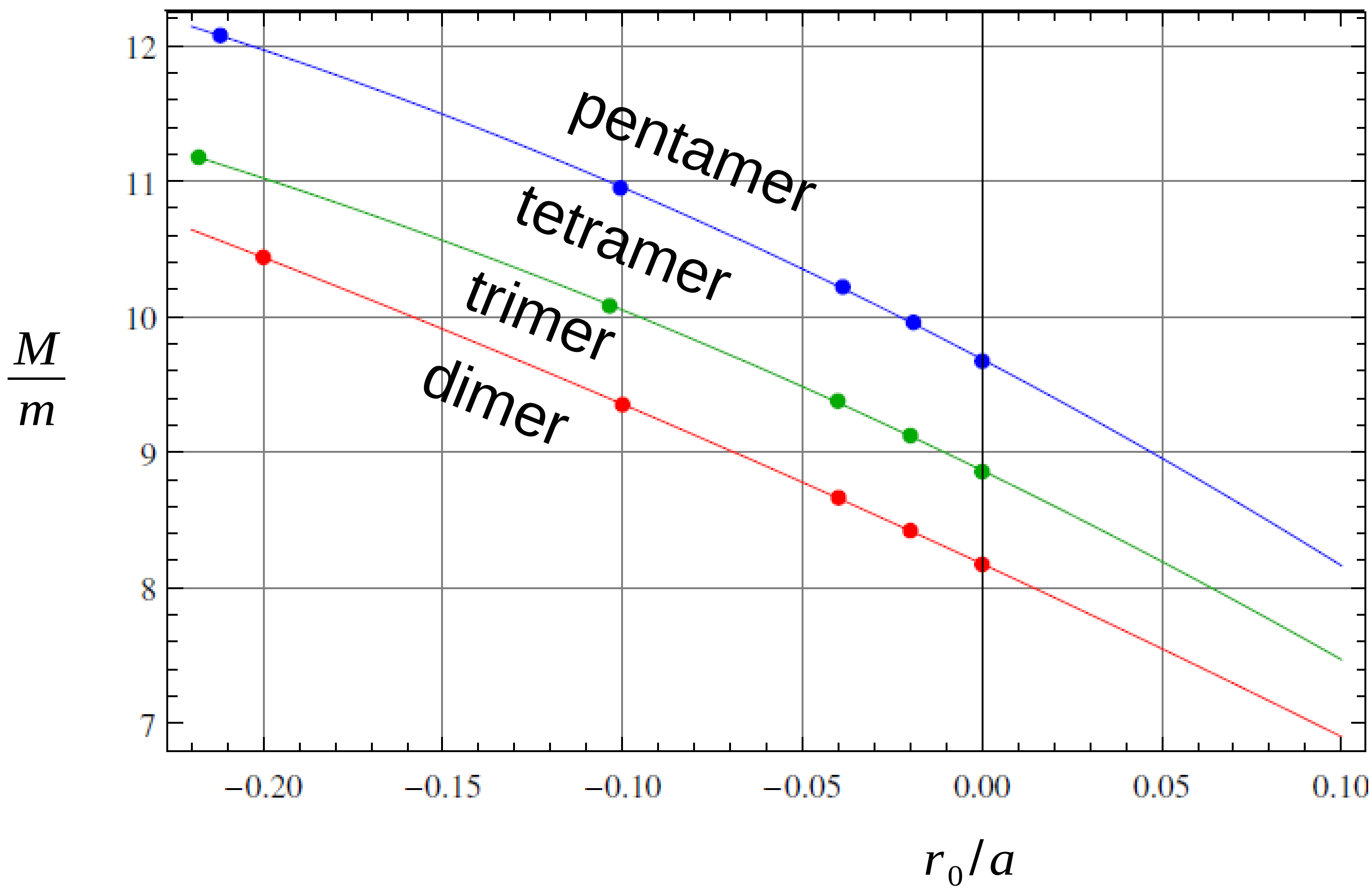
CONJECTURE:

No hexamer!

(requires justification)

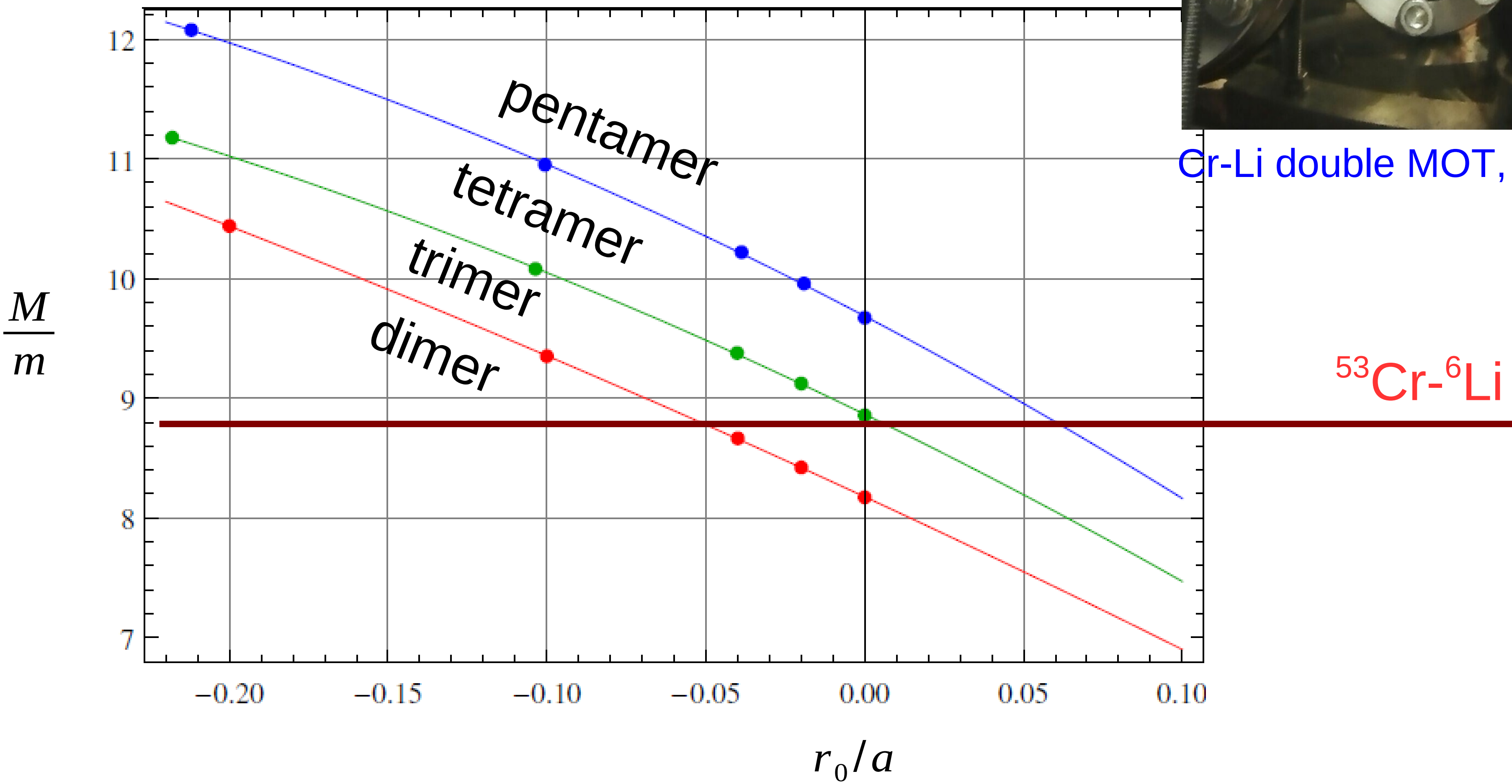
Effective-range effects

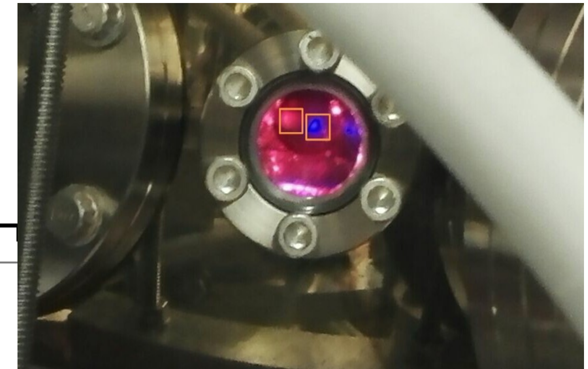
$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2} k^2$$



Effective-range effects

$$\frac{1}{a} \longrightarrow \frac{1}{a} - \frac{r_0}{2} k^2$$





Cr-Li double MOT, Florence

Summary

- History and basics of the zero-range few-body physics
- Efimovian and non-efimovian regimes of three-body systems with zero-range interactions
- Approach of Skorniakov and Ter-Martirosian and its comparison with the Born-Oppenheimer and hyperspherical methods
- Universal trimers, tetramers, and pentamers in mass-imbalanced fermionic mixtures