

Wuhan Institute of Physics  
and Mathematics  
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# Quantum Magnetism with Ultracold Fermions

Pedro Duarte, Russell Hart, Tsung-Lin Yang, Xinxing Liu

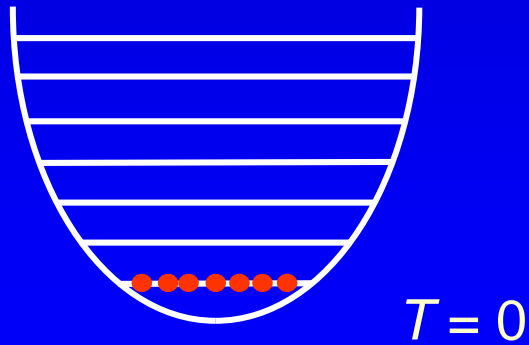
David Huse (Princeton), Thereza Paiva (Rio), Ehsan Khatami (San Jose),  
Richard Scalettar (UC Davis), Nandini Trivedi (Ohio State)



# Lithium: Non-identical Twins

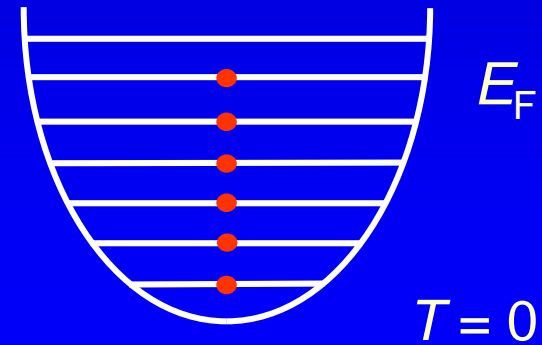
${}^7\text{Li}$

- 3 e's, 3 p's, 4 n's  
= 10 spin- $\frac{1}{2}$  particles  
 $\Rightarrow$  **Boson**
- 94% abundance

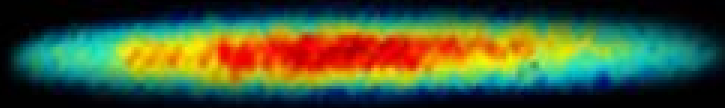


${}^6\text{Li}$

- 3 e's, 3 p's, 3 n's  
= 9 spin- $\frac{1}{2}$  particles  
 $\Rightarrow$  **Fermion**
- 6% abundance

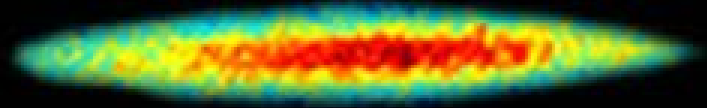


# Bosons



810 nK

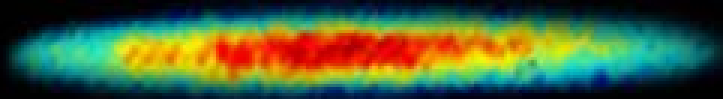
# Fermions



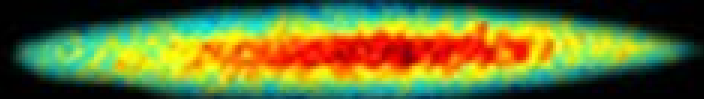
$T/T_F = 1.0$

# Bosons

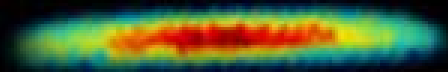
# Fermions



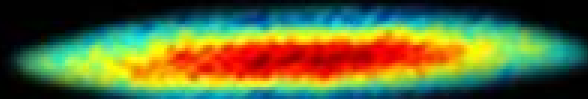
810 nK



$T/T_F = 1.0$



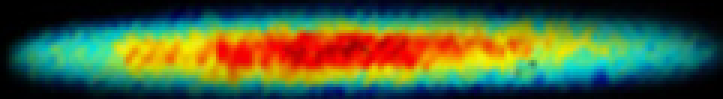
510 nK



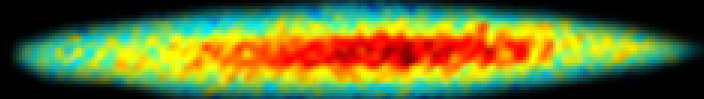
$T/T_F = 0.56$

# Bosons

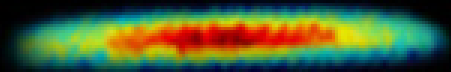
# Fermions



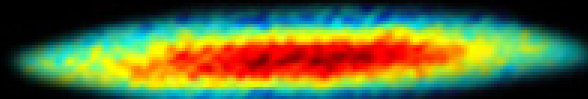
810 nK



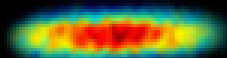
$T/T_F = 1.0$



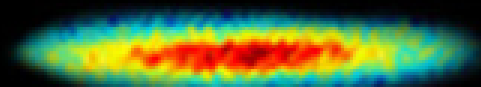
510 nK



$T/T_F = 0.56$



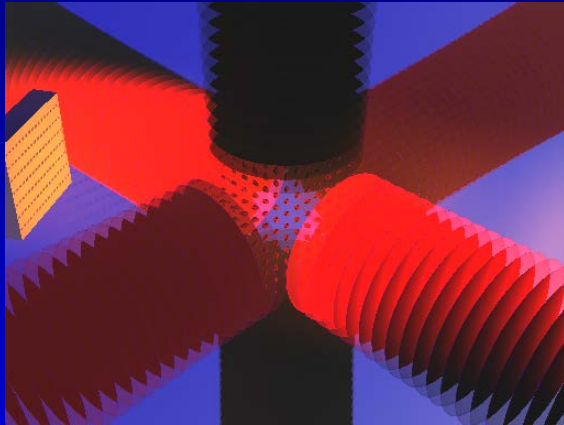
240 nK



$T/T_F = 0.25$

# Many-Body Physics with Ultracold Atoms

Ultracold atoms are being used to create strongly interacting many-body systems  
Relevant to: condensed matter, nuclear, quark matter

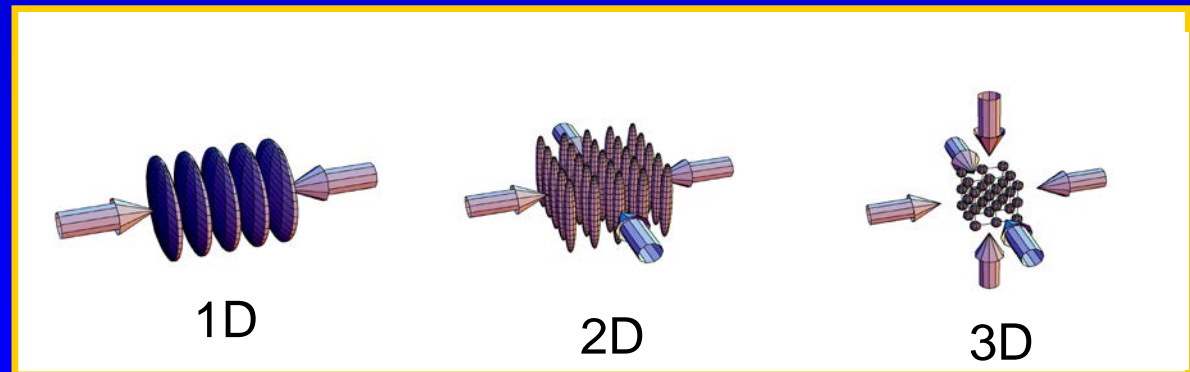


Examples underway:

- quantum magnetism ←
- exotic superconductors
- exactly solvable 1D systems ←
- quantum criticality
- disordered insulators
- topological matter

Tunable parameters:

- interactions ( $U$ )
- lattice hopping ( $t$ )
- temperature ( $T$ )
- density ( $n$ )
- dimensionality
- geometry
- spin polarization
- spin-orbit interaction
- disorder

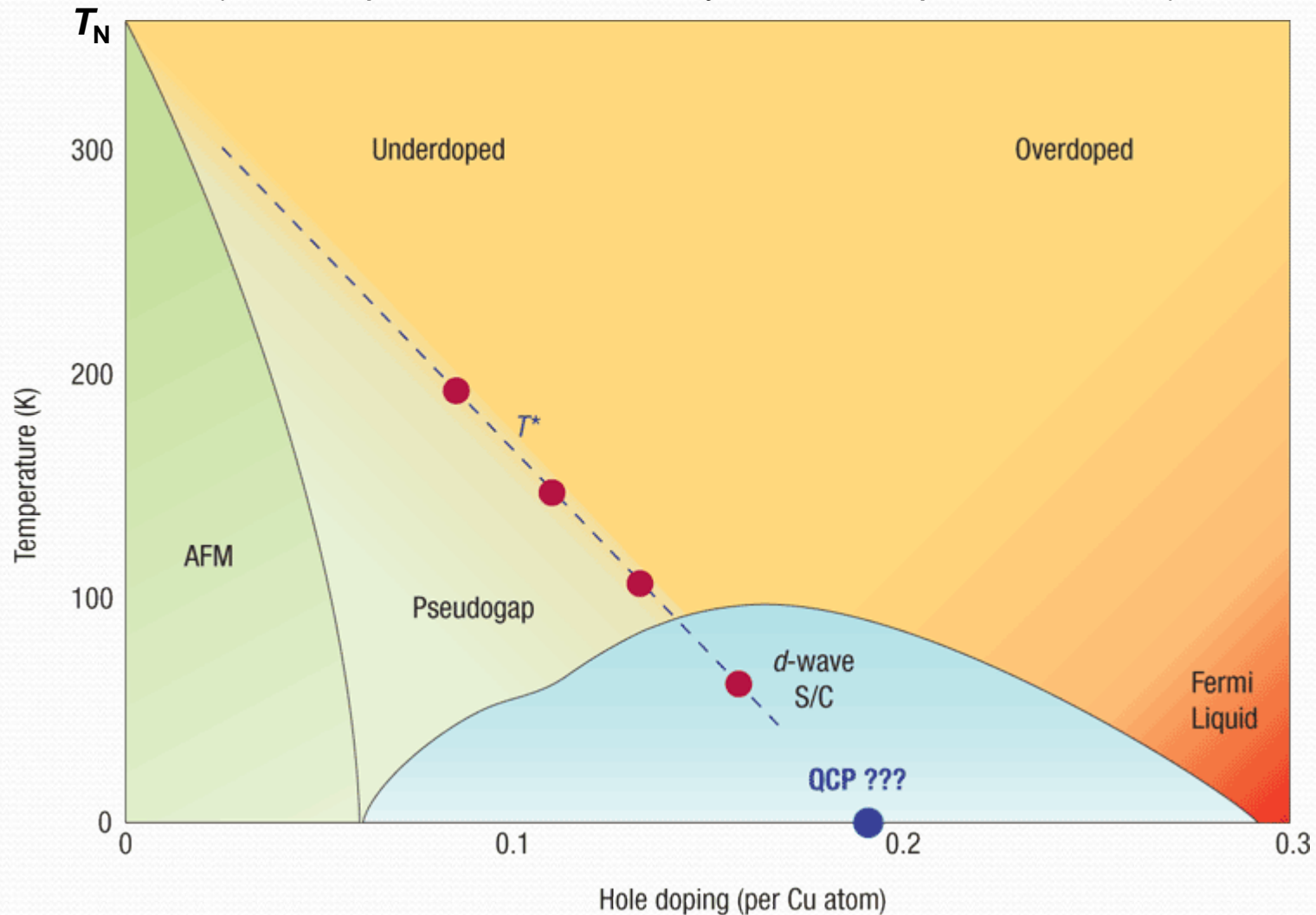


Optical lattice configurations



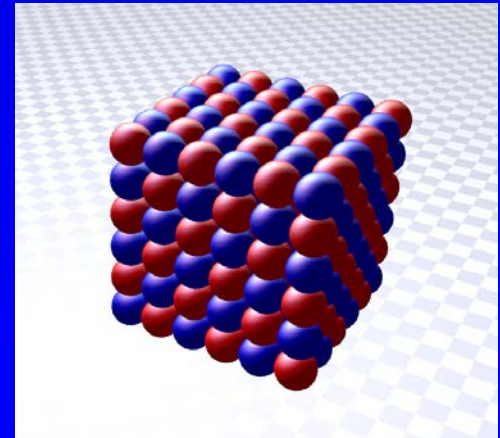
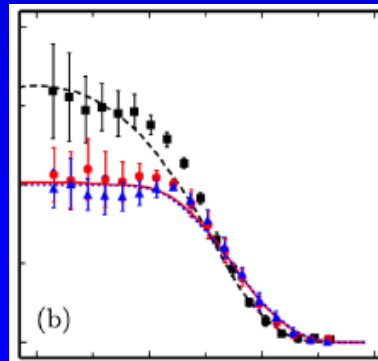
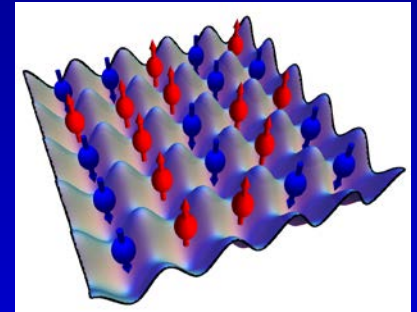
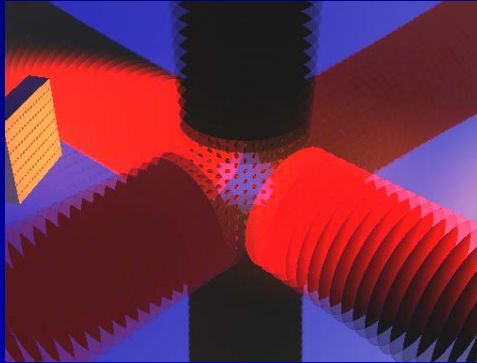
# Phase Diagram of a High- $T_c$ Superconductor

Example: cuprate superconductor  
(also Fe pnictides, and heavy fermion superconductors)



# Outline

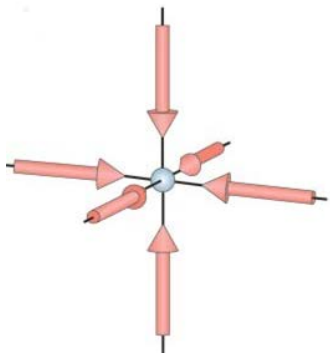
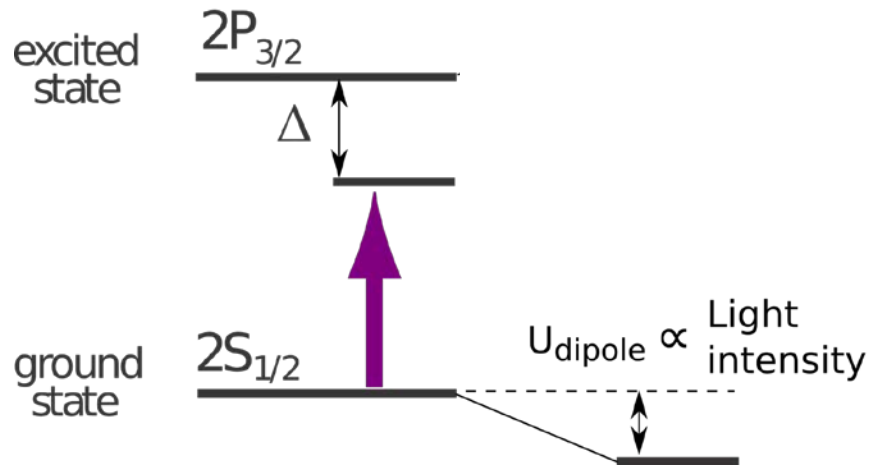
- Optical lattices
- Hubbard model
- Compensated optical lattice
- Mott insulator
- Detecting antiferromagnetic order



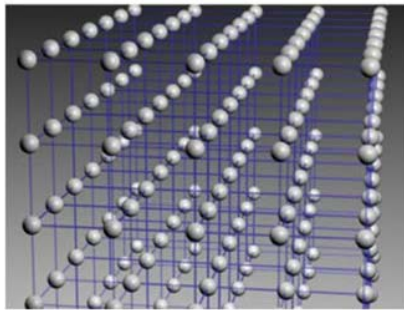


# Optical lattice

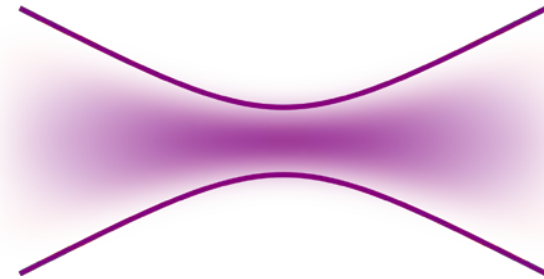
## AC Stark shift



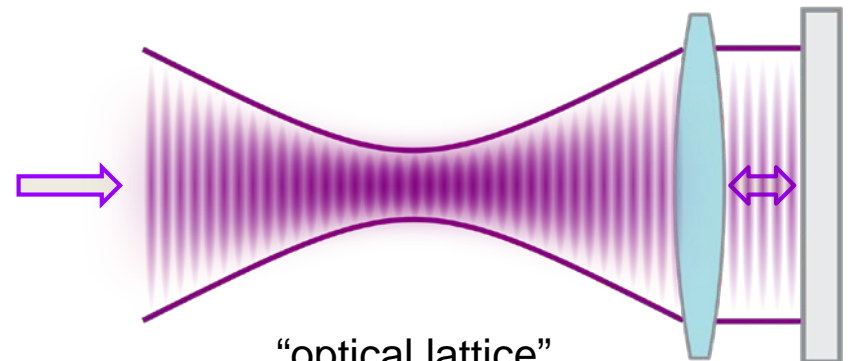
simple cubic lattice



Tightly focused laser beam has an intensity maxima at the waist and can trap atoms there

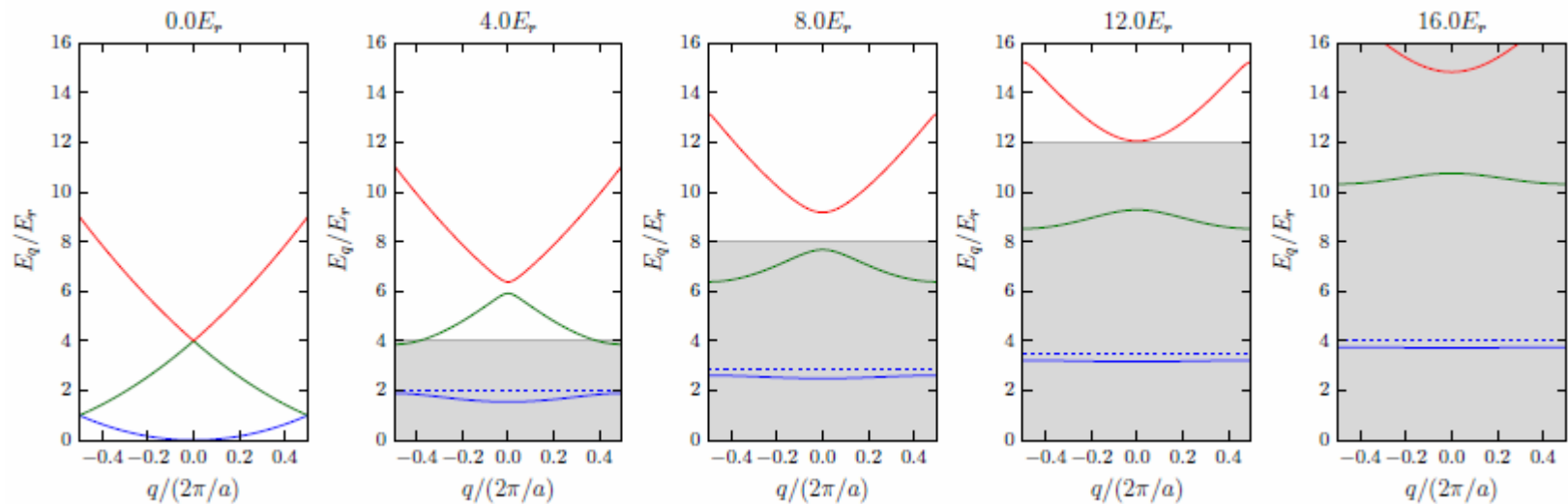


"optical trap"

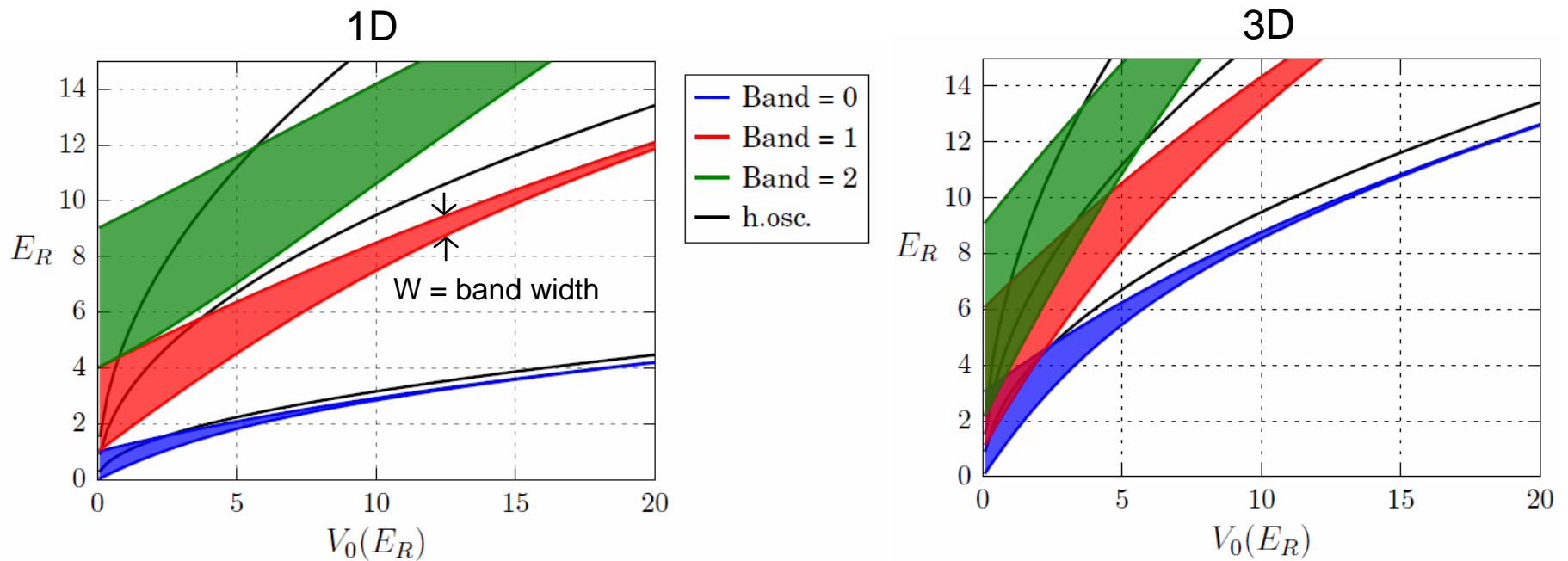


"optical lattice"

# Band Structure



shaded area: lattice depth, dotted line: harmonic approximation  $E_0 \approx v_o^{1/2}$



# Eigenstates

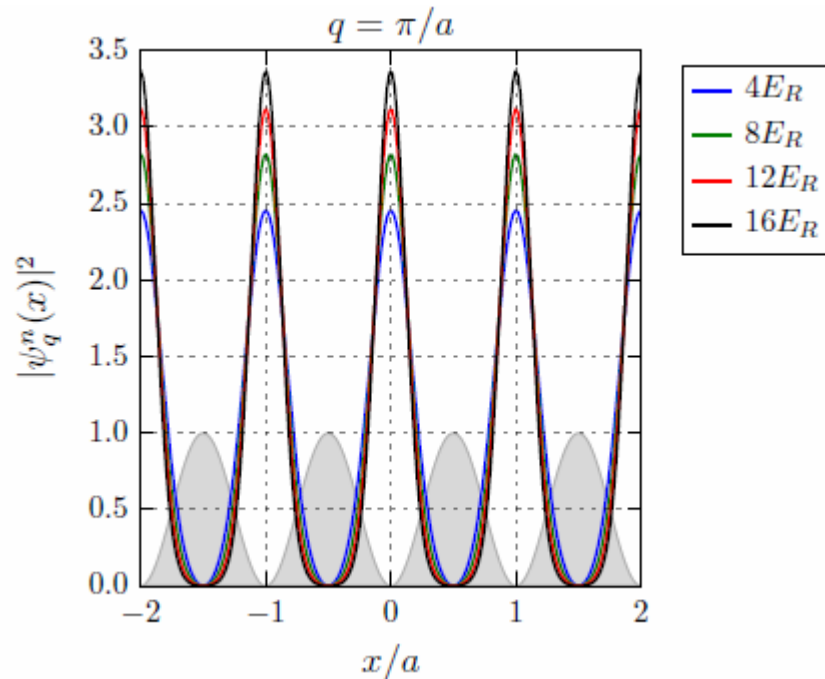
Tight binding limit:

For a deep lattice,  $V_0 \gg E_r$ , the eigenstates are harmonic oscillator functions

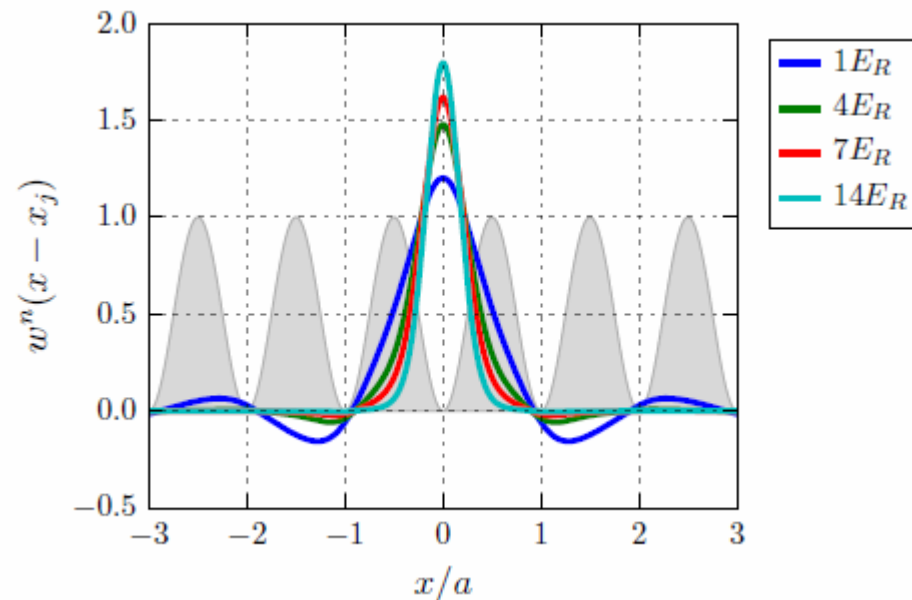
For a weaker lattice, the eigenstates are not tightly bound

A basis of states localized around a single site are known as *Wannier functions*  $w(x)$

Eigenstates of  $H_{1D}$

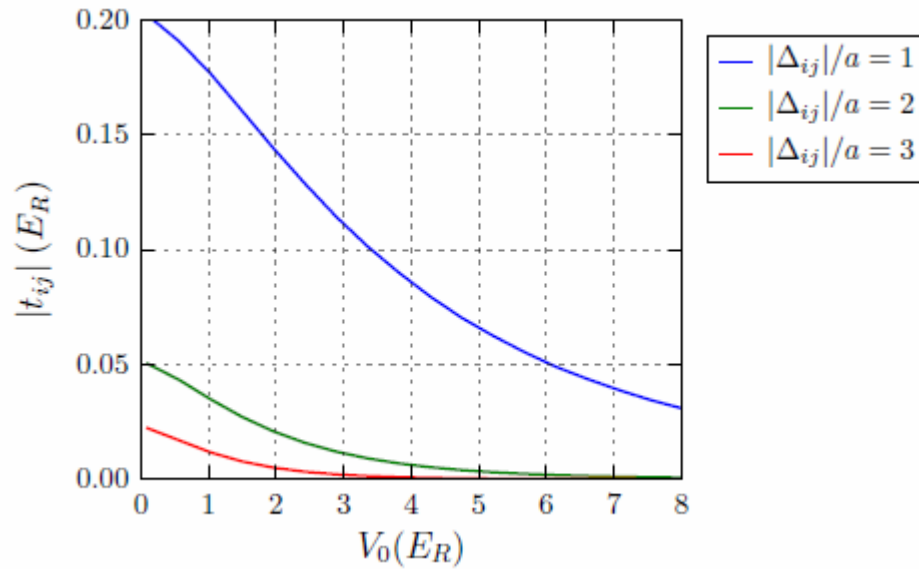


Wannier states



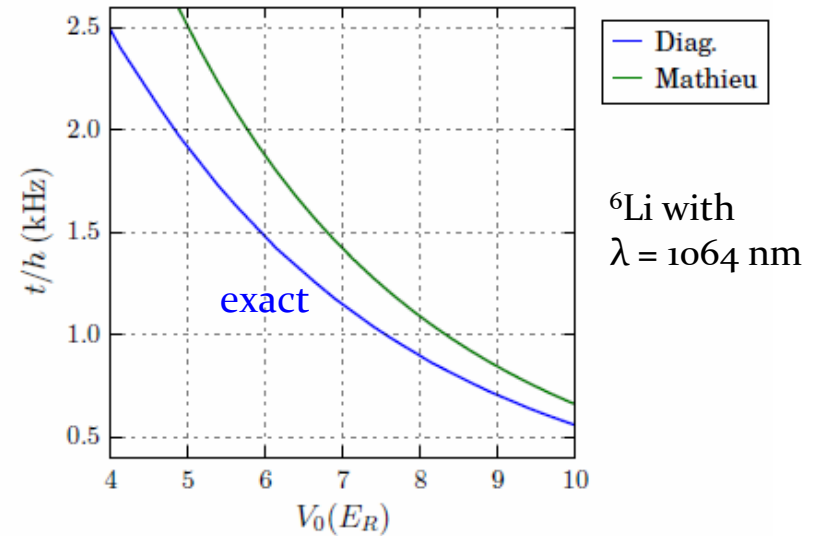
# Tunneling

Atoms can tunnel from site to site, but for  $V_o > 5 E_r$  only nearest neighbors contribute:



$t$  can be approximated in the tight-binding limit:

$$t/E_r \simeq \frac{4}{\sqrt{\pi}} v_0^{3/4} \exp(-2\sqrt{v_0})$$



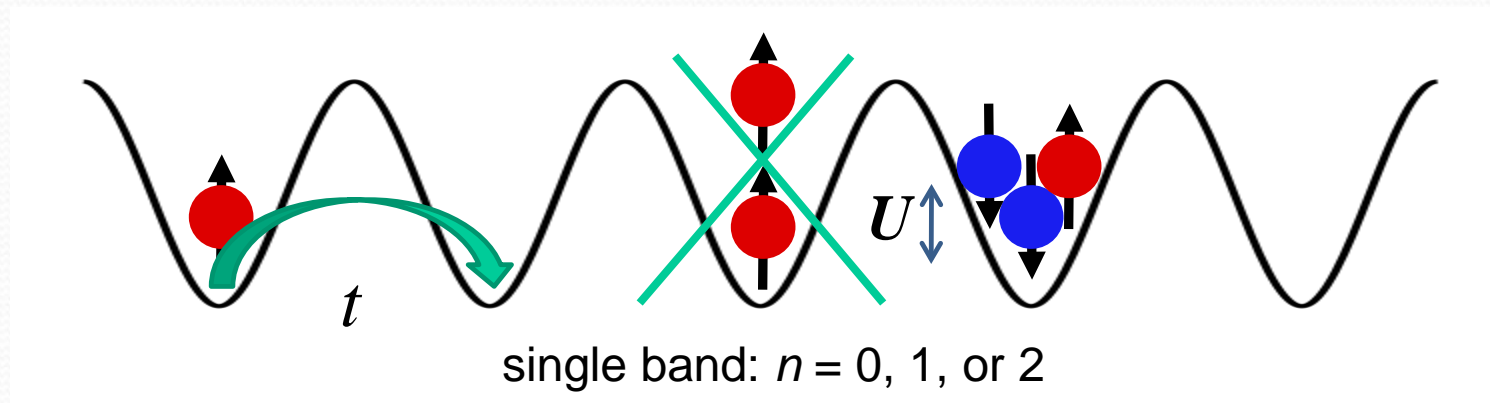
# Hubbard Model

- the hydrogen atom of condensed matter

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$t$  = hopping energy

$U$  = on-site interaction energy (repulsive)



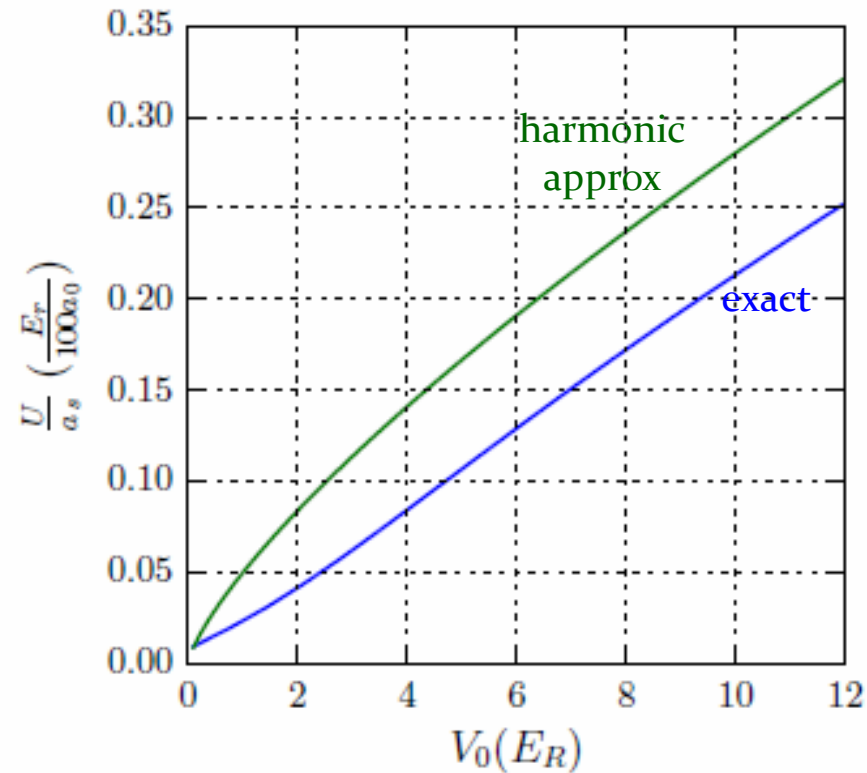
- Paradigm model of strongly correlated matter
- Proposed model for high- $T_c$  superconductors
  - but we don't know for sure if it has  $d$ -wave pairing

Cannot be solved exactly: basis size =  $2^N$

# On-Site Interactions

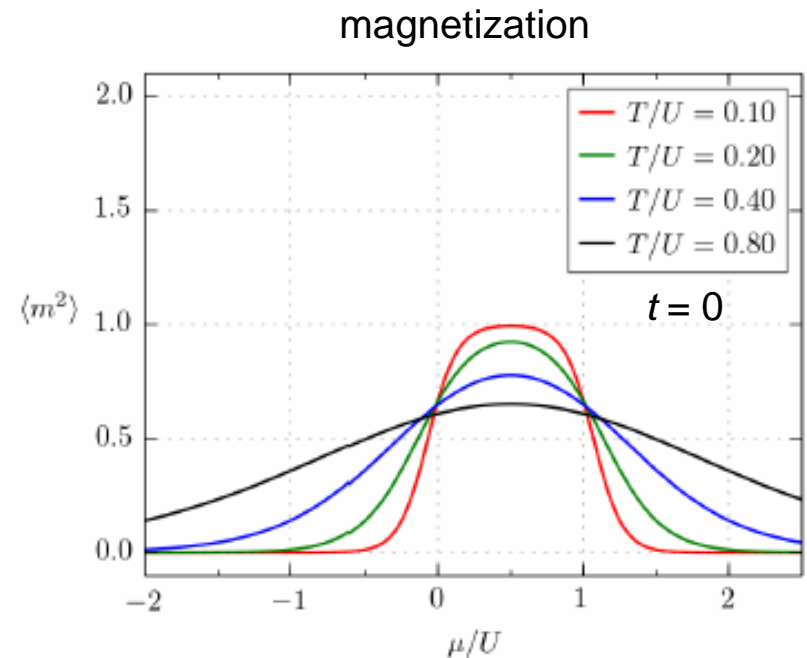
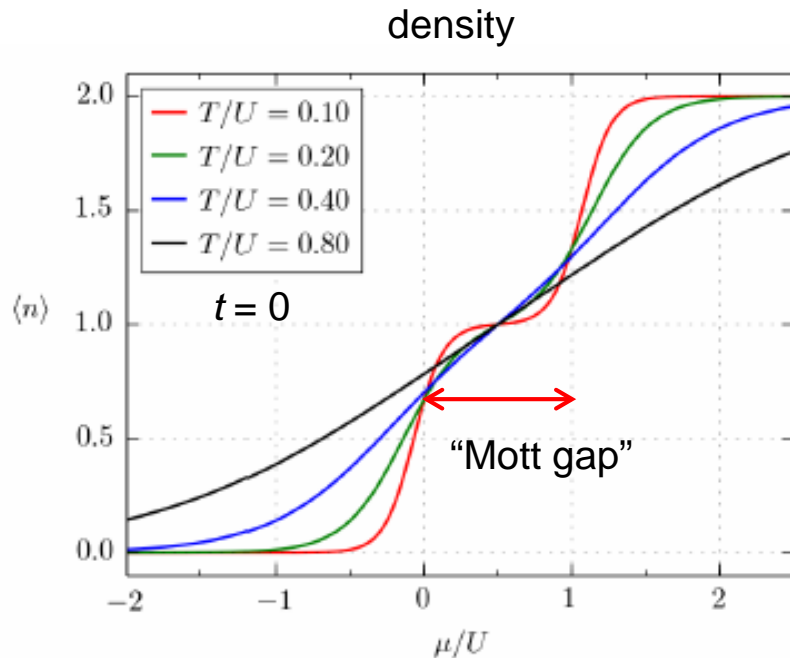
Harmonic approximation:

$$U/E_r \approx (8\pi)^{1/2} a_s/a v_0^{3/4}$$

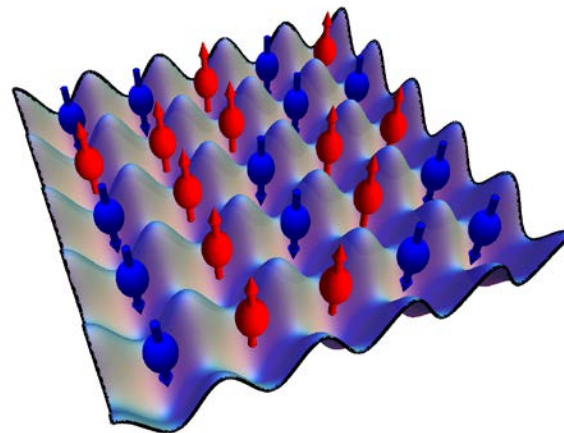




# Mott Insulator Develops for $T \ll U$



A Mott insulator develops  
at  $n = 1$  ("half-filling"):



# Fermi-Hubbard Model

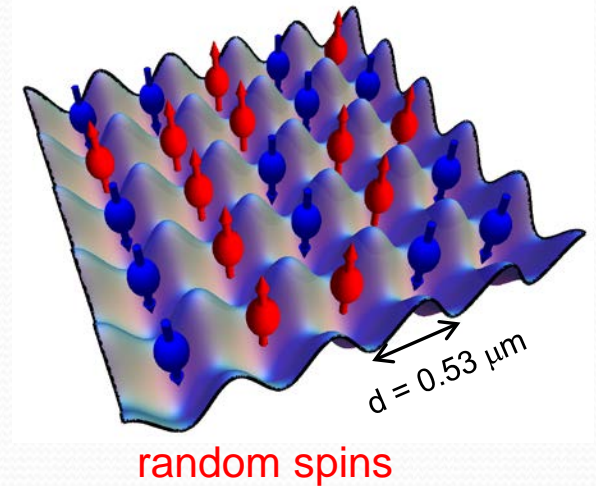
Special case:  $n = 1$  “half-filling”:  $H_{\text{AFM}} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j,$

with  $J = \hbar^2 / U$  “super-exchange”

- **Mott insulator** for  $T < U$ , with  $U/t \gg 1$

R. Jördens *et al.* (ETH), Nature, 2008:  
reduction of double occupancies

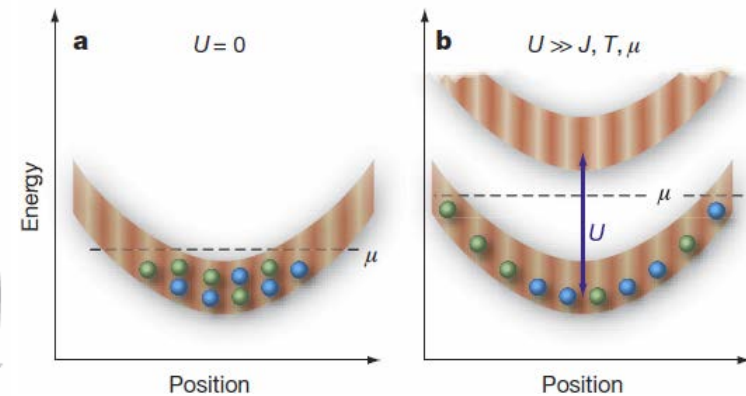
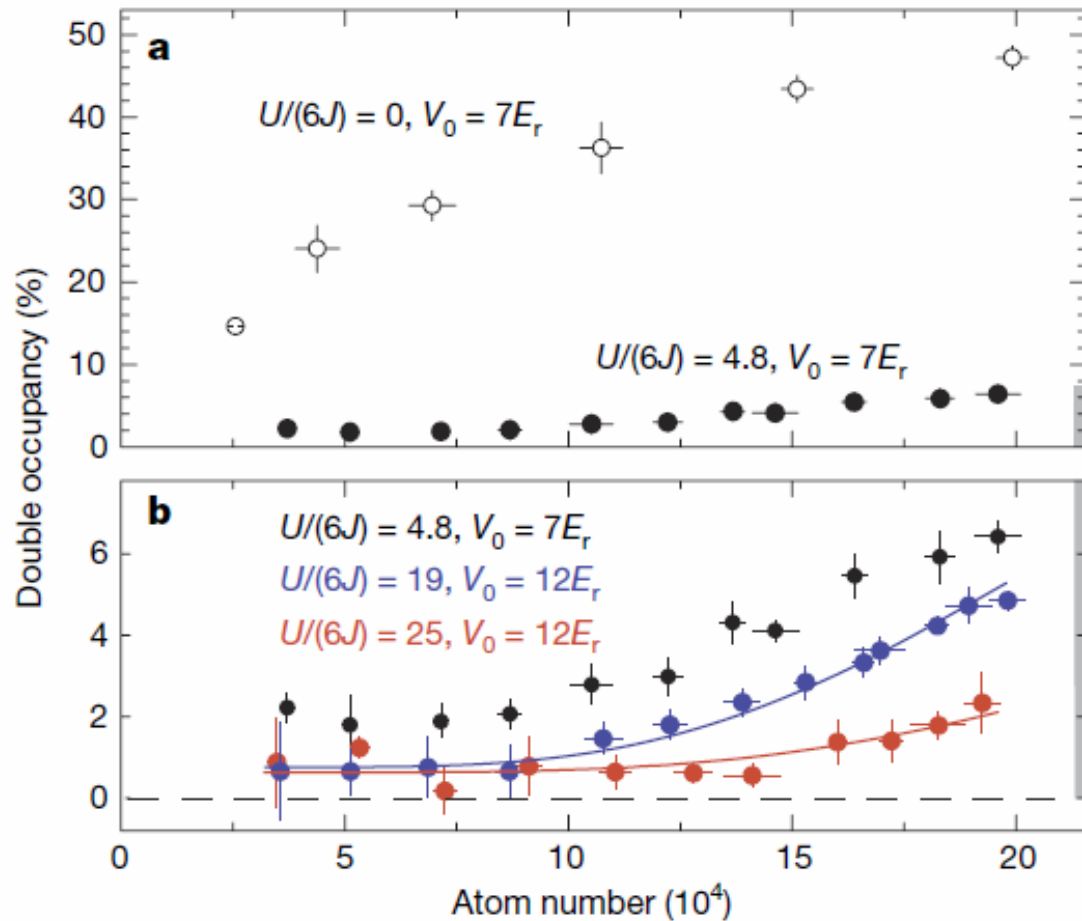
U. Schneider *et al.* (Munich), Science, 2008:  
reduction of compressibility



# Fermion Mott Insulator – ETH, 2008

## Double Occupancy

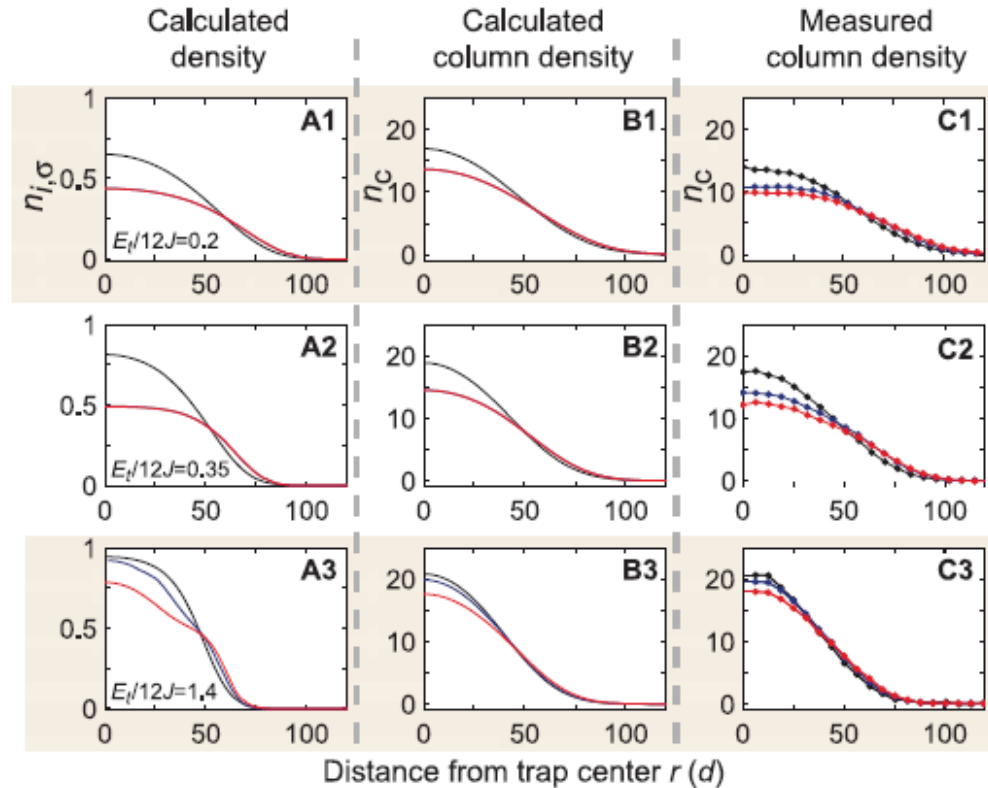
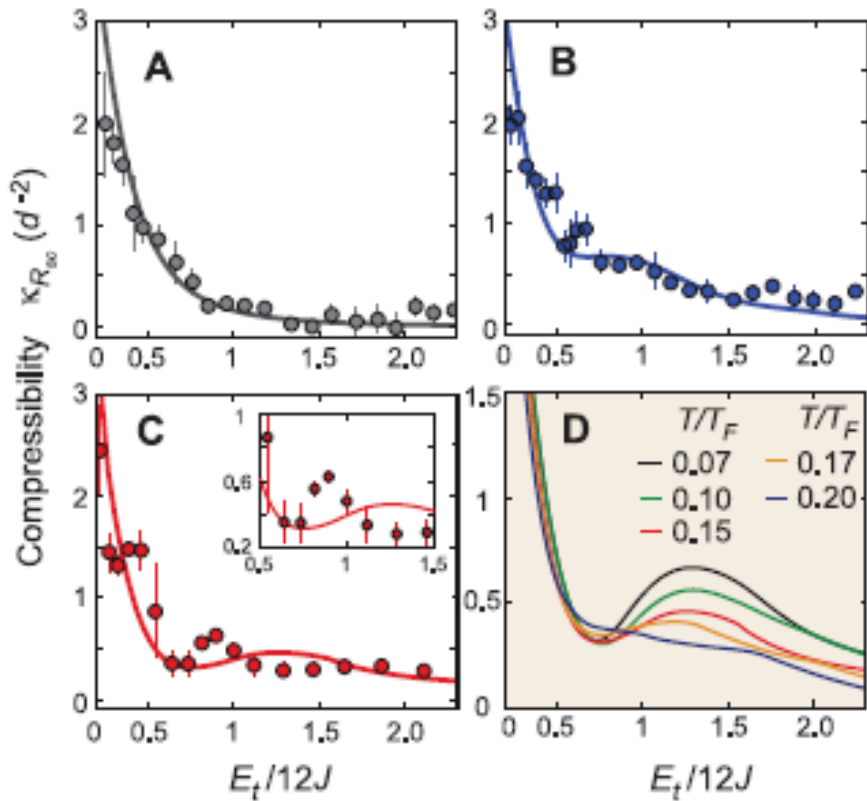
R. Jordans *et al*, *Nature* 455, 204, (2008)



# Fermion Mott Insulator – Munich, 2008

## Incompressibility

U. Schneider *et al*, *Science* 322, 1520 (2008)





# Fermi-Hubbard Model

Special case:  $n = 1$  “half-filling”:  $H_{\text{AFM}} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$ ,

with  $J = \hbar^2 / U$  “super-exchange”

- **Mott insulator** for  $T < U$ , with  $U/t \gg 1$

R. Jördens *et al.* (ETH), Nature, 2008:  
reduction of double occupancies

U. Schneider *et al.* (Munich), Science, 2008:  
reduction of compressibility

- **Antiferromagnet** for  $T < T_{\text{Néel}} \sim 4\hbar^2 / U$

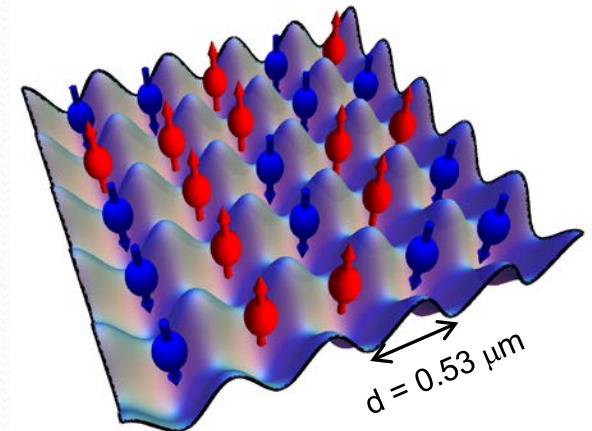
$$\rightarrow S/N < k_B \ln(2) \sim 0.7 k_B$$

## Fermions in an Optical Lattice:

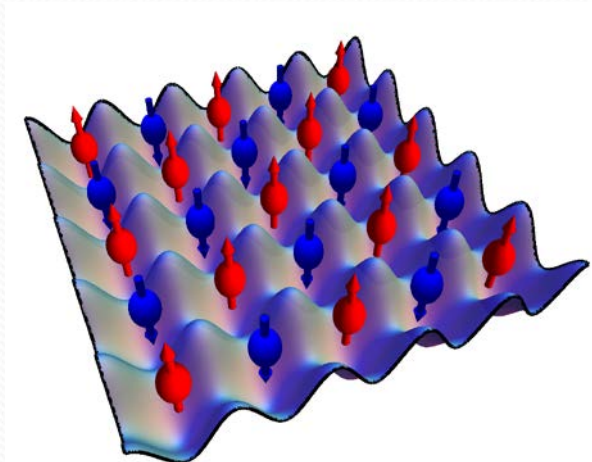
Greif *et al.* (ETH), Science 2013:

nearest neighbor singlet correlations along 1D chains

*$T_{\text{Néel}}$  not yet achieved in a 3D optical lattice  
– Poses a major challenge for realizing  
novel quantum materials w/ cold atoms*

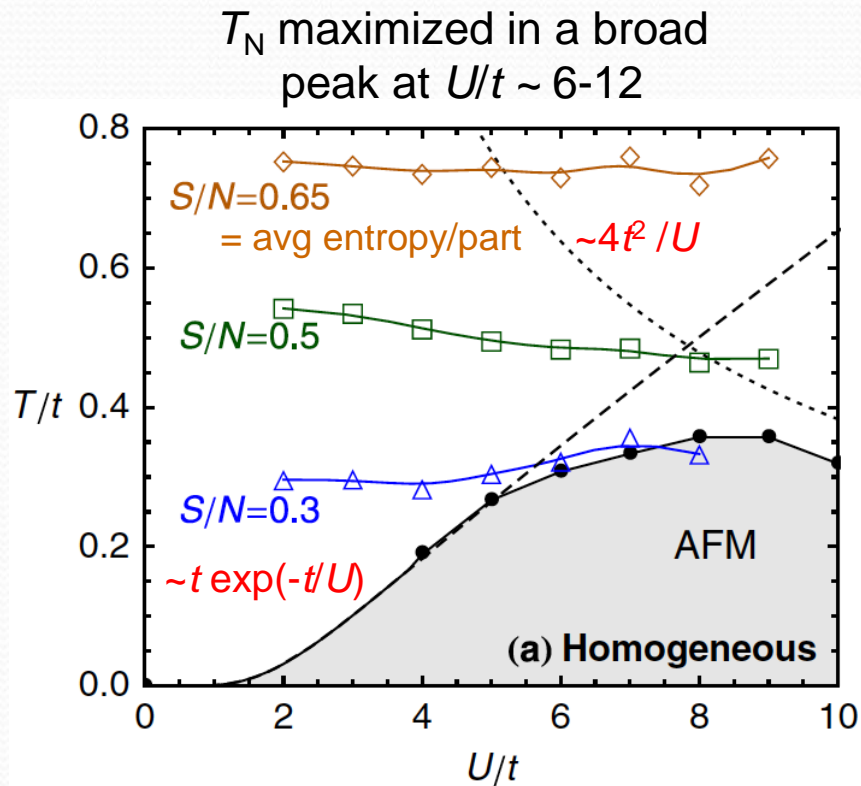


random spins



magnetically ordered

# Néel Transition

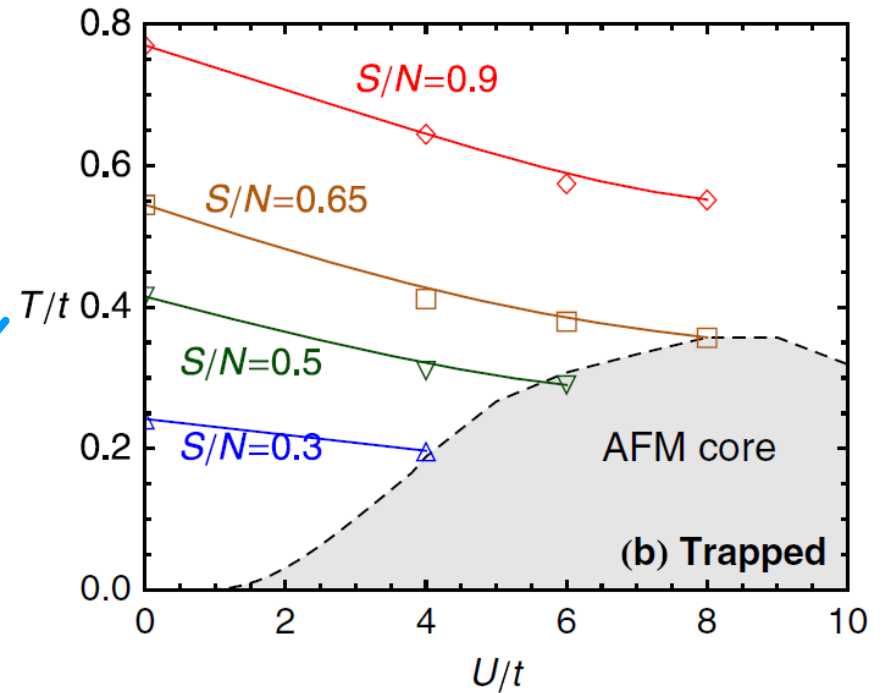


ETH Expt

$T/t = 1.0$

Imriska *et al*, PRL (2014)

$S/N = 1.6 k_B$

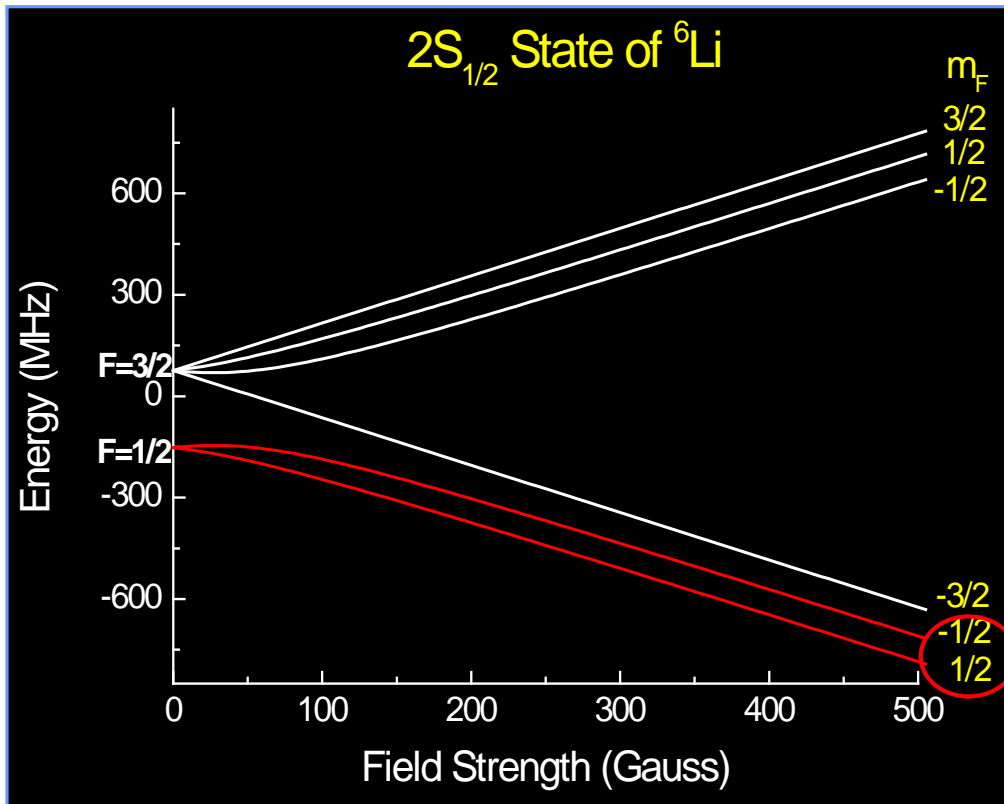


Quantum Monte Carlo

Paiva *et al*, PRL **107**, 086401 (2011)



# "Spin" States in ${}^6\text{Li}$



$$m_s = 1/2$$

Interactions tunable  
by a Feshbach resonance

$$m_s = -1/2$$

Typical experimental parameters  
for  $N \approx 2 \times 10^5$   ${}^6\text{Li}$  atoms:

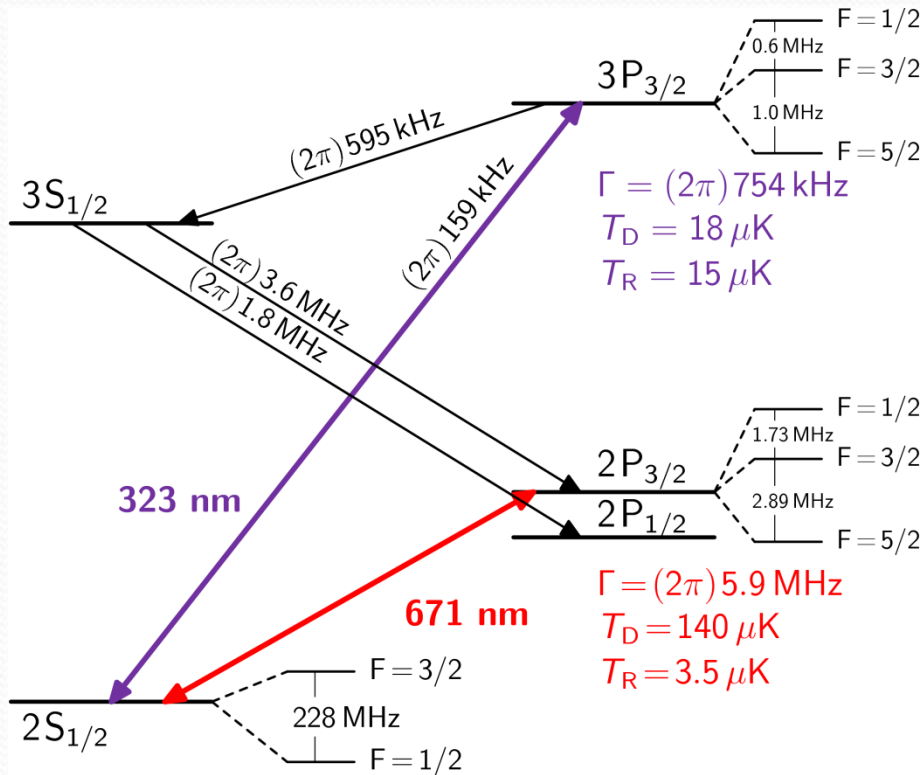
$$V_L \approx 7 E_R \quad (E_R = \hbar^2 k^2 / 2m = 1.4 \mu\text{K})$$

$$t \approx 0.038 E_R \approx 50 \text{ nK} \approx 1 \text{ kHz}$$

$$U \approx 0.38 E_R \text{ @ } 250 a_0$$

$$4\ell^2/U \approx 25 \text{ nK} \approx 0.025 T_F$$

# Narrow-Line Laser Cooling



P. Duarte, R. Hart *et al.*, PRA 84, 061406 (2011)

Also, in  $^{40}\text{K}$ : D. McKay (Toronto), PRA 84, 063420 (2011)

## Red MOT

$$N = 1 \times 10^9$$

$$T = 290 \mu\text{K}$$

$$n = 3 \times 10^{10} \text{ cm}^{-3}$$

$$\rho_{\text{ps}} = 2 \times 10^{-6}$$

## UV MOT

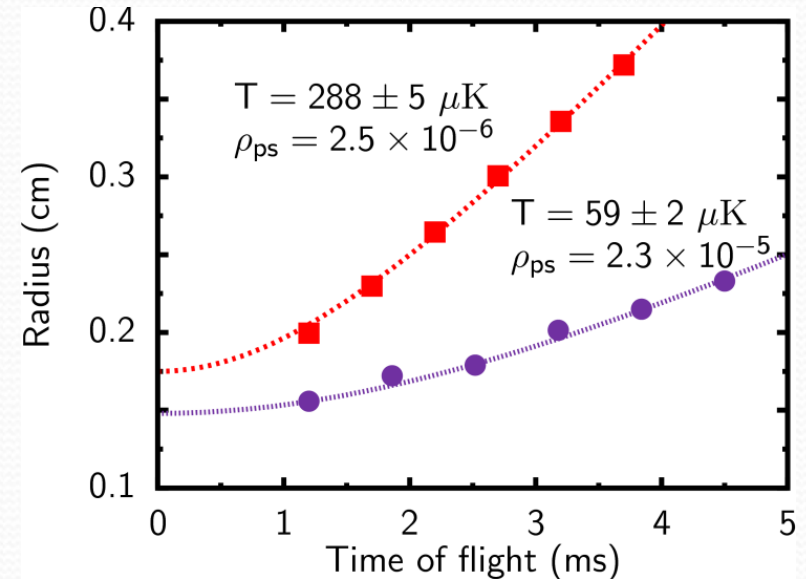
$$N = 5 \times 10^8$$

$$T = 59 \mu\text{K}$$

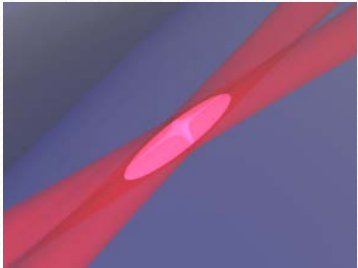
$$n = 3 \times 10^{10} \text{ cm}^{-3}$$

$$\rho_{\text{ps}} = 2 \times 10^{-5} \rightarrow \times 10 \text{ increase}$$

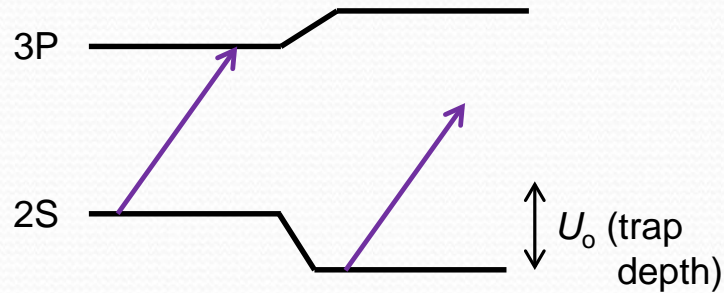
## Time-of-Flight



# Optical Trap - Magic Wavelength

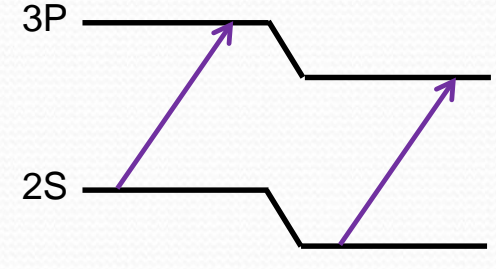


Crossed-beam trap at 1070 nm



Usual *non-magic* wavelength

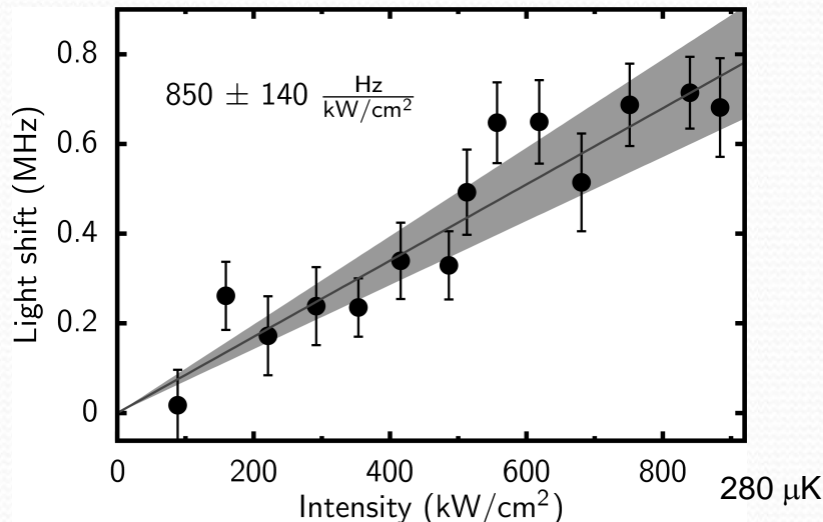
- Trap light shifts cooling transition out of resonance



Magic wavelength

- Light shifts of upper and lower levels are equal

Light shift measurement: Light shift  $\square \gamma$  at full trap depth



P. Duarte, R. Hart *et al.*, PRA 84, 061406 (2011)

Light shift calculation: M. Safranov

After laser cooling in trap:

$$N = 1 \times 10^7$$

$$T = 50 \mu\text{K}$$

$$n_0 = 4 \times 10^{13} \text{ cm}^{-3}$$

$$T/T_F = 2.5$$

$$\rho_{ps} = 2 \times 10^{-2} \rightarrow \times 1000 \text{ increase}$$

After 5 s of evaporation:

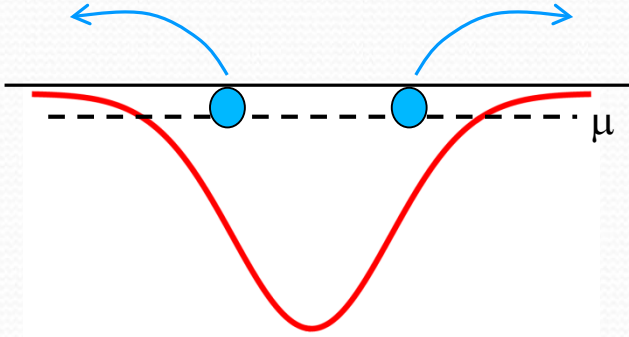
$$N = 4 \times 10^6 \text{ atoms}$$

$$T < 0.1 T_F$$



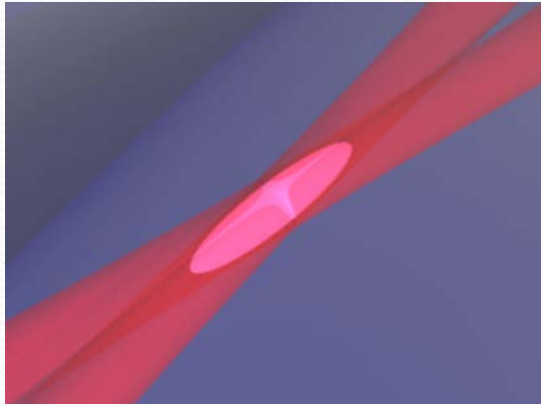
# Evaporative Cooling – Trap vs Lattice

Evaporative cooling in a trap:

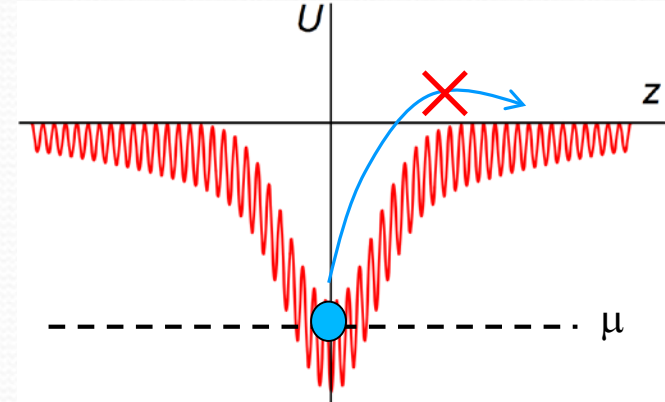


Very effective cooling:  $T < 0.05 T_F \rightarrow S < 0.7 k_B$

Crossed Beam Trap

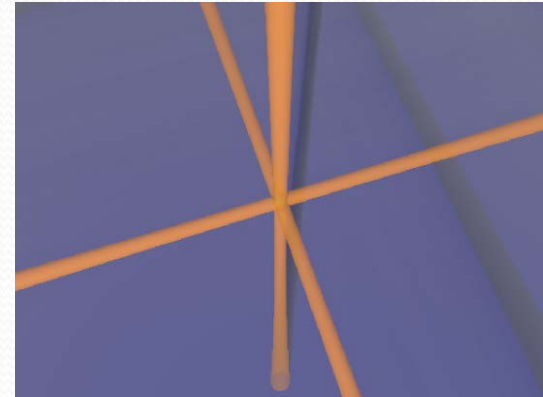


In a red-detuned 3D lattice:  
(looking along a lattice direction)



No cooling  
(but there is heating)

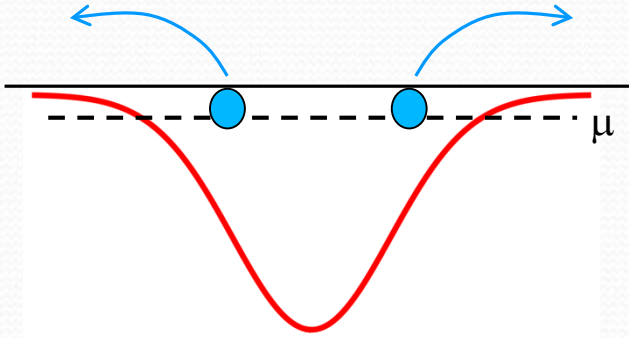
Optical Lattice



1064 nm

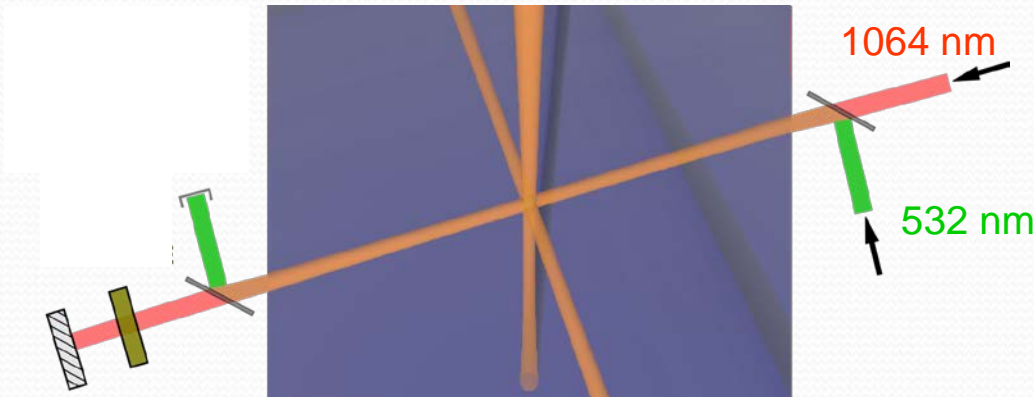
# Compensated Lattice – How to get colder

Evaporative cooling in a trap:

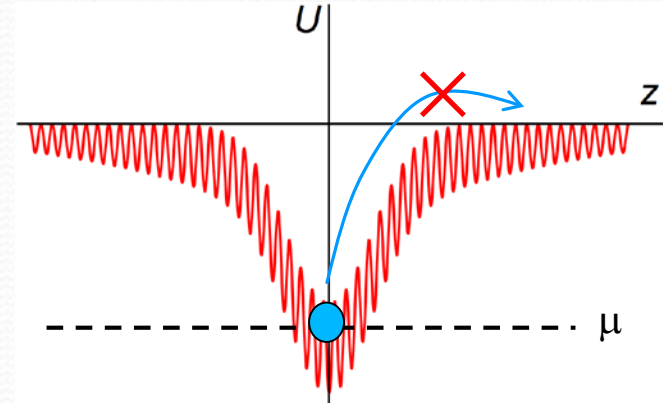


Very effective cooling:  $T < 0.05 T_F \rightarrow S < 0.7 k_B$

Proposed solution: *compensated lattice*  
with anti-confining green beams (not a lattice)

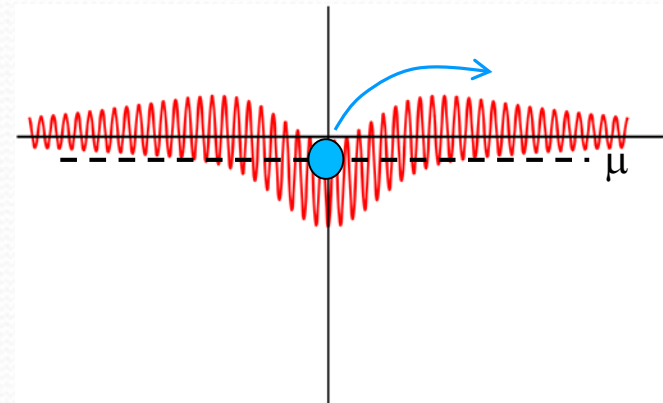


In a red-detuned 3D lattice:  
(looking along a lattice direction)



No cooling  
(but there is heating)

In a compensated lattice:



# Three Functions of Compensated Lattice

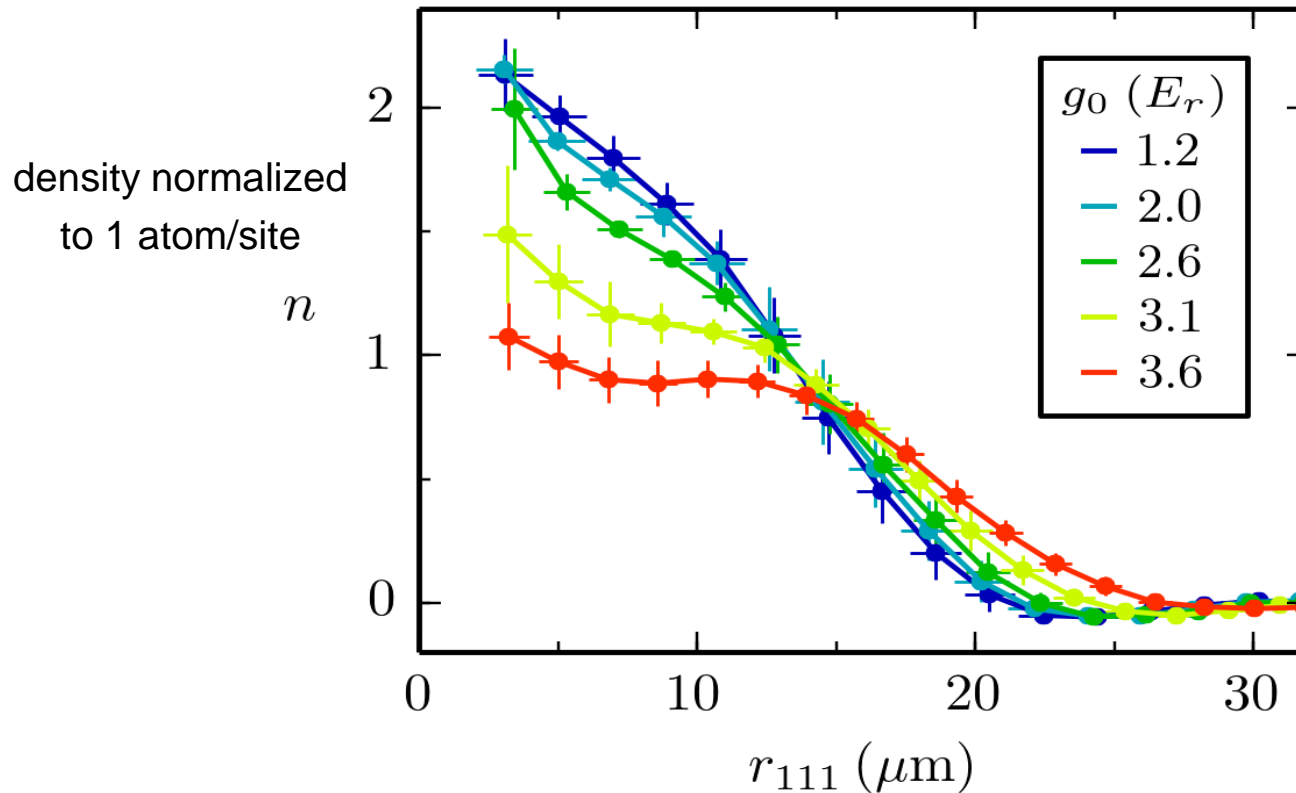
Mathy, Huse, Hulet, Phys. Rev. A **86**, 023606 (2012)

## Compensation

- 1) density control knob
- 2) flatten band
- 3) provide cooling

3D densities from Abel transform

Central density plateaus as Mott core is formed



$$V_L = 7 E_R$$

$$U/t = 15$$

$$N = 1.8 \times 10^5$$



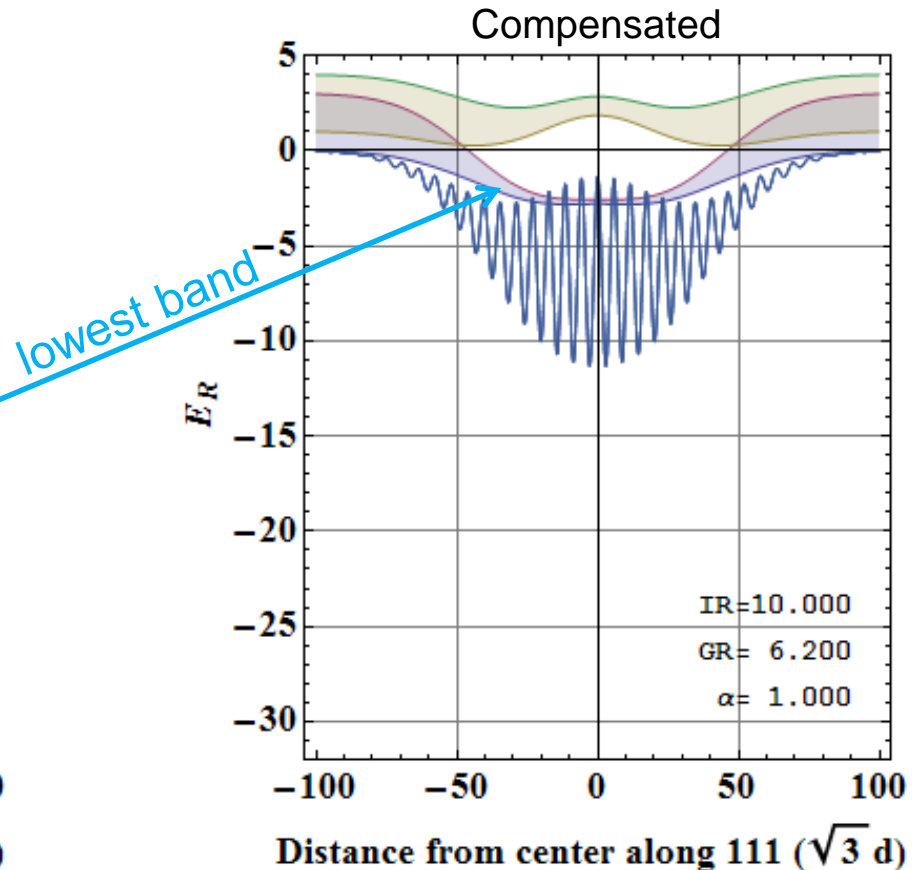
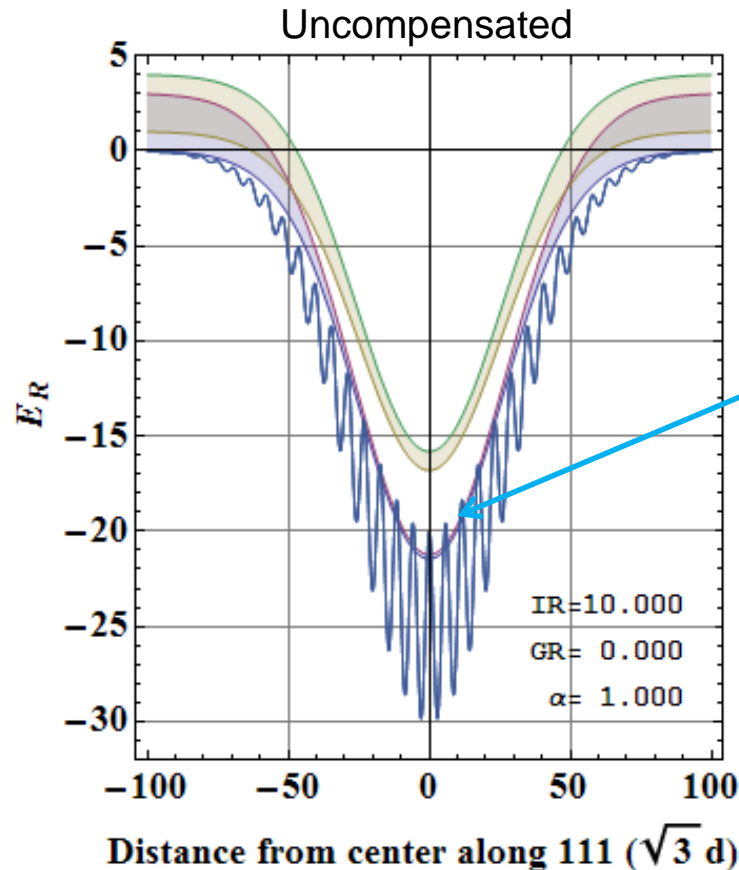
# Three Functions of Compensated Lattice

Mathy, Huse, Hulet, Phys. Rev. A **86**, 023606 (2012); also Ma *et al*, Phys. Rev. A **78**, 023605 (2008)

## Compensation

- 1) density control knob
- 2) flatten band
- 3) provide cooling

Compensation beams have smaller waists than lattice beams in order to flatten potential

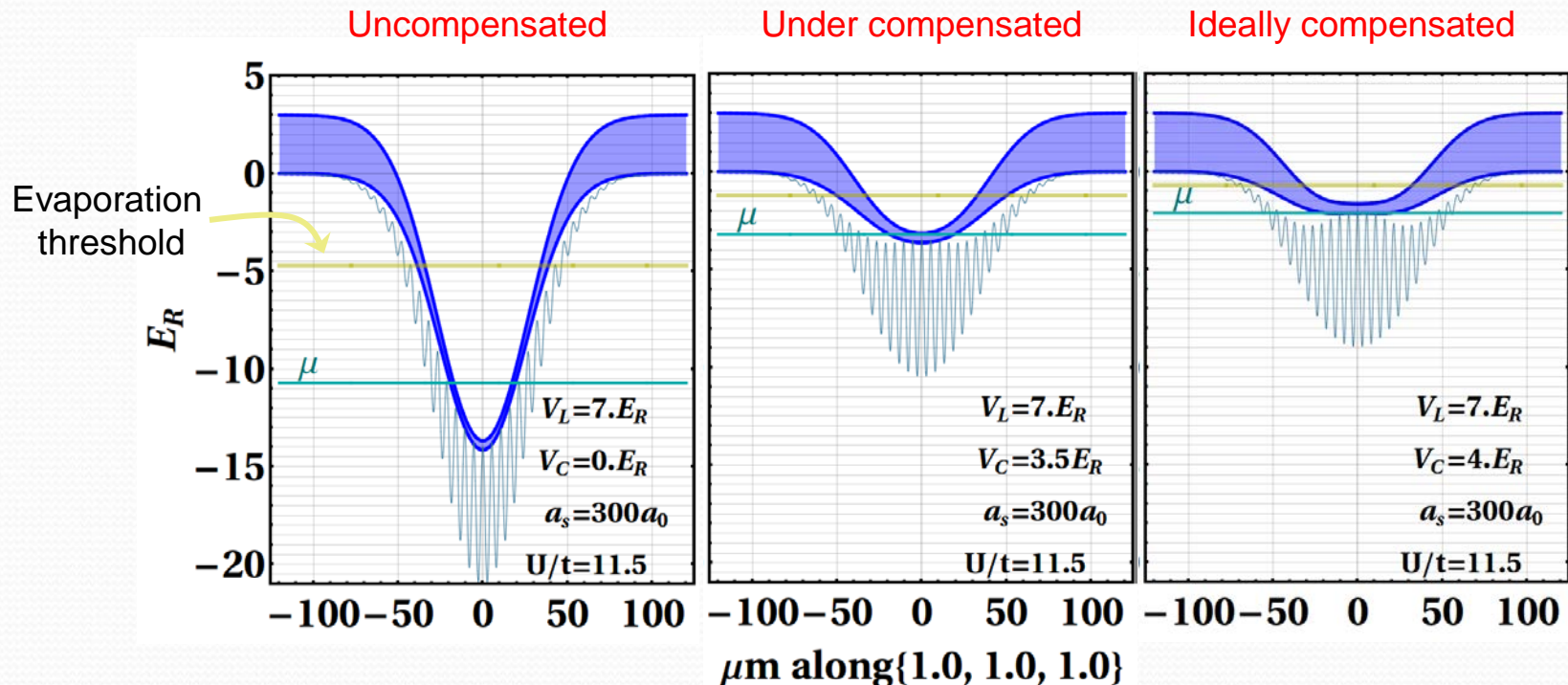


# Evaporative Cooling in Compensated Lattice

## Compensation

- 1) density control knob
- 2) flatten band
- 3) provide cooling

Compensation beams have smaller waists than lattice beams in order to flatten potential



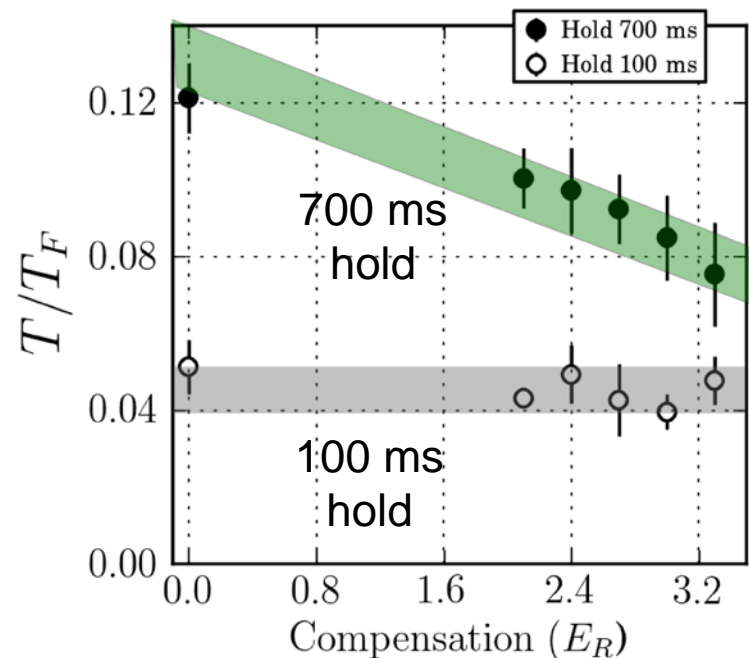
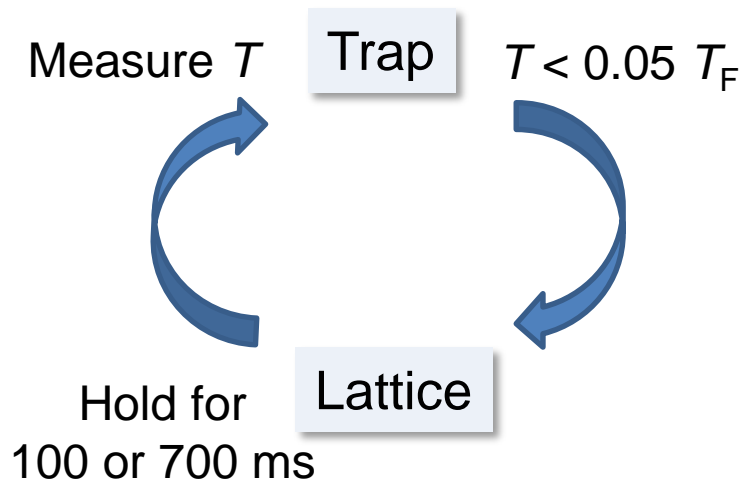
# Three Functions of Compensated Lattice

Mathy, Huse, Hulet, Phys. Rev. A **86**, 023606 (2012)

## Compensation

- 1) density control knob
- 2) flatten band
- 3) provide cooling

Current setup is not optimized for evaporative cooling,  
but we observe the suppression of heating:



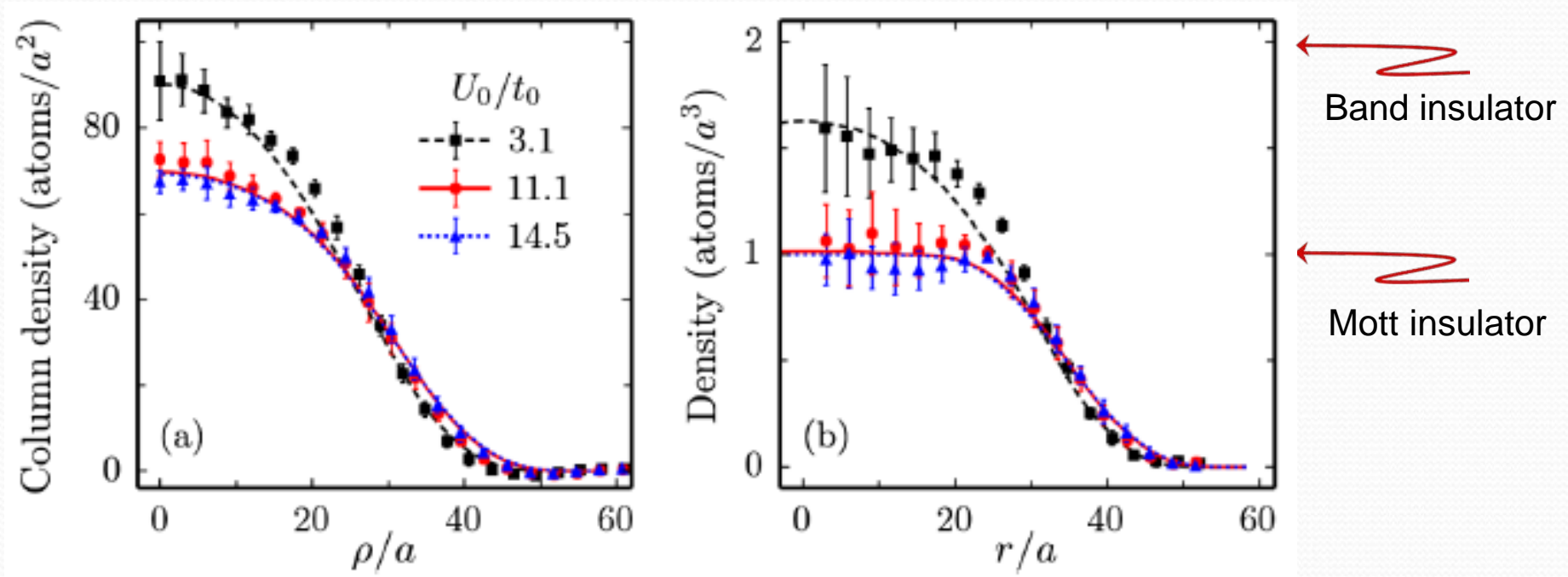
# Mott Insulator Develops for Large $U/t$

Abel  
Transform

Column density images



3D density distributions

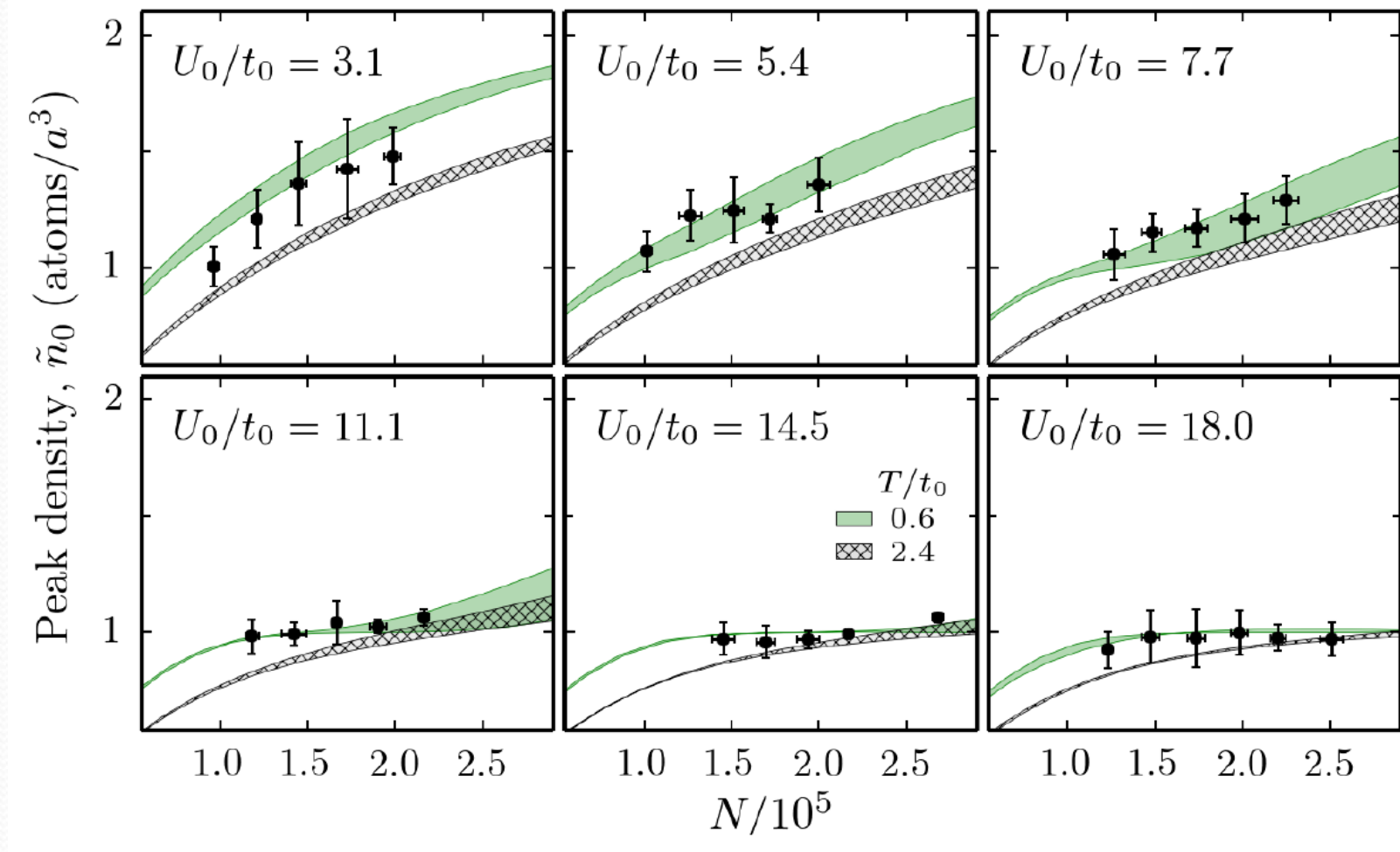


P.M. Duarte *et al*, Phys Rev Lett (2015)

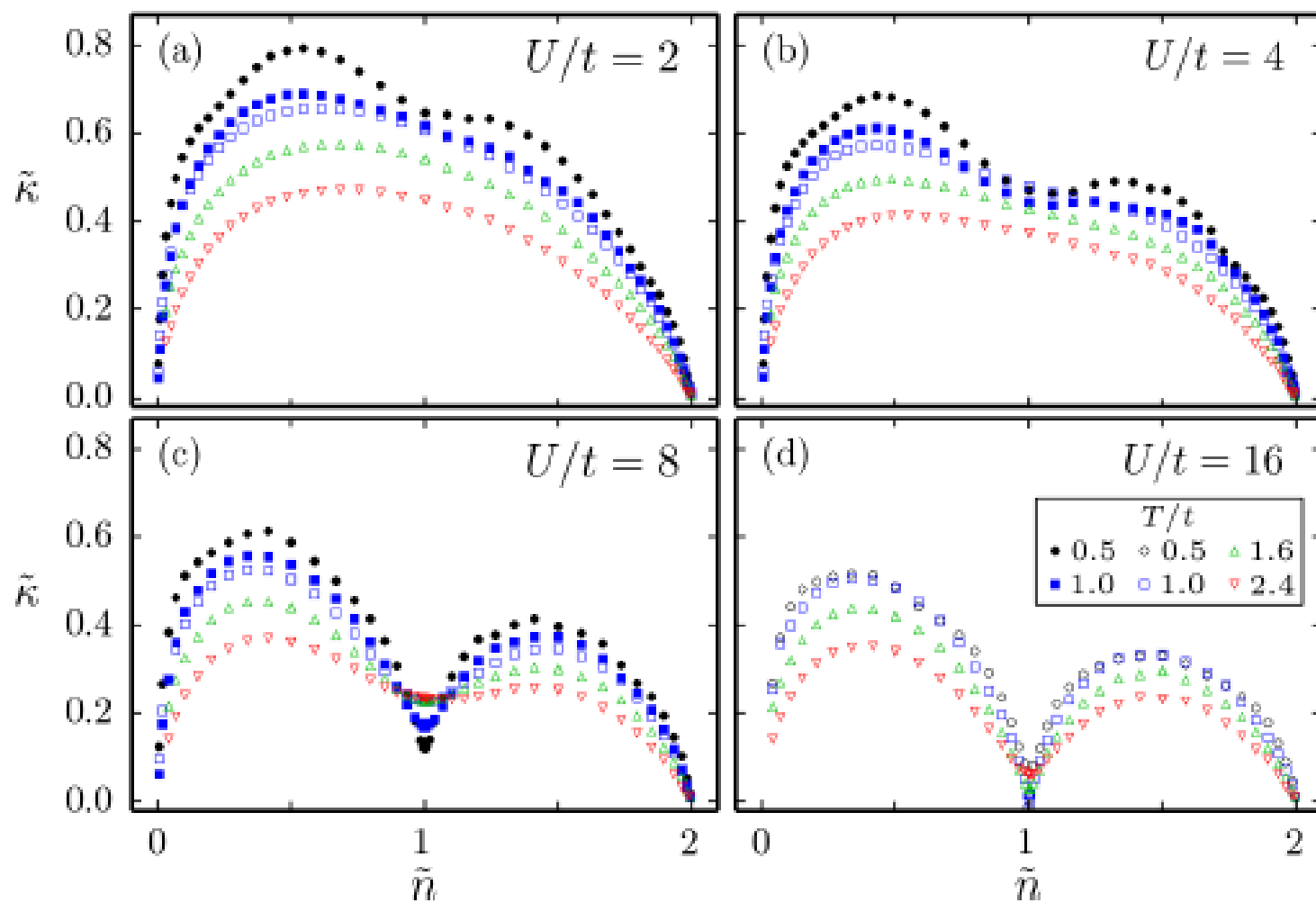
Solid lines are calculated using a numerical linked-cluster expansion



# Incompressibility of Mott Phase



$dn/dN$  is related to the compressibility:  $\kappa = (1/n^2) dn/d\mu$

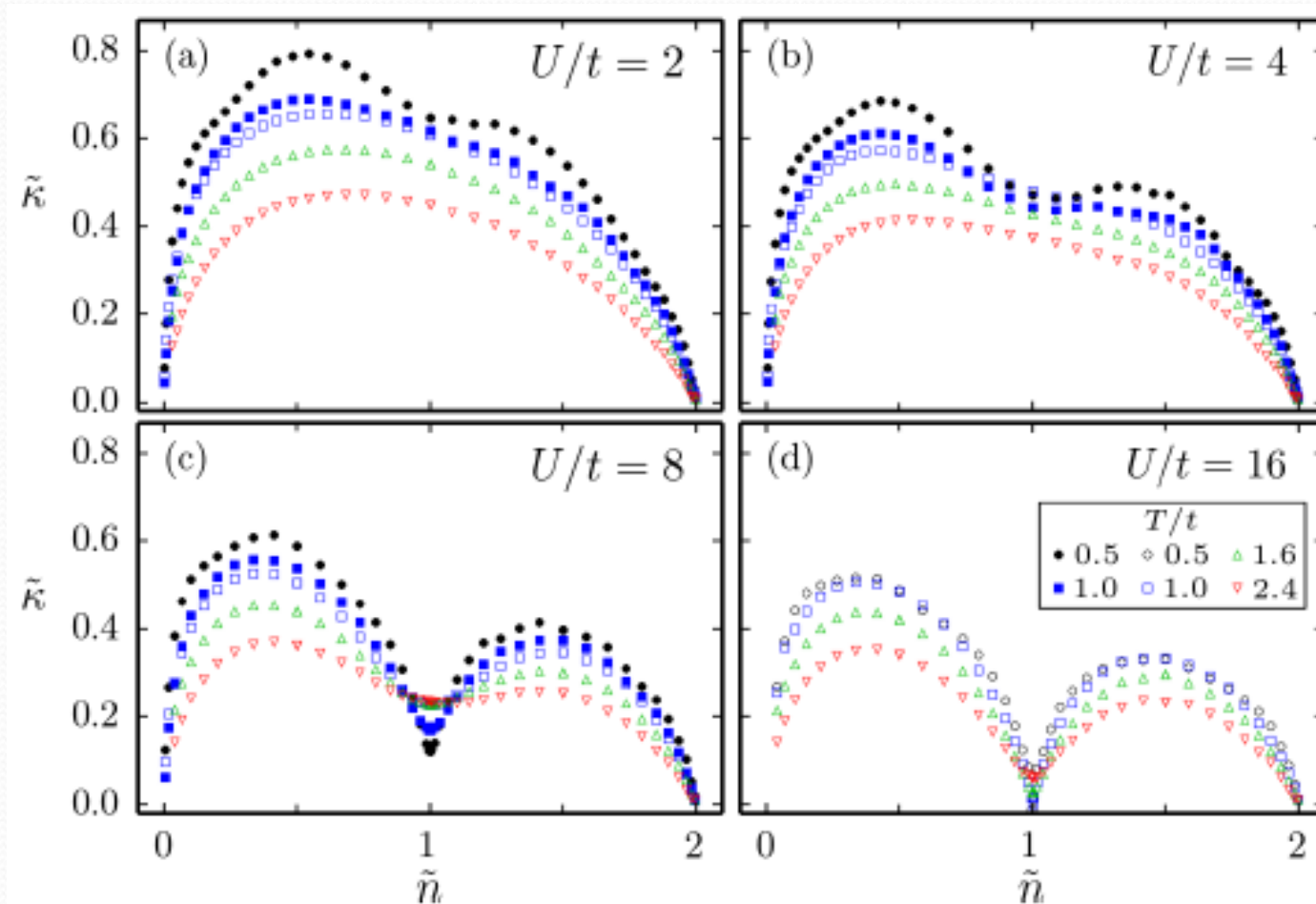




# Local Compressibility

The compressibility vanishes in the Mott phase, due to the presence of a Mott gap:

$$\kappa = \frac{1}{n^2} \frac{\partial n}{\partial \mu}$$

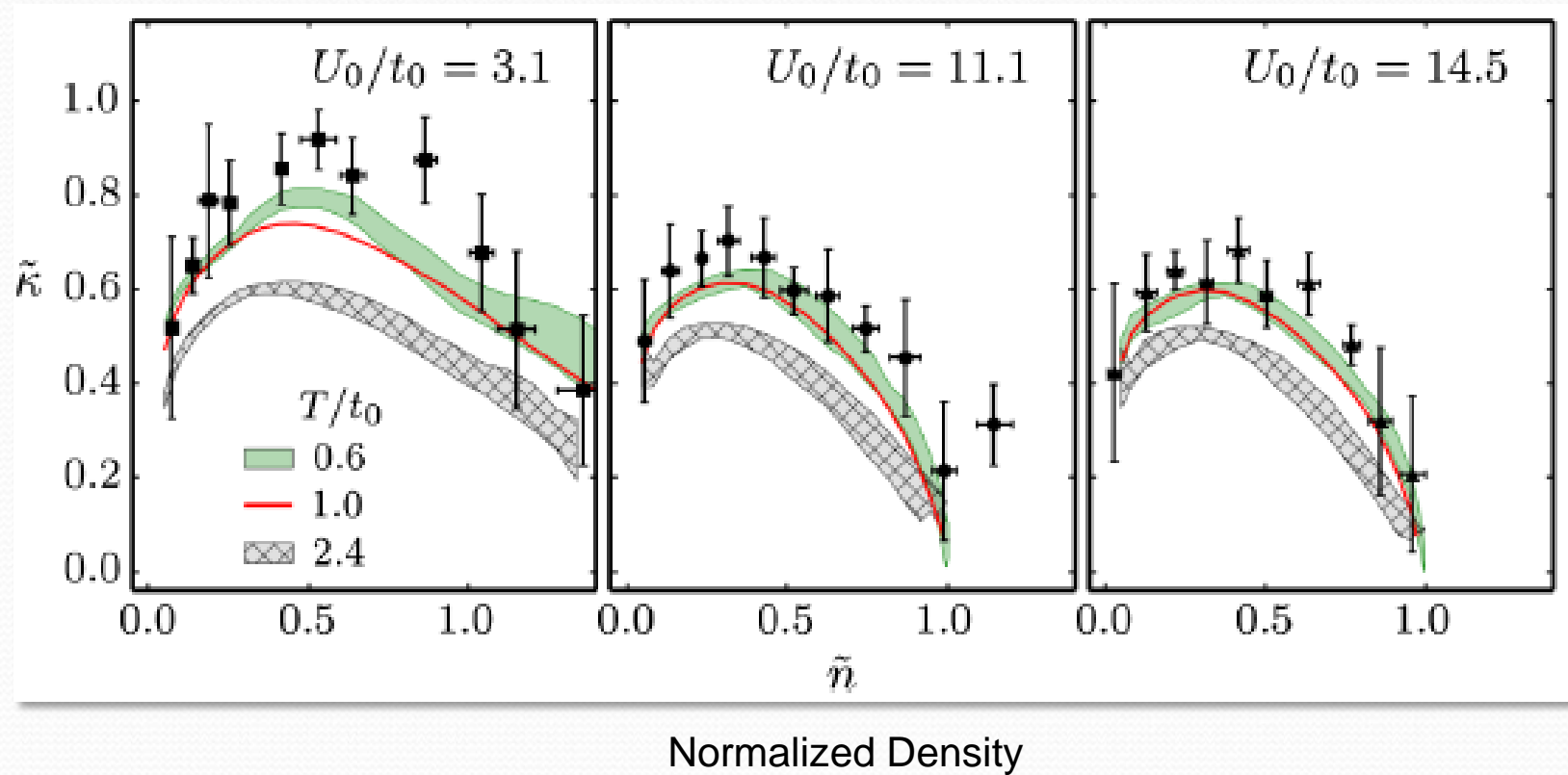


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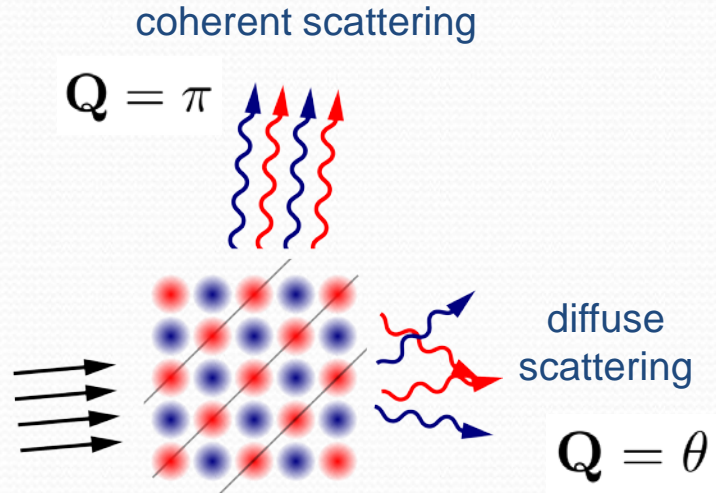
P.M. Duarte *et al*, Phys Rev Lett (2015)



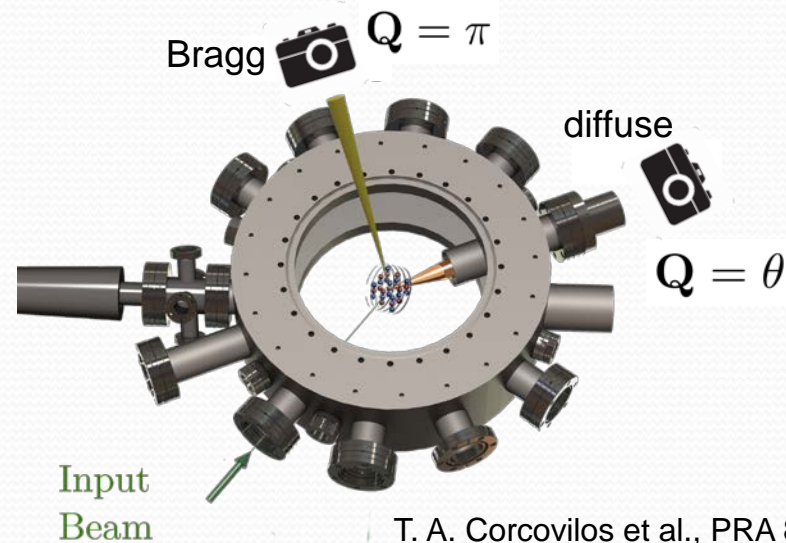
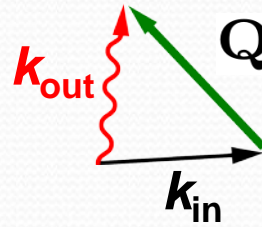
Lines and shaded regions: numerical calculations  $\rightarrow T/t \leq 1.0$

# Detect Order by Bragg Scattering

Bragg scattering of near resonant light:



Bragg condition:

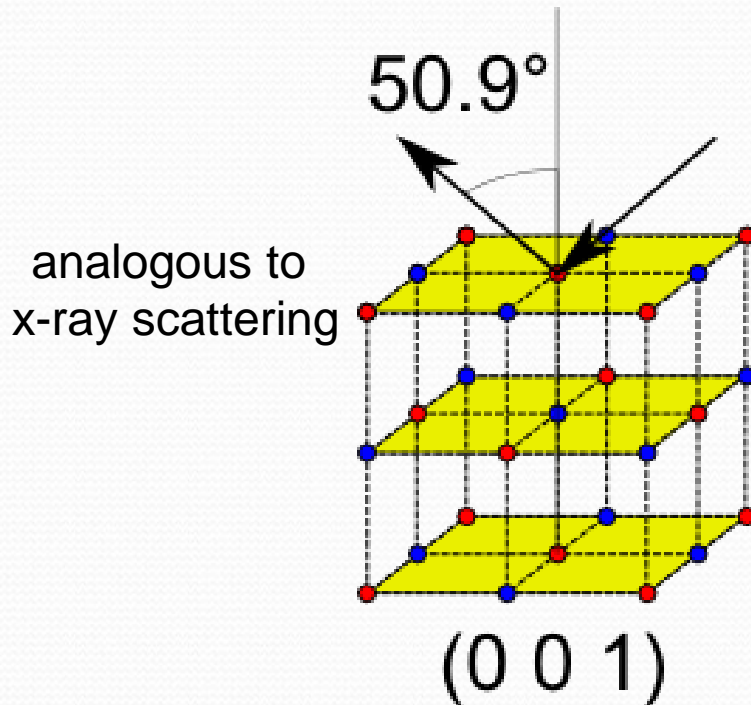




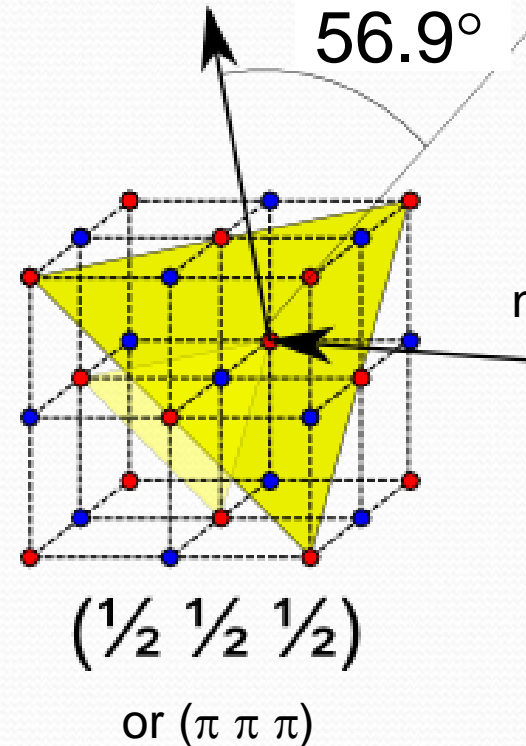
# Detect Order by Bragg Scattering

Bragg scattering of near resonant light:

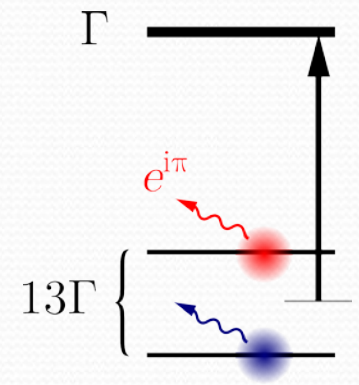
$(0\ 0\ 1)$  peak indicates presence of cubic lattice structure



$(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$  peak unambiguously indicates presence of AFM order



analogous to neutron scattering



T. A. Corcovilos et al., PRA **81**, 013415 (2010)

# Bragg Scattering – Crystalline Order

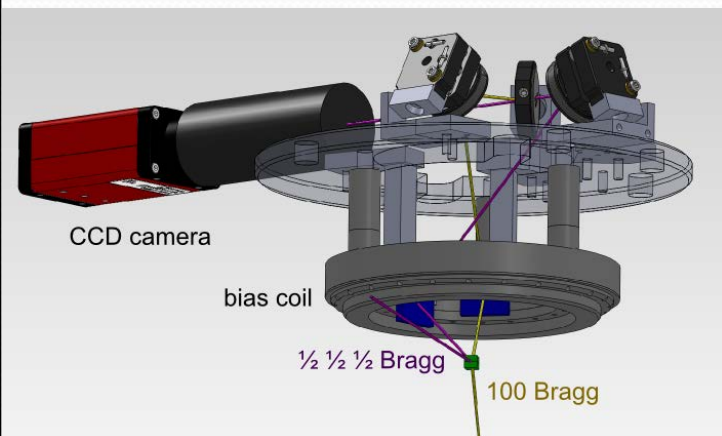
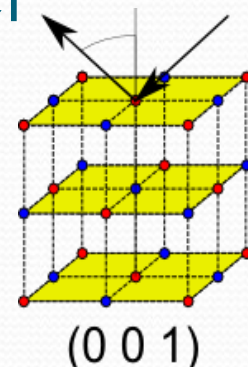


Image the scattered light onto a single pixel



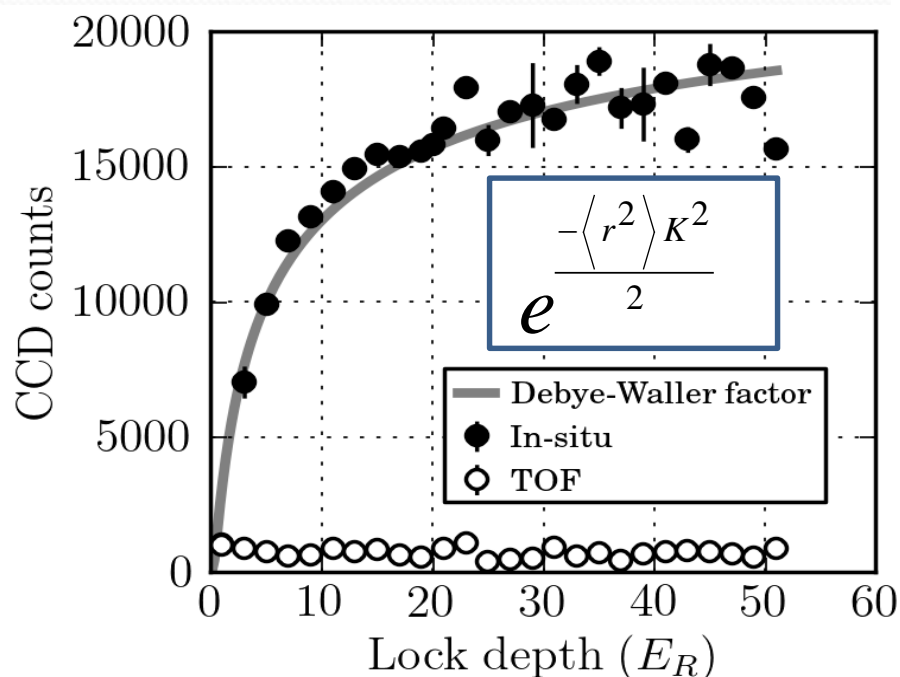
Also:

Birkl *et al* (NIST) PRL (1995)

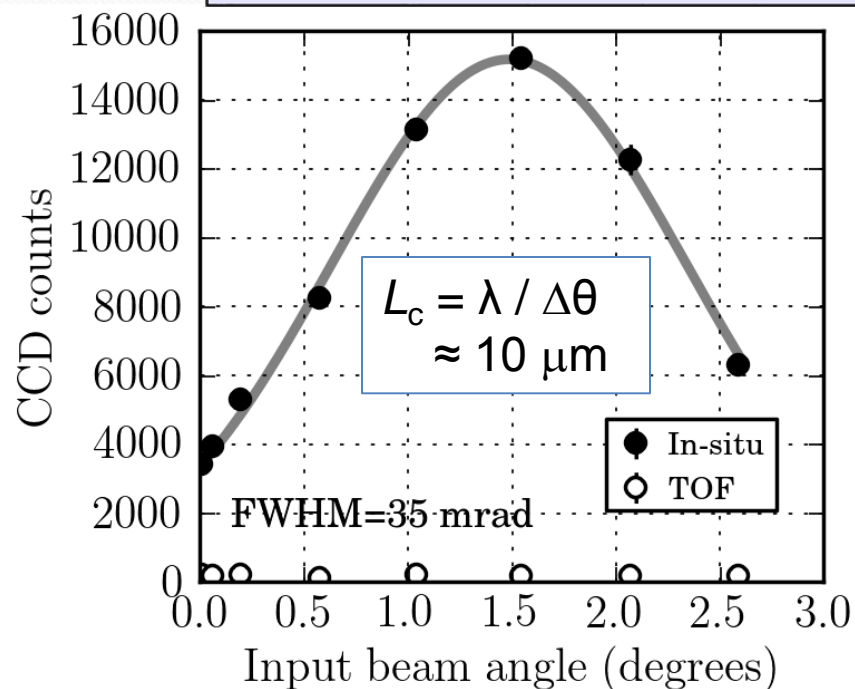
Weidemuller *et al* (MPQ), PRL (1995)

Miyake *et al* (MIT), PRL (2011)

Bragg signal vs lattice depth:



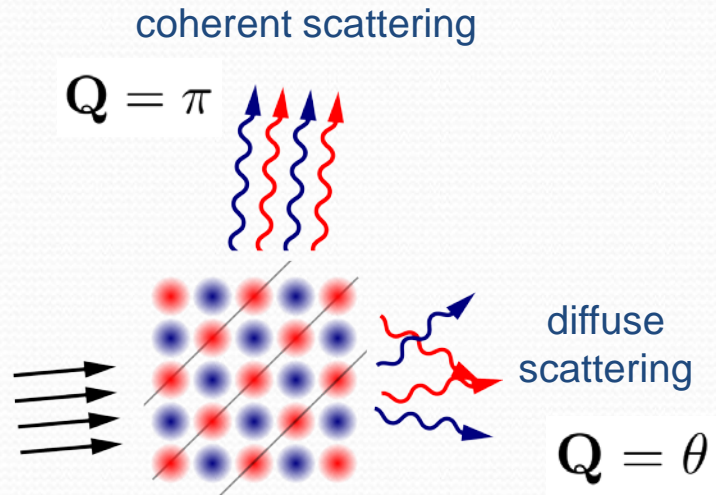
Bragg signal vs input angle:



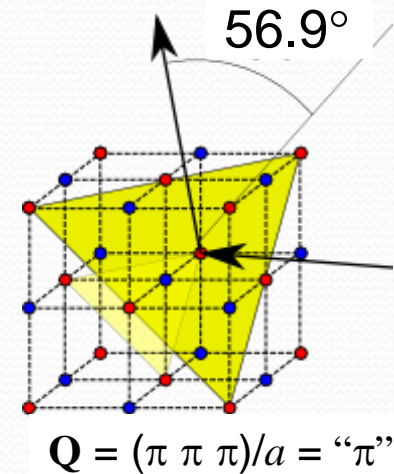
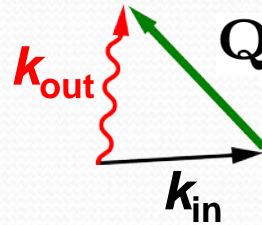


# Detect AFM Order by Bragg Scattering

Spin-sensitive Bragg scattering of near resonant light:

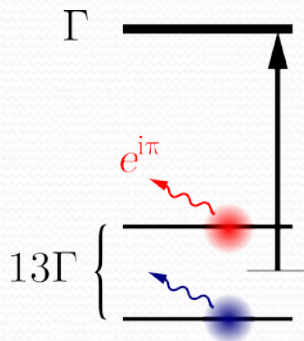


Bragg condition:

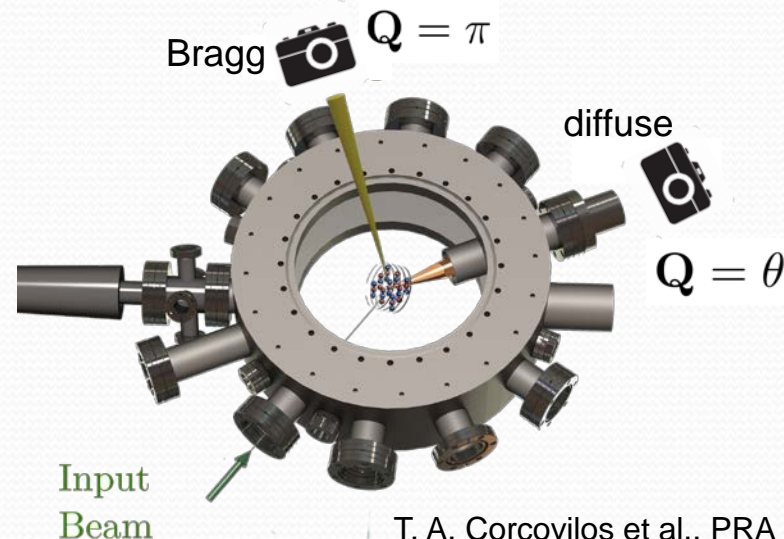


$(\pi \pi \pi)$  peak unambiguously indicates presence of AFM order

Spin-sensitive:



analogous to neutron scattering



# Bragg Signal / Spin-Structure Factor

$$S_Q = \frac{4}{N} \sum_{mn} e^{iQ(\mathbf{R}_m - \mathbf{R}_n)} S_{zm} S_{zn} = \text{spin structure factor} = 1 \text{ to } N$$

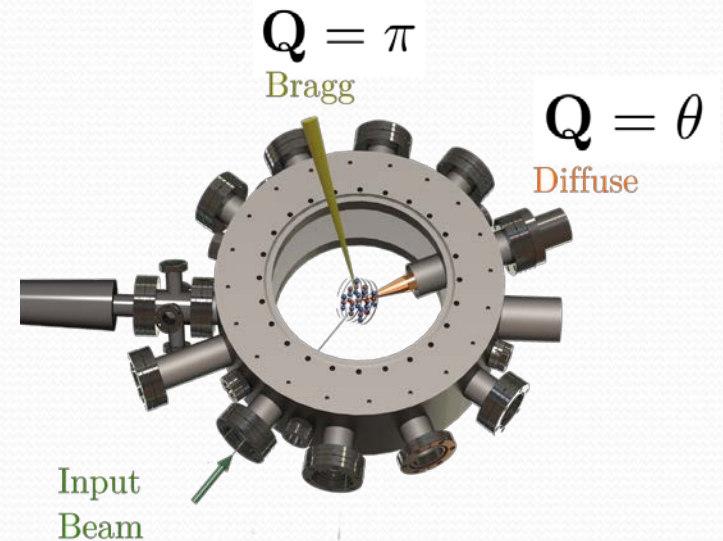
Bragg signal is proportional to  $S_Q$ :

$$I_Q \propto S_Q$$

## Normalization:

Bragg signal is normalized by the signal after long TOF  $\tau$ :

$$S_Q = I_Q (\tau=0) / I_Q (\tau=\infty)$$

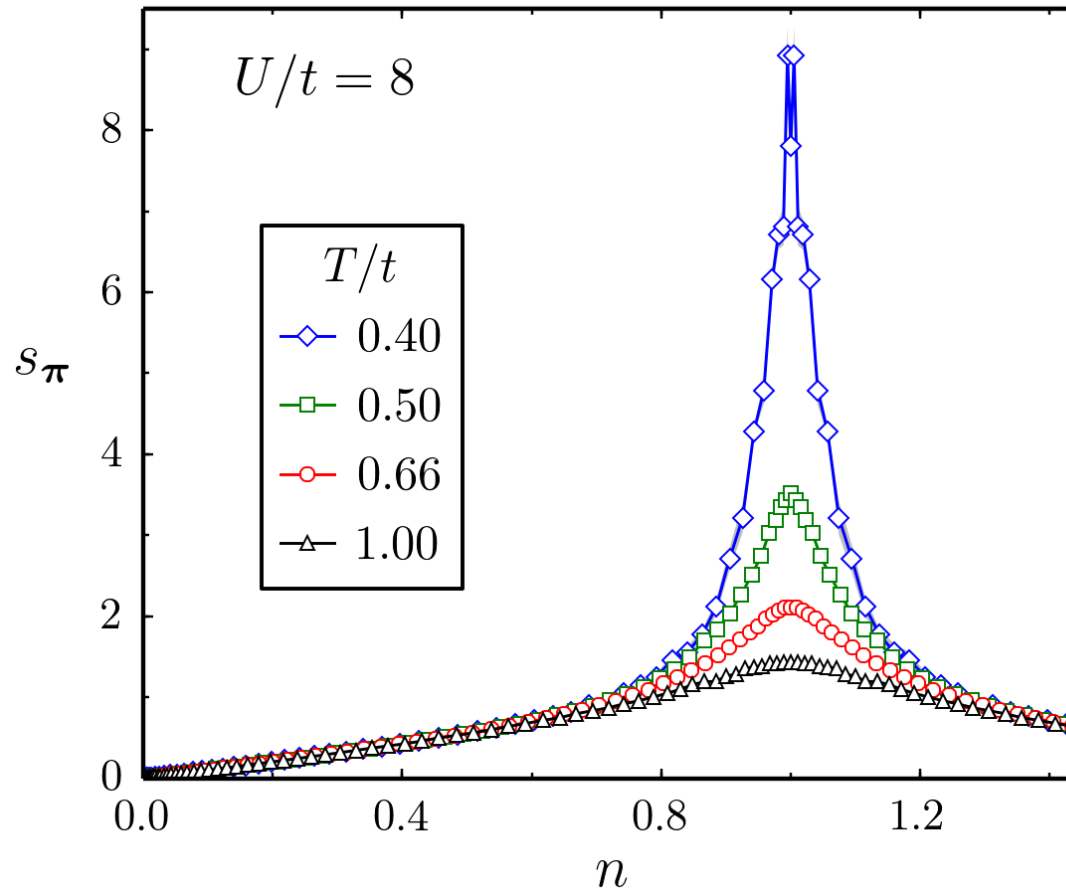




# Density Dependence - QMC

Bragg signal comes mainly from an  $n = 1$  shell

QMC:  $S(\pi)$  vs.  $n$



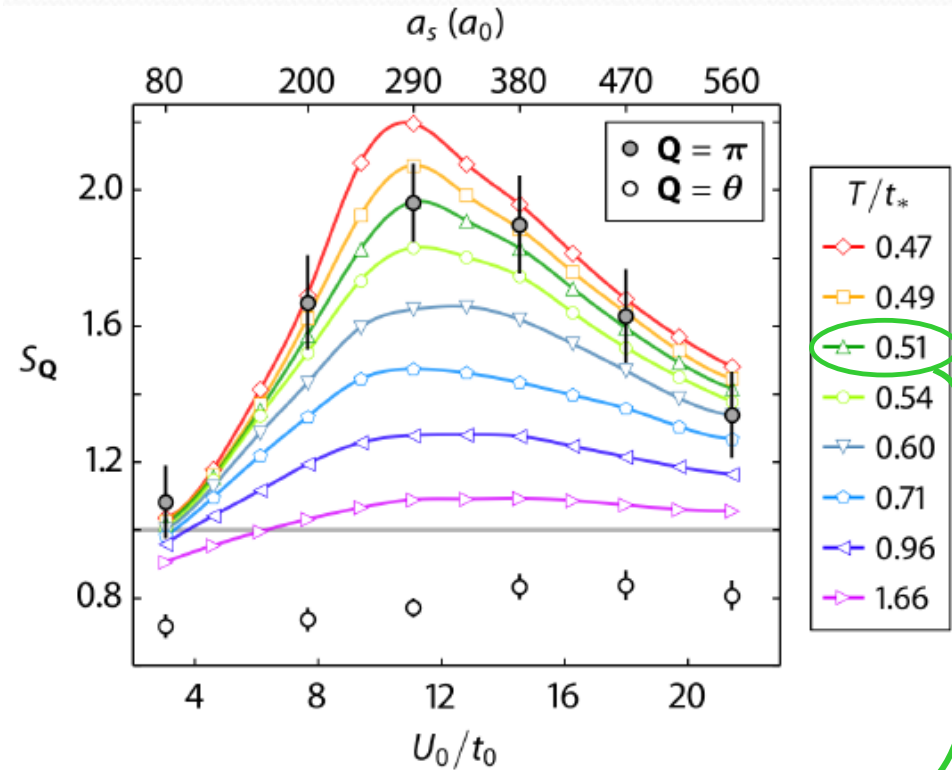
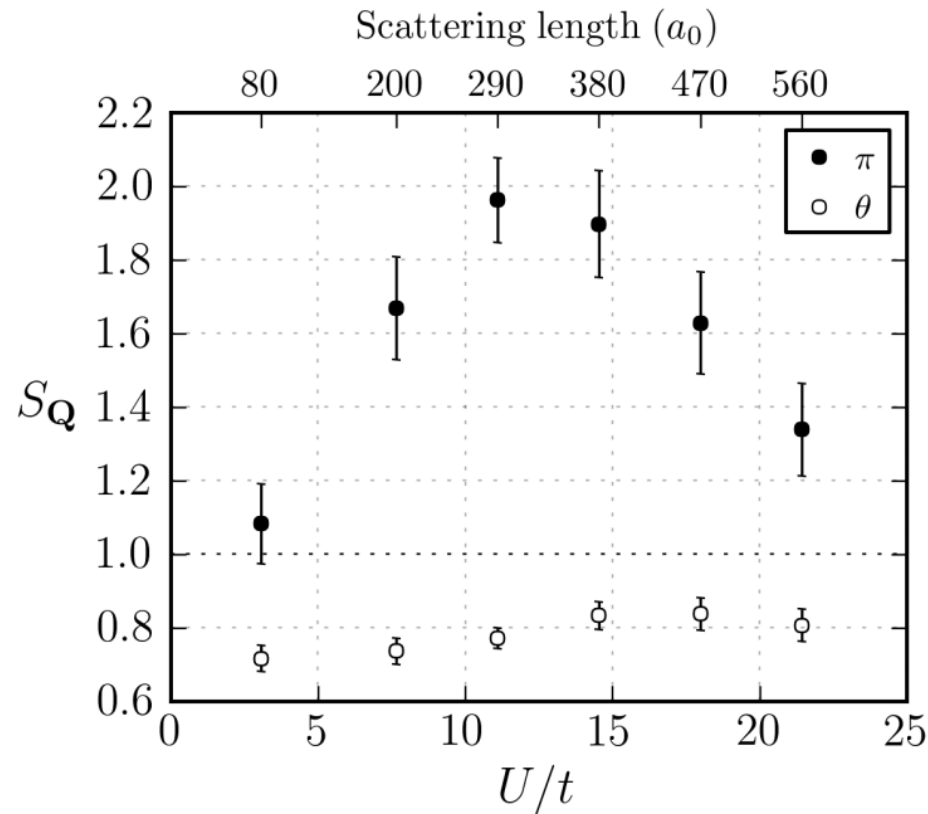
In the experiment, we vary  $N$  to maximize  $S_\pi$

# Interaction Dependence

Comparison with theory provides sensitive thermometry

Theory: Numerical Linked-Cluster Expansion and Quantum Monte Carlo

Theoretical  $S_Q$  is a trap average



Temperature fit:  $T/t = 0.50 (0.04) \rightarrow T/T_N = 1.4 (0.1)$

# Detection of Magnetic Order

If the atoms are released from the lattice the Bragg signal decays due to loss of coherence:

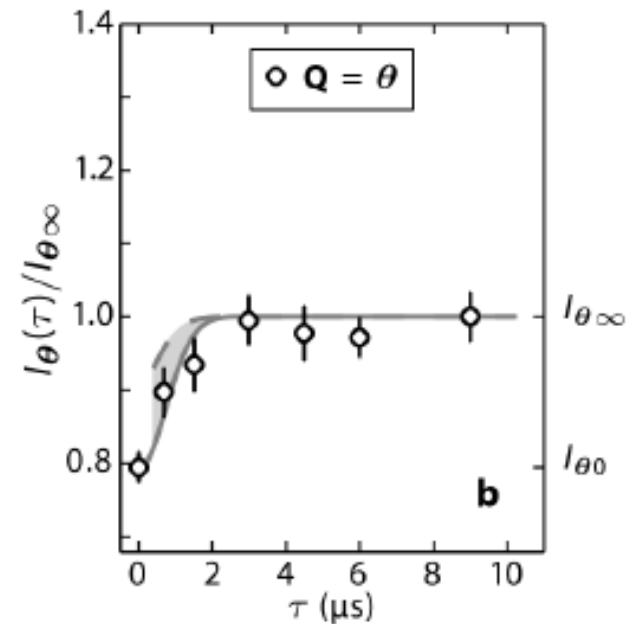
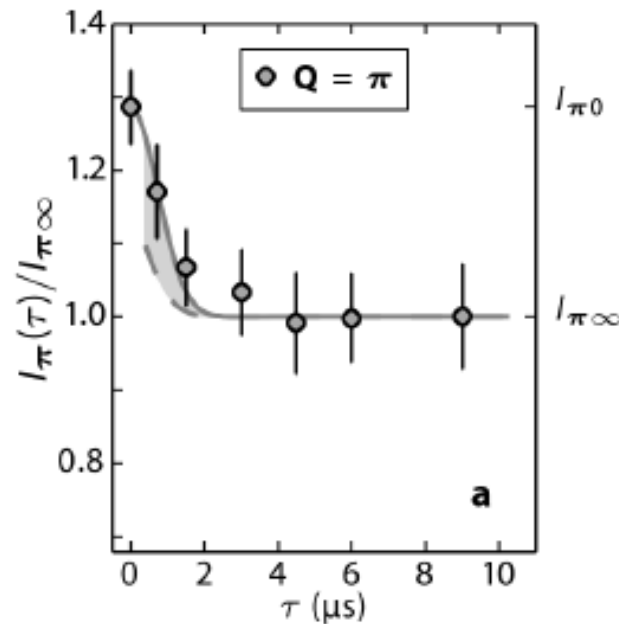
Debye-Waller factor: 
$$e^{\frac{-\langle r^2 \rangle K^2}{2}}, \quad \text{where} \quad \langle r^2 \rangle_t = \langle r^2 \rangle_0 + \frac{t^2}{m} \langle p^2 \rangle_0$$

$$S_Q = I_{Q_0} / I_{Q_\infty}$$

7 Er,  $U/t = 13.4$

$a = 350 a_0$

Shading indicates broadening by probe exposure time ( $1.7 \mu\text{s}$ )

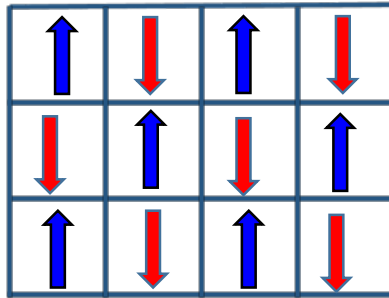




# Charge-Density Order for $U < 0$

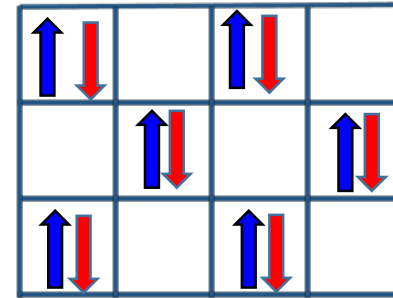
For  $T < t^2/|U|$ , the system will order according to the sign of  $U$ :

$U > 0$ : magnetic order



*or*

$U < 0$ : charge order



- Both types of order have symmetry planes in the  $\pi, \pi, \pi$  direction  
→ detect by Bragg scattering
- Determine dynamical response of system to sudden change in  $U$

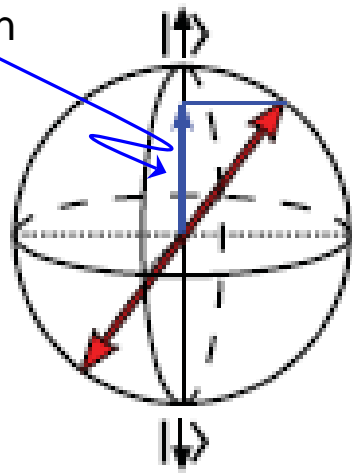
# Canted Antiferromagnetism

Bragg signal only sensitive to magnetism along z

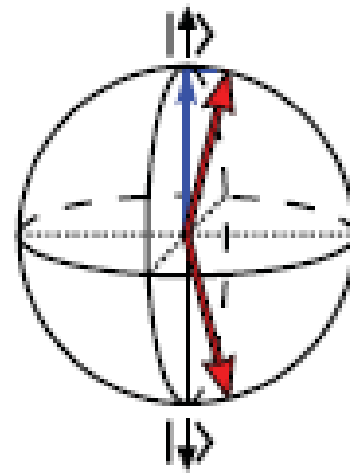
But small population imbalances favor AFM in the x-y plane:

Gottwald and van Dongen, PRA (2009) and E. Demler *et al.*, PRA (2010)

z-component of  
staggered spin



Isotropic AFM

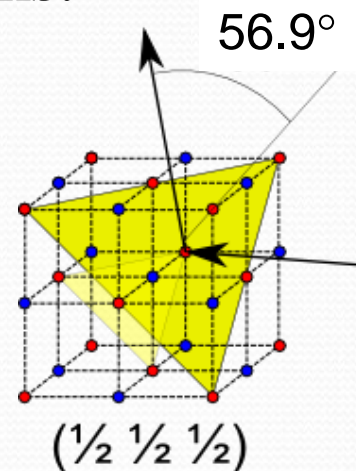


Canted AFM after  
 $\pi/2$  pulse

# Summary

- ***Detected short-range antiferromagnetic correlations:***

- *Spin-dependent Bragg scattering*
- *Thermometry by comparison with QMC*
- *Temperature:  $T = 1.4 T_N$*       *Limit of QMC!  
(away from  $n = 1$ )*



- ***Enabled by compensated lattice***

- *Density control*
- *Flatten band*
- *Mitigation of heating by evaporation*
- *Optimized compensation could lead to even lower  $T$*

Opens new avenues for exploring other novel states of matter such as non-Fermi liquids



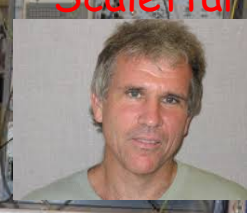
David  
Huse



Nandini  
Trivedi



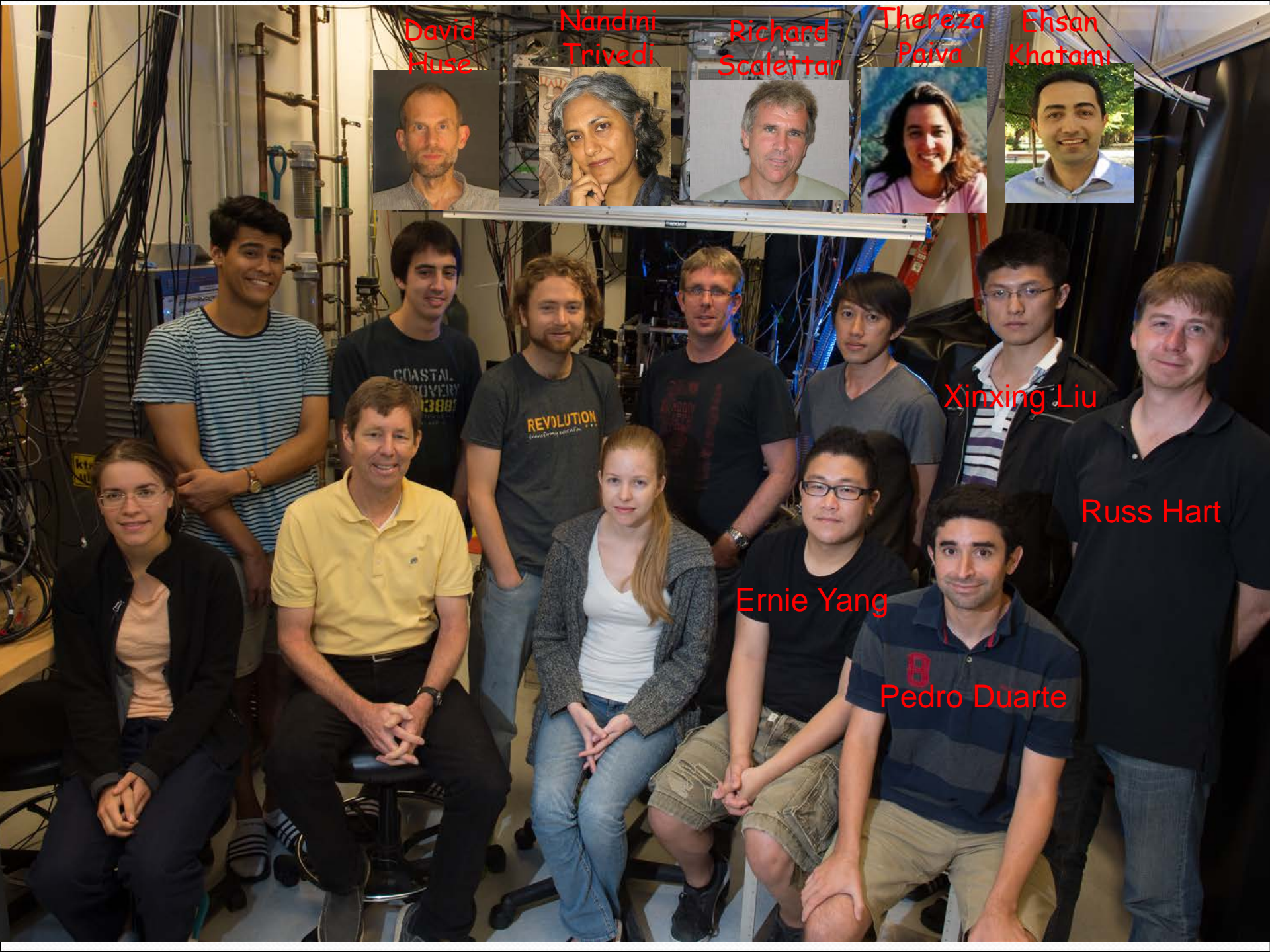
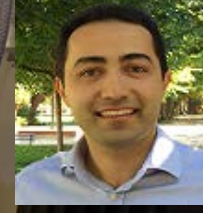
Richard  
Scalettar



Therеза  
Paiva



Ehsan  
Khatami



Xinxing Liu

Russ Hart

Ernie Yang

Pedro Duarte