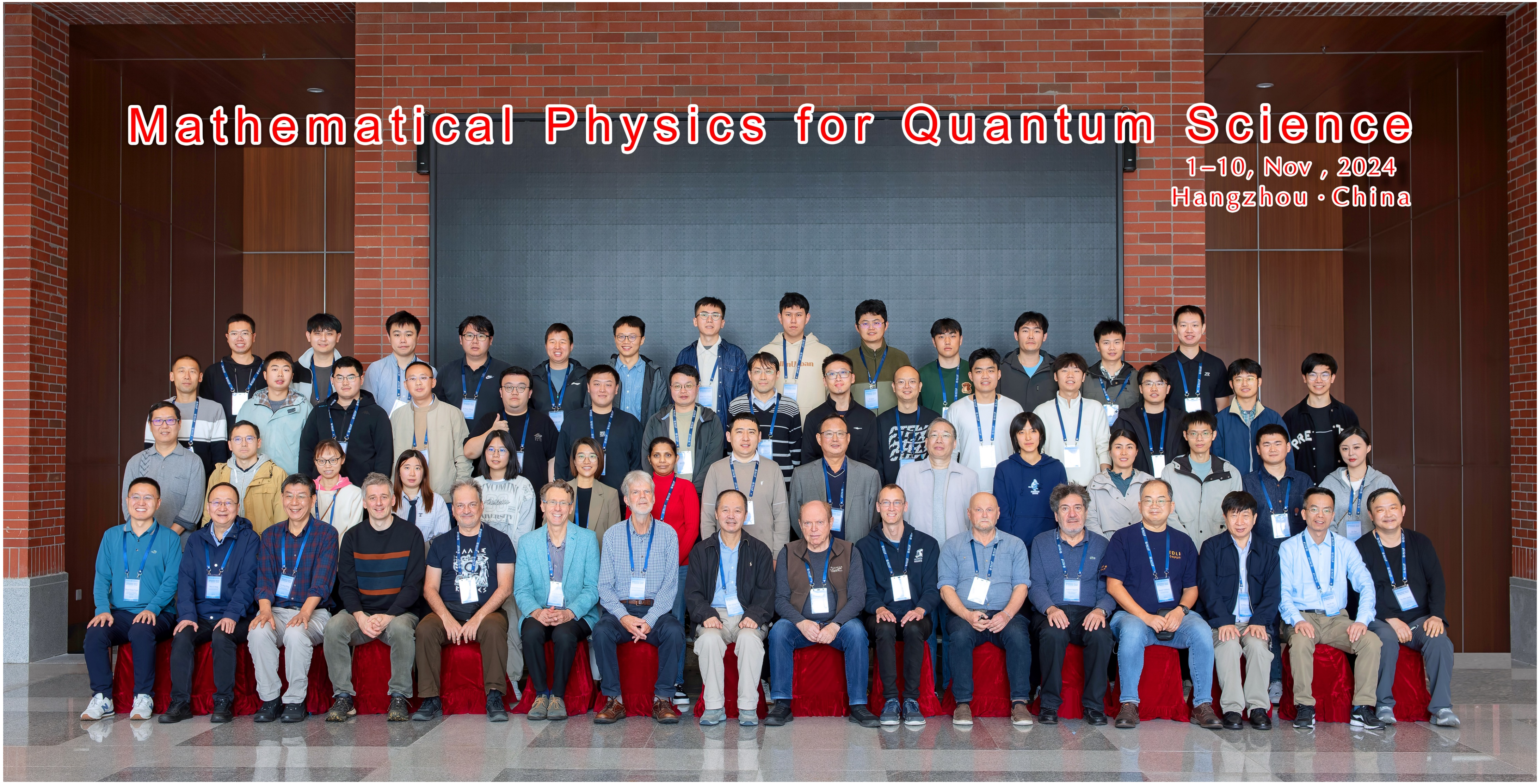
浙江大學 物理高等研究院
INSTITUTE FOR ADVANCED STUDY IN PHYSICS OF ZHEJIANG UNIVERSITY

中国科学院 精密测量科学与技术创新研究院
INNOVATION ACADEMY FOR PRECISION MEASUREMENT SCIENCE AND TECHNOLOGY, CAS

Brief Summary on

Mathematical Physics for Quantum Science

1-10, Nov, 2024 Hangzhou · China



- Thank all distiguished speakers, participants, workshop volunteers, students for your great contributions!

- **Thank the Dean of Physical School, Zhejiang University, professor Hai-Qing Lin, A/professor Huai-Yang Yuan, Secretariat: Ms Qi Fan, Fang-Yu Shi for their hard work and generousness which make our stay comfortable!**

感谢浙江大学！

感谢启真会展！

- Thank our collabrative company for proving wonderful food, transportation and accommodations!

- **Thank all staff and students of my group, secretary Ms Yan-Ping Zhu from APM, Chinese Academy of Science for their wonderful services!**

I. Bethe Ansatz & beyond: reaching new level of understanding “quantum”

Natan Andrei, Murray Batchelor, Andreas Klumper, Balazs S. Pozsgai, Paul Wiegmann

II. Confined & deconfined kinks: From gapless to massive quantum field theory & GHD, 3D QF

Marton Kormaos, Gabor Takacs, Wei Zhu, Jian-Da Wu

III. Boundary can be something: topology, boundary CFT, dissipative

Junpeng Cao, Masaki Oshikawa, Chihiro Matsui, Yu-Peng Wang, Wenli Yang

30 wonderful talks

Mathematical Physics for Quantum Science

Perhaps, involving 6 fields of research

IV. Quantum simulation: Thermodynamics in & out equilibrium, anyon, superTG, quantum holonomy

Xiao-Ling Cui, Marcos Rigol, Ovidiu Patu,

V. String theory, confrormal field theory, topology, anyon condnsation, supersymmetric Yang-Mills

Changrim Ahn, Jean Bourgine, Andrea Cappelli, Yun-Feng Jiang, Jian-Xin Lu, Thomas Quella, Yi-Dun Wan, Rui-Dong Zhu, Rui-Dong Zhu

VI. Cavity-QED, Quantum devices: From integrability to quasi-integrability, transport

Luigi Amico, Henrik Johannesson, Jose M. P. Carmelo, Qinghu Chen, Pedro Ribeiro

I. Bethe Ansatz & beyond: reaching new level of understanding "quantum"

Natan Andrei, Murray Batchelor, Andreas Klumper, Balazs S. Pozsgai, Paul Wiegmann

Paul Wiegmann, PEIERLS PHENOMENON VIA BETHE ANSATZ

Cnodial wave: Periodic solution of classical integrable equations can be seen in quantum integrable models

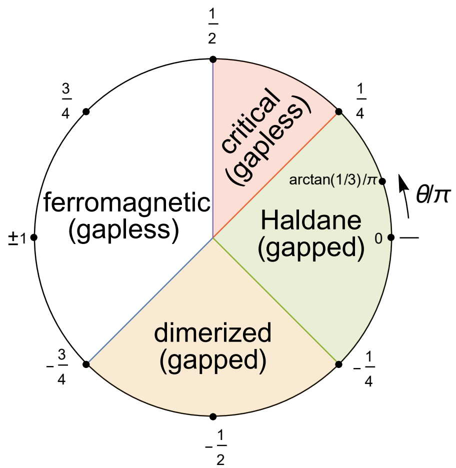
Gross-Neveu model recovers the Peierls model in the large limit of N

Large N as a semiclassical parameter: $\psi \to (\psi_1, \dots \psi_N)$

$$H = \sum_{1 \le k \le N} \overline{\psi}_k \sigma_2 i \partial_x \psi_k + \frac{\lambda}{2} \Big(\sum_{1 \le k \le N} \overline{\psi}_k \psi_k \Big)^2$$

Murray Batchelor,

N-state clock model, free parafermions, spin-1 model, diagrammatical Temperley-Lieb equivalence

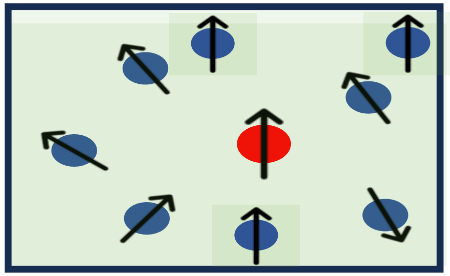


Spin-1 bilinear and biquadratic model

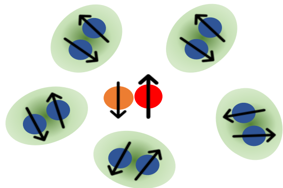
$$\mathcal{H} = \sum_{i=1}^{N} \cos\theta (\vec{S}_i \otimes \vec{S}_{i+1}) + \sin\theta (\vec{S}_i \otimes \vec{S}_{i+1})^2$$

Ferromagnetic
Critical trimerized
Haldane phase
Dimerized phase

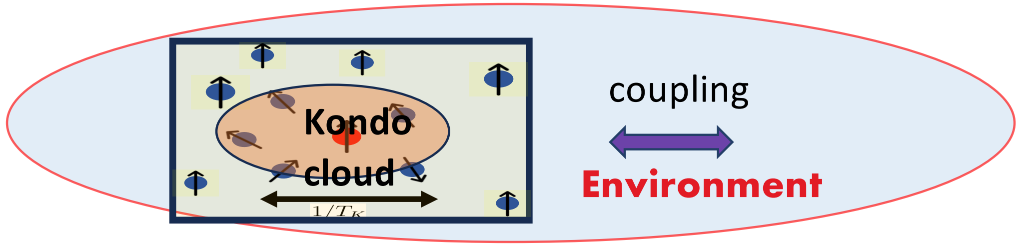
Natan Andrei, Kondo impurity problems in interacting environments



Kondo impurity

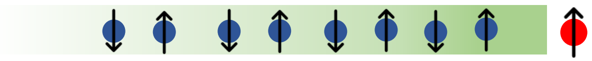


Kondo impurity in superconductor



Open Kondo System

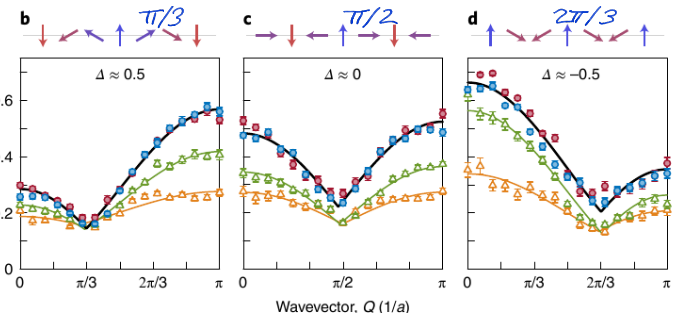
$$H_{eff} = \sum_{\mathbf{k},\sigma} \epsilon_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma\sigma'} (J_{\Re} + iJ_{\Im}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \sigma_{\sigma\sigma'} \cdot \mathbf{S}_{imp}$$



Kondo impurity sit at the edge of superconductor

A. Klumper, Spin helix: possible possesses topological

The temporal decay of the transverse polarization of a spin helix in the XX model



$$\gamma(Q) = -\lim_{t\to\infty} (t^{-1} \log\langle \sigma^x_n(t)\rangle_Q)$$

$$= \frac{8}{\pi} |\cos(Q)|$$

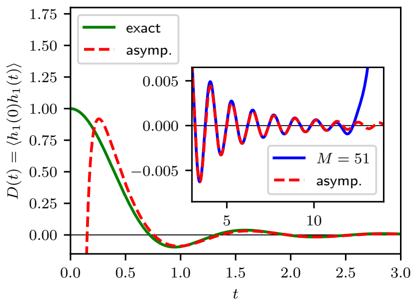
Balazs S. Pozsgai : Free fermions beyond Jordan & Wigner--giving dynamical correlations

$$H = \sum_{j=1}^{M} b_j h_j$$

$$h_1 = X_1,\ h_2 = Z_1 X_2$$

$$h_j = Z_{j-2} Z_{j-1} X_j, \quad j \geq 3$$

$$D(t) \approx \alpha \frac{\sin\left(3\sqrt{3}t + 3\pi/4\right)}{t^{13/6}}$$



$$D(t) = \langle h_1(t) h_1(0)\rangle$$

II. Confined & deconfined quasiparticles: massive quantum field theory

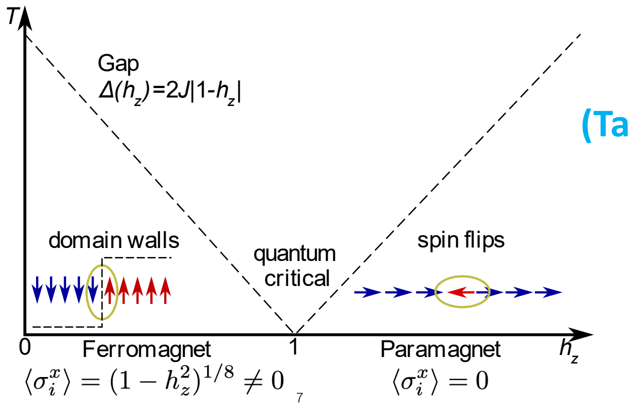
Marton Kormaos, Gabor Takacs, Wei Zhu, Jian-Da Wu

Transverse field Ising model

$$H_{TFIM} = -J\sum_{i=1}^{L}\left(\sigma_i^x\sigma_{i+1}^x + h_z\sigma_i^z\right)$$

Model exactly solvable in terms of free fermions

$$\epsilon(k) = 2J\sqrt{1+h_z^2-2h_z\cos(k)}$$



(Takacs, Wu)

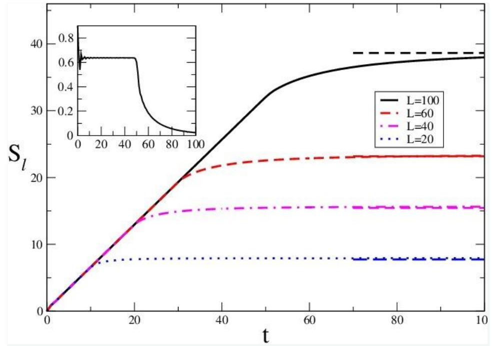
Entangment

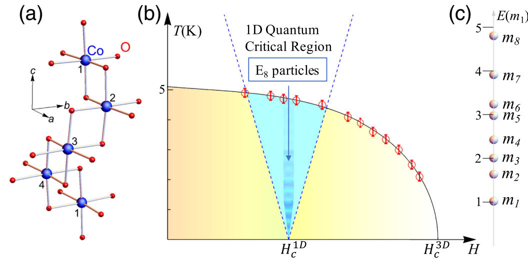
Confinement

Insights in QCD?
Meson, baryon in particle physics

future

Quantum metrology



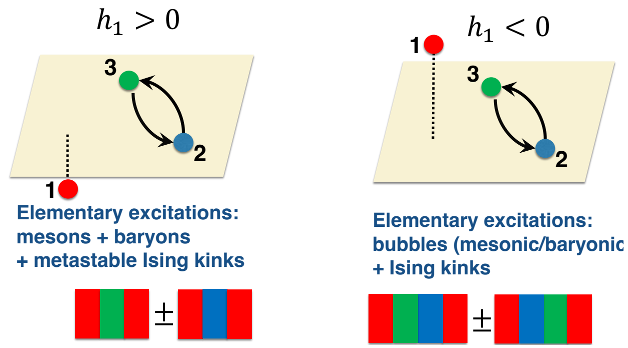


E_8 massive field theory

$$H_{1/2}^{\{1,2\}} = H_{1/2} + \mathrm{h}\int \sigma(x)d^2x$$

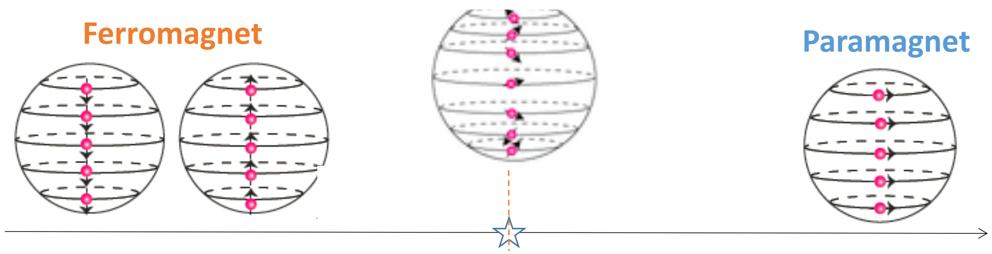
3-state Potts model

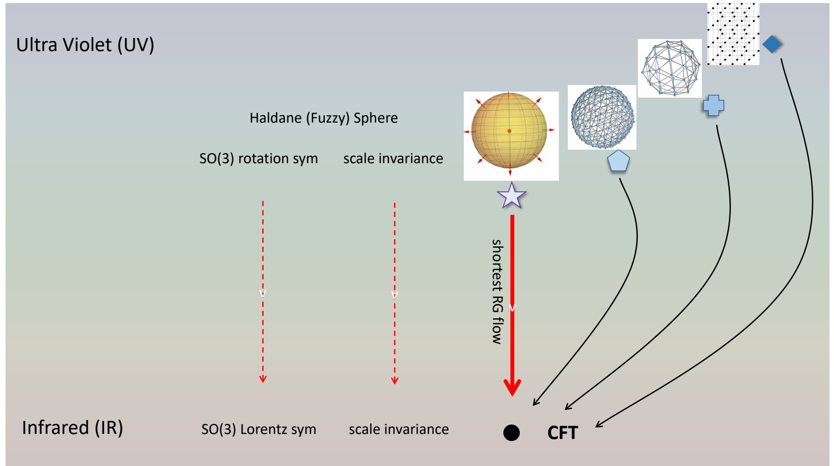
$$H_{trans} = -J\sum_i\left(\sum_{\mu=1}^{3} P_i^\mu P_{i+1}^\mu + g\tilde{P}_i\right)$$



3D Ising transition: 3D CFT from fuzzy sphere—new perspective of CFT

Wei Zhu





3D Ising conformal symmetry transition: spectra forms an almost conformal tower structure

Sine-Gordon nodel: favourate of theoretical physicists

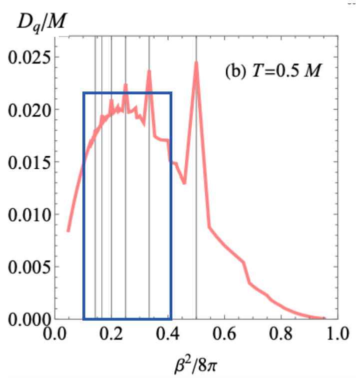
Marton Kormaos

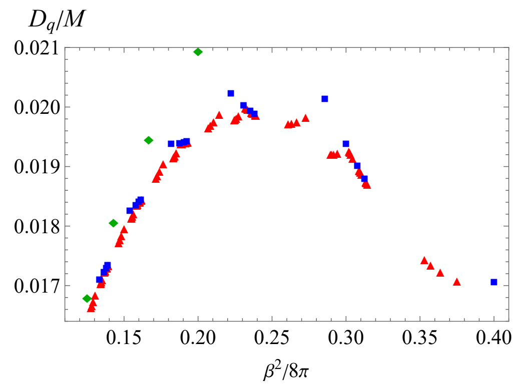
Quantum transport in sine-Gordon model: GHD appoach

$$\mathrm{Re}\,\sigma_{ik}(\omega) = 2\pi\tilde{D}_{ik}\delta(\omega) + \sigma_{ik}^{\mathrm{reg}}(\omega)$$

$$D_{ik} = \lim_{\tau\to\infty}\frac{1}{2\tau}\int_{-\tau}^{\tau}\mathrm{d}t\int\mathrm{d}x\,\langle j^{(i)}(x,t)j^{(k)}(0,0)\rangle^c$$

$$D_q = \sum_a\int\mathrm{d}\theta\rho_a^{\mathrm{tot}}(\theta)\vartheta_a(\theta)[1-\vartheta_a(\theta)]\left[v_a^{\mathrm{eff}}(\theta)q_a^{\mathrm{dr}}(\theta)\right]^2$$





Fractal structure in Drude weight

III. Boundary can be something: topology, boundary CFT, dissipative boundary…

Junpeng Cao, Masaki Oshikawa, Chihiro Matsui, Yu-Peng Wang, Wenli Yang

Many integrable models with general boundary conditions can be solved by T-Q relation, T-W relation, T-θ relation

XXZ model: $$H = -\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \cosh\eta\,\sigma_j^z\sigma_{j+1}^z\right]$$

Topological momentum $$\mathbf{P}_q = -i\ln \mathbf{t}(0)$$

$$\mathbf{t}(0) = \sigma_1^x P_{1,N}P_{1,N-1}\cdots P_{1,2} \qquad k = -i\ln\Lambda(0)$$

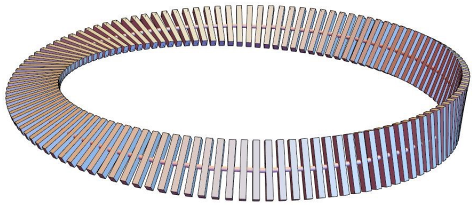
$$k = \frac{\pi l}{N} \bmod \{\pi\} \qquad l = \{-N, -N+1, \cdots, N-1\}$$

Heisenberg spain chain, t-J model, Hubbard model, G_2

- **Swapped entanglement exhibits a logarithmic scaling**
- **Entanglement Entropy in CFT: Boundary Scaling Dimension**

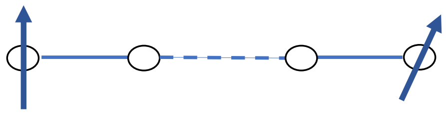
$$\frac{Z_n}{Z^n} \sim \frac{\mathrm{Tr}{\rho_A}^n}{(\mathrm{Tr}\rho)^n} \sim \langle\sigma(0)\sigma(l)\rangle \sim \left(\frac{1}{l}\right)^{2\Delta_\sigma}$$

(Oshikawa)



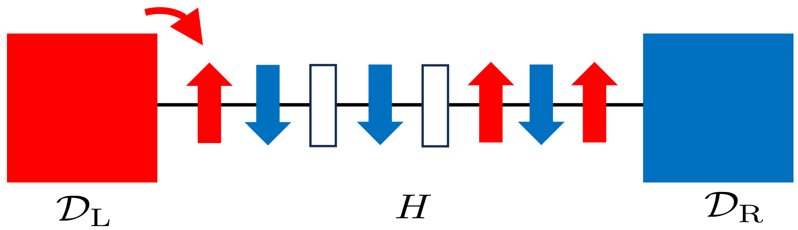
Topological

(Wang)



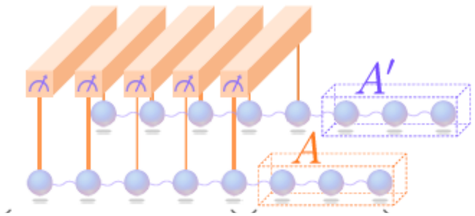
Open boundaries breaking U(1) symmetry

(Cao, Yang)



(Matsui)

Boundary dissipators



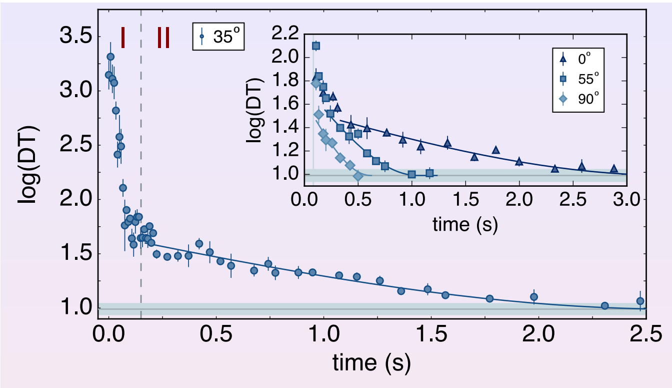
Bell-pair meansurement on a subsegment leads to entanglement beteen the unmeasured segments

IV. Quantum simulation: discovery of new physics in & out of equilibrium

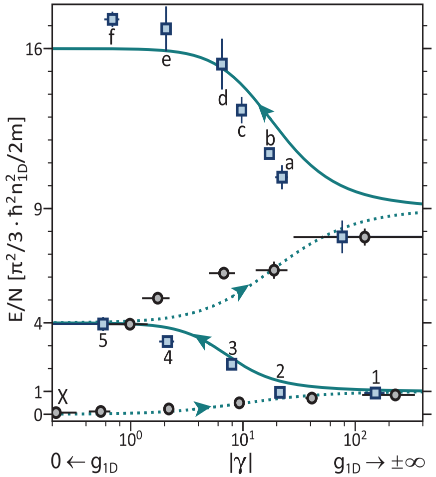
Xiao-Ling Cui, Marcos Rigol, Ovidiu Patu,

Thermalization, Prethermal lization, Generalized Gibbs Ensemble, Generalized hydrodynamic, dynamical thermolization, Quantum Scar, …

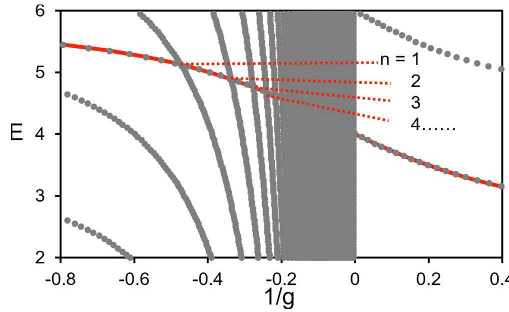
Far from equilibrium: Separation of the time scales



(Rigol)



Quantum holonomy (Cui)



Smaller interbranch coupling leads to a stable sTG state

Quickly relaxs to the thermal stat before eventually relaxing to the true thermal equilibrium:
Quench dynamycs, weakly breaking integrability, Periodically driven….

$$H = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + U_{\rm H}(x_i)\right] + \sum_{1\le i<j\le N}\left[g_{\rm 1D}^{\rm vdW}\delta(x_i-x_j) + U_{\rm DDI}^{\rm 1D}(\theta_{\rm B}, x_i-x_j)\right]$$

Quantum correlators in strongly interacting quantum gases (Patu)

$$\rho_\sigma^{SILL}(x,y) = \frac{1}{Z}\sum_{N=0}^{\infty}\sum_{q_1<\cdots<q_N}\sum_{N_1=0}^{N}\sum_{N_2=0}^{N-N_1}\cdots\sum_{N_{\kappa-1}=0}^{N-(N_1+\cdots+N_{\kappa-2})}\sum_{n=1}^{N!/[N_1!\cdots N_\kappa!]}$$
$$\times\, e^{-\sum_{j=1}^{N}\varepsilon(q_j)/T+\sum_{\sigma'=1}^{\kappa}\mu_{\sigma'}N_{\sigma'}/T}\langle\Phi_{\boldsymbol{N},\boldsymbol{q},n}|\Psi_\sigma^\dagger(x)\Psi_\sigma(y)|\Phi_{\boldsymbol{N},\boldsymbol{q},n}\rangle$$

V. String theory, confrormal field theory, topology: unifying physics

Changrim Ahn, Jean Bourgine, Andrea Cappelli, Yun-Feng Jiang, Jian-Xin Lu, Thomas Quella, Rui-Dong Zhu

- Boundary entropy and g-function(Jiang)

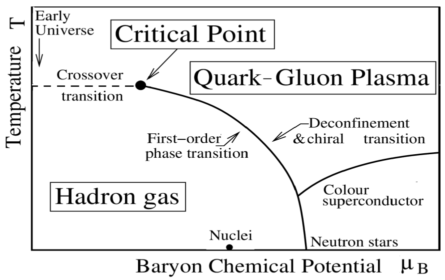
$$Z_{ab} = \mathrm{Tr}\, e^{-\beta H_{ab}} = e^{-\beta F_{ab}}$$
$$F_{ab} = F_{\text{bulk}} + f_a + f_b$$

$$s_a = (1 - \beta\partial_\beta)(-\beta f_a)$$

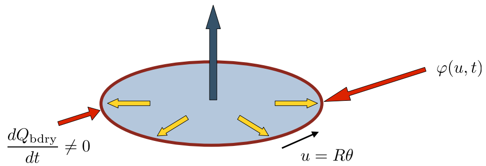
Boundary entropy

$$Z_{ab}(\beta, L) \approx \langle B_a|0\rangle\langle 0|B_b\rangle e^{-LE_0(\beta)}$$
$$\langle B_a|0\rangle = e^{-\varepsilon_a\beta}\, g_a$$

- Non-perturbative QFT (Ahn):



- Bosonization & Duality in 2+1 D:

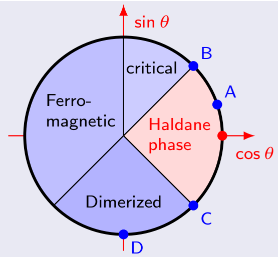


- M-thory: D-branes analogous to thre Schwinger pair production in QED

Pairing production = Weak-field stringy rate : $\mathcal{W}^{(\text{String})} = \mathcal{W}^{\text{QED}}$ (Lu)

- Symmetry-protected topological phases: from quantum group invariant

Spin-1



Analytic results:
A: AKLT model
B: SU(3) Uimin-Lai-Sutherland – $SU(3)_1$ WZW
C: Babujian-Takhtajan – $SU(2)_2$ WZW
D: Map to 9-state Potts model

Haldane phase:
Symmetry-protected topological order

(Quella)

- Fractional quantum Hall effect: from quantum group invariant (Bourgine)

$$\psi(\boldsymbol{x}) = \prod_{a<b}(x_a - x_b)^k e^{-\frac{eB}{4\hbar}\sum_a |x_a|^2} \qquad \boldsymbol{\nu = p/(k + pn)}$$

- Integrability in 2D Supersymmetric QFT: Bethe/Gauge correspondence (**Rui-Dong Zhu**)

3D N=2* (k) gauge field vacua eq. ⟺ XXZ spin chain with twist BC

VI. Cavity-QED & Quantum devices: From integrability to quasi-integrability

Luigi Amico, Qinghu Chen , Henrik Johannesson, Jose M. P. Carmelo, Pedro Ribeiro

Quantum Rabi model

$$H_R = \omega a^\dagger a + \Delta\sigma_z + g\sigma_x(a^\dagger + a)$$

photon frequency — level splitting (two-level atom) — Rabi coupling

Quantum Rabi-Stark model

A. L. Grimsmo and S. Parkins, Phys. Rev. A **87**, 033814 (2013)

$$H_{RS} = \omega a^\dagger a + \sigma_z(\Delta + \gamma a^\dagger a) + g\sigma_x(a^\dagger + a)$$

Stark coupling

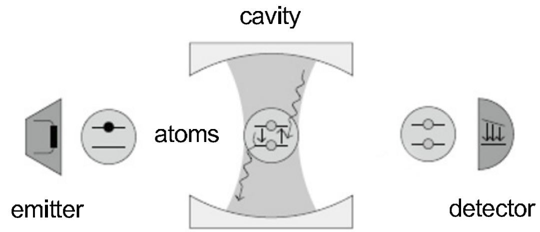
Quantum matter coupled with photons

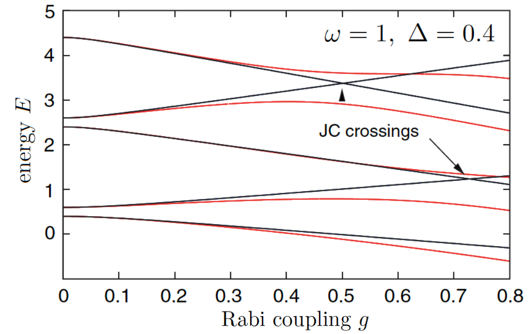
↓

Mathematical euqations describe the solutions of Cavity-QED

↓

Quantum metrology



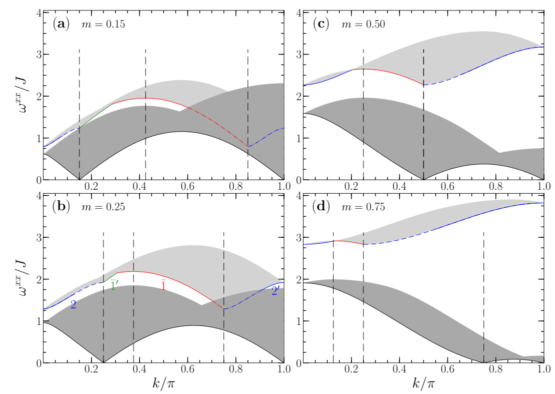


J. Carmelo, Quantum transport in spin XXZ model and Hubbard model:

Spectral function, dynamical structure factor, correlation lenth

$$S^{aa}(k,\omega) = \sum_{j=1}^{N} e^{-ikj} \int_{-\infty}^{\infty} dt\, e^{-i\omega t} \langle GS|\hat{S}_j^{aa}(t)\hat{S}_j^{a}(0)|GS\rangle$$

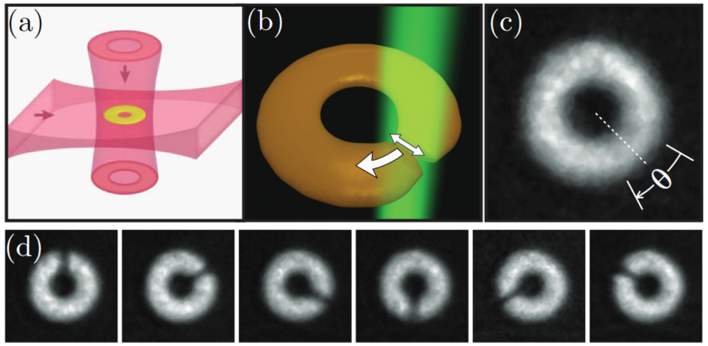
$$= \sum_{\nu} |\langle \nu|\hat{S}_k^{a}|GS\rangle|^2 \delta(\omega - \omega_\nu^\tau(k)) \ \ \text{for} \ \ a = x, y, z$$



Atomitronics: mesocopic systems of cold atoms can be possibly realized as quantum devices

Luigi Amico

Magneto-optical toroidal circuits



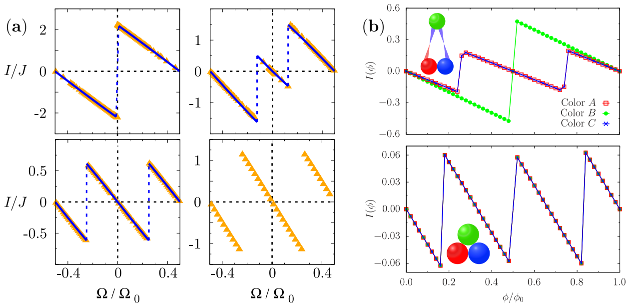
Experimental stiring of the ring

$$\mathcal{H} = \int \Psi^\dagger(\mathbf{r})\left[\frac{1}{2m}(-\imath\hbar\nabla + \mathbf{A}(\mathbf{r}))^2 + V_{\text{toroidal}}(\mathbf{r}) + V_{local}(\mathbf{r})\right]\Psi(\mathbf{r})\mathrm{d}\mathbf{r} + \frac{1}{2}\int \Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')\mathcal{V}(\mathbf{r}-\mathbf{r}')\Psi(\mathbf{r}')\Psi(\mathbf{r})\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}',$$

SU(N) fermionic & bosonic trapped in a ring

$$\mathcal{H}_{BH} = \sum_{j=1}^{N_s}\left[-J(b_j^\dagger b_{j+1}e^{\frac{2\imath\pi}{N_s}\frac{\phi}{\phi_0}} + \text{h.c.}) + \frac{U}{2}n_j(n_j-1)\right]$$

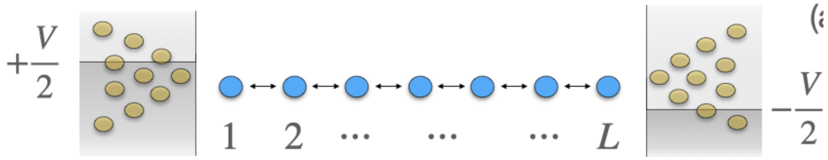
Persistent current for attractive Fermi gases

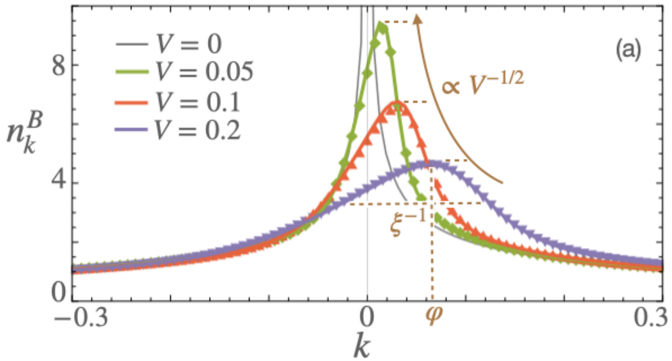


$$I(\phi) = -2N\left(\frac{2\pi}{L}\right)^2\sum_{a=1}^{n_N}\left[\frac{K_a}{N}+\phi\right]$$

1D quantum transport: Bias-driven quantum matter

Pedro Ribeiro






Thank all of you for your great contribuitions!
Welcome to Wuhan in 2025!