



中國研学院精密测量科学与技术创新研究院 INNOVATION ACADEMY FOR PRECISION MEASUREMENT SCIENCE AND TECHNOLOGY, CAS

# **Brief Summary on**

# Mathematical Physics for Quantum Science

1-10, Nov, 2024 Hangzhou · China



• Thank all distiguished speakers, participants, workshop volunteers, students for your great contributions!



• Thank the Dean of Physical School, Zhejiang University, professor Hai-Qing Lin, A/professor Huai-Yang Yuan, Secretariat: Ms Qi Fan, Fang-Yu Shi for their hard work and generousness which make our stay comfortable!







# 感谢浙江大学!

# 感谢启真会展!



• Thank our collabrative company for proving wonderful food, transportation and accommodations!



• Thank all staff and students of my group, secretary Ms Yan-Ping Zhu from APM, Chinese Academy of Science for their wonderful services!

I. Bethe Ansatz & beyond: reaching new level of understanding "quantum" Natan Andrei, Murray Batchelor, Andreas Klumper, Balazs S. Pozsgai, Paul Wiegmann II. Confined & deconfined kinks: From gapless to massive quantum field theory & GHD, 3D QF

Marton Kormaos, Gabor Takacs, Wei Zhu, Jian-Da Wu

III. Boundary can be something: topology, boundary CFT, dissipative

Junpeng Cao, Masaki Oshikawa, Chihiro Matsui, Yu-Peng Wang, Wenli Yang

### 30 wonderful talks

Mathematical Physics for Quantum Science

### Perhaps, involving 6 fields of research

IV. Quantum simulation:

Thermodynamics in & out equilibrium, anyon, superTG, quantum holonomy

Xiao-Ling Cui, Marcos Rigol, Ovidiu Patu,

V. String theory, confrormal field theory, topology, anyon condusation, supersymmetric Yang-Mills

Changrim Ahn, Jean Bourgine, Andrea Cappelli, Yun-Feng Jiang, Jian-Xin Lu, Thomas Quella, Yi-Dun Wan, Rui-Dong Zhu, Rui-Dong Zhu VI. Cavity-QED, Quantum devices: From integrability to quasi-integrability, transport

Luigi Amico, Henrik Johannesson, Jose M. P. Carmelo, Qinghu Chen, Pedro Ribeiro

### I. Bethe Ansatz & beyond: reaching new level of understanding "quantum"

Natan Andrei, Murray Batchelor, Andreas Klumper, Balazs S. Pozsgai, Paul Wiegmann

#### Paul Wiegmann, PEIERLS PHENOMENON VIA BETHE ANSATZ

**Cnodial wave:** Periodic solution of classical integrable equations can be seen in quantum integrable models

*Large N* as a semiclassical parameter:  $\psi \rightarrow (\psi_1, \dots, \psi_N)$ 

Gross-Neveu model recovers the Peierls model in the large limit of N

$$H = \sum_{1 \le k \le N} \overline{\psi}_k \sigma_2 i \partial_x \psi_k + \frac{\lambda}{2} \left( \sum_{1 \le k \le N} \overline{\psi}_k \psi_k \right)^2$$

Murray Batchelor,

N-state clock model, free parafermions, spin-1 model, diagrammatical Temperley-Lieb equivalence





#### Natan Andrei, Kondo impurity problems in interacting environments





Kondo impurity in

superconductor

Kondo impurity





$$H_{eff} = \sum_{\mathbf{k},\sigma} \epsilon_k c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma\sigma'} (J_{\Re} + iJ_{\Im}) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \sigma_{\sigma\sigma'} \cdot \mathbf{S}_{imp}$$

Kondo impurity sit at the edge of superconductor

**Balazs S. Pozsgai : Free fermions beyond Jordan** & Wigner--giving dynamical correlations

$$H = \sum_{j=1}^{M} b_j h_j$$

$$h_1 = X_1, h_2 = Z_1 X_2$$

$$h_j = Z_{j-2} Z_{j-1} X_j, \quad j \ge 3$$

$$D(t) \approx \alpha \frac{\sin\left(3\sqrt{3}t + 3\pi/4\right)}{t^{13/6}} \qquad D(t) = \langle h_1(t)h_1(0) \rangle$$

#### A. Klumper, Spin helix: possible possesses topological

The temporal decay of the transverse polarization of a spin helix in the XX model



$$\begin{array}{c} h_z < 1 \\ \langle \sigma_i^x \rangle = (1 - h_z^2)^{1/8} \neq 0 \end{array}$$



 $\overline{\bullet} m_8$ 

 $4 - m_7$ 

 $3 - \frac{m_6}{m_5} m_5$  $\oint m_4$  $2 - m_3$ 

 $\phi m_2$ 

 $1 \rightarrow m_I$ 

#### **Entanglement entropy between** interval and rest of the system II. Confined & deconfined quasiparticles: massive quantum field theory

#### Marton Kormaos, Gabor Takacs, Wei Zhu, Jian-Da Wu **Transverse field Ising model** - L=100 --- L=60 --- L=40 --- L=20 $H_{TFIM} = -J \sum_{i=1}^{L} \left( \sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z \right) \qquad \begin{array}{l} \text{Model exactly solvable in} \\ \text{terms of free fermions} \end{array}$ $\mathbf{S}_{i}$ $\epsilon(k) = 2J\sqrt{1 + h_z^2 - 2h_z \cos(k)}$ Entangment Gap $\Delta(h_{-})=2J|1-h_{-}|$ (Takacs, Wu) (b) ↑<sub>*T*(K)</sub> $(\mathbf{C}) = E(m_1)$ 1D Quantum Critical Region Confinement spin flips domain walls E8 particles quantum critical Ferromagnet Paramagnet $\langle \sigma_i^x \rangle = (1 - \tilde{h}_z^2)^{1/8} \neq 0$ $\langle \sigma_i^x \rangle = 0$ H<sup>3D</sup><sub>c</sub> H **Insights in QCD?** E<sub>8</sub> massive field theory Meson, baryon in $h_1 > 0$ $h_1 < 0$ $H_{1/2}^{\{1,2\}} = H_{1/2} + \mathbf{h} \int \sigma(x) d^2 x$ 1 particle physics

**3-state Potts model** 

$$H_{trans} = -J \sum_{i} \left( \sum_{\mu=1}^{3} P_{i}^{\mu} P_{i+1}^{\mu} + g \tilde{P}_{i} \right)$$

future Quantum metrology

### 3D Ising transition: 3D CFT from fuzzy sphere—new perspective of CFT

 $H(\{s\}) = -J\sum_{x,y} s_x s_{x+\hat{\mu}} - h\sum_{x,y} s_x$ 



dressed quantities:  $\frac{\partial}{\partial \delta u} \int \mathrm{d}\zeta \, \mathrm{j}(\zeta) \Big|$ 

### III. Boundary can be something: topology, boundary CFT, dissipative boundary...

Junpeng Cao, Masaki Oshikawa, Chihiro Matsui, Yu-Peng Wang, Wenli Yang

Many integrable models with general boundary conditions can be solved by T-Q relation, T-W relation, T- $\theta$  relation

XXZ model:

$$H = -\sum_{j=1}^{N} \left[ \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \cosh \eta \sigma_j^z \sigma_{j+1}^z \right]$$

**Topological momentum**  $\mathbf{P}_q = -i \ln \mathbf{t}(0)$ 

$$\mathbf{t}(0) = \sigma_1^x P_{1,N} P_{1,N-1} \cdots P_{1,2} \qquad k = -i \ln \Lambda(0)$$
$$k = \frac{\pi l}{N} \mod \{\pi\} \qquad l = \{-N, -N+1, \cdots, N-1\}$$

Heisenberg spain chain, t-J model, Hubbard model,  $G_2$ 

- Swapped entanglement exhibits a logarithmic scaling
- Entanglement Entropy in CFT: Boundary Scaling Dimension

$$\frac{Z_n}{Z^n} \sim \frac{\mathrm{Tr}\rho_A{}^n}{\left(\mathrm{Tr}\rho\right)^n} \sim \left\langle \sigma(0)\sigma(l) \right\rangle \sim \left(\frac{1}{l}\right)^{2\Delta_\sigma}$$



Bell-pair meansurement on a subsegment leads to entanglement beteen the unmeasured segments

(Oshikawa)

at times past the intrinsic dynamical e. Systems in which thermalization are of great fundamental interest they violate equilibrium statistical cs, and of technological interest beme quantum information in these des decoherence. Nonthermal excited st in integrable (2, 3) and many-body (3, 4) systems. More recently, it has ized that even nonintegrable systems ve special excited initial states for ermalization is thent; these states and antum many-body scars (3-10). Both

of a dipolar Bose gas confined in one dimension. The cycles are made possible through dipolar stabilization of the gas. In a conventional topological pump (12), the Hamiltonian returns to itself after one cycle, but the state is translated by one lattice site. In the present setup, by contrast, the state is translated up the many-body energy spectrum; thus, this protocol maps each eigenstate to an eigenstate with an extensively higher energy. This effect is called a "quantum holonomy" (1). A toy

We implement the following protocol. First, we create a low-temperature dipolar 1D Bose gas in a regime with weak repulsive contact interactions. We then tune the scattering length across confinement-induced resonances (CIRs) of colliding atoms (23-26) in the following stages. First, we ramp up the contact interactions toward the resonance, so the gas adiabatically enters the strongly antibunched Tonks-Girardeau (TG) state (27-29). At this point, we quenche hese interactions across the on: discoverv

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unstable regime  $\rightarrow$ 

10<sup>-2</sup> 10<sup>-1</sup> 10<sup>0</sup>

 $A^2 = N a_{1D}^2 / a_{1}^2$ 

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onomy, CUNY College of Staten Island.

 $^{\circ}$  unstable regime  $\rightarrow$ 

(Rigol)

 $\theta = 55^{\circ}$ 

 $\theta = \dot{0}^{\circ}$ 

 $\theta = 90^{\circ}$ 

10<sup>1</sup> 10<sup>2</sup>

Ξ/N [π²/3 · ħ²n<sup>1</sup>D/2m]

x mole that must Set this elect is pointed subject to a 8-function potential. The nth evenlity and scars are fine-tuned, and can parity eigenstate for an infinitely repulsive pobe approximately realized in actual ex-s, in the form of long-lived prethermal tential is identical to the (n + 1)th even-parity-eigenstate for an infinitely attractive potential. Stanford, CA 94305, USA. <sup>2</sup>E. L. Ginzton Laboratory, anticipated in (10, 11), approximate Hence, by cycling the potential from zero to lity can give rise to states that closely . infinitely repulsive to infinitely attractive and. Staten Island, NY 10314, USA. <sup>5</sup>Physics Program and scars: i.e., av Action and Comparison of the state o **SEARCH** REPORT edly long relaxation times. In such | the wave function and create a sequence of

active, to create the sTG. As the attractive eriments a for anet away from this unitary, estiffness R versus intertate hegendented strapsas usually becomes The proving the bosons ofernies all the bound cluster states ight stoangle flato Been observed in a nonholonomy cycle for the Mar 🕲 Jas (19) eBy contrast, our dig endeabbears donatifically stable for stinis allows as to then ramp the active teoma entitle faction strength toward Sagara Vogerferate a weakly attractive Bose Thiad We shill establish the state. That stystem temans althaidfically stable throughthis procesule is a consequence of the Alsheckieling interactions, as we will diseasekomente paratiabeledeb zvuen perseands inte

ther CIR produces even higher excited states. se claims are supported through gas still s and energy-per-particle measurements at ous stages in the protocol.

Ve begin our experiments by preparing a rly pure Bose-Einstein condensate (BEC) of hly magnetic Dy atoms at 26.69 G, just of the CIRs employed. (162Dy's magnetic ment of  $\mu = 10$  Bohr magnetons is 10 times of, e.g., CsQudick WI relaxs whose first transverse excited-state energy strengths, indicated by gr  $\omega_{\perp}/k_{\rm B} = 11$  Oct energy strengths, indicated by gr  $\omega_{\perp}/k_{\rm B} = 11$  oct energy strengths, indicated by gr ncy is  $\omega_{\perp} = 2\pi \times 24.6(4)$  kHz;  $h = 2\pi\hbar$  is nck's constance rei @ othe alta wand r stant. The 00 1D optical traps with about 40 atoms he central tube and 30 atoms per tube on rage; see Fig. 1A and (22). Each tube apximates a 1D channel of finite length, ere the natic O longitalinal versus cross scillation measurements are to be

ective formed; see (22) for details. These measurents are consistent with zero-temperature und state predictions (22), which implies t the temperature is sufficiently low to obve the sTG gas (31).

he system may be described with a Liebger (LL) Hamiltonian (32, 33) augmented the magnetic DDI:



al states, othermoliza\* onasi@uantum Scarbow Fig. 2. Post-quench gas action parameter A<sup>2</sup>. Shown is the attractive  $g_{1D} < 0$  regime of the first

3.6

3.2-

4.8

3.2

4.8

 $10^{-5}$   $10^{-4}$   $10^{-3}$ 

 $-\infty \leftarrow q_{1D}$ 

(A) nondipolar (θ = 55°) and (B) attractive DDI (0°) system, and for (C) the repulsive, 90° DDI-stabilized excited gas. In (A) and (B), ~≏4.4 Э an sTG gas exists in the ~~4.0 ℃ unitary regime of  $A^2 \lesssim 10^{-3}$ . Beyond, however, the gas softens before collapsing Υ. ▲ near  $A^2 \approx 10^{-1}$  and  $10^{-2}$ . respectively. For comparison, the dashed green curve in (A) plots data from the nondipolar variational Monte Carlo simulation of (15). Unexpectedly, the repulsive DDI system in (C) remains stable beyond the

unitary regime. This allows correlated prethermal states to emerge around strengths, indicated by gray over into the R = 4 weakly attractive, excited Bose gas regime beyond  $A^2 \approx 10$ . The solid curve is ansatz prediction of (18). The vertical dotted line indicates where the contact and the short-range regularized DDI contributions become approximately equal (21). Numbers refer to points in Figs. 10 and 3. The error bars here and in subsequent figures represent the standard error.

strength  $g_{1D}$  is independently controlled by setting the field magnitude B to be near a CIR d is then further ramped (in a few milli-onds) to the destruction of the second state of the

transverse ground state. It modifies the contact interaction strength as follows:

$g_{1D}(B) = -\frac{2m}{ma_{rn}}$	( <u>R</u> )	
$=rac{2\hbar^2 a_{ m 3D}}{ma_{\scriptscriptstyle \perp}^2}$	$\frac{\overset{(D)}{(B)}}{1-Ca_{\rm 3D}(B)/a_{\perp}}$	(2

Here,  $C \approx 1$  and  $a_{3D}$  and  $a_{1D}$  are the 3D and 1D scattering lengths, respectively (24). We tune  $g_{1D}$  by controlling  $a_{3D}$  with a Feshbach recommended the fixed a Feetback recommende

(22). Indeed, their positions are adequ  $\theta$ -independent. We implement the holor cycle(s) by sweeping B up to a desired hi field value, thereby preparing a state a particular  $g_{1D}$ . The second holonomy begins after point 5 in Fig. 1B, where  $g_{1D}$ 

turns positive again, and continues to point fwhere  $g_{1D}$  crosses zero again.

We measure gas stiffness through observations of collective oscillations of the atoms along the 1D trap axis (18, 37, 38). The frequency  $\omega_B$  of the breathing mode of the gas is sensitive to its inverse compressibility (stiffness), and thus contains information about correlations Normalizing on by the frequence

 $E_{sTG} = E_0 - \frac{3J}{2a},$ 

where  $\Psi_0$  is the fermionalized wave function in the hardcore limit with total energy  $E_0$ , and  $\Psi_1$  is from the first order correction when a  $\uparrow \downarrow$  pair comes close together [21]. For later comparisons, we have transformed Eqs. (5) and (6) into the c.m. frame [21]. Figure 2(b) shows that

# f new physics in Eqs. (3) and (5) can indeed well approximate the two Importantly, Eqs. (3) and (5) suggest qualitatively differ-

ent real-space distributions between sTG and atom-dimer states. To be concrete, all atom-dimer states have a dominant weight when one  $\uparrow \downarrow$  pair comes close to each is Ensemble, Generalized hydrodynamic, dynamical  $\rightarrow 0$ , given that

they contain very localized dimer components. In contrast, the sTG state is dominated by the  $\Psi_0$  part which is much more extended in real space while it only has a little weight along the dimer lines  $(\sim P_1/g)$ . Such difference is have plotted real-sp results are consiste Eqs. (3) and (5) sho The above wave-

standing the loss me Fig. 2(b), at certain dimer branch have p strongly and open a level crossing is eigenstate inherits a [Fig. 2(c2)]. Therefore tends to develop a mulate great possib This leads to the ir easily undergo an i cause atom loss. Sin excited molecular st othis in anharmon



theoretical prediction to  $1/g_c$  by comparing (4) and (6) is shown in (e1), and the wave function

(e2). In all plots we take  $\omega$  and l as the units of energy (E) and length  $(r, \rho)$ . The units

## <sup>*T* = 1</sup><sup>*V*</sup>. String theory, confrormal field theory, topology: unifying physics

Changrim Ahn, Jean Bourgine, Andrea Cappelli, Yun-Feng Jiang, Jian-Xin Lu, Thomas Quella, Rui-Dong Zhu

• Boundary entropy and g-function(Jiang)

 $Z_{ab} = \operatorname{Tr} e^{-\beta H_{ab}} = e^{-\beta F_{ab}}$   $F_{ab} = F_{\text{bulk}} + f_a + f_b$   $s_a = (1 - \beta \partial_\beta)(-\beta f_a)$   $\gg 1 \text{Boundary entropy}$   $Z_{ab}(\beta, L) \approx \langle B_a | 0 \rangle \langle 0 | B_b \rangle e^{-LE_0(\beta)}$   $\langle B_a | 0 \rangle = e^{-\varepsilon_a \beta} g_a$ 

• Non-perturbative QFT (Ahn):



• M-thory: D-branes analogous to thre Schwinger pair production in QED

Pairing production = 
$$(\vec{s}_k \vec{s}_l)^2 Lu$$
  
Weak-field string rate  $(\vec{s}_k \vec{s}_l)^2 Lu$ 

Symmetry-protected topological phases: from quantum group invariant



• Fractional quantum Hall effect: from quantum group invariant (Bourgine)

$$\psi(\boldsymbol{x}) = \prod_{a \leq b} (x_a - x_b)^k e^{-\frac{eB}{4\hbar} \sum_a |x_a|^2} \qquad \boldsymbol{\nu} = \boldsymbol{p}/(\boldsymbol{k} + \boldsymbol{p}\boldsymbol{n})$$

 Integrability in 2D Supersymmetric QFT: Bethe/Gauge correspondence (Rui-Dong Zhu)

3D N=2\* (k) gauge field vacua eq. XXZ spin chain with twist BC

$$S_{\text{bdry}}[\varphi] = \frac{k}{4\pi} \int_{\partial \mathcal{M}} dt du \; \partial_u \varphi \; \dot{\varphi} - \left( \partial_u \varphi \right)^2 \qquad \longrightarrow \; \varphi = \varphi(u+t)$$

### VI. Cavity-QED & Quantum devices: From integrability to quasi-integrability

Luigi Amico, Qinghu Chen, Henrik Johannesson, Jose M. P. Carmelo, Pedro Ribeiro



#### J. Carmelo, Quantum transport in spin XXZ model and Hubbard model:

Spectral function, dynamical structure factor, correlation lenth

$$S^{aa}(k,\omega) = \sum_{j=1}^{N} e^{-ikj} \int_{-\infty}^{\infty} dt \, e^{-i\omega t} \langle GS | \hat{S}_{j}^{aa}(t) \hat{S}_{j}^{a}(0) | GS \rangle$$
$$= \sum_{\nu} |\langle \nu | \hat{S}_{k}^{a} | GS \rangle|^{2} \delta(\omega - \omega_{\nu}^{\tau}(k)) \text{ for } a = x, y, z$$



cavity

detector

0.7

0.8

tor, which allows for direct computation of the canonical partition function at low temperatures by summing over all the spin eigenstates and only some of the charge excitations. For a dilute system (n < 0.1)the relevant temperature scales are  $T_F = \pi^2 n^2$  for the charge degrees of freedom and  $T_s = \pi^2 n^3/U$  for the Atomitronics: mesocopic systems of freedom. For temperatures  $T \ll T_F$  be possibly realized as quantum devices  $\sum_{\text{relevant sets } I} \sum_{\text{all sets } J} \exp\{-E(\{k_j\}, \phi)/T\}$  and gives the PC. This approach requires the knowledge of the all  $C_{N_{\downarrow}}^{N}$  states of the Heisenberg spin-chain with with rest for attractive Fermi gases Luigi Amico Magneto-optical toroidal circuits and  $N_{\downarrow}$  spins down, which can be found in [49–51] FIG. 4. ized by  $I_0(N_1) + I_0(N_1)$ ) for N = 6 and N = 10. Note that 1 = 1  $\left[\frac{K_a}{N} + \phi\right]$ Using this method we were able to investigate the PC for (a) (b)  $(\mathbf{c})$  $N \leq 10, N_{\downarrow} \leq N/2$  and  $T \leq 0.06 T_{F}$ focus on dilute systems, we note that hile belov for  $N \stackrel{\text{def}}{=} 6, N_{\downarrow} \stackrel{\text{def}}{=} 3$ , and  $N \mid = 10, N_{\downarrow} = 5$ , there is an interval ain valid for all densities  $0 < n \stackrel{!}{\triangleleft} 1$  if in which the amplitude is increasing with temperature. For all cases  $n \neq 0.01$  and U = 100. nce of the PC on tomperature in the € 0.00 ing Hubbard model<sup>®</sup> is very complex with with spin, region both  $N_{\downarrow,\uparrow}$  even, paramagnetic with peof the system playing an important role. riod 1. This general pattern can be understood by noting th N = 8 and  $N_{\perp} = 2$ , Fig. 3 shows that  $\Omega_0$ that an increase in temperature is qualitatively similar to temperatures the current is diamagnetic  $\operatorname{erv} l$ with period 1/8, at higher temperatures the periodicity changes to 1/4. At even higher temperatures the current the decrease in U and therefore the evolution of the cur-rent with T mimics the evolution of the current at T = 0, **Experimental stiring of the ring** becomes paramagnetic with period 1. The evolution of the current with increasing temperature for a system with  $\mathcal{H} = \int \Psi^{\dagger}(\mathbf{r}) \left[ \frac{1}{2m} (-\imath \hbar \nabla + \mathbf{A}(\mathbf{r}))^2 + V_{\text{torman}} \mathbf{r} \text{ and } N_{\downarrow} = \mathbf{2} \text{ is similar (but note paramagnetic bound of a system with the second of the system with the second of the system with the system with the second of the system with th$ period 1/10, paramagnetic with period 1/5, and param- $+\frac{1}{2}\int \Psi^{\dagger}(\mathbf{r})\Psi^{\dagger}(\mathbf{r})\Psi^{\dagger}(\mathbf{r})\Psi(\mathbf{r}$ temperatures, when the two spin-down electrons become located symmetrically in the system. with periodicity 1/N, and is followed at high educed expoatures by the current with  $N_{\parallel}/N$  periodicity V = 0quantitative (a) -V = 0.05is the same as the one at zero temperature, w  $\propto V^{-1/2}$ is plotted in -V = 0.1SU(N) fermionic & bosonic trapped from (Fingt very high temperatures, should have the same characteristics as for fr  $n_k^B$ ), and shows -V = 0.2ited with the  $\mathcal{H}_{BH} = \sum_{i=1}^{N_s} \left[ -J(b_j^{\dagger} b_{j+1} e^{\frac{2i\pi}{N_s} \frac{\phi}{\phi_0}} + \text{h.c.}) + \frac{U}{2} n_j (n_j - 1) \right]$ -0.30 φ 0.3 k

tor is much smaller than the energy of the charge sec-

# Thank all of you for your great contribuitions!

# Welcome to Wuhan in 2025!