Quantum dynamics of many-body scars in near-integrable 1D dipolar gases

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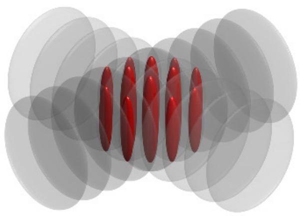
Yang, Zhang, Li, Lin, Gopalakrishnan, MR & Lev, Science **385**, 1063 (2024).
Li, Zhang, Yang, Lin, Gopalakrishnan, MR & Lev, PRA **107**, L061302 (2023).

Outline

Outline

1D regime: Equilibrium and far from Equilibrium



Effective one-dimensional δ potential

Olshanii, PRL **81**, 938 (1998).

$$U_{1D}(x) = g_{1D}\delta(x)$$

where

$$g_{1D} = \frac{2\hbar a_s \omega_\perp}{1 - C a_s \sqrt{\frac{m\omega_\perp}{2\hbar}}}$$

Lieb & Liniger '63, Girardeau '60 ($g_{1D} = \infty$)

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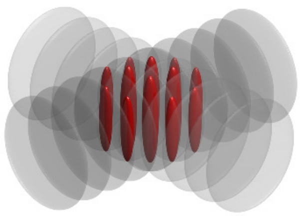

Kinoshita, Wenger & Weiss, Science **305**, 1125 (2004).

Kinoshita, Wenger & Weiss, PRL **95**, 190406 (2005).

$$g^{(2)}(x) = \frac{\langle \hat{\Psi}^{\dagger 2}(x) \Psi^2(x) \rangle}{n_{1D}^2(x)}$$

and

$$\gamma = \frac{m g_{1D}}{\hbar^2 n_{1D}}$$

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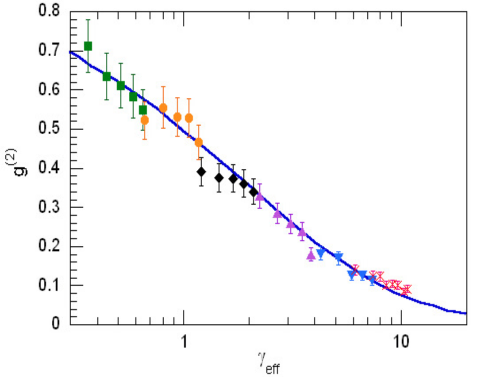

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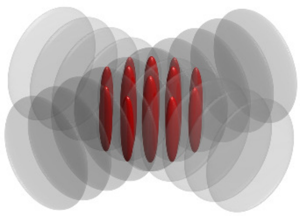

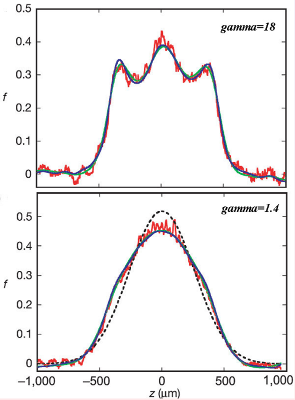

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Quantum Newton's cradle quench

Kinoshita, Wenger & Weiss, Nature **440**, 900 (2006).

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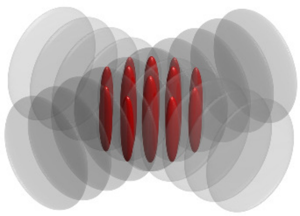
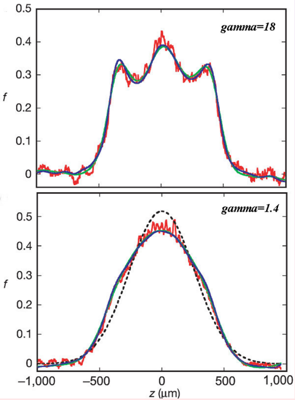
$\gamma \gtrsim 1$ correlated Bose gas

$\gamma \gg 1$ fermionized Tonks-Girardeau gas





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Generalized Gibbs ensemble (GGE):

Vidmar & MR, J. Stat. Mech. 064007 (2016).

Outline

Bose-Fermi mapping in a 1D lattice

Hard-core boson Hamiltonian in an external potential

$$\hat{H} = -J \sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \right) + \sum_i v_i \, \hat{n}_i$$

Constraints on the bosonic operators

$$\hat{b}_i^{\dagger 2} = \hat{b}_i^2 = 0$$

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$$\Downarrow$$

Map to spins and then to fermions (Jordan-Wigner transformation)

$$\hat{\sigma}_i^+ = \hat{f}_i^\dagger \prod_{\beta=1}^{i-1} e^{-i\pi \hat{f}_\beta^\dagger \hat{f}_\beta}, \quad \hat{\sigma}_i^- = \prod_{\beta=1}^{i-1} e^{i\pi \hat{f}_\beta^\dagger \hat{f}_\beta} \hat{f}_i$$

$$\Downarrow$$

Non-interacting fermion Hamiltonian

$$\hat{H}_F = -J \sum_i \left(\hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.} \right) + \sum_i v_i \ \hat{n}_i^f$$

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Constraints on the bosonic operators

$$\hat{b}_i^{\dagger 2} = \hat{b}_i^2 = 0$$

$$\Downarrow$$

Set of conserved quantitites

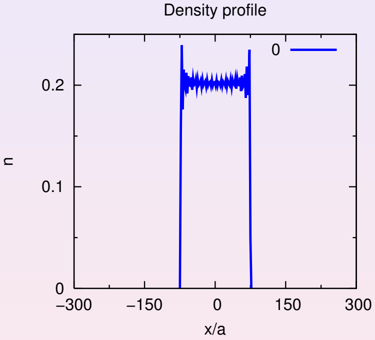
(Occupations of the single-particle energy eigenstates of the noninteracting fermions)

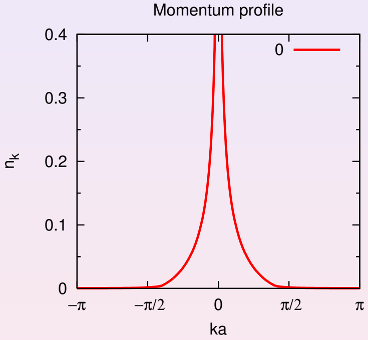
$$\hat{H}_F \hat{\gamma}_m^{f\dagger} |0\rangle = E_m \hat{\gamma}_m^{f\dagger} |0\rangle$$
$$\left\{ \hat{I}_m^f \right\} = \left\{ \hat{\gamma}_m^{f\dagger} \hat{\gamma}_m^f \right\}$$

Efficient computational approach using properties of Slater determinants:

MR and Muramatsu, PRL **93**, 230404 (2004).

Relaxation dynamics in an integrable system





MR, Dunjko, Yurovsky, and Olshanii, PRL **98**, 050405 (2007).

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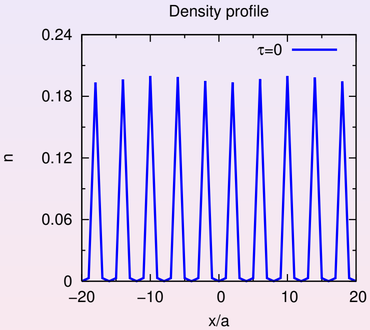

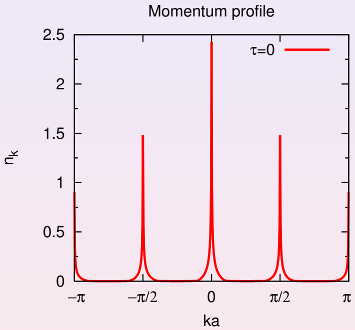

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Generalized Gibbs ensemble (GGE)

Thermal equilibrium

$$\hat{\rho} = Z^{-1} \exp\left[-\left(\hat{H} - \mu\hat{N}\right)/k_B T\right]$$

$$Z = \mathrm{Tr}\left\{\exp\left[-\left(\hat{H} - \mu\hat{N}\right)/k_B T\right]\right\}$$

$$E = \mathrm{Tr}\left\{\hat{H}\hat{\rho}\right\}, \quad N = \mathrm{Tr}\left\{\hat{N}\hat{\rho}\right\}$$

MR, PRA **72**, 063607 (2005).

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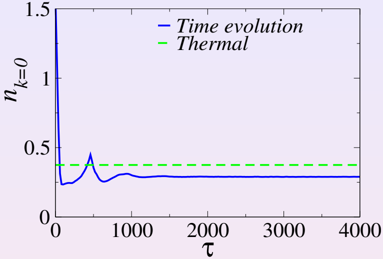
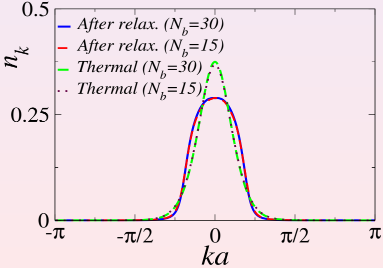
Evolution of $n_{k=0}$



n_k after relaxation



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Conserved quantities

(underlying noninteracting fermions)

$$\hat{H}_F \hat{\gamma}_m^{f\dagger}|0\rangle = E_m \hat{\gamma}_m^{f\dagger}|0\rangle$$
$$\left\{\hat{I}_m\right\} = \{\hat{\gamma}_m^{f\dagger}\hat{\gamma}_m^f\}$$

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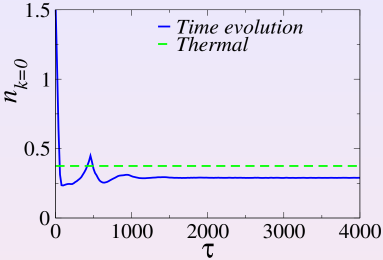

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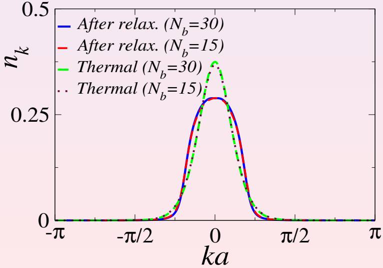

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Generalized Gibbs ensemble

$$\hat{\rho}_{\text{GGE}} = Z_c^{-1} \exp\left[-\sum_m \lambda_m \hat{I}_m\right]$$

$$Z_c = \mathrm{Tr}\left\{\exp\left[-\sum_m \lambda_m \hat{I}_m\right]\right\}$$

$$\mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_{\text{GGE}}\right\} = \langle \hat{I}_m \rangle_{\tau=0}$$

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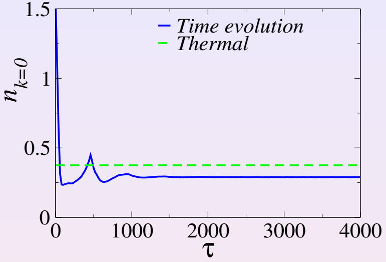

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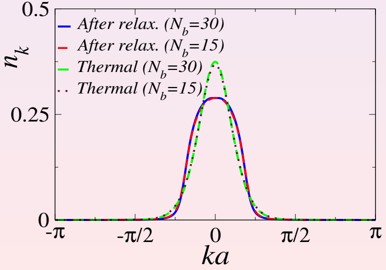

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The constraints

$$\mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_{\text{GGE}}\right\} = \langle \hat{I}_m \rangle_{\tau=0}$$

result in

$$\lambda_m = \ln\left[\frac{1 - \langle \hat{I}_m \rangle_{\tau=0}}{\langle \hat{I}_m \rangle_{\tau=0}}\right]$$

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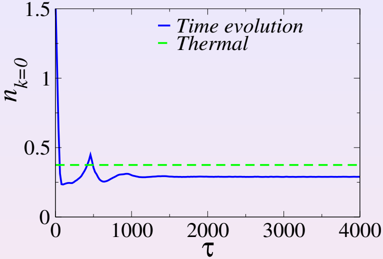

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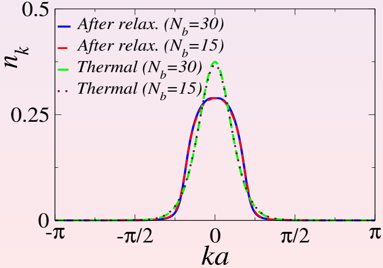

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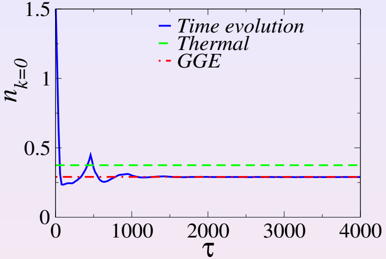

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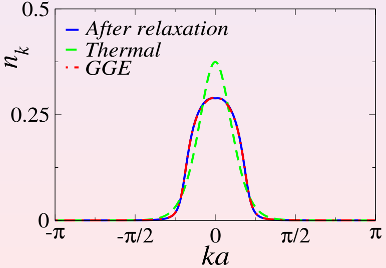

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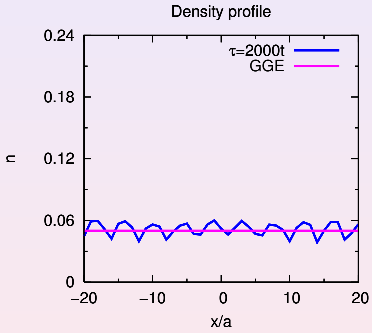

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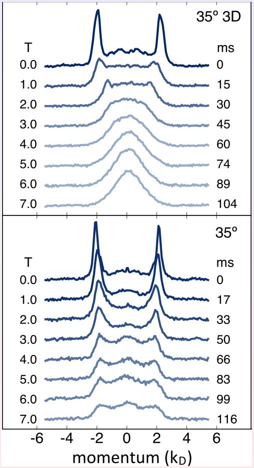
Prethermalization & thermalization (QNC Dysprosium)



Tang, Kao, Li, Seo, Mallayya, MR, Gopalakrishnan & Lev, PRX **8**, 021030 (2018).

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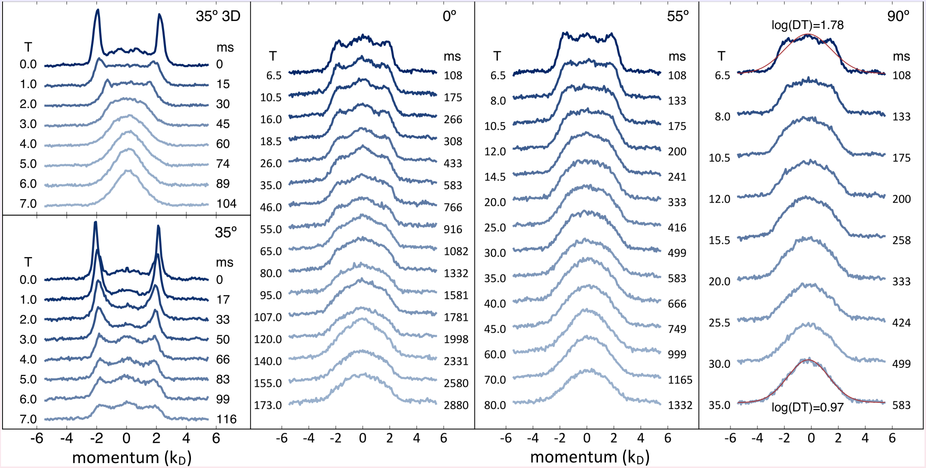

$$\text{DT} = \sqrt{\sum_k \left[n(k) - n_G(k)\right]^2}$$

Tang, Kao, Li, Seo, Mallayya, MR, Gopalakrishnan & Lev, PRX **8**, 021030 (2018).

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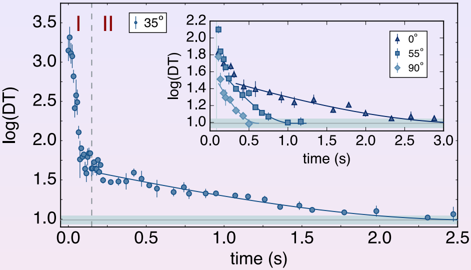
Approach to thermal predictions:



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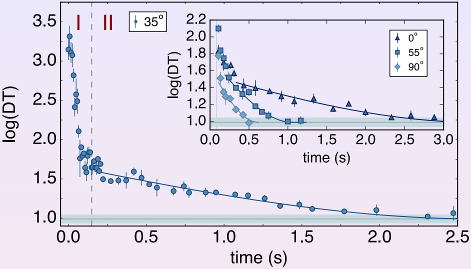
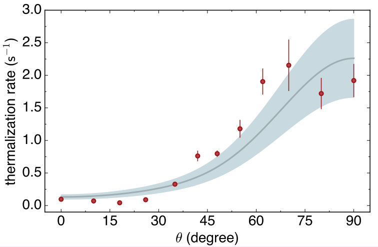


Consistent with FGR $\propto U^2_{\text{total}}(\theta)$



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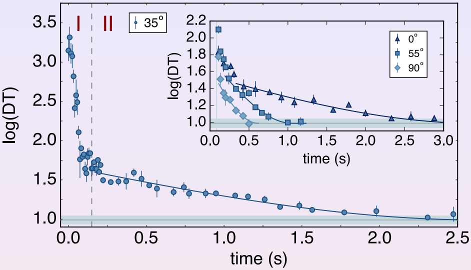

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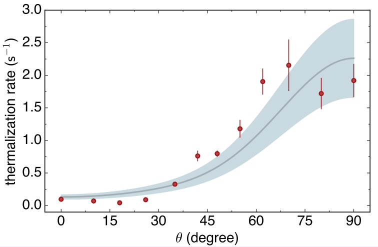
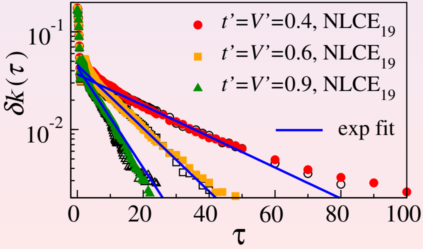


Breaking integrability in the XXZ model (NLCEs)



Mallayya & MR, PRL **120**, 070603 (2018).

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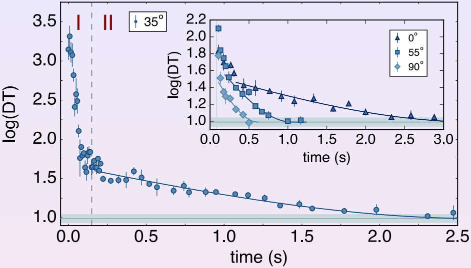

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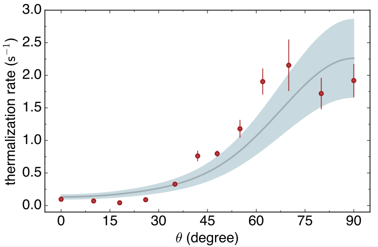

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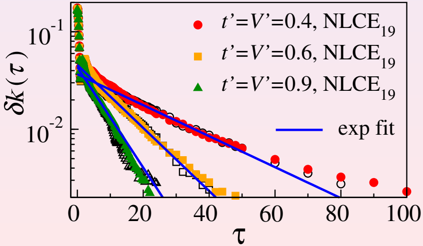

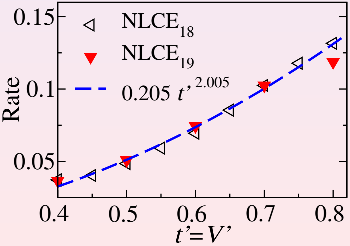

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Particle-number conservation (nonintegrable model)

Fermi's golden rule:

$$\dot{n}(\tau) = \frac{2\pi g_1^2}{L} \sum_{i,j} \delta(E_j^0 - E_i^0)\,(N_j - N_i)\,P_i^0(\tau) \times \left| \langle E_j^0 | \hat{U}_1 | E_i^0 \rangle \right|^2,$$

where

$$N_i = \langle E_i^0 | \hat{N} | E_i^0 \rangle$$
$$P_i^0(\tau) = \langle E_i^0 | \hat{\rho}(\tau) | E_i^0 \rangle$$

Thermalization rate

$$\Gamma^{\text{Fermi}}(g_1) = -\frac{\dot{n}(\tau)}{n(\tau) - 0.5}$$

Mallayya, MR & De Roeck, PRX **9**, 021027 (2019).

Mallayya & MR, PRB **104**, 184302 (2021).

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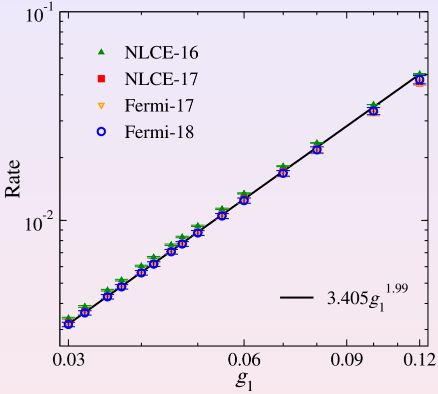

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Outline

Dynamical fermionization during expansion in 1D

Dynamics after turning off the confining potential

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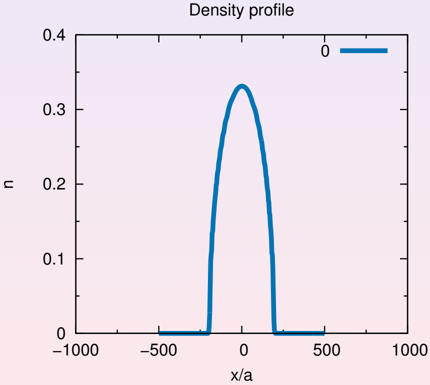

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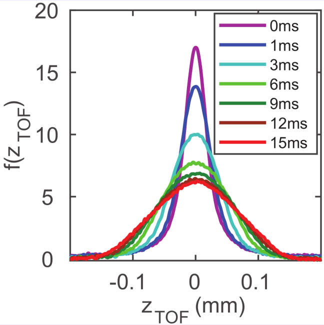
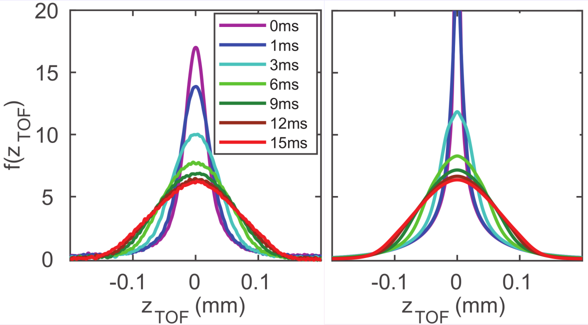
1D expansion of TG gases (fermionization)



Wilson, Malvania, Le, Zhang, MR & Weiss, Science **367**, 1461 (2020).

1D expansion of TG gases (fermionization)



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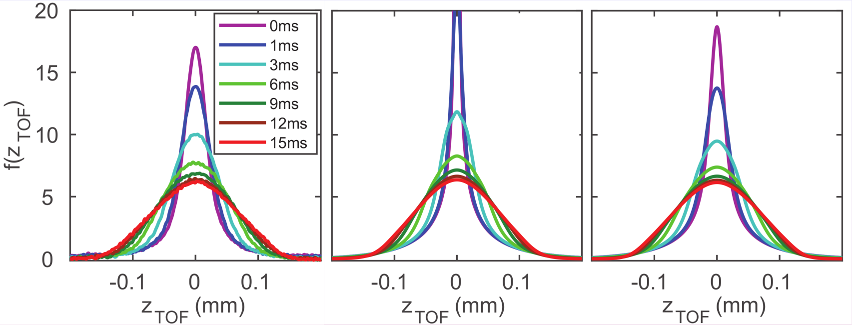

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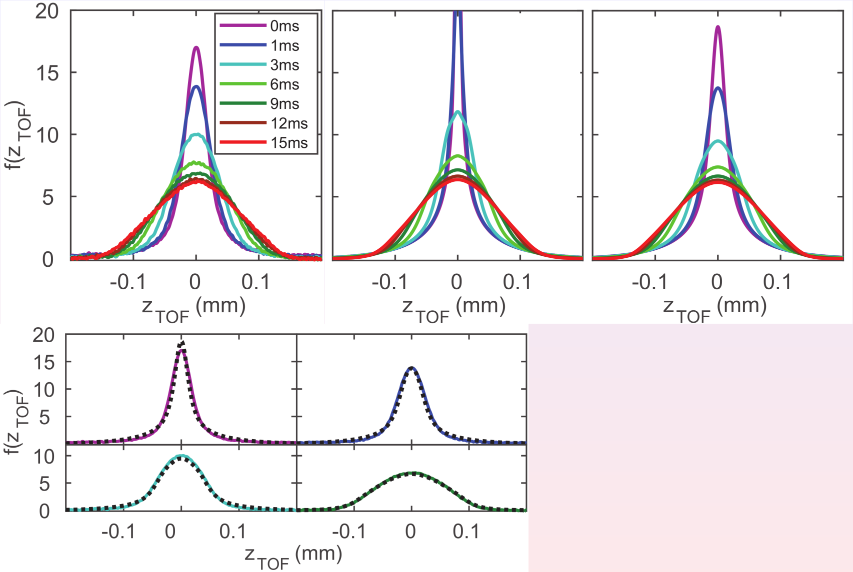

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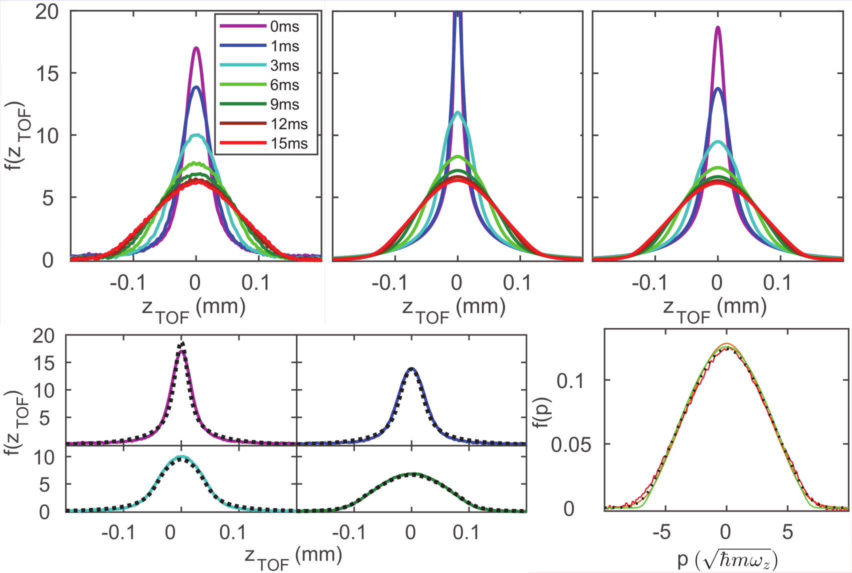

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Outline

Theoretical description of 1D dysprosium gases

1D Hamiltonian (neglecting the intertube DDI):

$$H = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^2} + U_{\mathrm{H}}(x_i) \right] + \sum_{1 \leq i < j \leq N} \left[g_{\mathrm{1D}}^{\mathrm{vdW}} \delta(x_i - x_j) + U_{\mathrm{DDI}}^{\mathrm{1D}}(\theta_{\mathsf{B}}, x_i - x_j) \right].$$

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Intratube DDI in the single-mode approximation:

$$U_{\mathrm{DDI}}^{\mathrm{1D}}(\theta_{\mathsf{B}}, x) = \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3\cos^2\theta_{\mathsf{B}}}{\sqrt{2} a_\perp^3} \left[V_{\mathrm{DDI}}^{\mathrm{1D}}(u) - \frac{8}{3}\delta(u) \right], \quad \text{where} \quad u = \sqrt{2} x / a_\perp$$

Deuretzbacher, Cremon & Reimann, PRA **81**, 063616 (2010).

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We replace $V_{\rm DDI}^{\rm 1D}(u) \to A\delta(u)$, where $A = \int_{-\infty}^{\infty} V_{\rm DDI}^{\rm 1D}(u) du = 4$:

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Our 1D near-integrable Hamiltonian:

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where $g_{\rm 1D}=g_{\rm 1D}^{\rm vdW}+g_{\rm 1D}^{\rm DDI}$.

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Modeling the initial state preparation

For the loading of the 3D BEC into the 2D optical lattice ($U_{\rm 2D}$), we assume:

(i) At $U^*_{\rm 2D}$, the entire 3D system decouples into individual 1D tubes with N_l atoms, and all tubes are in thermal equilibrium at a temperature T^*.

- ★ We determine N_l and the entropy S_l of each tube l using the exact solution of the homogeneous Lieb-Liniger model at finite temperature plus the LDA.

Modeling the initial state preparation

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(ii) As the loading proceeds beyond $U^*_{2\mathrm{D}}$, the process is adiabatic, i.e., the entropy for each tube remains constant.

- ★ Using the experimental parameters at the end of the loading process, we obtain the temperature T_l of each tube.

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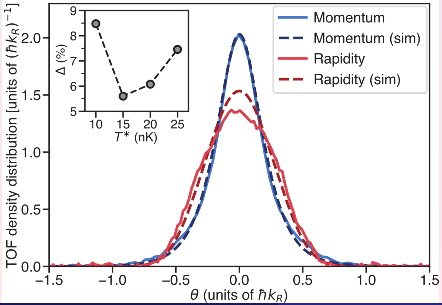
$\gamma_{\rm T} \approx 420$, $N \approx 6 \times 10^3$ & $U^*_{\rm 2D} = 5E_{\rm R}$



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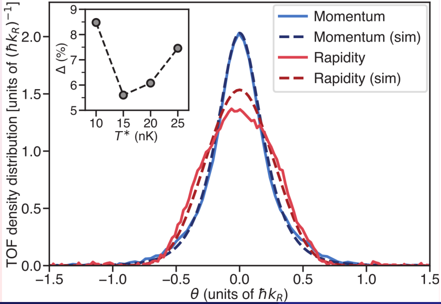
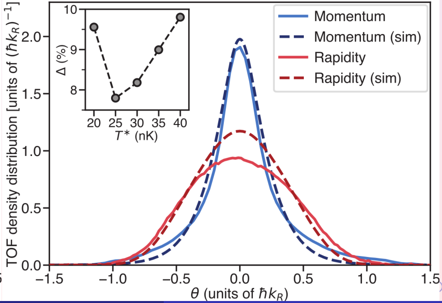
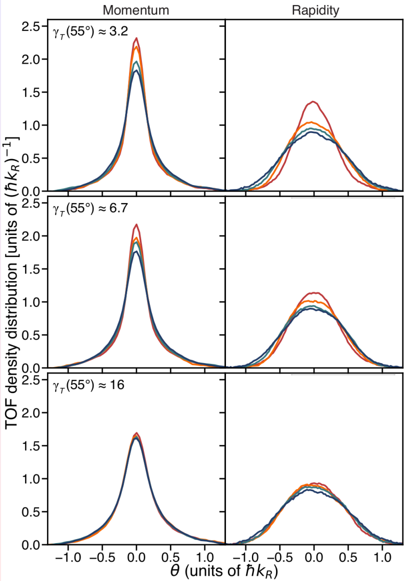


$\gamma_T \approx 6.7$, $N \approx 2.3 \times 10^4$ & $U^*_{2D} = 5E_R$



Effect of the dipolar interactions ($N \approx 2.3 \times 10^4$)

Momentum and rapidity dist.: $\theta_B = 0°$ (red), $35°$ (orange), $55°$ (green), and $90°$ (blue):



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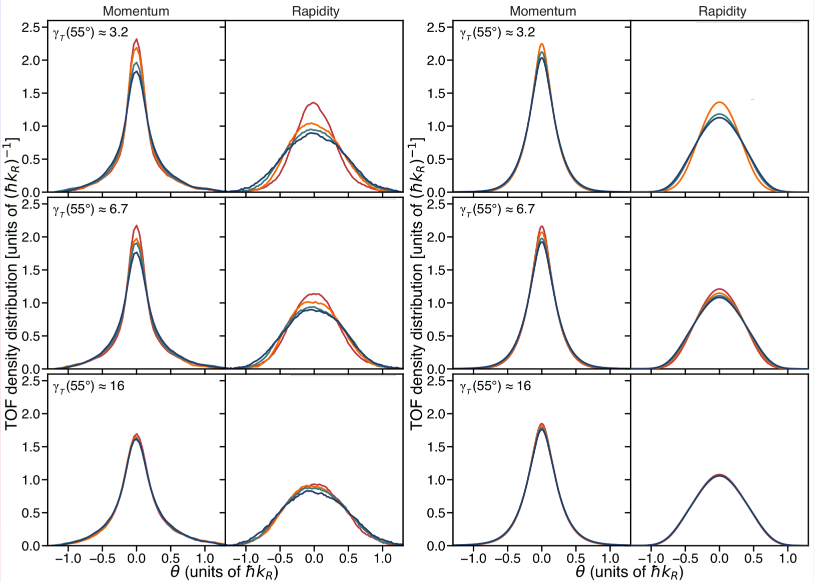

Outline

Scar states of 1D dysprosium gases

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Make $g_{\rm 1D}^{\rm vdW}$ attractive with $U_{\rm DDI}^{\rm 1D}$ repulsive.

Kao, Li, Lin, Gopalakrishnan & Lev, Science **371**, 296 (2021).

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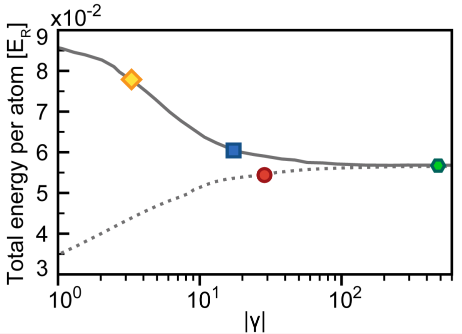
Repulsive DDI stabilized scar states ($N \approx 10^4$):



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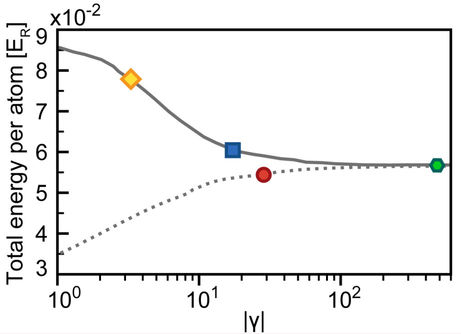

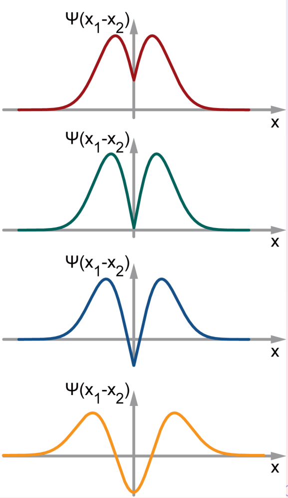

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Rapidity and momentum distributions: Initial states

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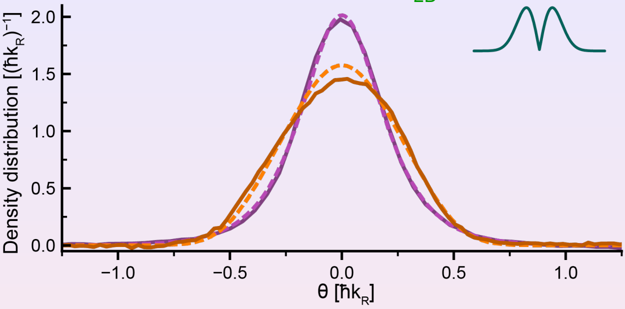

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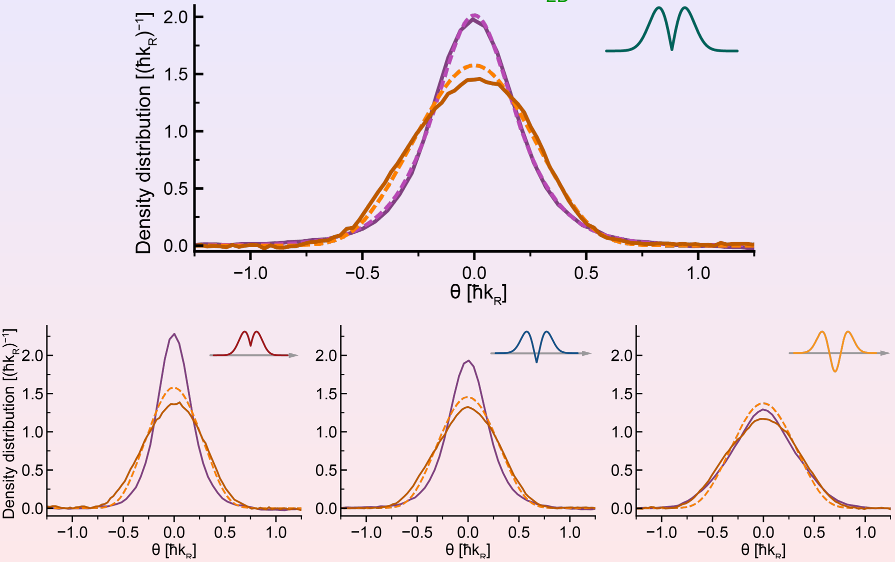
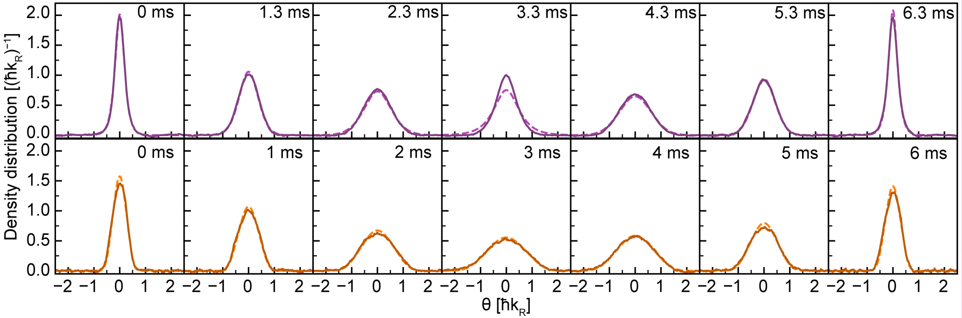


Outline

Trap quench: $U_{\mathrm{H}}(x) \to 10 \times U_{\mathrm{H}}(x)$

Experiments ($\gamma_{\mathrm{T}} \approx -480$) vs TG exact solution: Momentum (top), Rapidity (bottom)



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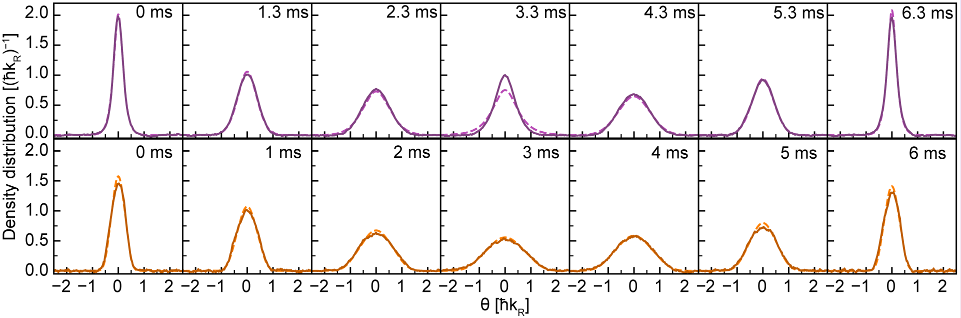
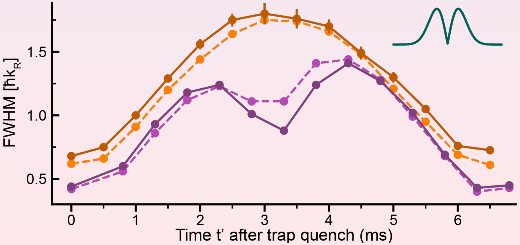


Time evolution of the FWHM



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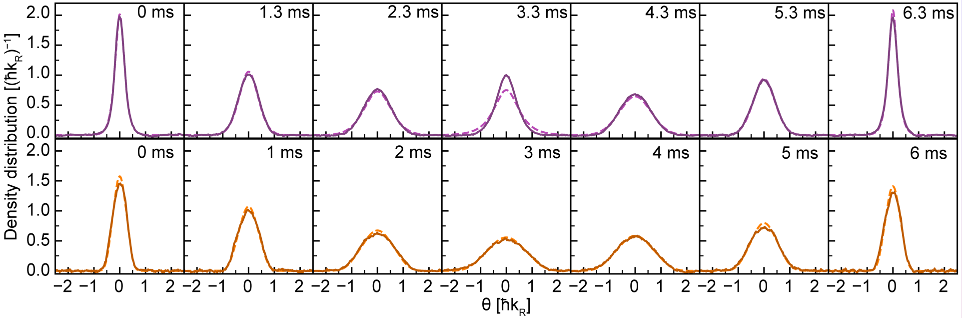

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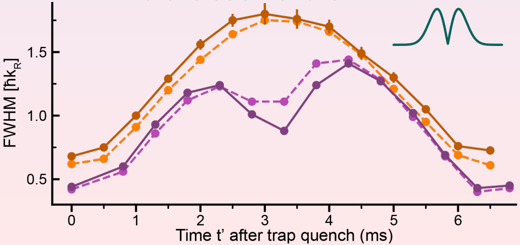

Bose-Fermi oscilations TG

Minguzzi & Gangardt, PRL **94**, 240404 (2005).

Wilson, Malvania, Le, Zhang, MR & Weiss, Science **367**, 1461 (2020).

Outline

Generalized hydrodynamics

Large-distances long-times dynamics:

- View the system as a continuum of fluid cells, each of which is spatially homogeneous, integrable, and contains many particles.
- Slow evolution of local quantities $\Rightarrow$ each fluid cell is locally equilibrated to a GGE parameterized by the distribution of rapidities.

Castro-Alvaredo, Doyon & Yoshimura, PRX **6**, 041065 (2016).
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GHD equation in an external potential $V(x)$:

$$\partial_t \rho_{\mathrm{qp}}(\theta, x, t) + \partial_x [v_{\mathrm{eff}}(\theta, x, t) \rho_{\mathrm{qp}}(\theta, x, t)] = \left(\frac{\partial_x V(x)}{m} \right) \partial_\theta \rho_{\mathrm{qp}}(\theta, x, t).$$

where θ are the rapidities, $\rho_{\mathrm{qp}}(\theta, x, t)$ is the density of quasi-particles with rapidity θ, at position x, and time t, and $v_{\mathrm{eff}}(\theta, x, t)$ is the effective velocity of the quasi-particles.

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$v_{\text{eff}}(\theta, x, t)$ depends on $\rho_{\text{qp}}(\theta, x, t)$:

$$v_{\text{eff}}(\theta, x, t) = \frac{\theta}{m} + \int d\theta' \varphi(\theta - \theta')\rho_{\text{qp}}(\theta', x, t)[v_{\text{eff}}(\theta', x, t) - v_{\text{eff}}(\theta, x, t)].$$

It encodes all the interaction effects [two-body scattering $\rightarrow \varphi(\theta - \theta')$].

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Densities of conserved quantities:

$$q(x, t) = \int d\theta \, h_q(\theta) \rho_{\text{qp}}(\theta, x, t)$$

where $h_q(\theta)$ depends on the quantity q, for the particle density $h_n(\theta) = 1$.

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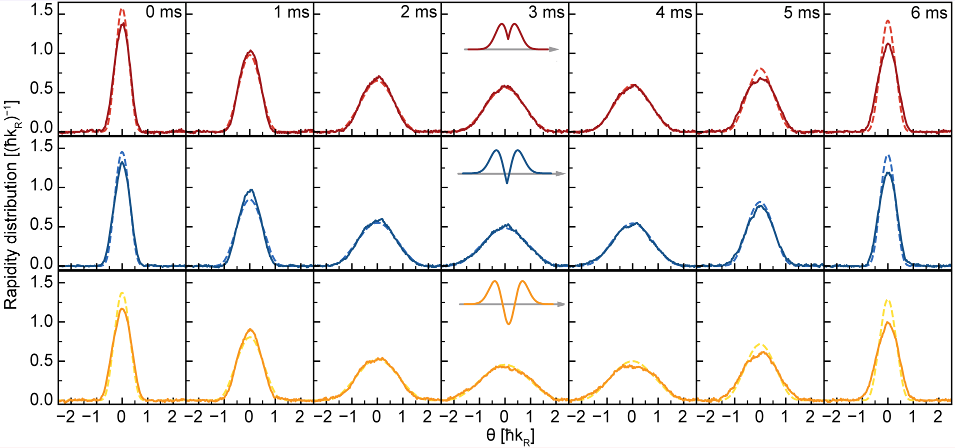
Trap quench: $U_{\mathrm{H}}(x) \to 10 \times U_{\mathrm{H}}(x)$

Experiment (continuous lines) vs GHD calculations (dashed lines):
$\gamma_{\mathrm{T}} = 26.3$ (top), $\gamma_{\mathrm{T}} = -15.5$ (center) & $\gamma_{\mathrm{T}} = -3.4$ (bottom)



“Total” (kinetic+interaction) and kinetic energy vs time

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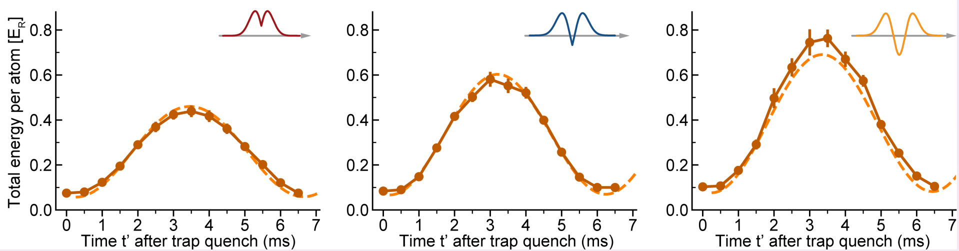

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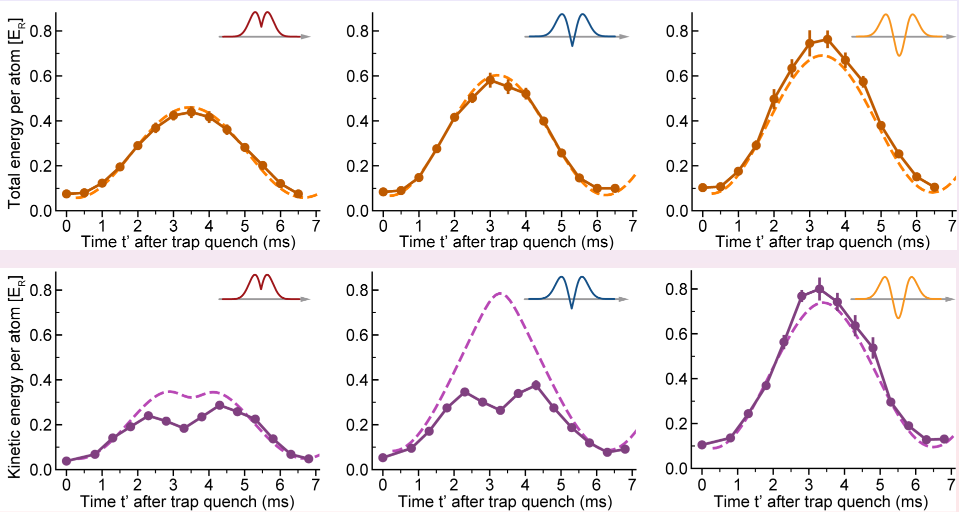

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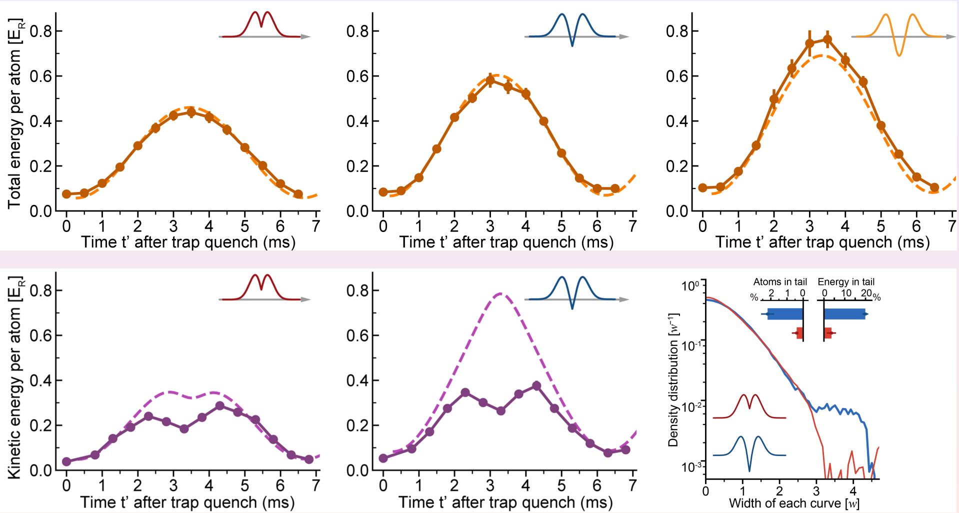
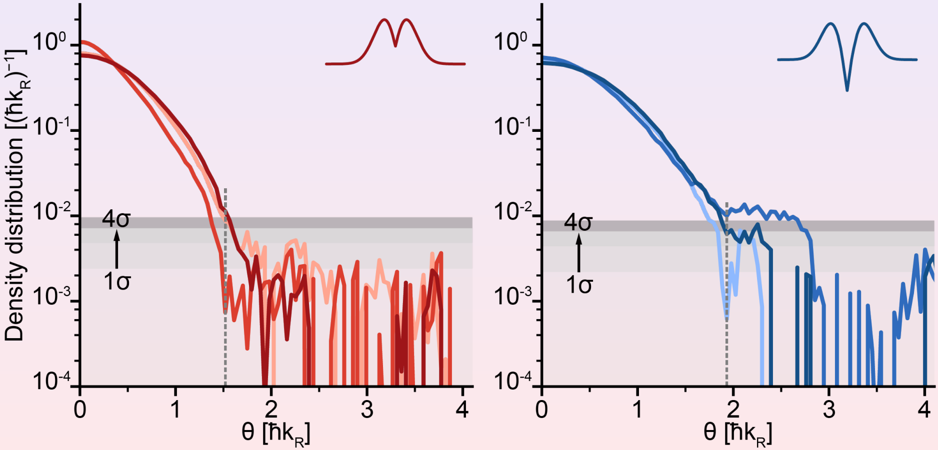


Momentum distributions close to maximal compression

Momentum distributions shortly before (lightest color) and shortly after (darkest color) maximal compression



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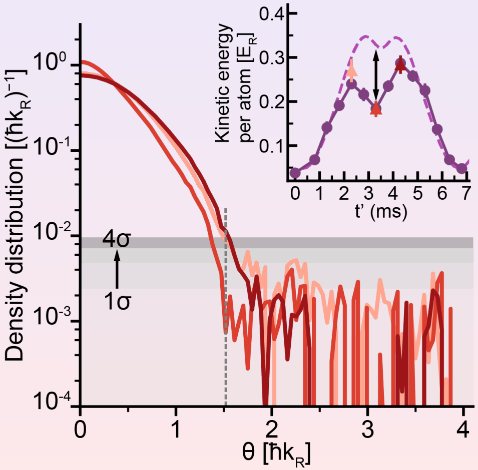

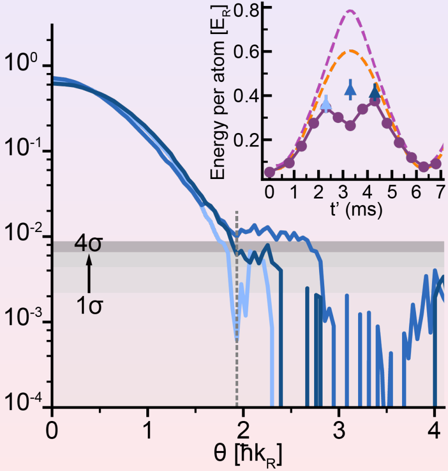

Summary

- Dipolar quantum gases in 1D allow one to controllably study the effect of integrability breaking in quantum dynamics.
  ★ Prethermalization: Short-time integrable dynamics + slow thermalization.
- Unique setup for precision many-body physics with long-range interactions.
  ★ The Lieb-Liniger model is a good starting point, but much still needs to be understood about the effect of the DDI.
- Novel scars states that can be studied far from equilibrium.
  ★ Generalized hydrodynamics accurately describes the rapidity distributions. A description of the correlations is still needed.

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Collaborators

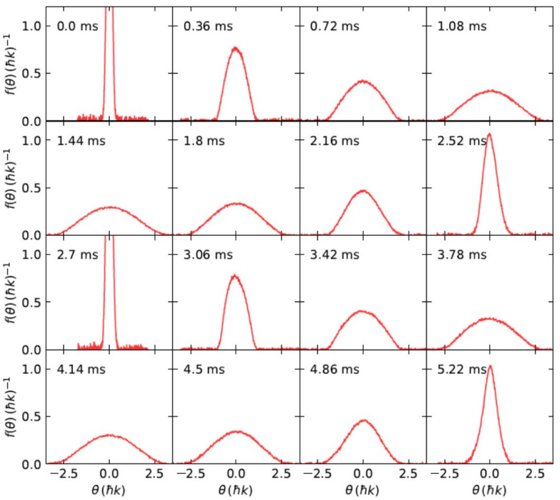
- *Yicheng Zhang* ($\to$ U Oklahoma)
- David Weiss & group (Penn State)
- Krishna Mallayya ($\to$ Cornell)
- Sarang Gopalakrishnan ($\to$ Princeton)
- Benjamin Lev & group (Stanford)
- Jerome Dubail (Lorraine, CNRS)
- Wojciech De Roeck (KULeuven)

Generalized hydrodynamics: trap-quench dynamics

Dynamics after increasing the strength of the confining potential

$$H_{\mathrm{LL}} = \sum_{j=1}^{N} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_j^2} + U(x_j) \right] + g \sum_{1 \le j < l \le N} \delta(x_j - x_l) \, ,$$

Evolution of the rapidity distributions for $U(x) \to 100U(x)$:



Malvania, Zhang, Le, Dubail,
MR & Weiss, Science **373**, 1129 (2021).

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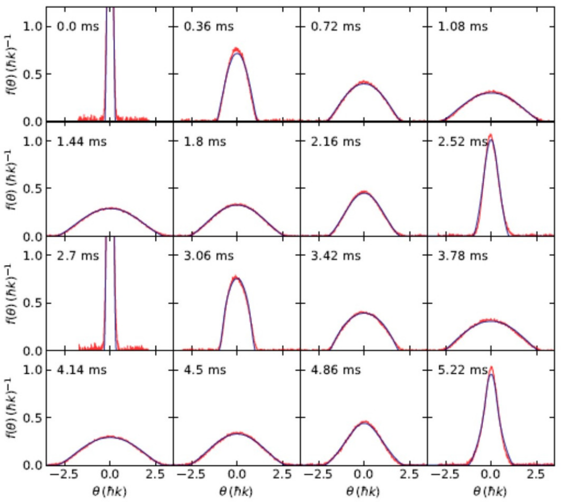

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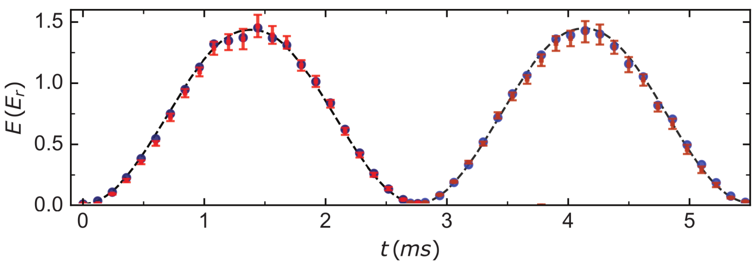

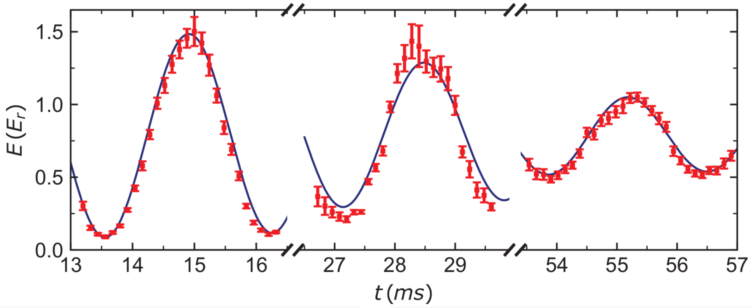

Oscillations

1st
and
2nd

6th
11th
and
21st

Generalized hydrodynamics: trap-quench dynamics

Evolution of the kinetic and interaction energies: