Wuhan Institute of Physics
and Mathematics
April 25, 2017

RICE UNIVERSITY

Soliton Collisions on the Edge of Integrability

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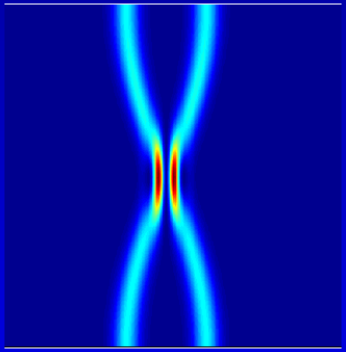
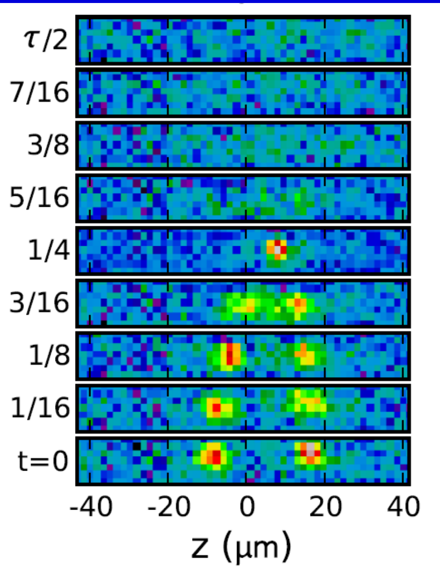
Boris Malomed (Tel Aviv)

Outline

- **Intro to BEC with *a*<0**
- **Intro to solitons**



- **Phase-dependent collisions**
- **Role of integrability**

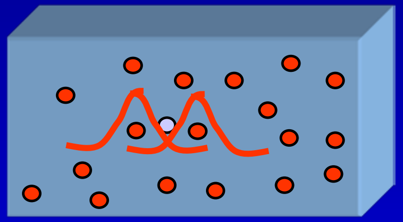


Quantum Gases

- Quantum regime $n\Lambda^3 \geq 1$

n = density
Λ = de Broglie wavelength

Identical particles!

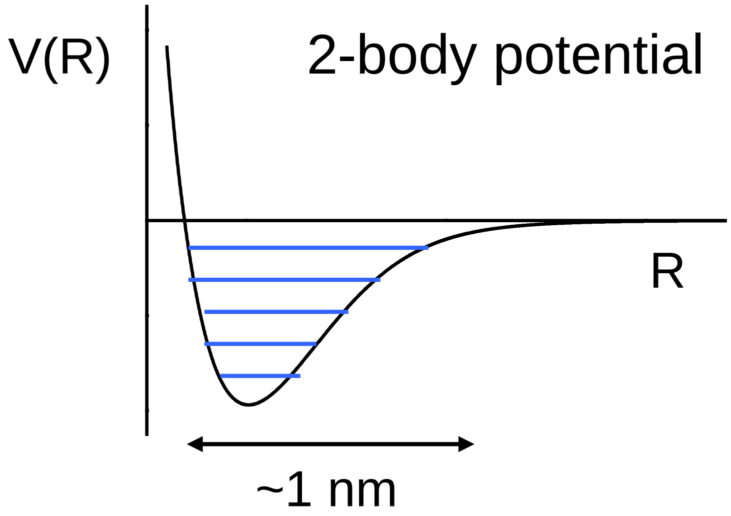


- Gas phase $n \approx 10^{12}$ cm^{-3}
- Low temperature $T \approx 100$ nK

$\Rightarrow \Lambda \approx 1$ µm

- Phase transitions
  - Bosons (^{7}Li): Bose-Einstein condensation
  - Fermions (^{6}Li): Fermion pairing

Interactions – Generic Discussion



Λ_{dB} ~ 1 μm

⇒ detailed shape of V(R) unimportant

Characterize interaction by *s*-wave scattering length *a*:

• mean-field interaction energy $nU_o = 4\pi\hbar^2 n a/m$

a < 0 *attractive*

a > 0 *repulsive*

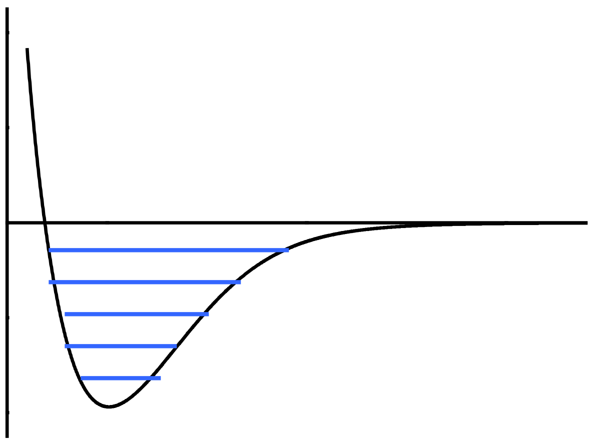
Implications of Interactions

- Bosons
  - stability of condensate
  - bright or dark solitons
  - healing length: vortices, speed of sound
  - excitation spectrum
  - Mott insulator: on-site interactions U
  - miscibility or immiscibility of spinor condensates

- Fermions
  - Cooper pairing (BCS) or molecules (BEC)
  - Hubbard model: U/t

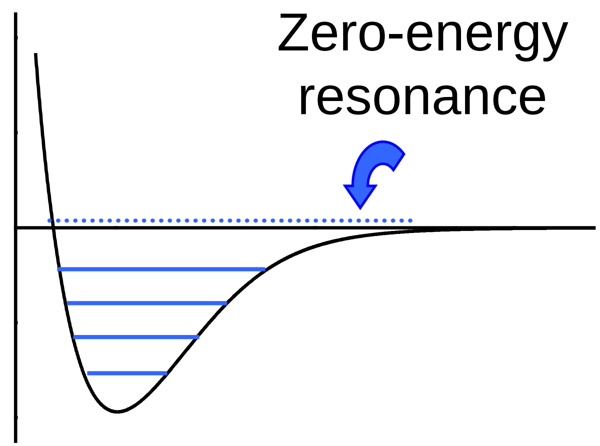
What Determines *a*?

^{7}Li



$a = -27\ a_o$

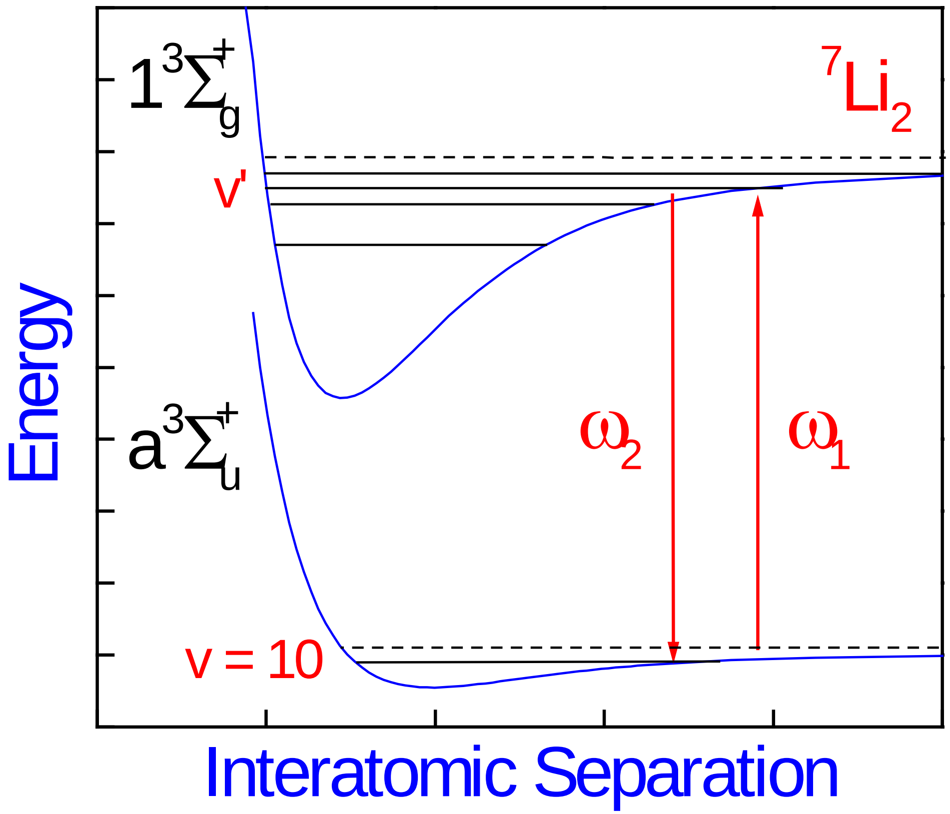
^{6}Li



$a = -2300\ a_o$

Answer: The last bound state!

Measuring *a* by Photoassociation



Abraham *et al*.,
PRL **74**, 1315 (1995)

2-photon photoassociation: $E_B = \omega_2 - \omega_1$

Li potentials well characterized: 2-body physics known *precisely*

Implications of Interactions for BEC

Mean-field interaction energy $U = 4\pi\hbar^2 a n/m$

a is the *s*-wave scattering length

$a > 0$ (eg H, ^{23}Na, ^{87}Rb): repulsive

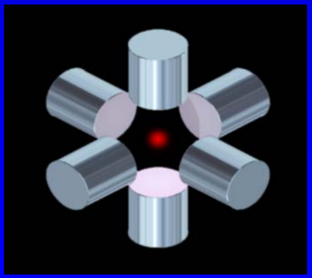
- Stable BEC

$a < 0$ (eg ^{7}Li, ^{85}Rb): attractive

- $dU/dn < 0 \Rightarrow$ mechanically unstable

$\therefore$ BEC in a gas thought not possible

(Bogolubov, 1947; Landau and Lifshitz, 1958; Stoof, 1994)



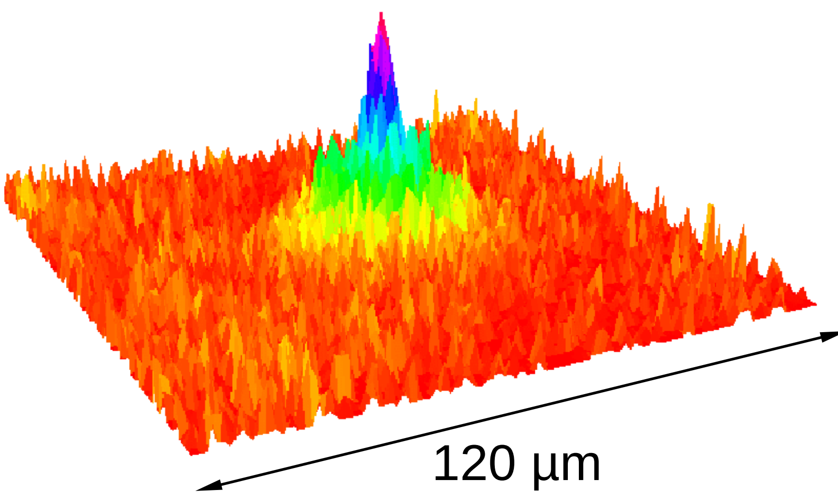
Permanent Magnet Trap 1995

Attractive condensate predicted to implode

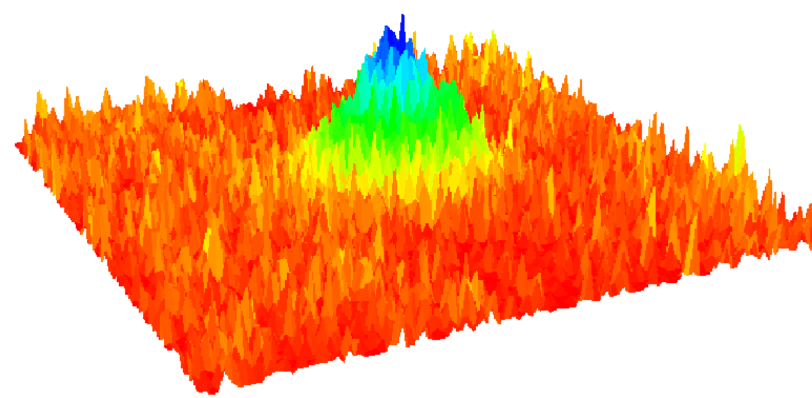
Result

There are condensates!

(although they are really puny)



N = 23k, N_o = 1050

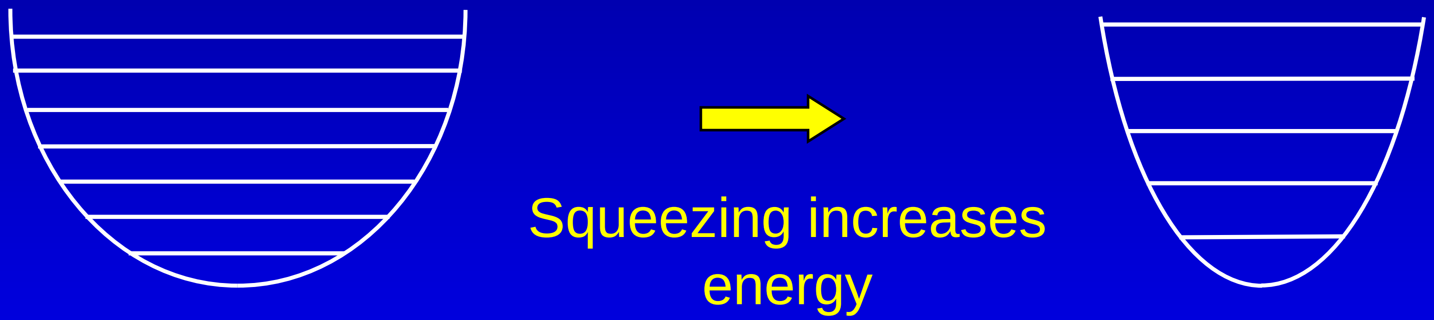


N = 23k, N_o = 65

BEC with Attractive Interactions – ~~3D~~ 0D

Mean-field interaction energy $U = 4\pi\hbar^2 an/m$

- Condensate mechanically unstable – no BEC in free space
- Stabilized by quantum pressure in a trap



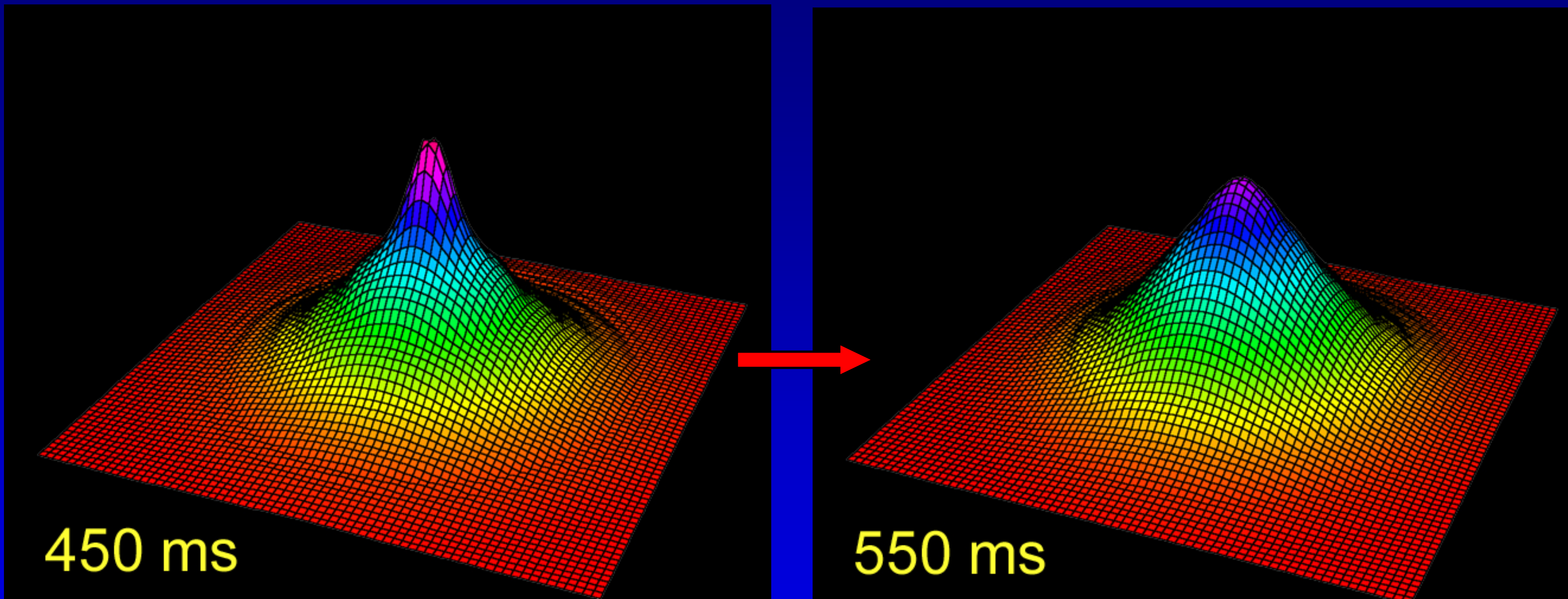
- Attraction balanced by zero-point energy

BEC is possible with $a < 0$, but N_o limited:

$U < \hbar\omega_{r,z}$ ⟶ "0D" limit

C.C. Bradley *et al.*, *PRL* **75**, 1687 (1995) and *PRL* **78**, 985 (1997)

Quench Cool and Attractive Condensate



Collapse!

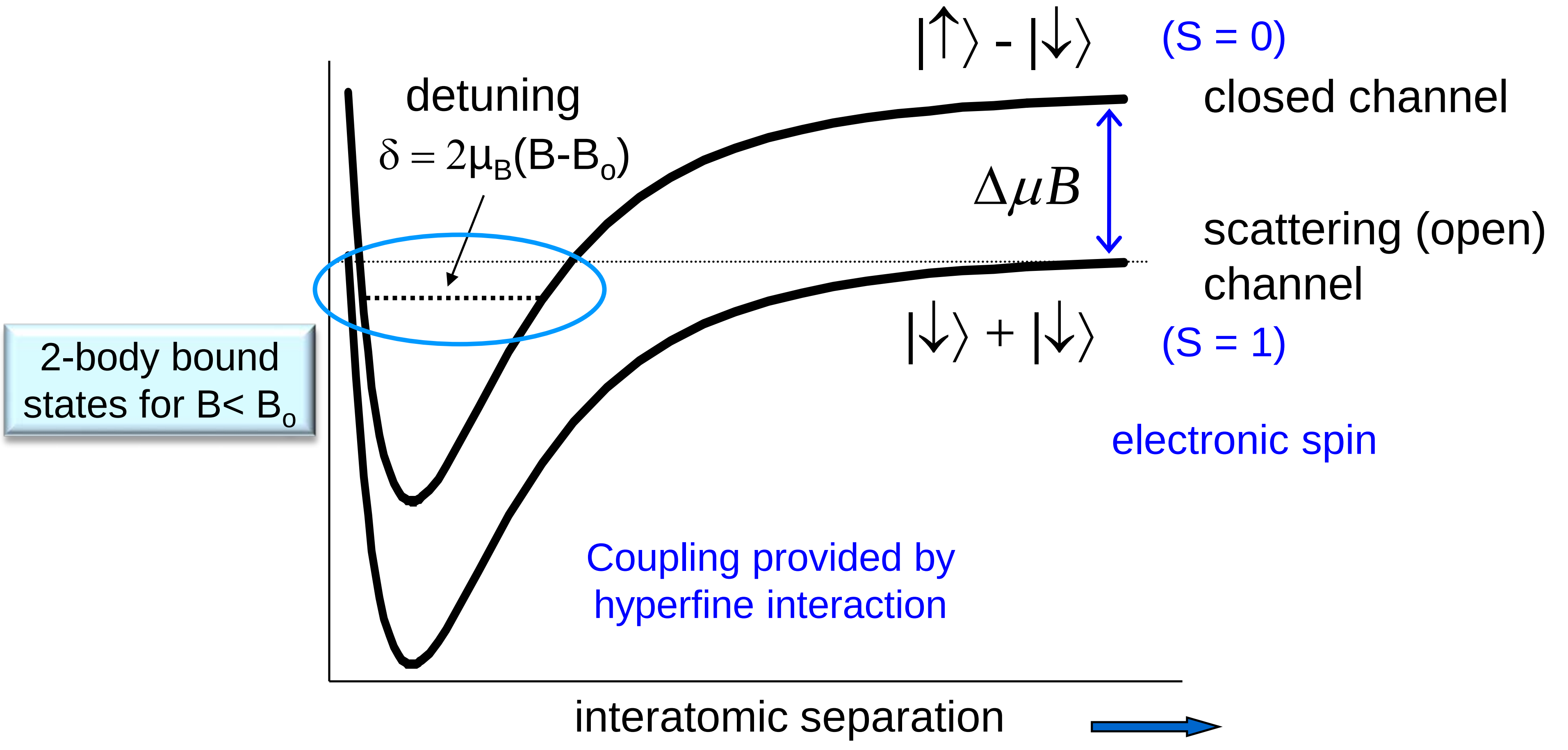
"Bosenova"

Gerton *et al.*, *Nature* **408**, 692 (2000) - Rice
Donley *et al.*, *Nature* **412**, 295 (2001) - JILA

Tunable Interactions - Feshbach Resonance

Magnetically tune free atoms into resonance with a bound molecular state:

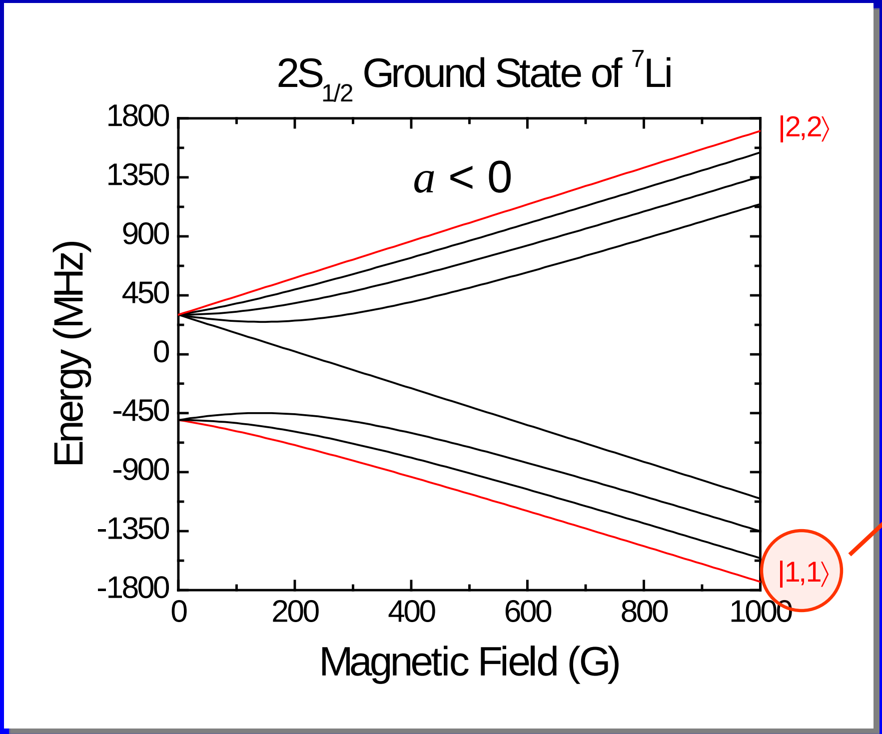
Alkali metal atoms interact via singlet or triplet potential

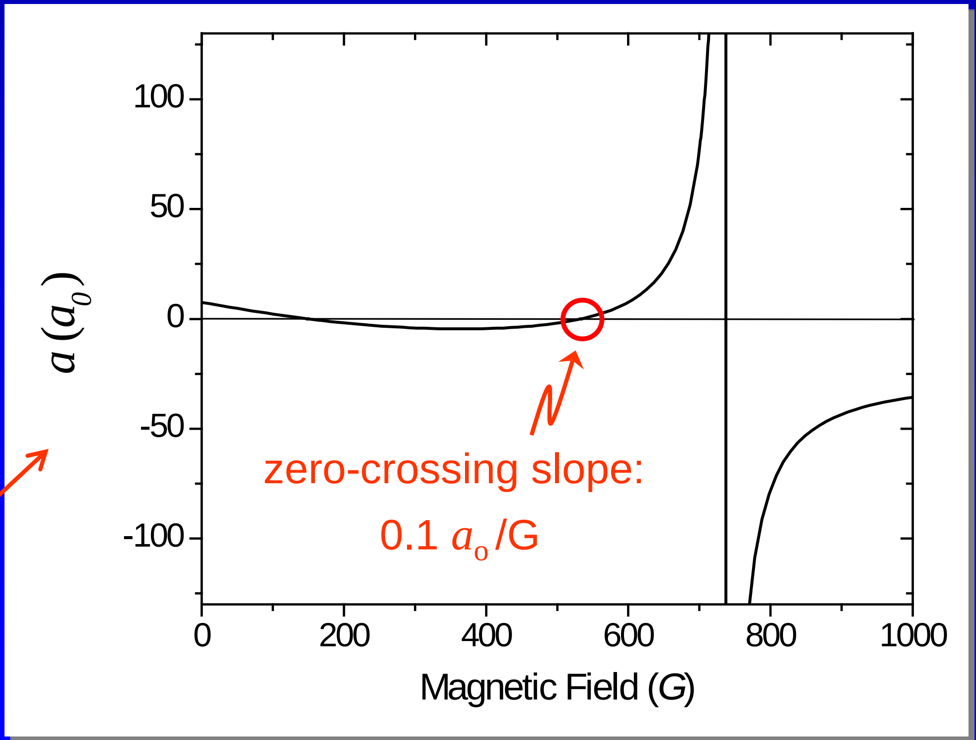


Feshbach Resonance in ^{7}Li (Boson)

Hyperfine sublevels of ^{7}Li

Coupled channels calculation of the scattering length of ^{7}Li |1,1⟩ state





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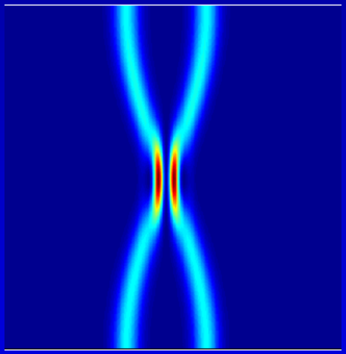
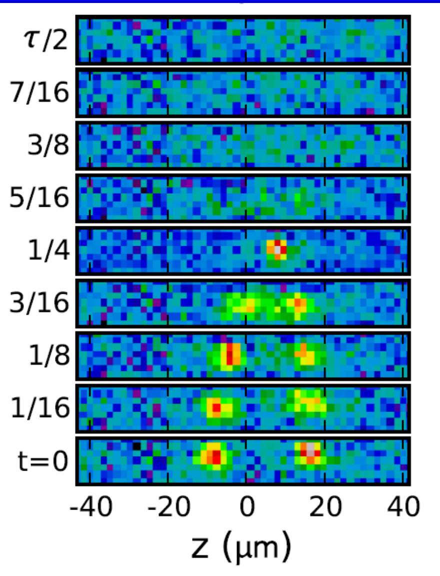


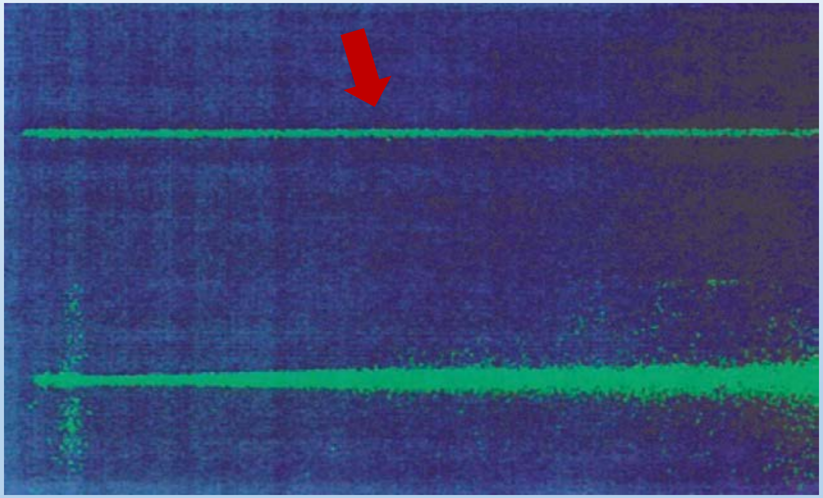
- Phase-dependent collisions
- Role of integrability



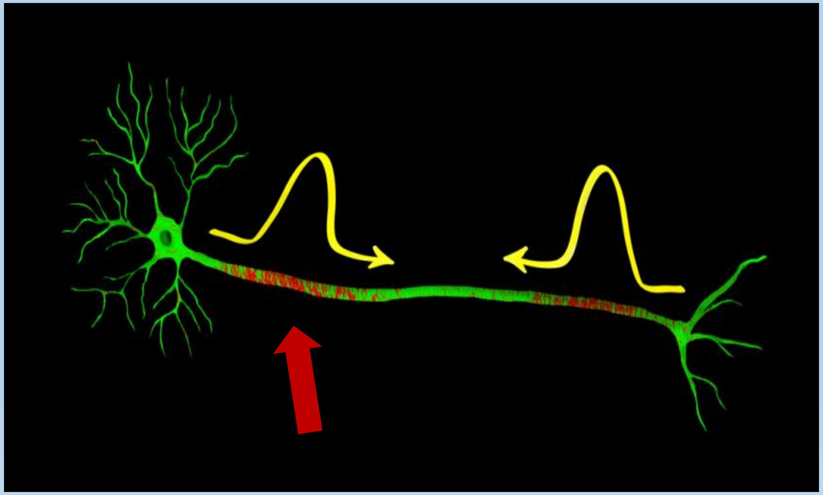
Solitons are Everywhere!

Scott Russell Aqueduct - Edinburgh

Photorefractive Crystal



Gulf of Carpentaria, Australia



Nerve Impulses – soliton collisions?

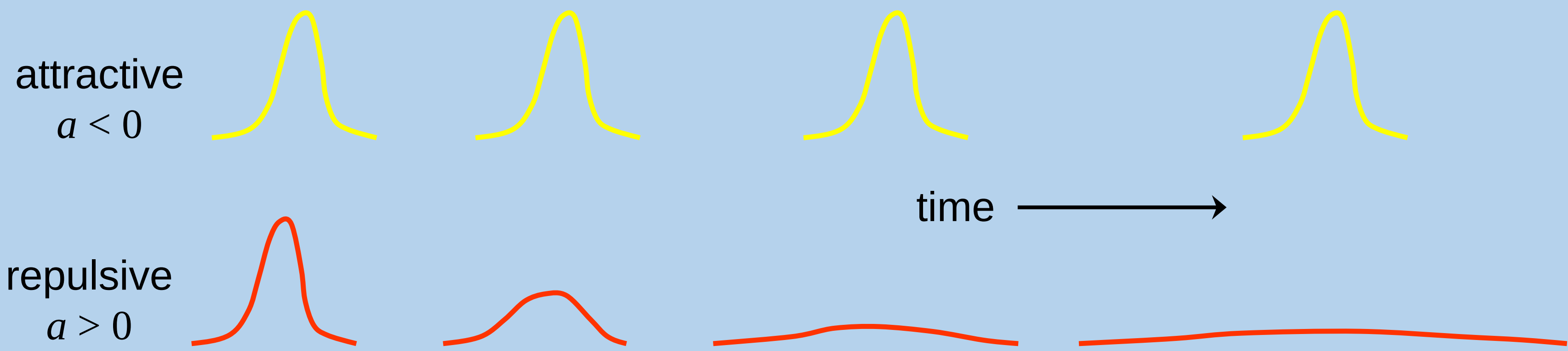
General Properties of Solitons

- **mathematically described by 1D nonlinear partial differential equations that are integrable, e.g**

$$i\hbar \frac{d}{dt}\Psi = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + g\left|\Psi\right|^2\right)\Psi$$

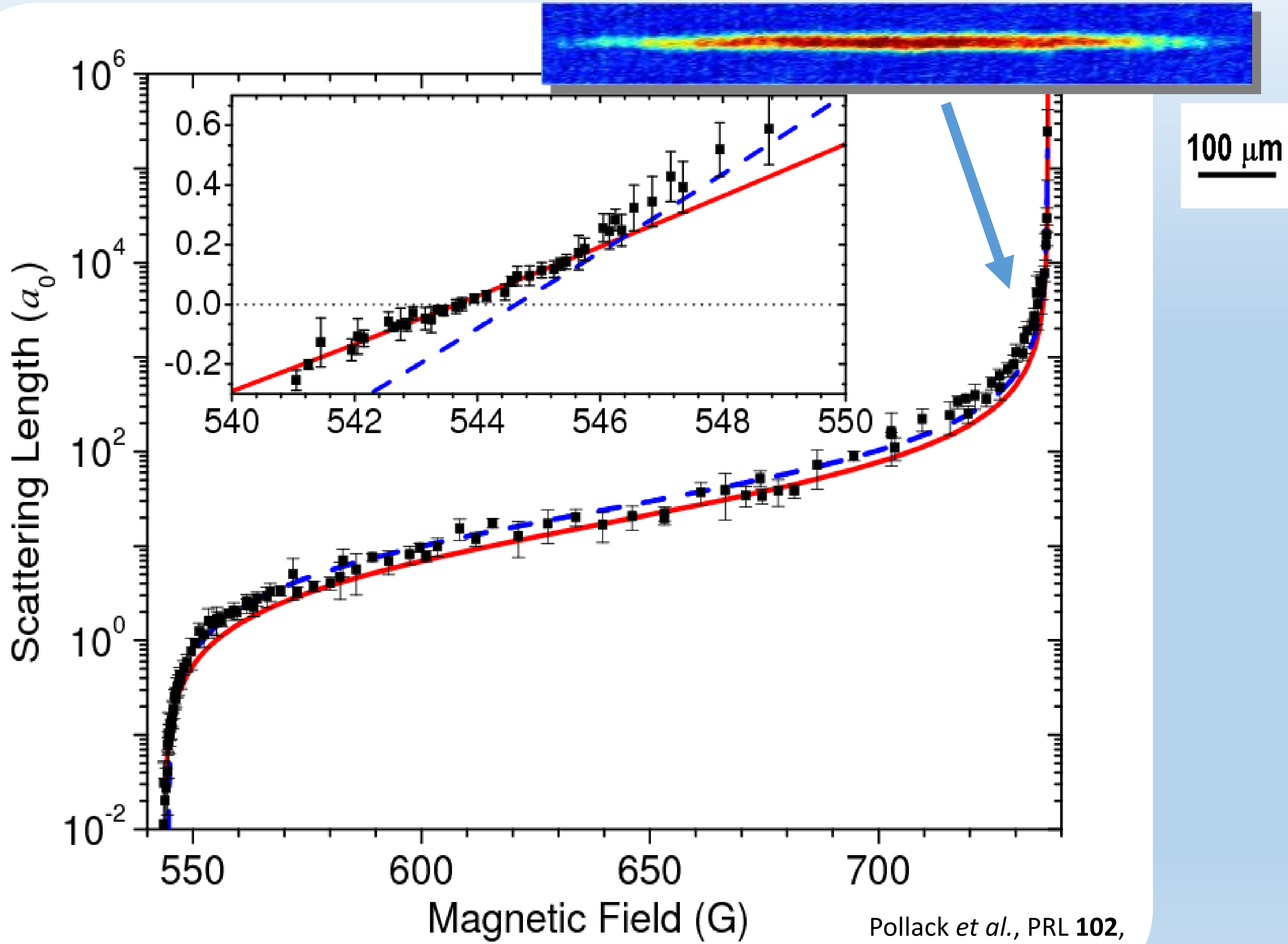
where $g = 2\hbar^2/ma_{1D} < 0$
and $a_{1D} = a_r/a$

- **dispersion compensated by nonlinear interaction – no spreading**



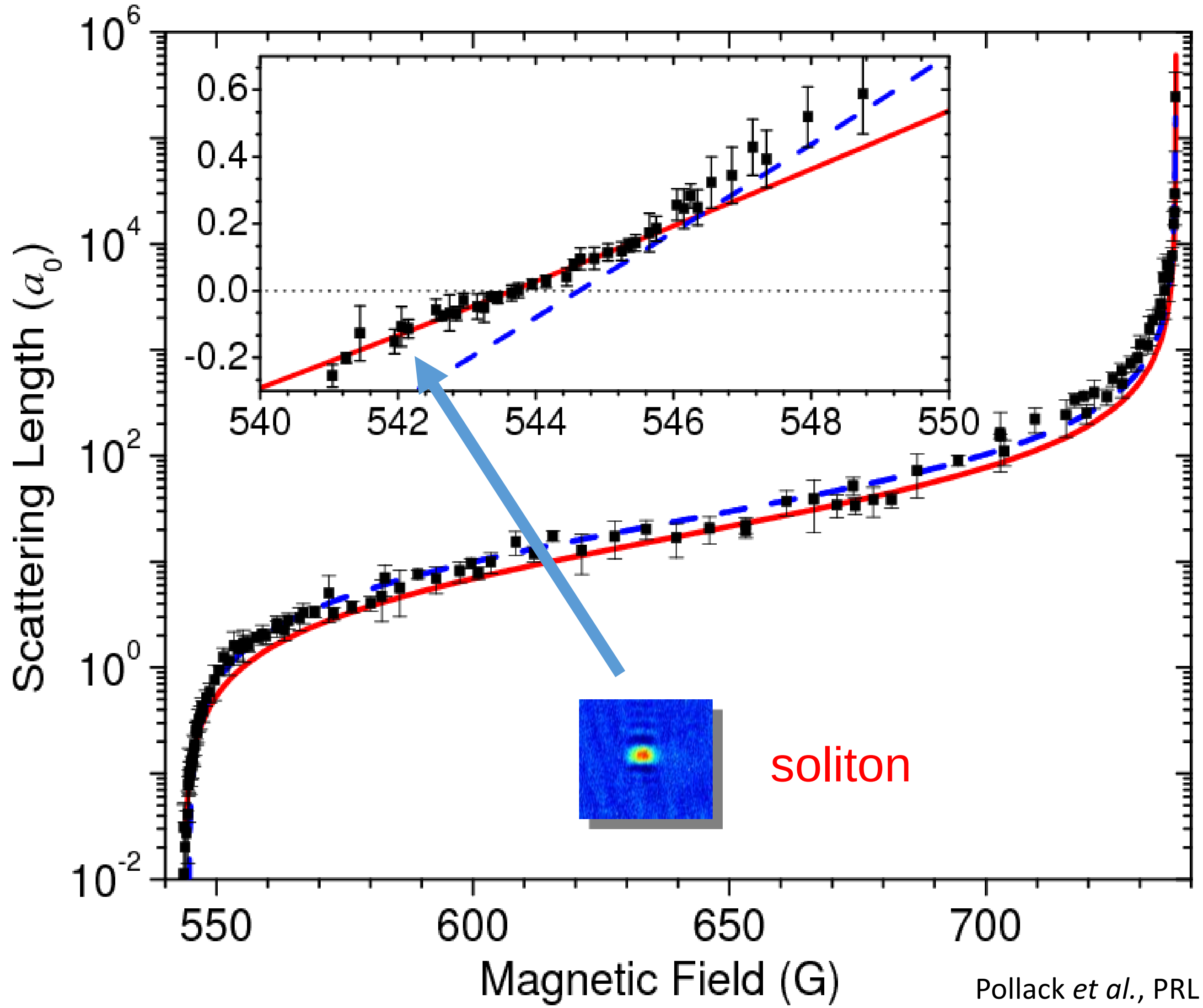
- **survives collisions without change in shape, amplitude or velocity (except for a possible phase jump)**
  - **independent of the number of collisions**
  - **system does not *thermalize***

Feshbach Resonance in ^{7}Li – Stable BEC



Pollack *et al.*, PRL **102**, 090402 (2009)

Sweep Through the Zero-Crossing to Make Soliton(s)



Pollack *et al.*, PRL **102**, 090402 (2009)

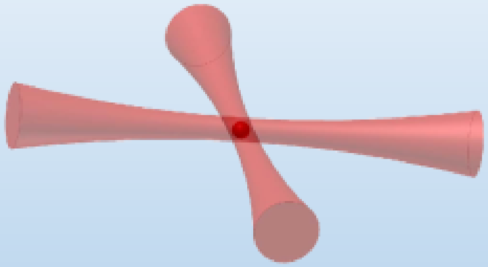
c.f. Khaykovich *et al.*, Science **296**, 1290 (2002) *ENS*;
Strecker *et al.*, Nature **417**, 150 (2002) *Rice*

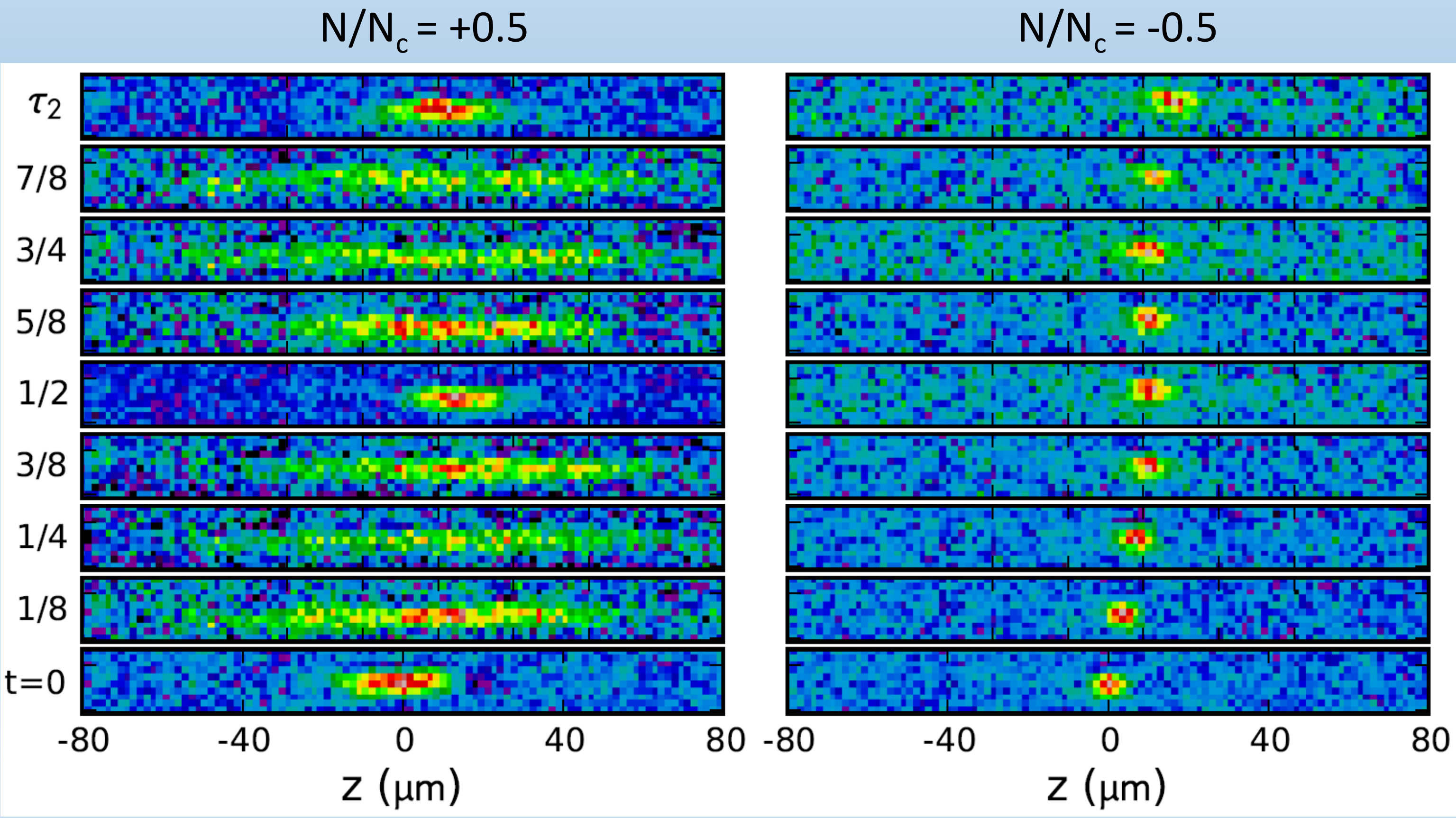
Expansion into 1D Waveguide – Nondispersive

Formed in crossed beam trap and transferred into single beam:

Crossed beam: $\omega_z = 2\pi \times 31$ Hz ($\tau = 32$ ms)

Single beam: $\omega_z = 2\pi \times 8$ Hz ($\tau = 125$ ms)





Stable only in quasi-1D → critical number:

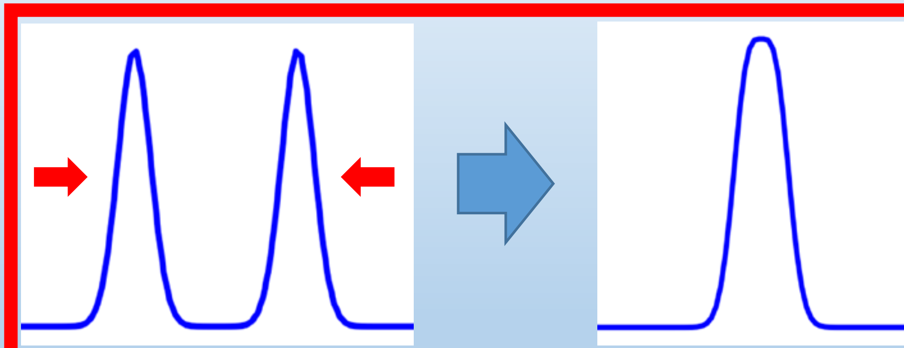
$$N_c = 0.67\,\frac{a_r}{|a|}$$

$$a_r = \sqrt{\frac{\hbar}{m\,\omega_r}}$$

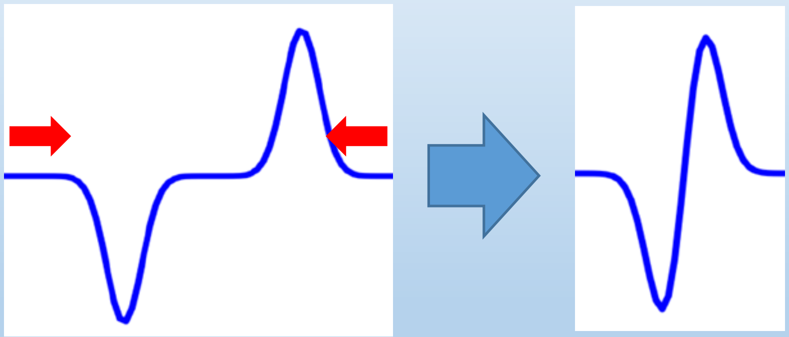
Nguyen, Dyke, Luo, Malomed & Hulet., Nat. Phys. **10**, 918 (2014)

Phase-dependent Interactions

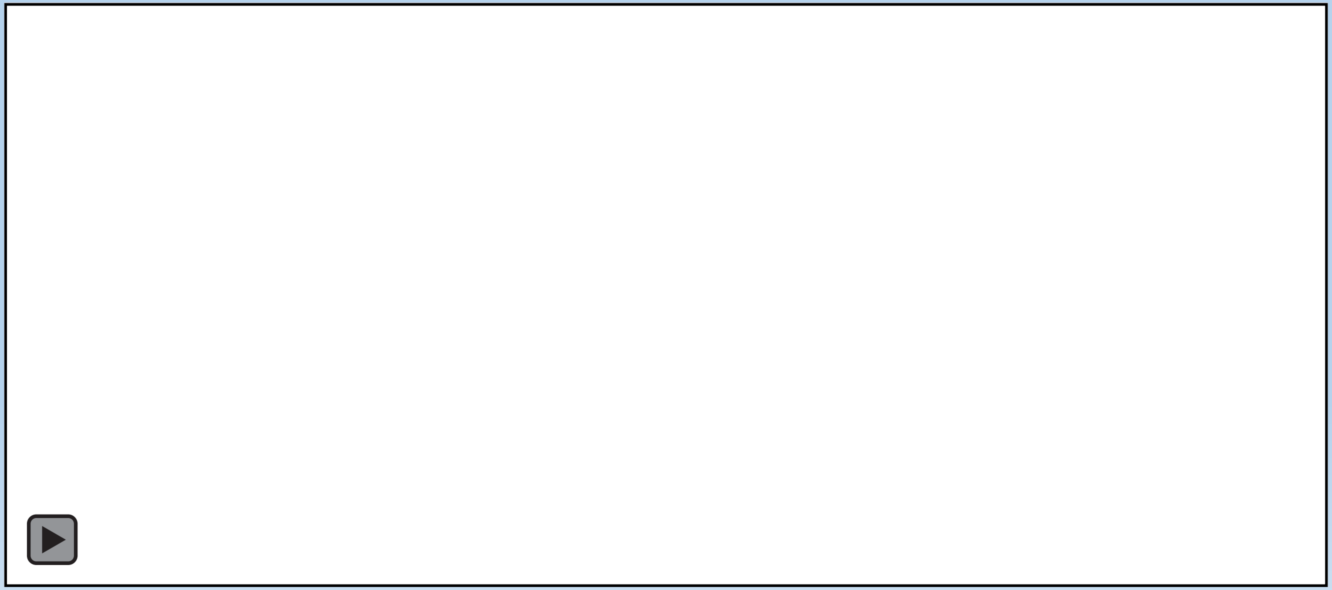
- **Gordon-Haus Effect:** **J. P. Gordon, Opt. Lett. 8, 596 (1983)**



($\Delta\phi$=0) in-phase collision



($\Delta\phi = \pi$) out-of-phase collision



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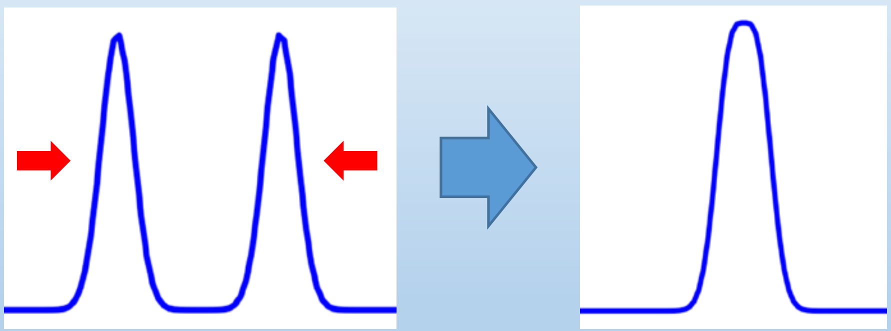

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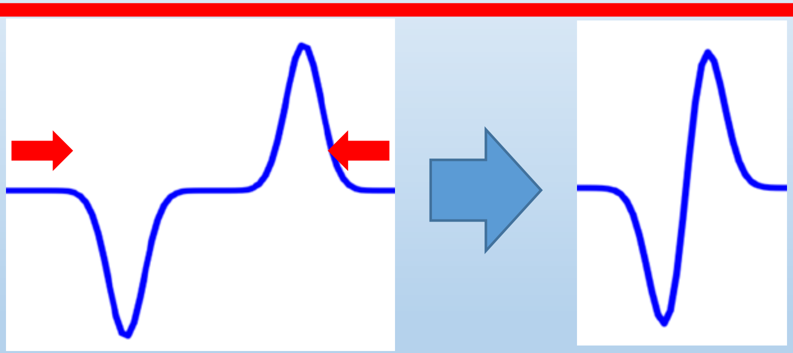
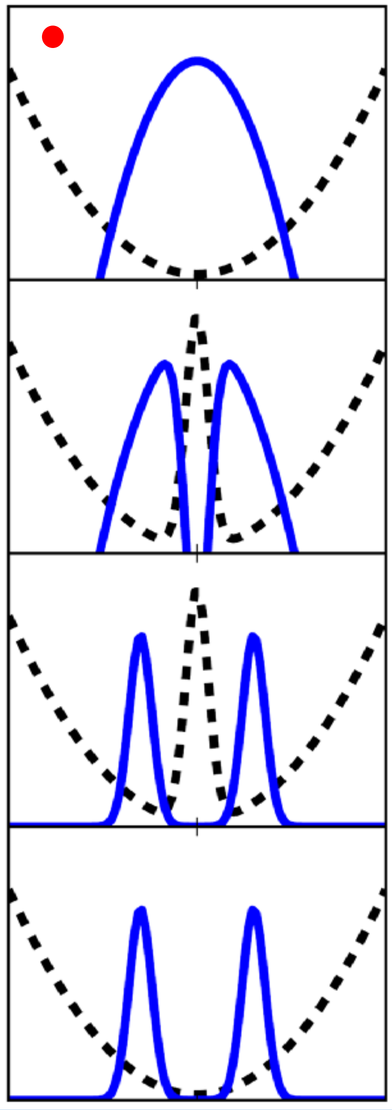
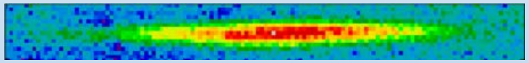


($\Delta\phi = \pi$) out-of-phase collision

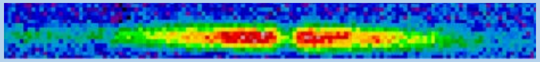
Formation of Soliton Pair for Collision Study



- form BEC by evaporation at +140a_o



- turn on barrier

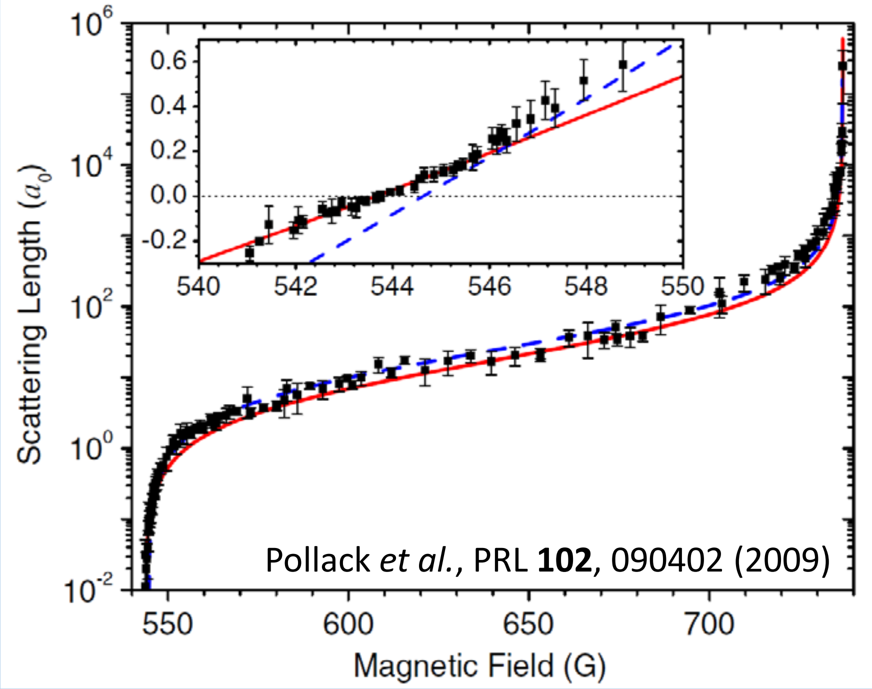


- ramp magnetic field to -0.57a0

- quickly turn barrier off

$\Delta z = 26\ \mu m$

^{7}Li Feshbach Resonance

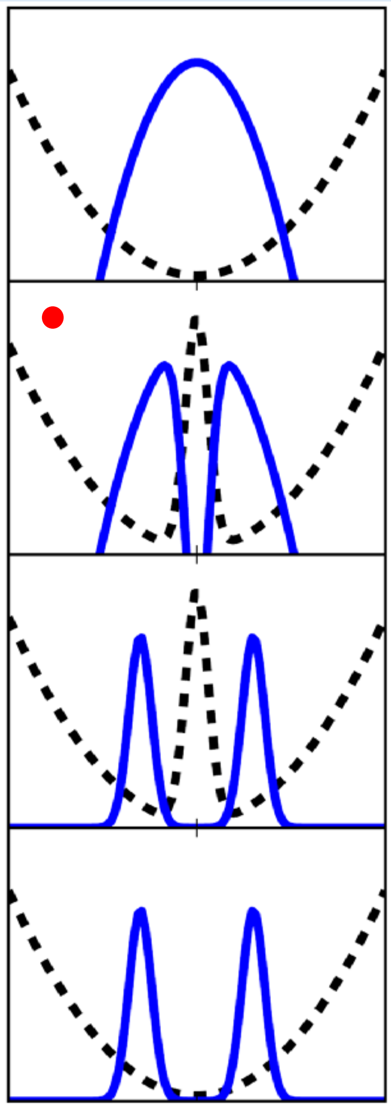


Atoms in $|F = 1, m_F = 1\rangle$ state

$\omega_z = 2\pi \times 31$ Hz

$\omega_r = 2\pi \times 254$ Hz

Formation of Soliton Pair for Collision Study



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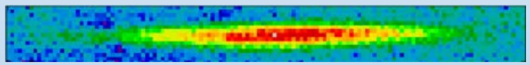
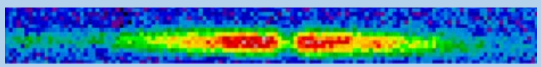


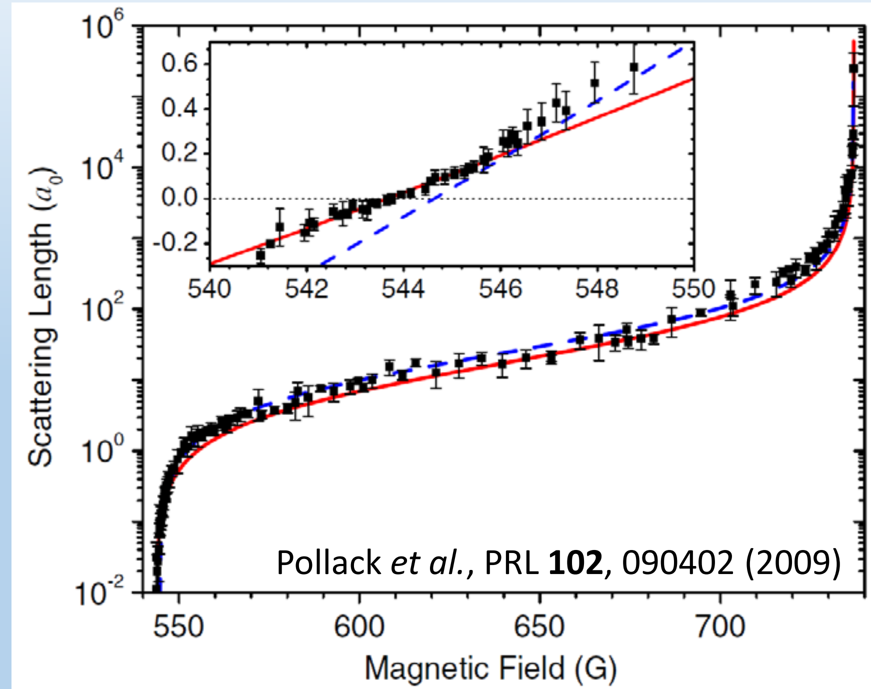
- **turn on barrier**



- **ramp magnetic field to -0.57a0**

- **quickly turn barrier off**

^{7}Li Feshbach Resonance



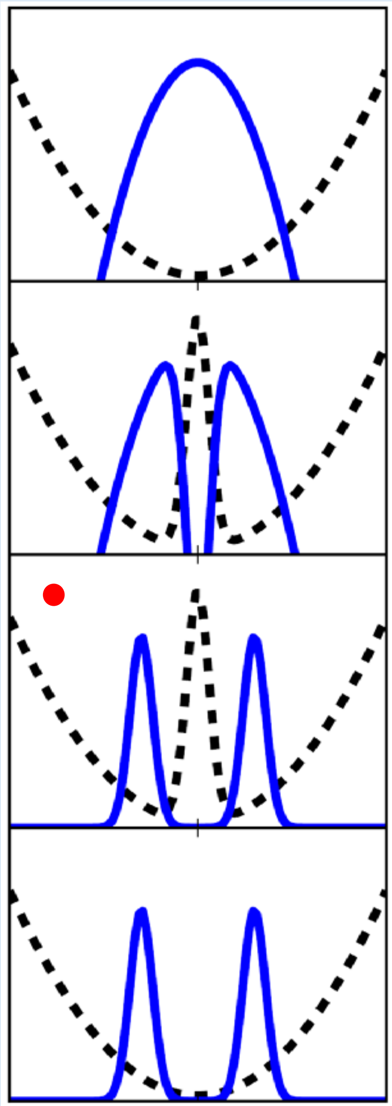
Atoms in $|F=1, m_F=1\rangle$ state

$w_x = 2.2$ mm

$w_z = 5.6\ \mu$m

900 GHz blue-detuned light sheet

Formation of Soliton Pair for Collision Study



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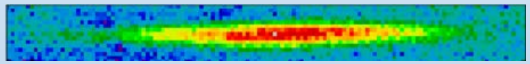
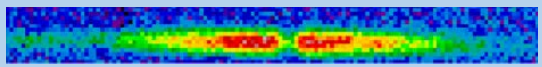


- **turn on barrier**



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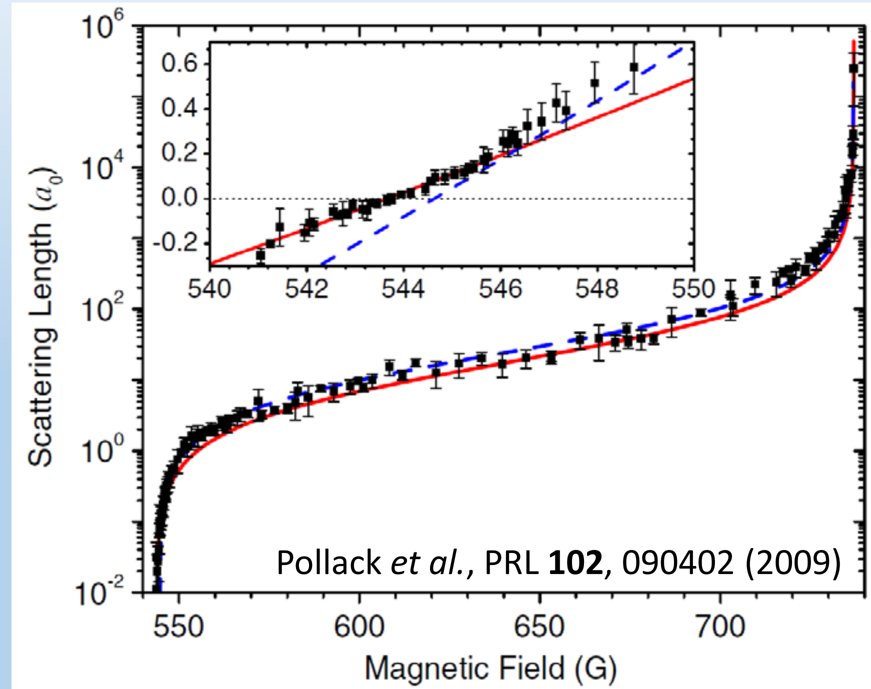

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$N \sim 28\,000$, N/Nc=-0.53

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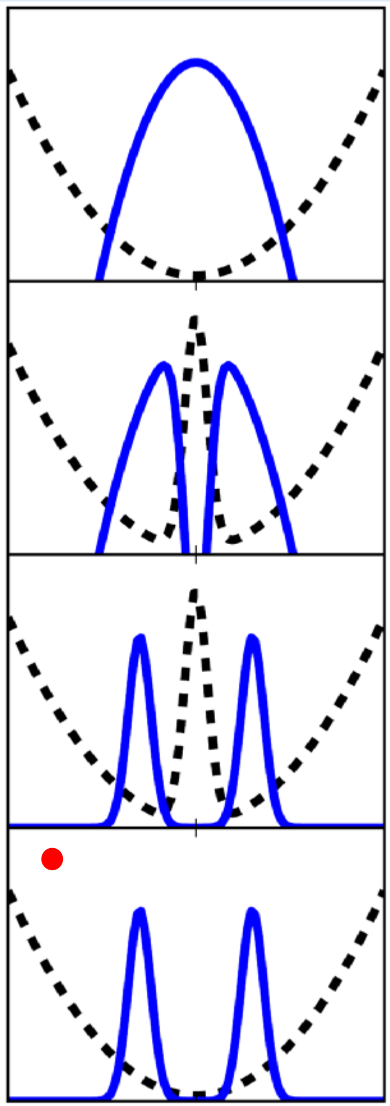

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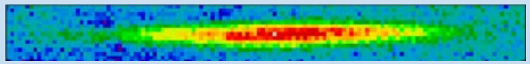

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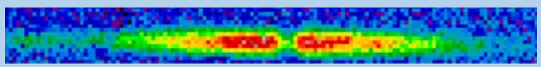

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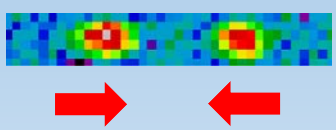

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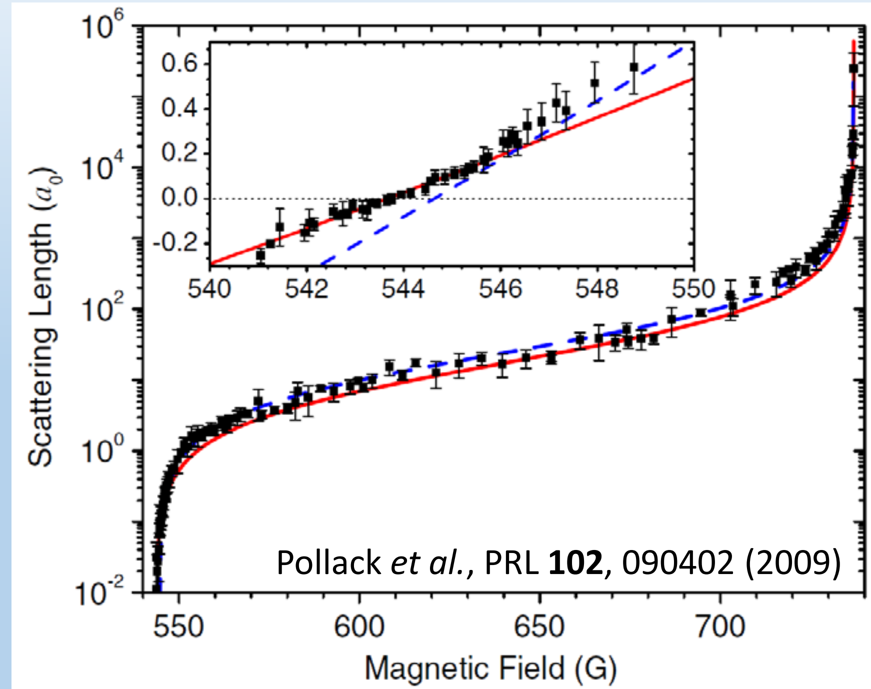

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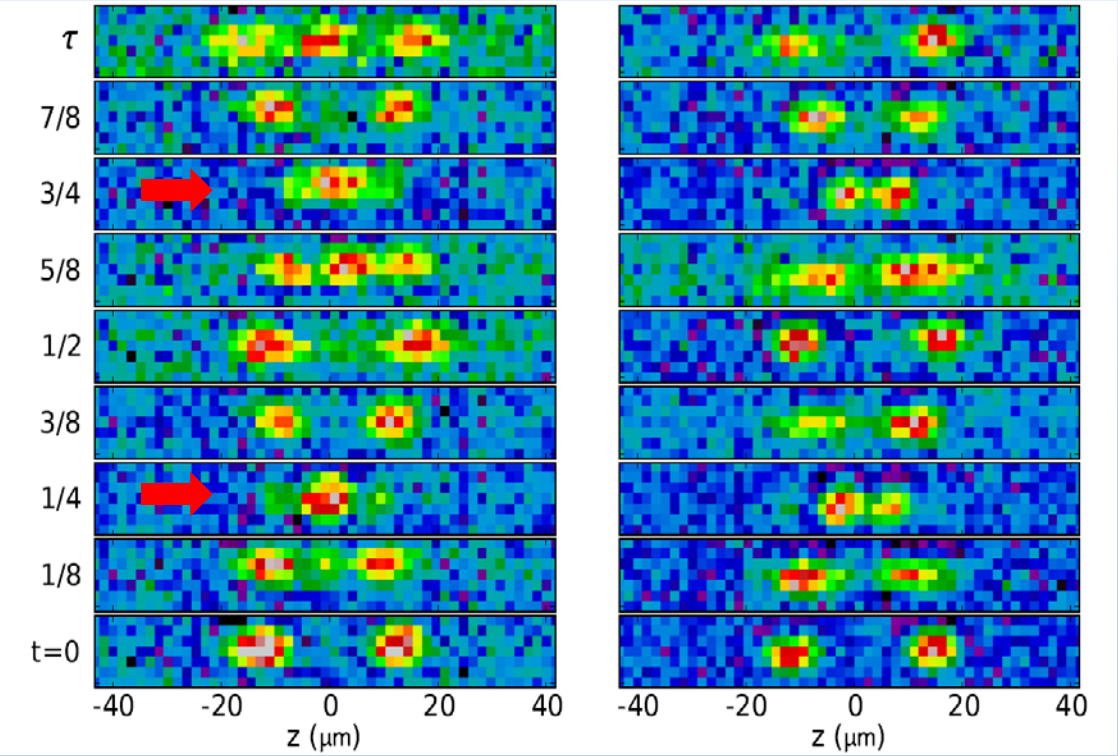
Phase-Dependent Collisions



$\Delta\phi \approx 0$ $\Delta\phi \approx \pi$

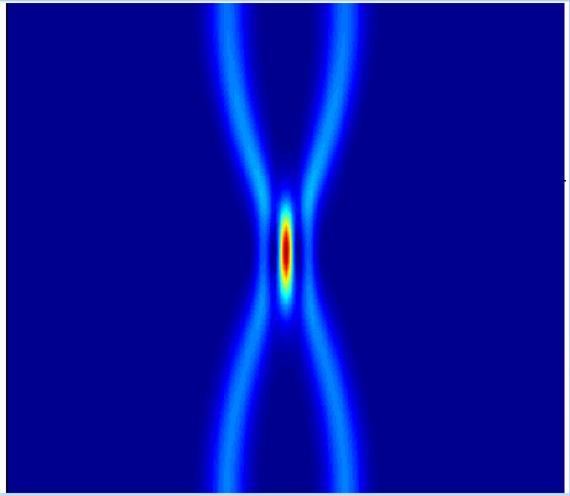
Nguyen, Dyke, Luo, Malomed & Hulet., Nat. Phys. **10**, 918-922 (2014)

Also, Parker, Martin, Cornish, Adams,
J Phys B 41, 045303 (2008)

- **collisions for a full trap period** $(\tau = 32\ ms)$
- **multiple images using phase contrast**
- **N/ N_c = -0.53**
- $\Delta\phi$ **inferred from simulations:**

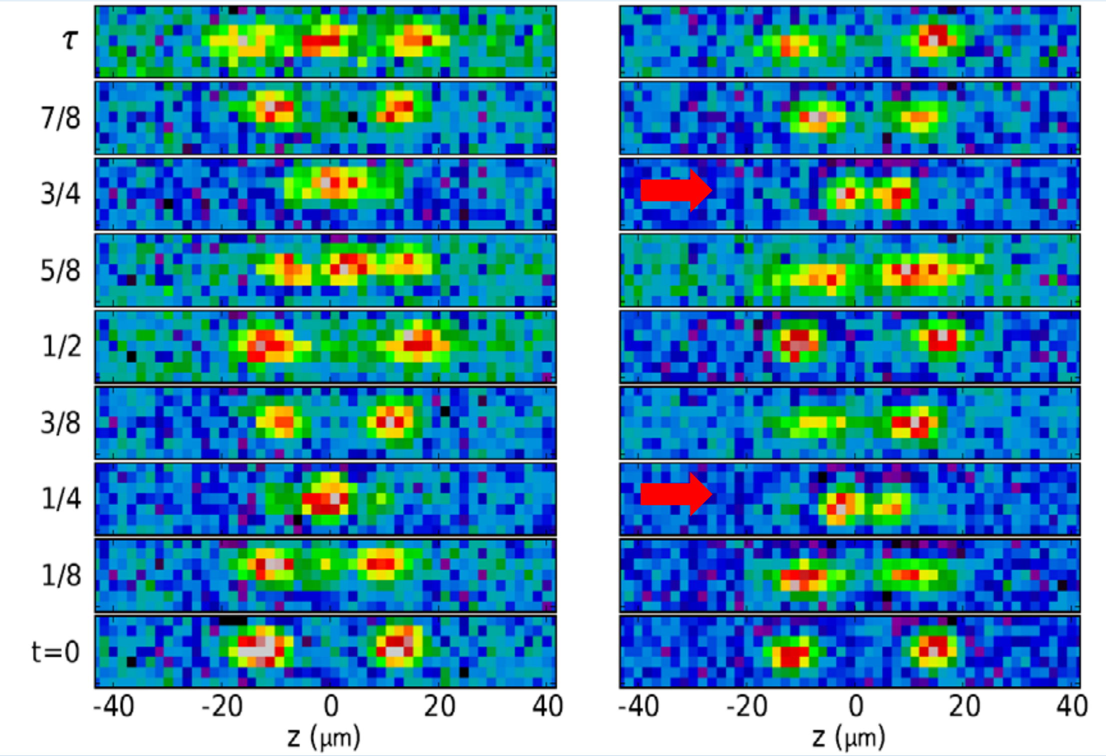
$\Delta\phi = 0$ **collision**

t



z

Phase-Dependent Collisions



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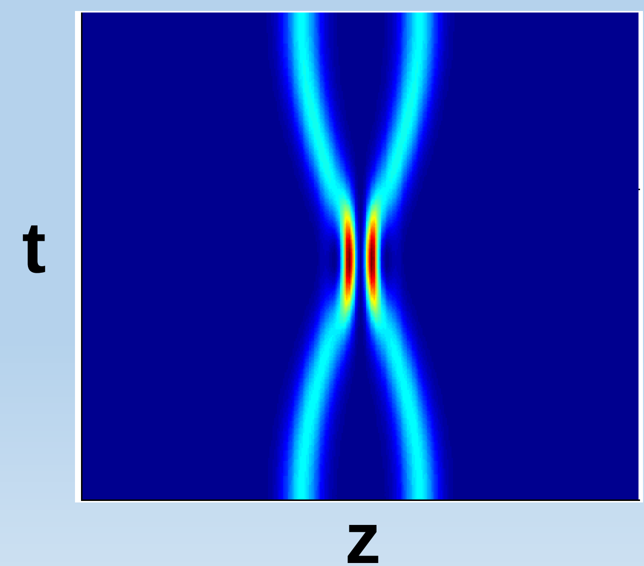
Nguyen, Dyke, Luo, Malomed & Hulet., Nat. Phys. **10**, 918-922 (2014)

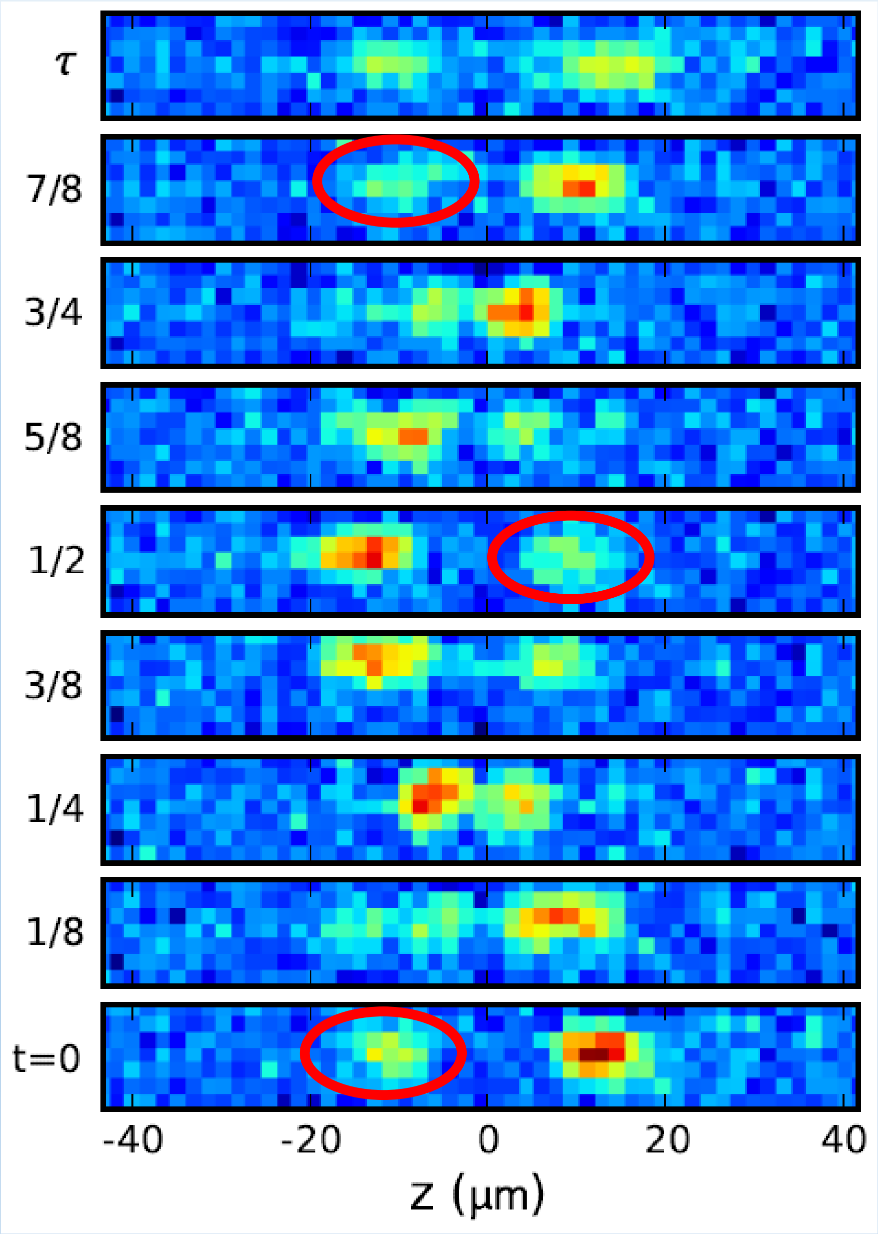
- collisions for a full trap period ($\tau = 32\ ms$)
- multiple images using phase contrast
- $N/N_c = -0.53$
- $\Delta\phi$ inferred from simulations:

$\Delta\phi = \pi$ collision

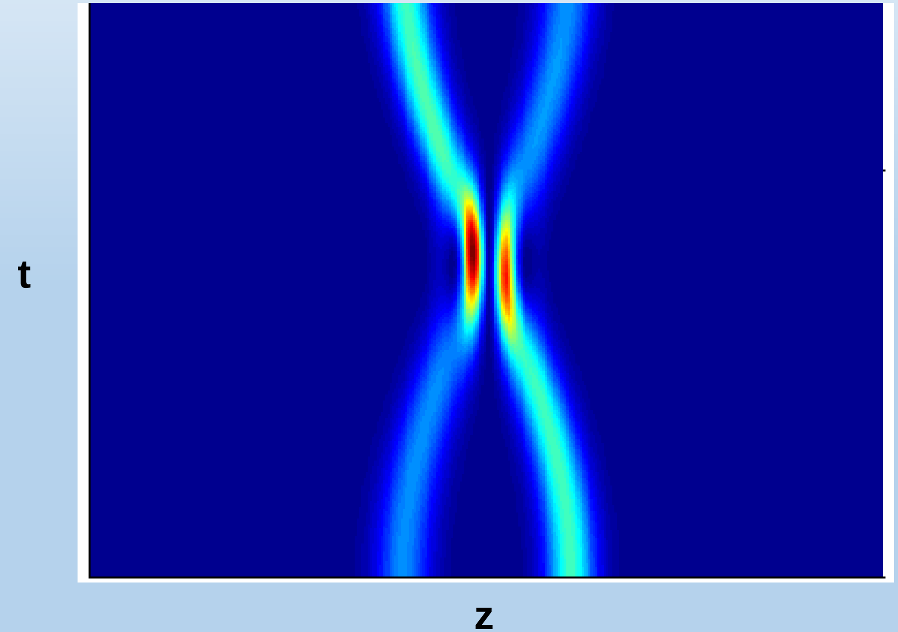


But, integrability → solitons must pass through one another (?)

Do They Cross? Tagged Collisions



- **numerical simulation (2:1 ratio):**



- **use resonant beam to remove atoms from only one side**
  - **$\Delta\phi = \pi$: appear to repel**
  - **solitons pass through one another**

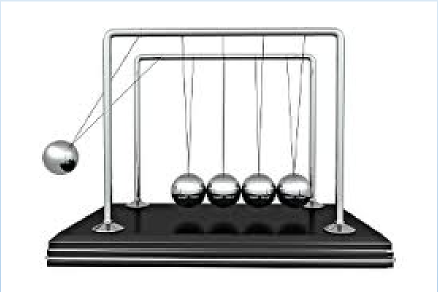
Consistent with integrability!

Examples of Quantum Integrability

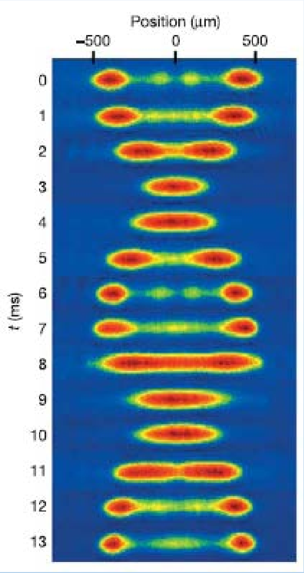
- Lieb-Liniger Model: 1D bosons with point interactions

"Quantum Newton's Cradle":

Kinoshita, Wenger, Weiss, Nature (2006)



100's of collisions without thermalization



- 1D solitons: $g|\psi|^3$ nonlinearity with $g < 0$, e.g. BEC with $a < 0$

Quasi-1D if $\mu > \frac{1}{2}\hbar\omega_r$, otherwise unstable to *collapse*

The critical number for collapse is: $N_c = 0.7\, a_r / a$, where $a_r = (\hbar/m\omega_r)^{1/2}$

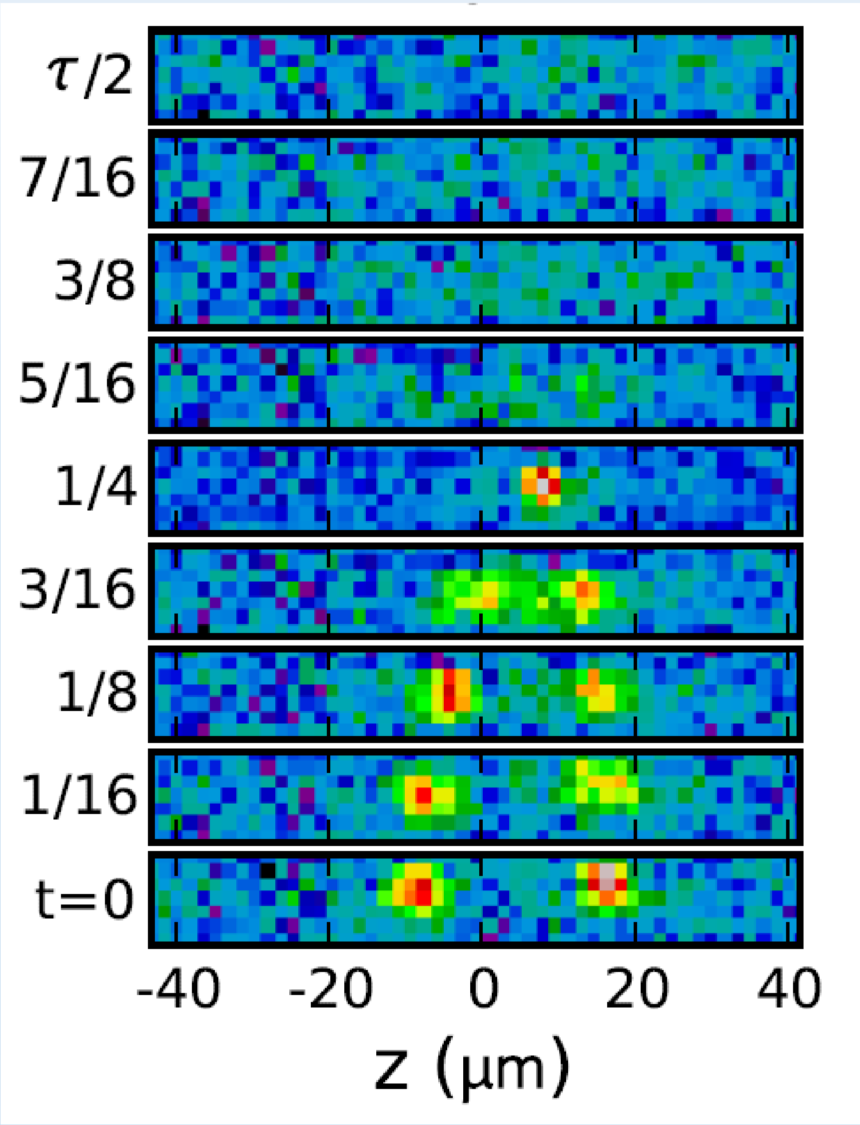
N / N_c is a measure of the strength of the nonlinearity and

$N / N_c = 1$ defines an *integrability edge*

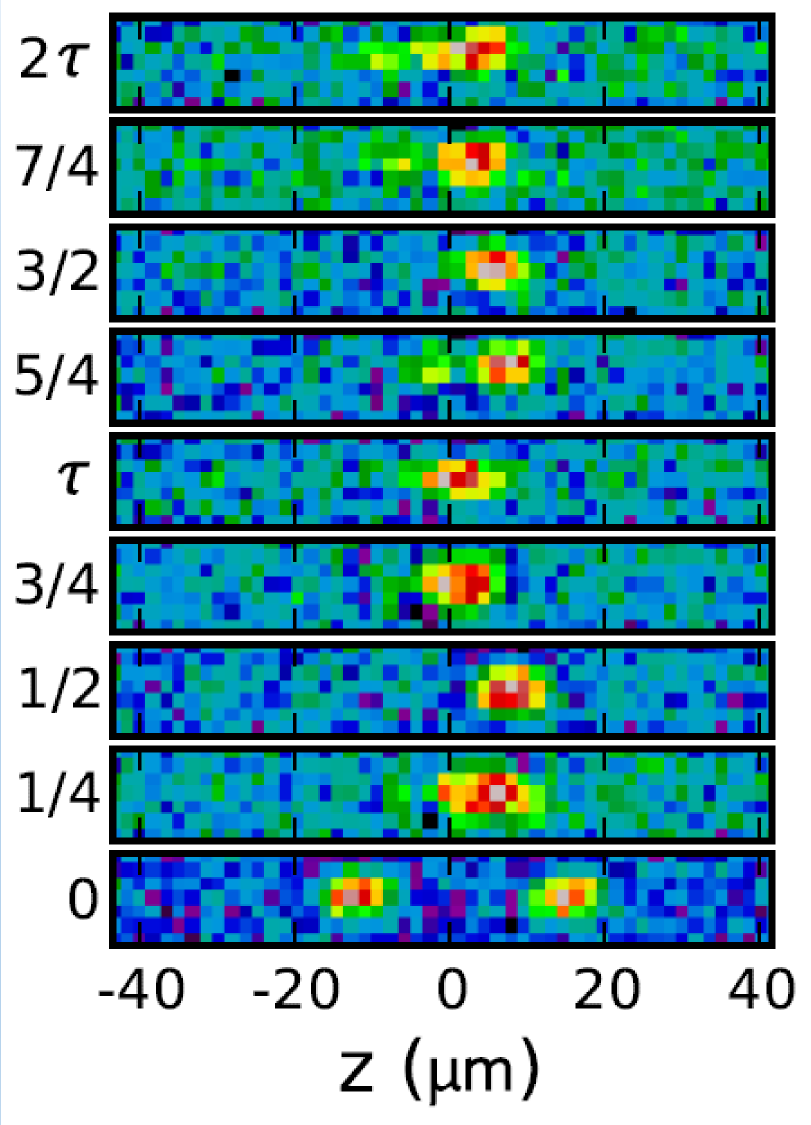
What are the implications of this boundary?

On the Edge of Integrability: N/ N_c = -0.53, $\Delta\phi \approx 0$

Annihilation



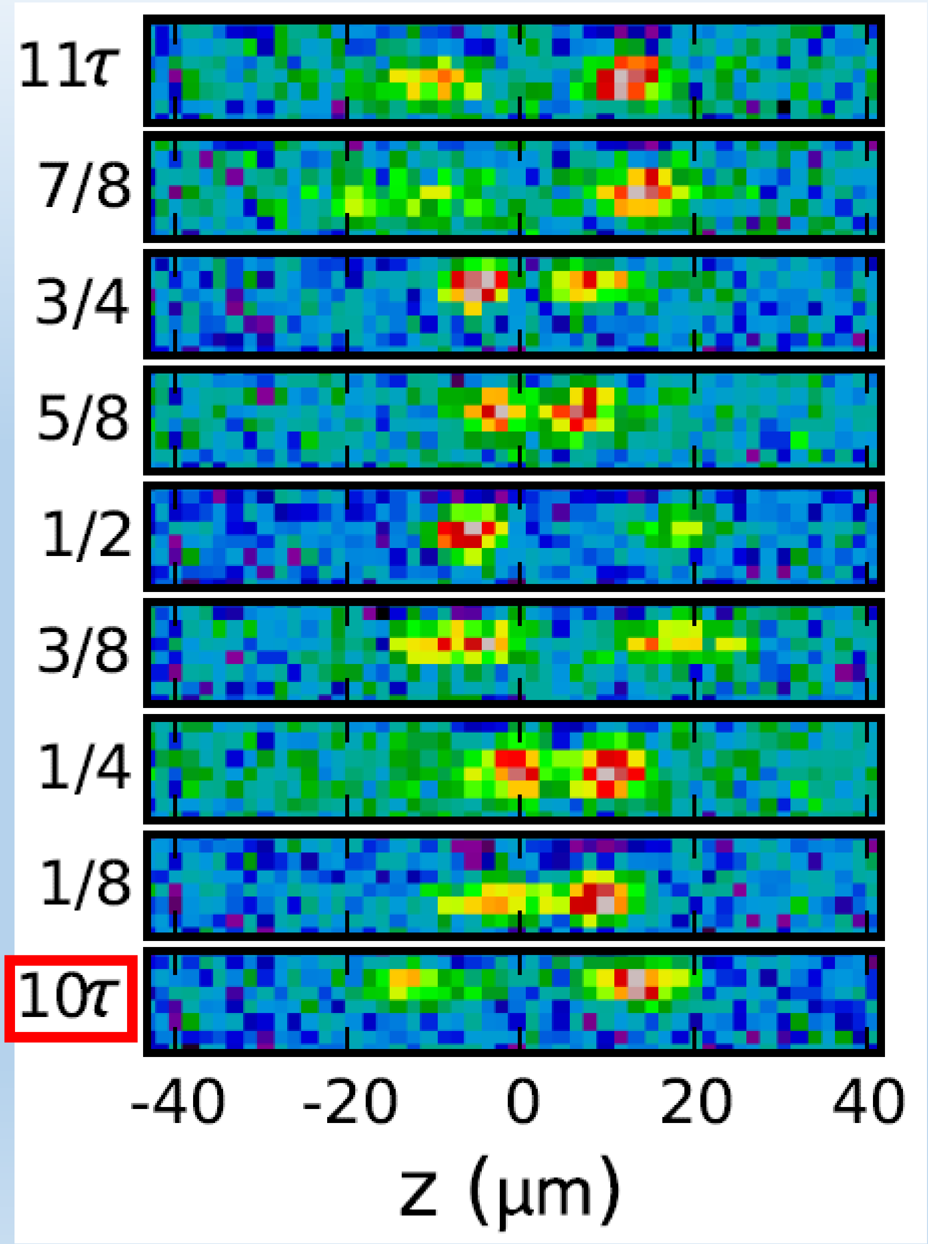
Merger



Nguyen, Dyke, Luo, Malomed & Hulet.,
Nat. Phys. **10**, 918-922 (2014)

Simulations, Durham: Parker et al, JPB (2008)
Billam et al, PRA (2011)

On the Edge of Integrability: $N/N_c = -0.53$, $\Delta\phi \approx \pi$

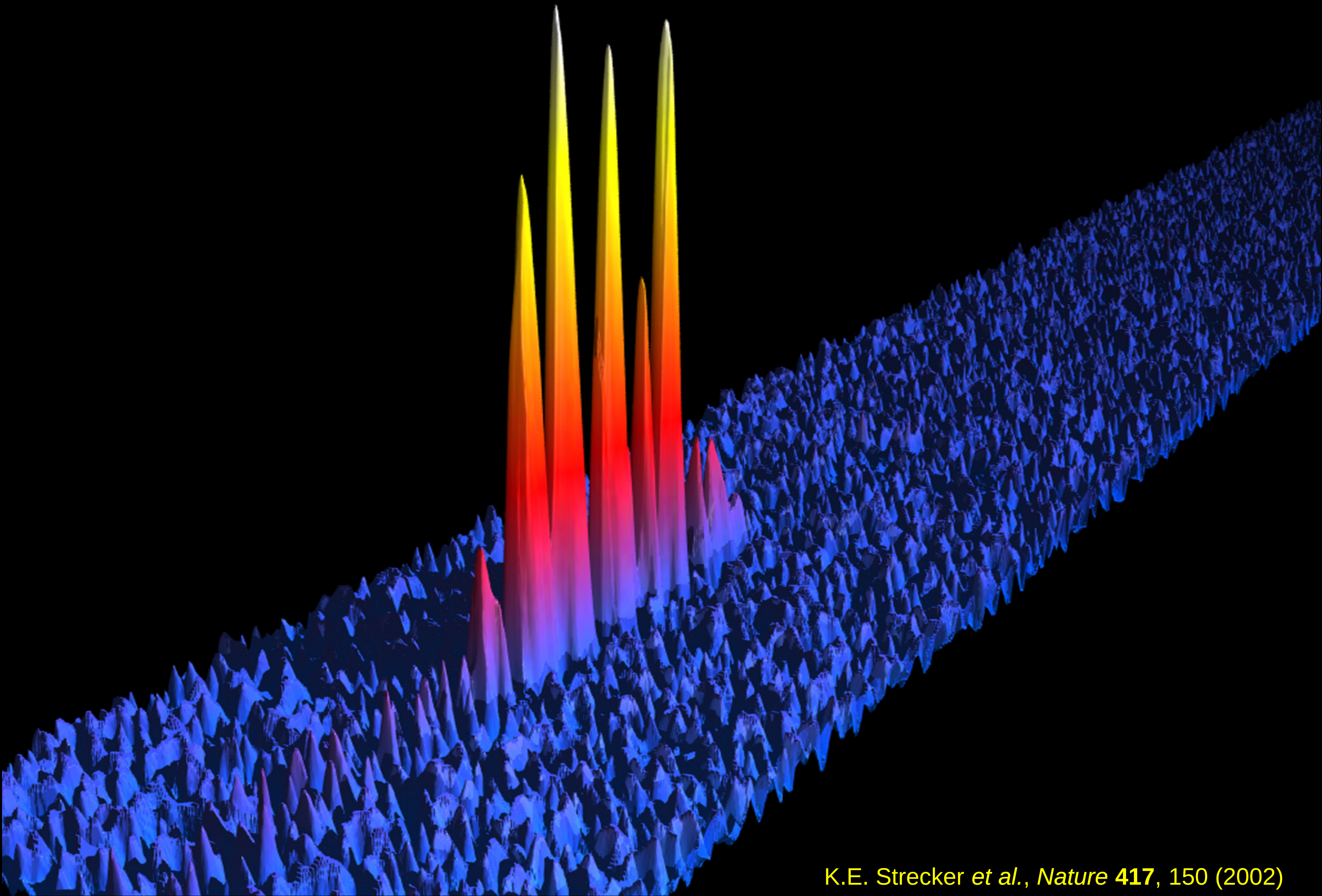


Behaves as if it were *integrable:*

survives for > 20 collisions

Nguyen, Dyke, Luo, Malomed & Hulet., Nat. Phys. **10**, 918-922 (2014)

Fast quench → Soliton Train



K.E. Strecker *et al.*, *Nature* **417**, 150 (2002)

Soliton Train Formed by Modulational Instability

- Soliton train formed by modulational instability (MI) following a quench:
  → exponential growth of amplitude and phase fluctuations at

$$k_{mg} = \frac{1}{\xi}$$

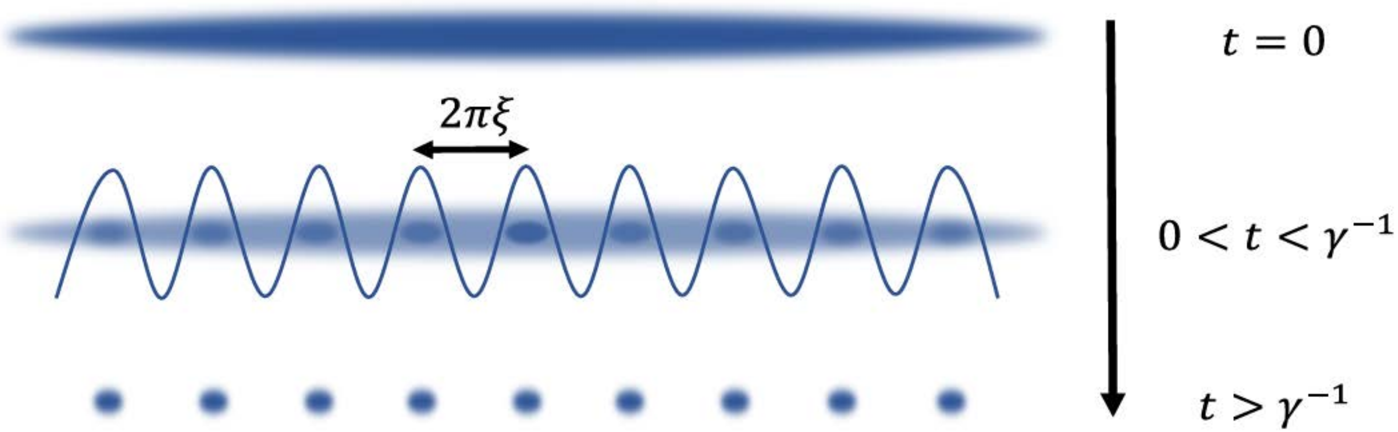
wavevector of maximal growth

$$\xi = \frac{a_r}{\sqrt{4|a_f|n_{1D}}}$$

length scale

$$\gamma = 2\omega_r |a_f| n_{1D}$$

characteristic rate



MI is seen in many wave contexts involving a self-focusing nonlinearity:

- deep water waves (Benjamin-Feir instability) – rogue waves
- plasma instabilities
- optical fibers with anomalous dispersion

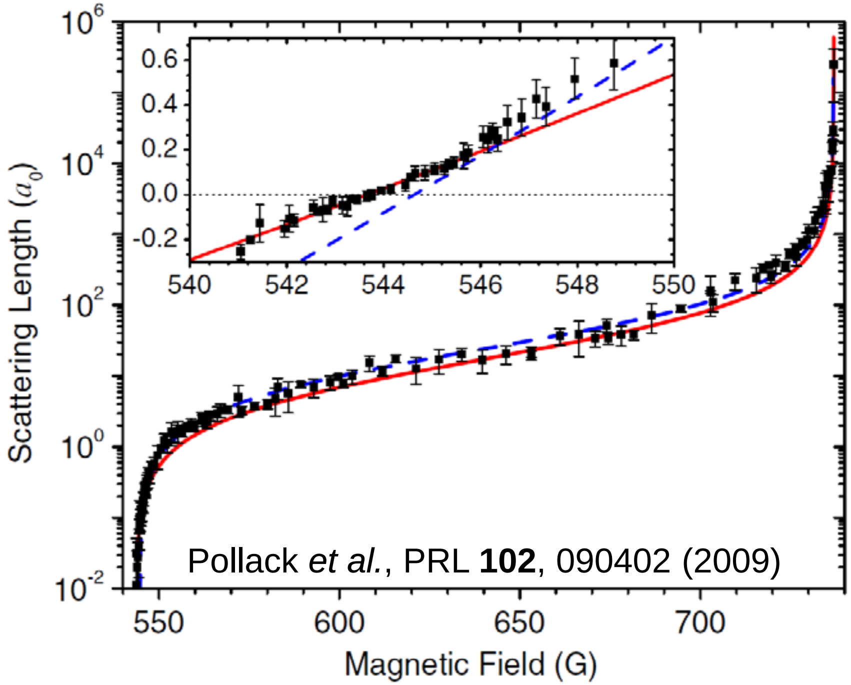
Formation of Soliton Trains

- **trap frequencies**

$$\omega_r = 2\pi \times 346\ Hz$$
$$\omega_z = 2\pi \times 7.4\ Hz$$

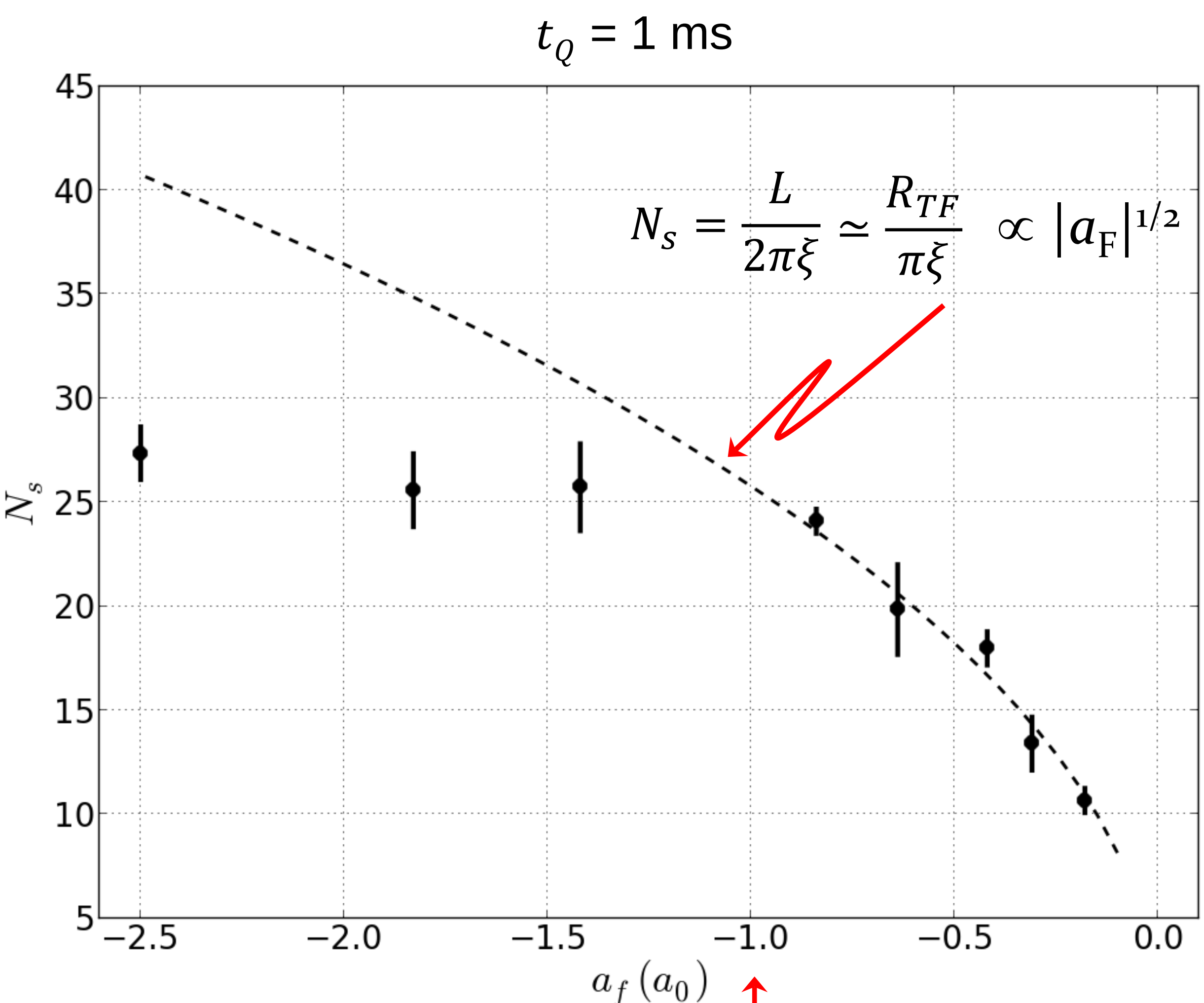
- **start with BEC with a = +3a_0**
- **quench to *a* < 0 in t_Q = 1 ms**
- **hold for t_h, take *in-situ* image**

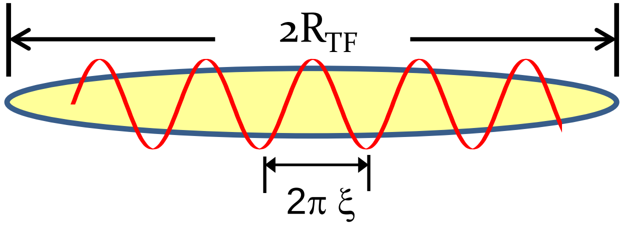
^{7}Li Feshbach Resonance



Pollack *et al.*, PRL **102**, 090402 (2009)

Number Solitons vs Interaction





$\gamma^{-1} < t_Q$

$\gamma^{-1} = t_Q$

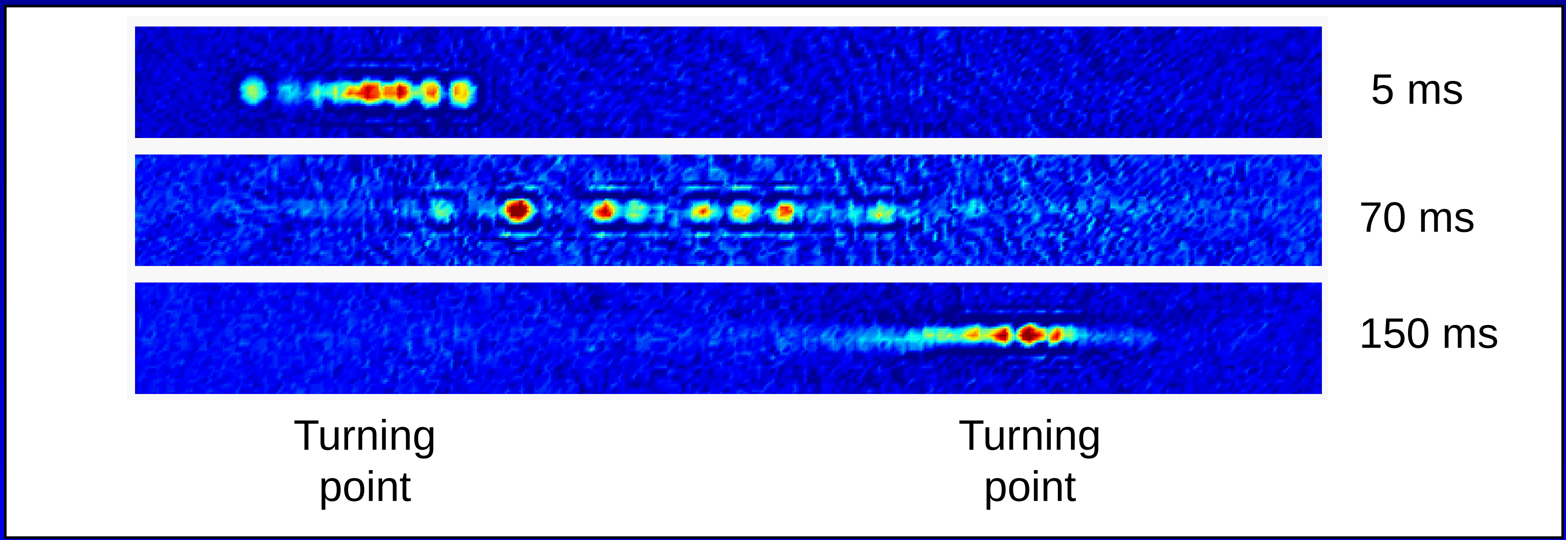
$\gamma^{-1} > t_Q$

where the freeze-out time = quench time

Solitons Appear to Repel

Create soliton train, then set into harmonic oscillation

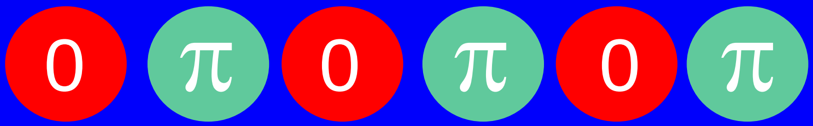
Distance between solitons increases at bottom of well, while bunching at turning points → repulsive interaction



K.E. Strecker et al., Nature 417, 150 (2002)

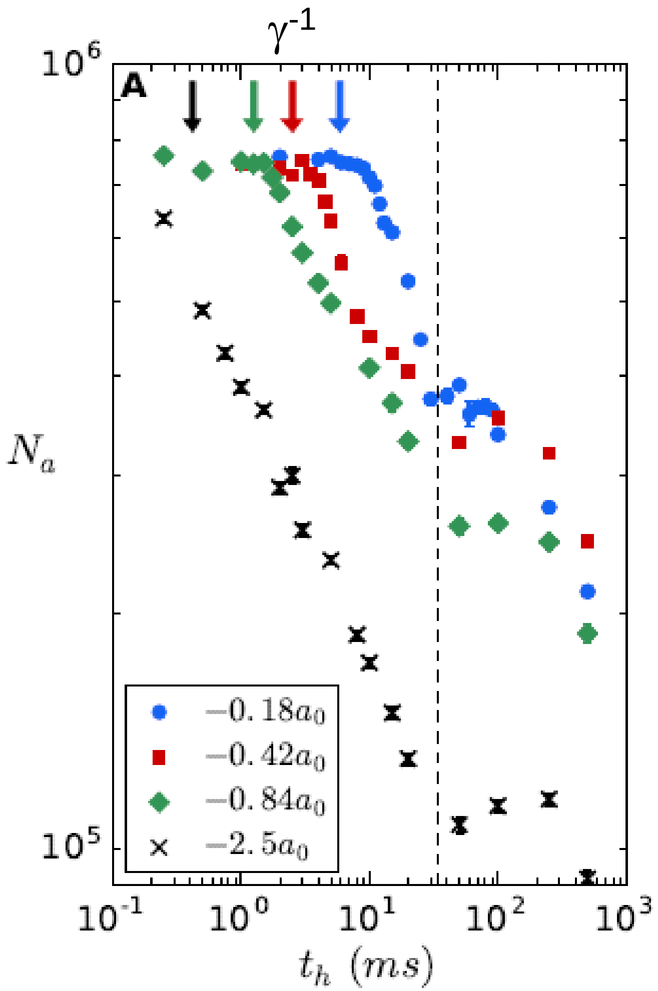
Also, S. Cornish *et al*., PRL 96, 170401 (2006) (JILA)

Gordon-Haus Effect: Repulsive interaction for out-of-phase solitons



How does this phase structure arise? Are they born that way, or does it develop?

Atom Loss During Hold Time



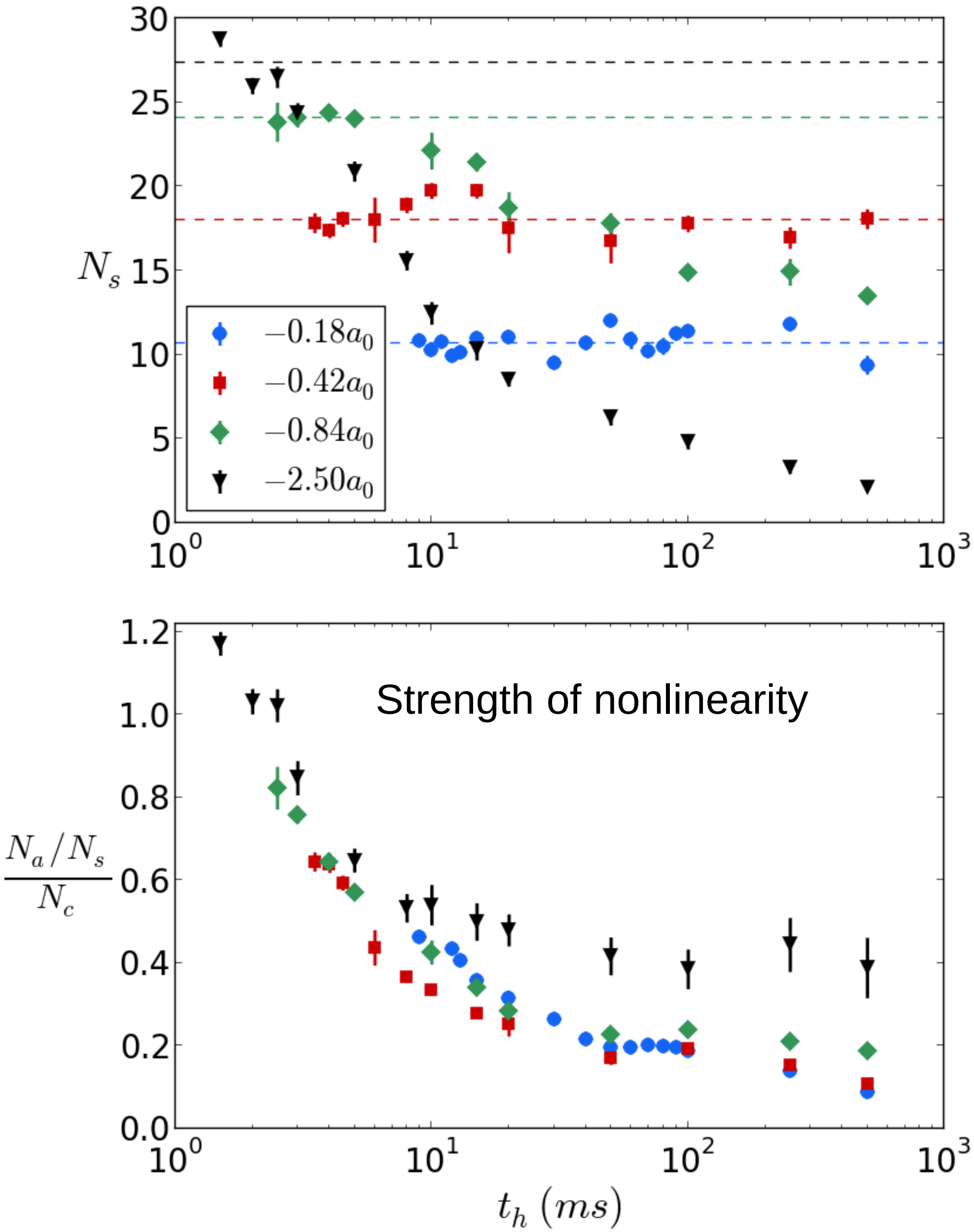
Data collapse by plotting N_a vs. γ^{-1}

$$N_a \propto (t_h \gamma)^\kappa$$

with $\kappa = 0.35(1)$

t_Q = 1 ms: $\gamma^{-1} < t_Q$ for $a_f = -2.5a_0$

Soliton Number vs Hold Time



- Small $|a_f|$: soliton number is constant
- Large $|a_f|$: soliton number decreases

- Particle loss without loss of solitons:

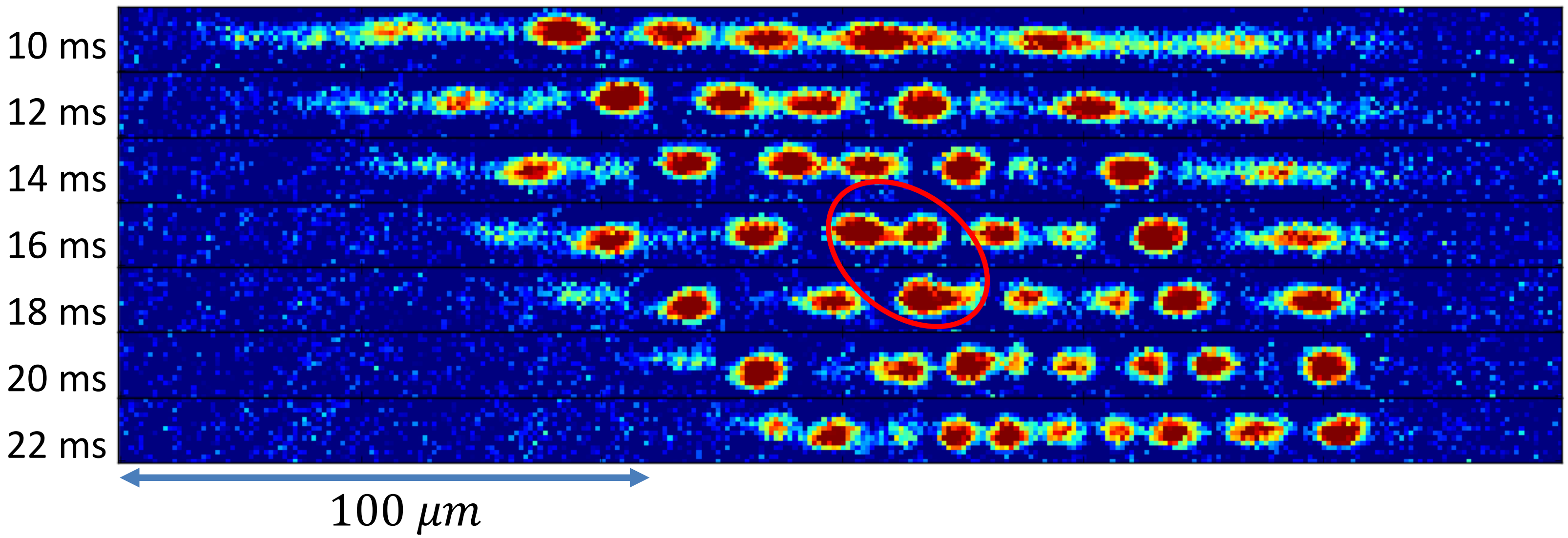
  → driven by partial, local collapse

(Papers by: Saito/Ueda, Kagan/Shlyapnikov)

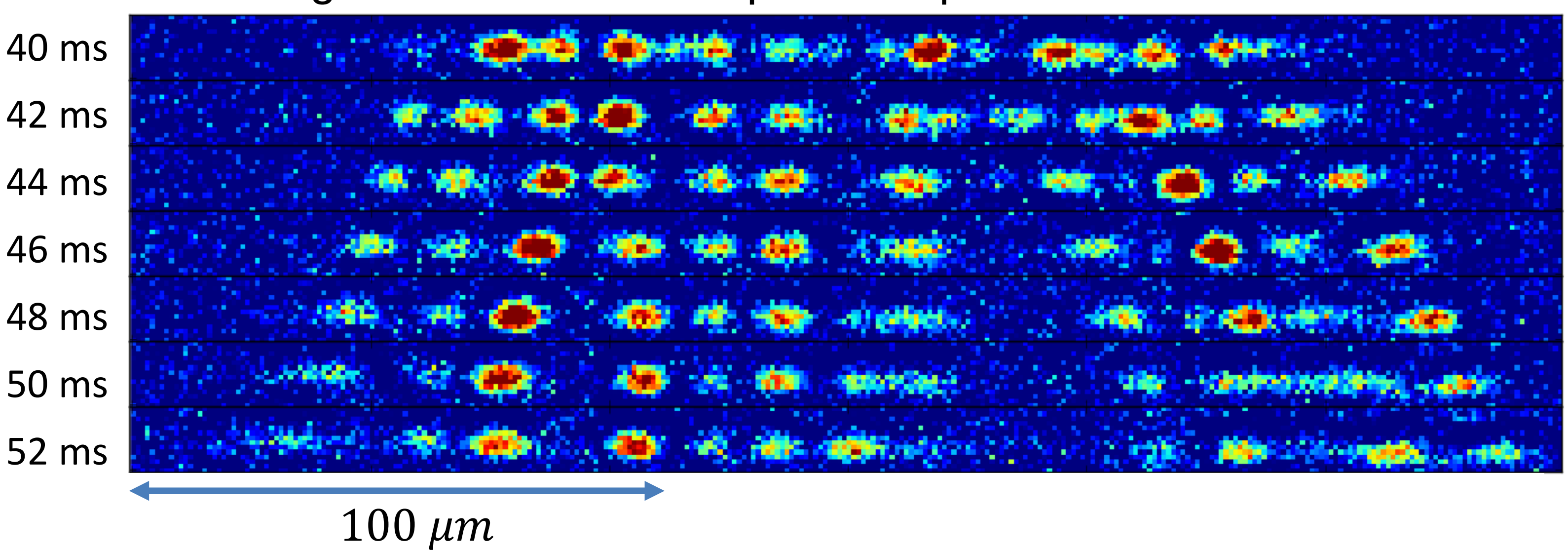
Soliton Train Dynamics

$a_f = -0.18a_0$

Mainly see repulsion between solitons, with just a few cases of attraction



Breathing mode excited: compression phase at ~34 ms



Conclusions

- System on the edge of integrability:
    - Integrability → solitons pass through one another without changing shape, speed, or amplitude
    - Yet, they undergo phase-dependent collisions
    - Breakdown of integrability is sudden and depends on $\Delta\phi$: annihilation and merger

- Soliton train is "born" with an alternating phase, whereas the notion of self-assembly by soliton annihilations is not supported by observation (L. Salasnich; H. Stoof)

- Freeze-out time agrees with modulational instability (L. Carr and J. Brand)

- After the freeze-out time, there is a continuous loss of atoms driven by partial local collapses (Saito/Ueda and Kagan/Shlyapnikov)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