The SO(5) Deconfined Phase Transition: Conformality and Pseudo-criticality under the Fuzzy Sphere Microscope

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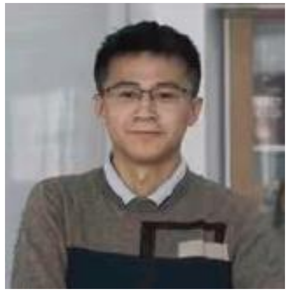
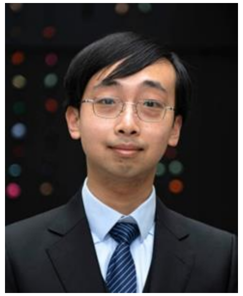
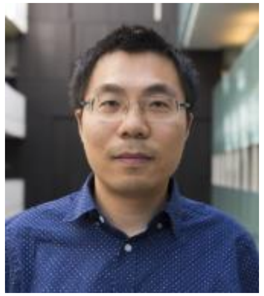
Nov 03th, 2024 @ Zhejiang University



Liangdong Hu
(Westlake)

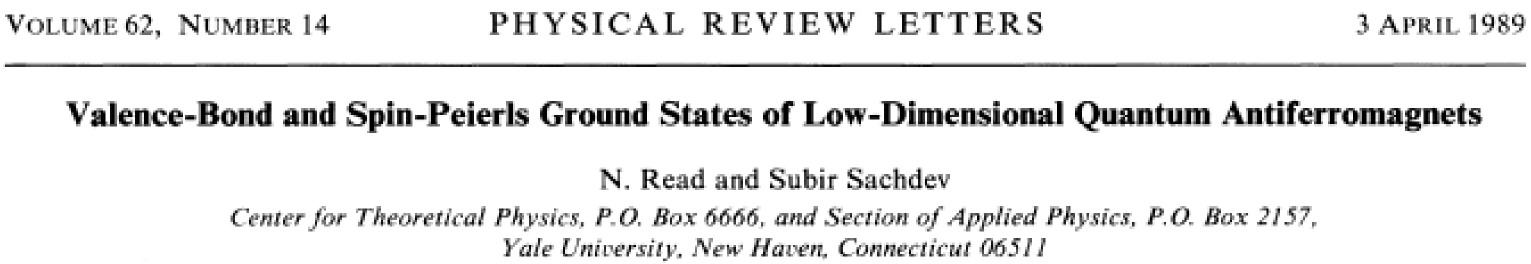


Zheng Zhou
(Perimeter)



Yin-chen He
(Perimeter)

Deconfined quantum criticality


VOLUME 62, NUMBER 14 PHYSICAL REVIEW LETTERS 3 APRIL 1989

Valence-Bond and Spin-Peierls Ground States of Low-Dimensional Quantum Antiferromagnets

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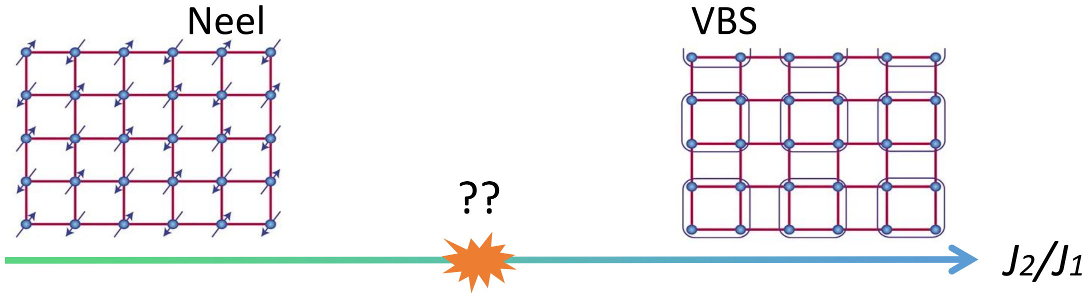
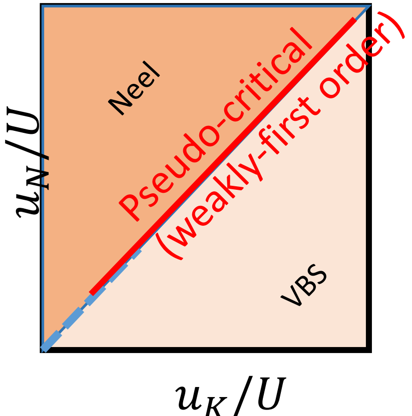


Deconfined quantum critical points, T. Senthil, *et. al*, Science 303, 1490 (2004).
Evidence for Deconfined Quantum Criticality in a Two-Dimensional Heisenberg Model with Four-Spin Interactions, A. Sandvik, PRL 97 228202 (2007)
Emergent SO(5) symmetry at the Néel to valence-bondsolid transition, A. Nahum, et. al, Phys. Rev. Lett. 115, 267203 (2015).
Deconfined quantum critical points: Symmetries and dualities, C. Wang, et. al, Phys. Rev. X 7, 031051 (2017).
…

Is a continuous transition possible? What is the nature of this transition?



Fuzzy sphere solution:

1) The Neel-VBS has approximately conformal symmetry; 2) DQCP transition is pseudo-critical.

l	P	Rep.	Δ	Operator	
0	−	**5**	0.584	$\phi \sim \mathcal{M}_{2\pi}$	SO(5) order parameter
0	+	**14**	1.454	$T \sim \phi^2 \sim \mathcal{M}_{4\pi}$	control Neel-VBS transition
1	+	**10**	2.000	J^μ	flavor current
0	−	**30**	2.565	$\mathcal{M}_3$	6π monopole
0	+	**1**	2.845	S	Parity even singlet → Pseudo-criticality
0	+	**55**	3.885	$\mathcal{M}_4$	8π monopole → VBS on C_4 lattice
0	−	**1**	5.354	S^-	Parity odd singlet → chiral spin liquid

The SO(5) Deconfined Phase Transition under the Fuzzy Sphere Microscope: Approximate Conformal Symmetry, Pseudo-Criticality, and Operator Spectrum, PRX 14,021044 (2024) arXiv.2306.16435

Outline

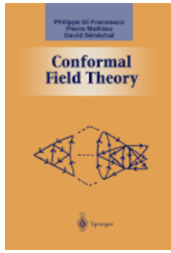
- Fuzzy sphere regularization
  a. Motivation from the CFT and State-operator correspondence
  b. Spherical Landau level regularization as a solution of the space-time geometry $S^2 \times R$

- Example of the 3D Ising transition
  a. Emergent conformal symmetry
  b. Scaling dimensions, operator product expansion coefficients, etc.

- Deconfined Quantum Critical Point
  a. Emergent (approximate) conformal symmetry
  b. Pseudo-criticality

- Outlook and discussion

Global conformal symmetry



Transformation $x_\mu \to x_\mu' = x_\mu + \epsilon_\mu$

- translation: $\epsilon_\mu = a_\mu$, i.e. ordinary translations independent of x.
- rotaion: $\epsilon_\mu = \omega_{\mu\nu} x_\nu$
- dilatation: $\epsilon_\mu = \lambda x_\mu$
- special conformal transformation: $\epsilon_\mu = b_\mu x^2 - 2x_\mu b_\nu x_\nu$

Generators

(translation) $P_\mu = -i\partial_\mu$

(rotation) $L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$

(dilatation) $D = -i x_\mu \partial_\mu$

(SCT) $K_\mu = -i(2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu)$

Commutation relation

$[D, P_\mu] = P_\mu$, $[D, K_\mu] = -K_\mu$ ...

simple harmonics

$$H = a^+ a + E_0$$
$$[H, a] = -a$$
$$[H, a^+] = a^+$$

analog ⇔

CFT generators

$$H \sim D$$
$$a^+ \sim P_\mu$$
$$a \sim K_\mu$$

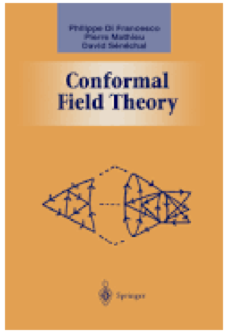
CFT states

$D|\phi> = \Delta_\phi |\phi>$ Scaling dims

$P_\mu |\phi> = |\phi + 1>$ Descendent fields

$K_\mu |\phi> = 0$ Primary fields

Conformal symmetry and conformal field theory



Higher symmetry, more constrains.

<u>Global conformal symmetry fixes the form of correlators.</u>

Correlators

Primary fields $O(z_1) = |w'(z_1)|^{\Delta}\ O(w(z_1))$

$$< O_i(x_1) >= \delta_{\Delta_i,0}$$

$$< O_i(x_1) O_j(x_2) >= \delta_{\Delta_i,\Delta_j} \frac{1}{|x_1 - x_2|^{2\Delta_i}}$$

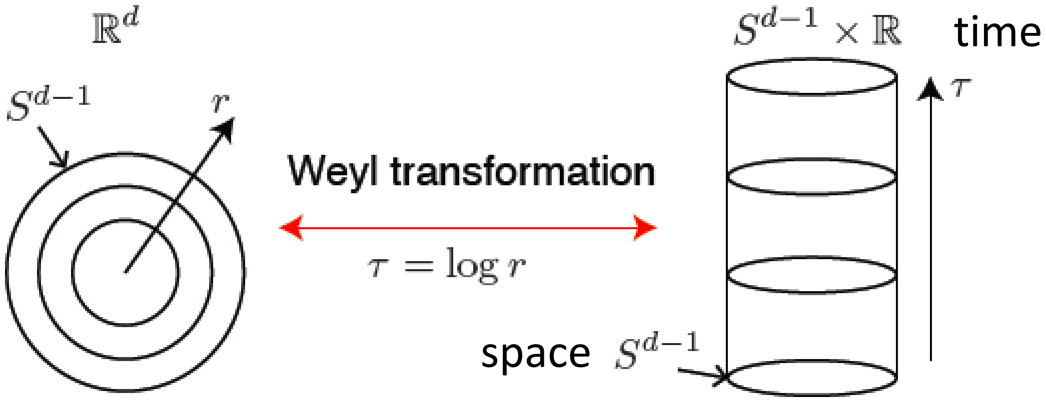
$$< O_i(x_1) O_j(x_2) O_k(x_3) >= \frac{f_{ijk}}{|x_{12}|^{\Delta_i+\Delta_j-\Delta_k}\ |x_{23}|^{\Delta_j+\Delta_k-\Delta_i}\ |x_{13}|^{\Delta_k+\Delta_i-\Delta_j}}$$

Conformal data (Δ_i, f_{ijk})
- scaling dimension Δ_i
- operator product expansion f_{ijk}

Radial quantization/State-operator correspondence

Radial quantization

John Cardy



$$< \phi(z_1)\phi(z_2) >_{R^d} \sim |z_1 - z_2|^{-2\Delta} \quad \longleftrightarrow \quad < \phi(z_1)\phi(z_2) >_{S^{d-1}\times R} \sim e^{-\Delta|\tau_1-\tau_2|}$$

State-operator correspondence (1984,1985)

Eigenstates of the quantum Hamiltonian defined on $S^{d-1} \times R$ are in one-to-one correspondence with CFT's operators

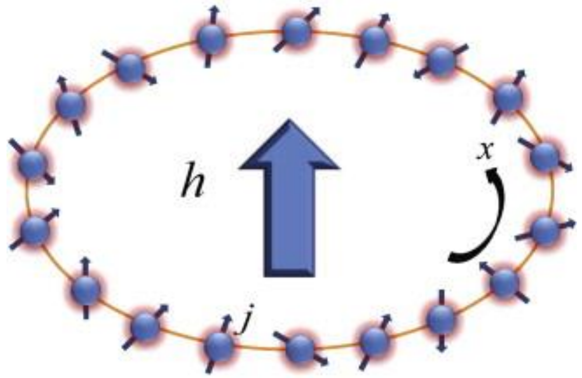
Energy gaps on $S^{d-1} \times R$ ~ scaling dimensions $\quad \delta E_n = E_n - E_0 \sim \frac{1}{\xi_n} \sim \Delta_n$

State operator correspondence: Example in 2D

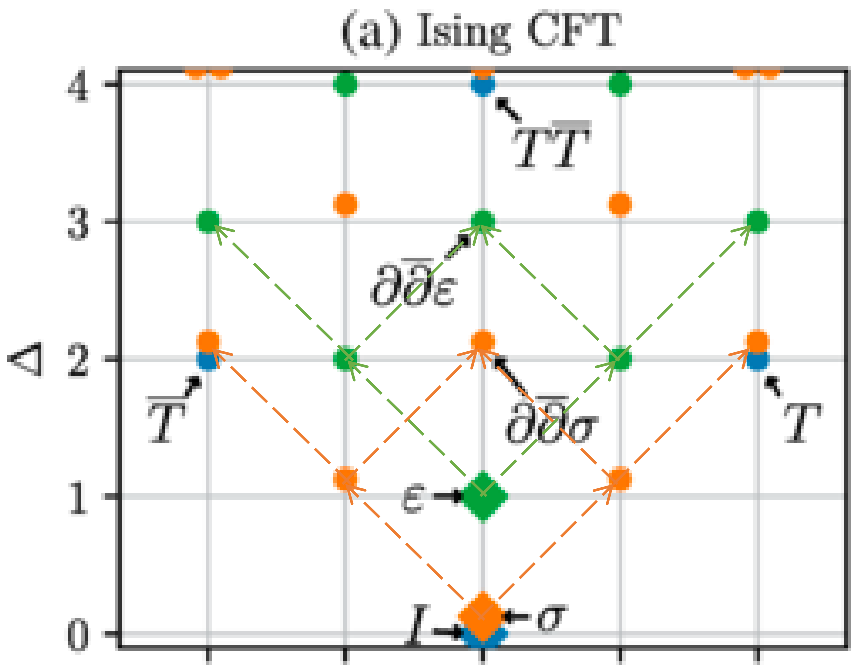
Energy gaps on $S^{d-1} \times R$ ~ scaling dimensions $\delta E_n = E_n - E_0 \sim \frac{1}{\xi_n} \sim \frac{2\pi}{L}(\Delta_n - \Delta_0)$

2D CFT on a quantum Hamiltonian on a circle $S^1 \times R$

$$H = -J \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=1}^{N} \sigma_i^x$$



$= S^1$



Cardy 1986; Milsted and Vidal 2017

$DP_\mu|\phi >= (\Delta_\phi + 1)|\phi + 1 >$

conformal tower in energy spectra !!

conformal symmetry!!

Conformal symmetry ⟺ State operator correspondence (conformal tower in energy spectra)

State operator correspondence in 3D

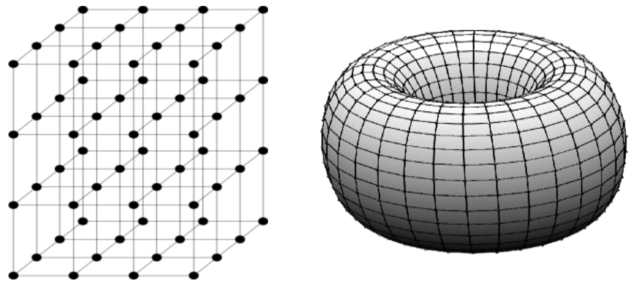
Energy gaps on $S^{d-1} \times R$ ~ scaling dimensions $\delta E_n = E_n - E_0 \sim \frac{1}{\xi_n} \sim \frac{2\pi}{L}(\Delta_n - \Delta_0)$

Higher dimensional quantum Hamiltonian $S^2 \times R$

But a regular lattice won't fit due to the topology of sphere

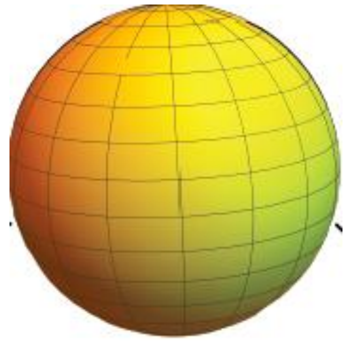
3D CFT on T^3 , $T^2 \times R$

Flat space-time geometry: LATTICE



Geometry	Theory	Simulation
$\mathbb{R}^3$	Yes	No
$\mathbb{S}^3$	Yes	No
$\mathbb{S}^2 \times R$	Yes	No
$\mathbb{T}^3$	No	Yes
$\mathbb{T}^2 \times R$	No	Yes

3D CFT on $S^2 \times R$



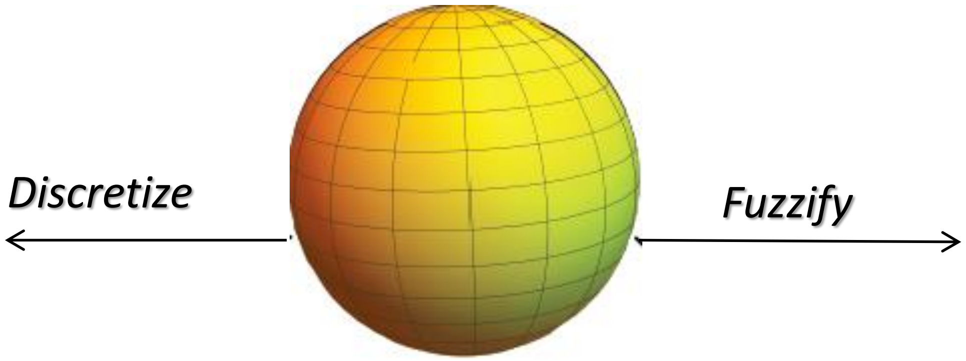
millions of papers ...

very few paper ...

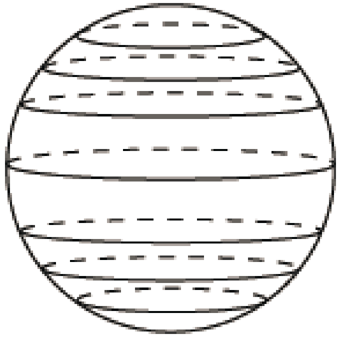
State operator correspondence in 3D

Higher dimensional quantum Hamiltonian $S^2 \times R$

discretized geometry $S^2 \times R$



fuzzy (non-commutative) sphere
Lowest Landau level projection



R. C. Brower, G. T. Fleming, and H. Neuberger, “Lattice radial quantization: 3D Ising,” Physics Letters B 721, 299–305 (2013)
M Weigel and W Janke, “Universal amplitude-exponent relation for the ising model on spherelike lattices,” Europhysics Letters (EPL) 51, 578–583 (2000).

Haldane 1983; J. Madore 1992

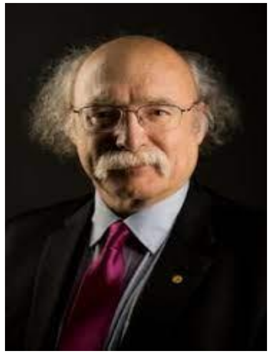
But a regular lattice won’t fit due to the topology of sphere!

This is a more practical way.

Haldane (fuzzy) sphere

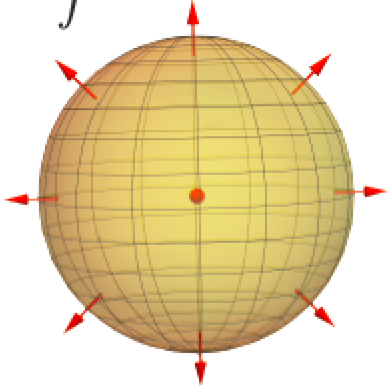
How to realize a phase transition on S^2 ?

Our solution: ***Fuzzy sphere regularization***



F. D. M. Haldane 1983

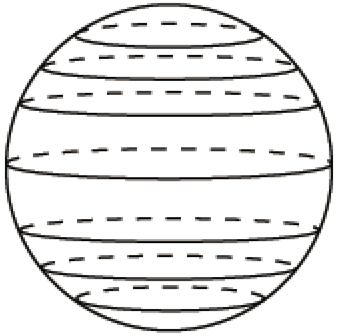
$$\int \vec{B} \cdot d\vec{r} = 4\pi \cdot s$$

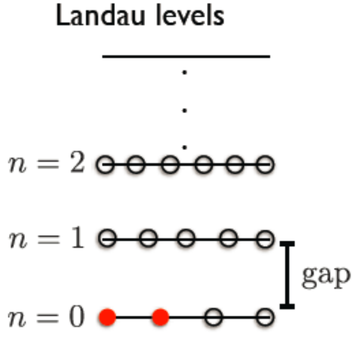


LLL projection

$$\hat{\psi}(\theta, \varphi) = \sum_{m=-s}^{s} \Phi_m^* \hat{c}_m.$$

$2s + 1$ orbitals





Monopole Harmonics

Wu & Yang, 1979

$$H = \frac{1}{2Mr^2}(\partial_\mu + iA_\mu)^2$$

Landau level: n = 0, 1, ...

(degeneracy: 2n + 2s + 1)

$$E_{s_0,l,m} = \hbar\omega_c[n + \frac{1}{2} + \frac{n(n+1)}{2|s_0|}]$$

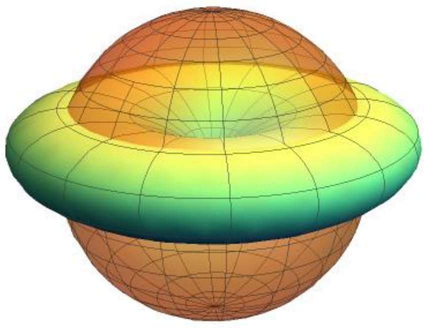
Wu-Yang monopole harmonics

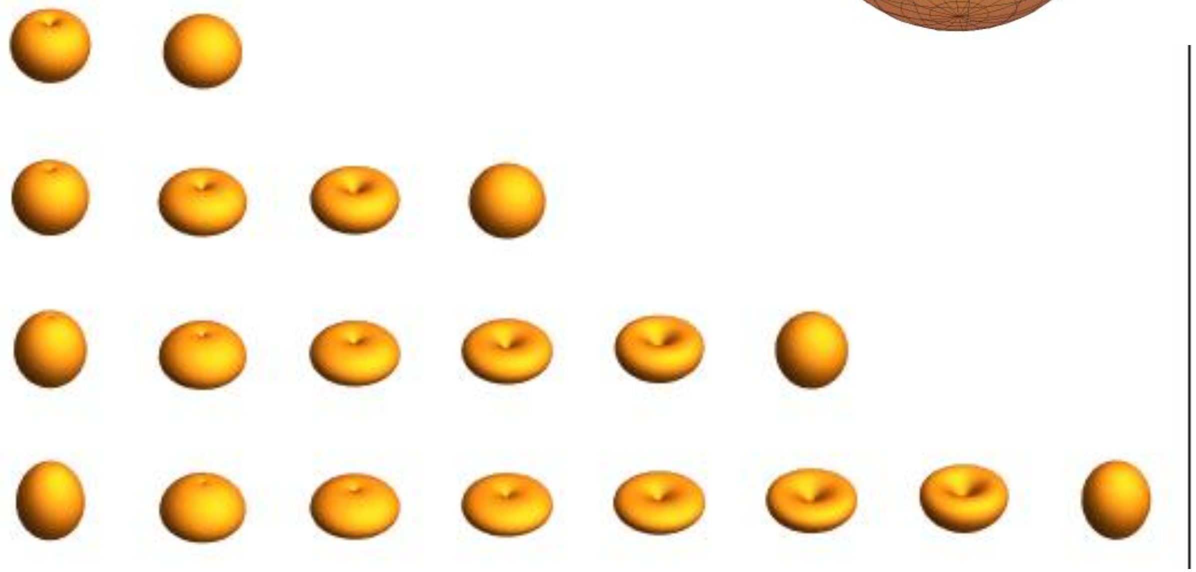
$$\Phi_{m,s}(\theta, \varphi) = \sqrt{\frac{(2s+1)!}{4\pi(s+m)!(s-m)!}} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$

Haldane (fuzzy) sphere

Spherical Harmonics

Monopole Harmonics





m=-l, ... , m=l

m=-s, ... , m=s

Fuzzy sphere

Hamiltonian

$$H_0 = \frac{\hbar^2}{2mR^2}|\mathbf{\Lambda}|^2,$$
$$\mathbf{\Lambda} = \mathbf{R} \times (-i\nabla + e\mathbf{A})$$

$$[\Lambda_i, \Lambda_j] = i\varepsilon_{ijk}(\Lambda_k - s_0\vec{r}_k)$$

angular momentum algebra

$$\mathbf{L} = \mathbf{\Lambda} + s_0\vec{r}$$

$$[L_i, L_j] = i\varepsilon_{ijk}L_k$$

LL projection →

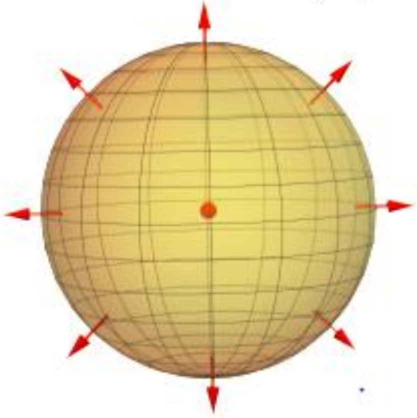
Fuzzy sphere (*J. Madore 1992*)
non-commutative electrons on sphere

$$[\tilde{x}_\mu, \tilde{x}_\nu] = i\frac{R}{s}\epsilon_{\mu\nu\rho}\tilde{x}_\rho.$$

$L_\mu \sim s\tilde{x}_\mu$, denotes the coordinates in the projected LLL

The fuzziness comes from the monopole.

Electrons see a fuzzy sphere.



Ising spin:

$$\sigma^z = c_\uparrow^\dagger c_\uparrow - c_\downarrow^\dagger c_\downarrow$$

Ising spins see a normal sphere.

Outline

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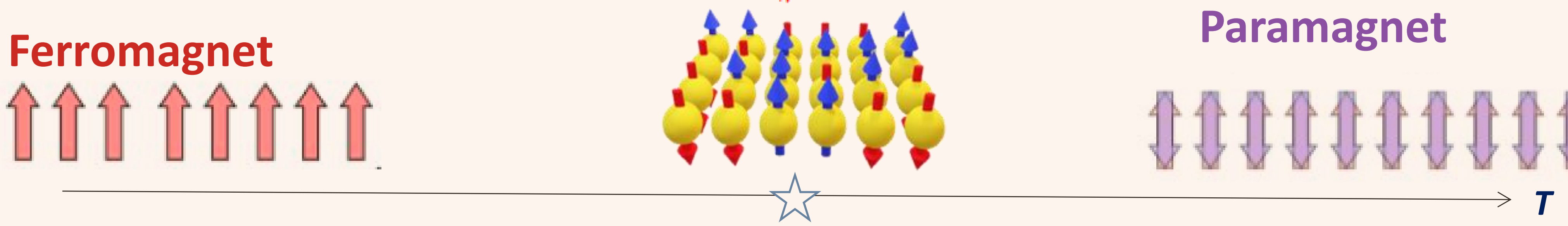
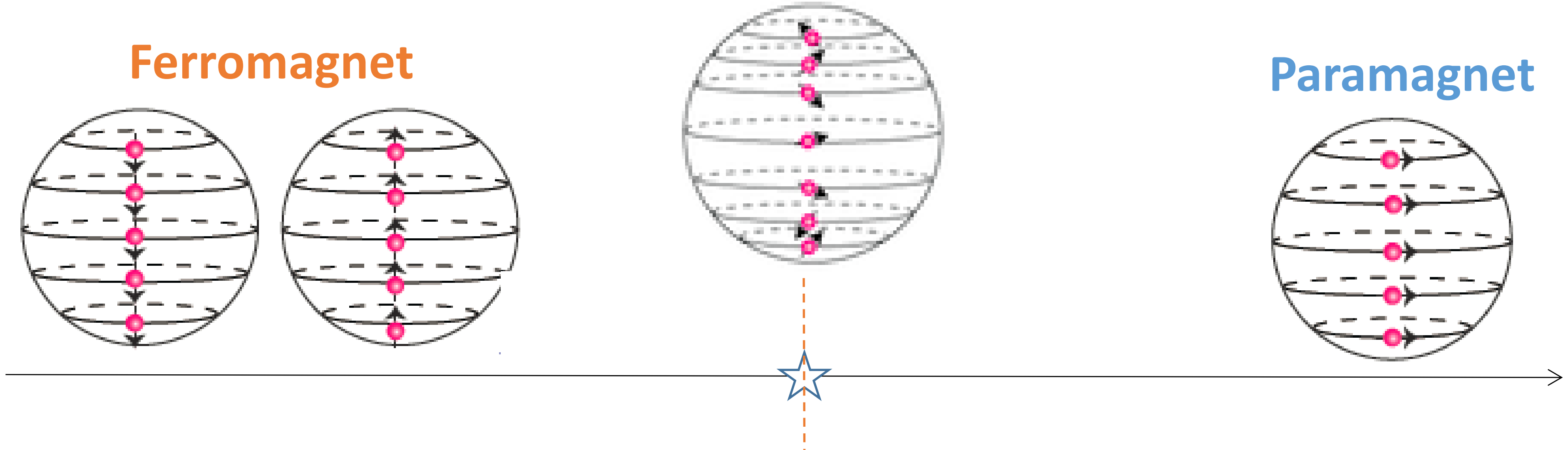
  a. Emergent conformal symmetry
  b. Scaling dimensions, operator product expansion coefficients, etc.
- Deconfined Quantum Critical Point
  a. Emergent (approximate) conformal symmetry
  b. Pseudo-criticality
- Outlook and discussion

3D Ising phase transition

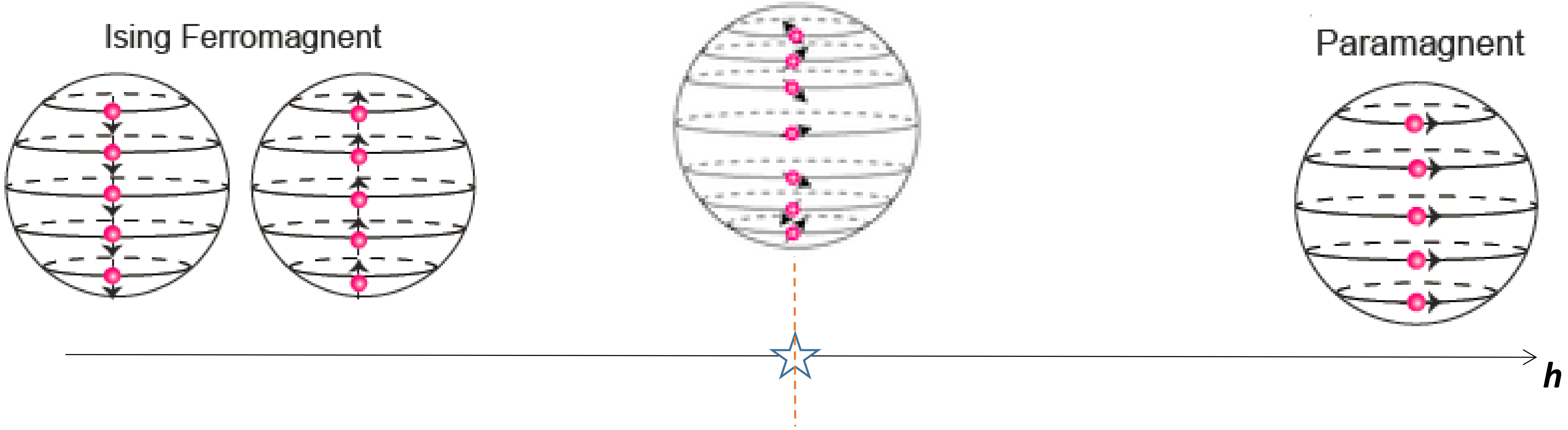
$$H(\{s\}) = -J\sum_{x,\mu} s_x s_{x+\hat{\mu}} - h\sum_x s_x$$



Explore a different path: 3D Ising transition on spherical geometry



2+1 D quantum Ising model on the spherical geometry



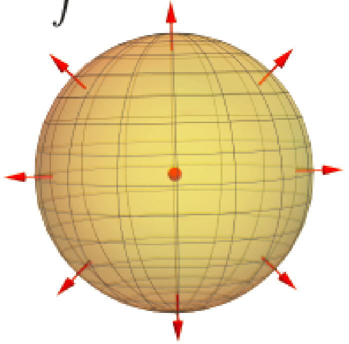
$$H = \int d\Omega_a d\Omega_b \, U(\Omega_{ab}) \left[n^0(\theta_a, \varphi_a) n^0(\theta_b, \varphi_b) - n^z(\theta_a, \varphi_a) n^z(\theta_b, \varphi_b) \right] - h \int d\Omega \, n^x(\theta, \varphi).$$

$$U(\Omega_{ab}) = g_0 \delta(\Omega_{ab}) + g_1 \nabla^2 \delta(\Omega_{ab}) \qquad n^\alpha(\theta, \varphi) = (\hat{\psi}^\dagger_\uparrow(\theta, \varphi), \hat{\psi}^\dagger_\downarrow(\theta, \varphi)) \, \sigma^\alpha \begin{pmatrix} \hat{\psi}_\uparrow(\theta, \varphi) \\ \hat{\psi}_\downarrow(\theta, \varphi), \end{pmatrix}$$

A similar model on the torus/infinite cylinder has been studied in *Ippoliti, Mong, Assaad, Zaletel* 2018

Symmetries and order parameter

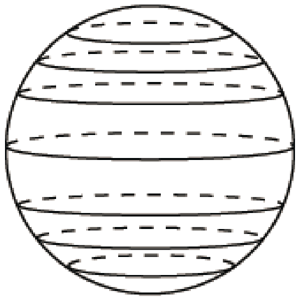
$\int \vec{B} \cdot d\vec{r} = 4\pi \cdot s$



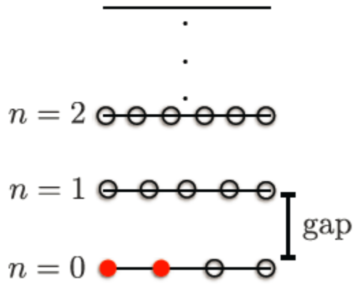
LLL projection

$$\hat{\psi}(\theta, \varphi) = \sum_{m=-s}^{s} \Phi_m^* \hat{c}_m .$$

$2s+1$ orbitals



Landau levels



$$H = \sum_{m_{1,2,3,4}=-s}^{s} V_{m_1,m_2,m_3,m_4}[(\mathbf{c}_{m_1}^\dagger \mathbf{c}_{m_4})(\mathbf{c}_{m_2}^\dagger \mathbf{c}_{m_3}) - (\mathbf{c}_{m_1}^\dagger \sigma^z \mathbf{c}_{m_4})(\mathbf{c}_{m_2}^\dagger \sigma^z \mathbf{c}_{m_3})] - h \sum_{m=-s}^{s} \mathbf{c}_m^\dagger \sigma^x \mathbf{c}_m$$

Total orbitals: N=2s+1 is the space volume

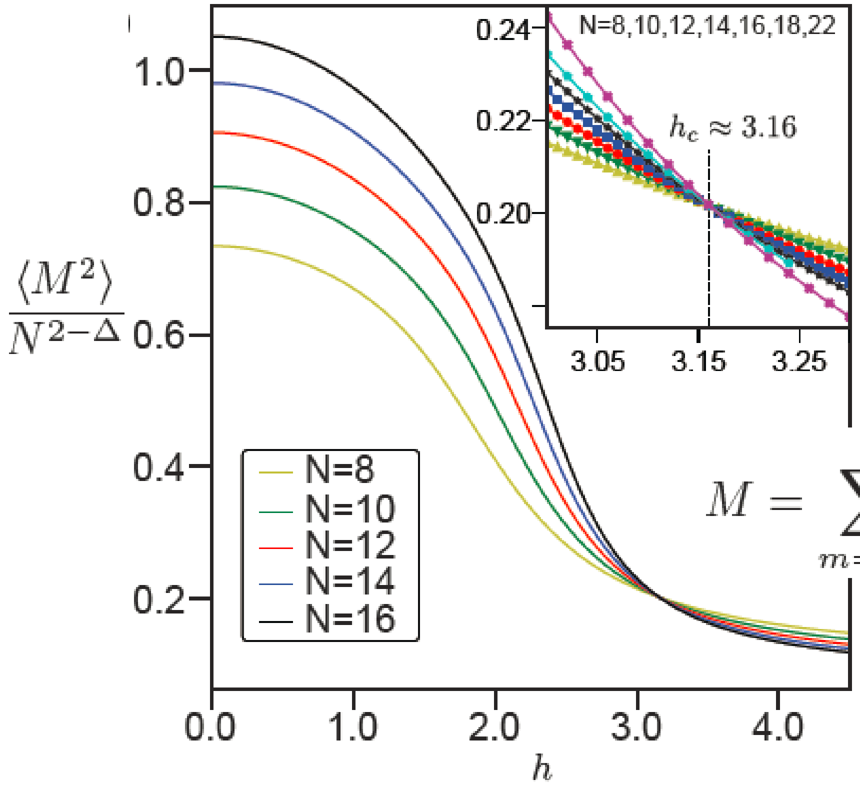
$R = \sqrt{s} \sim \sqrt{N}$, $(l_\phi = \sqrt{\frac{h}{2\pi e B}} = 1, s \times 2\phi_0 = 4\pi R^2 B)$

UV model	IR Ising transition
$\mathbf{c}_m \to \sigma^x \mathbf{c}_m$	Z2 Ising symmetry
SO(3): $\mathbf{c}_{m=-s,\cdots,s}$ spin-s irrep	SO(3) Lorentz rotation
Particle-hole symmetry $\mathbf{c}_m \to i\sigma^y \mathbf{c}_m^\dagger$, $i \to -i$	Space-time parity symmetry

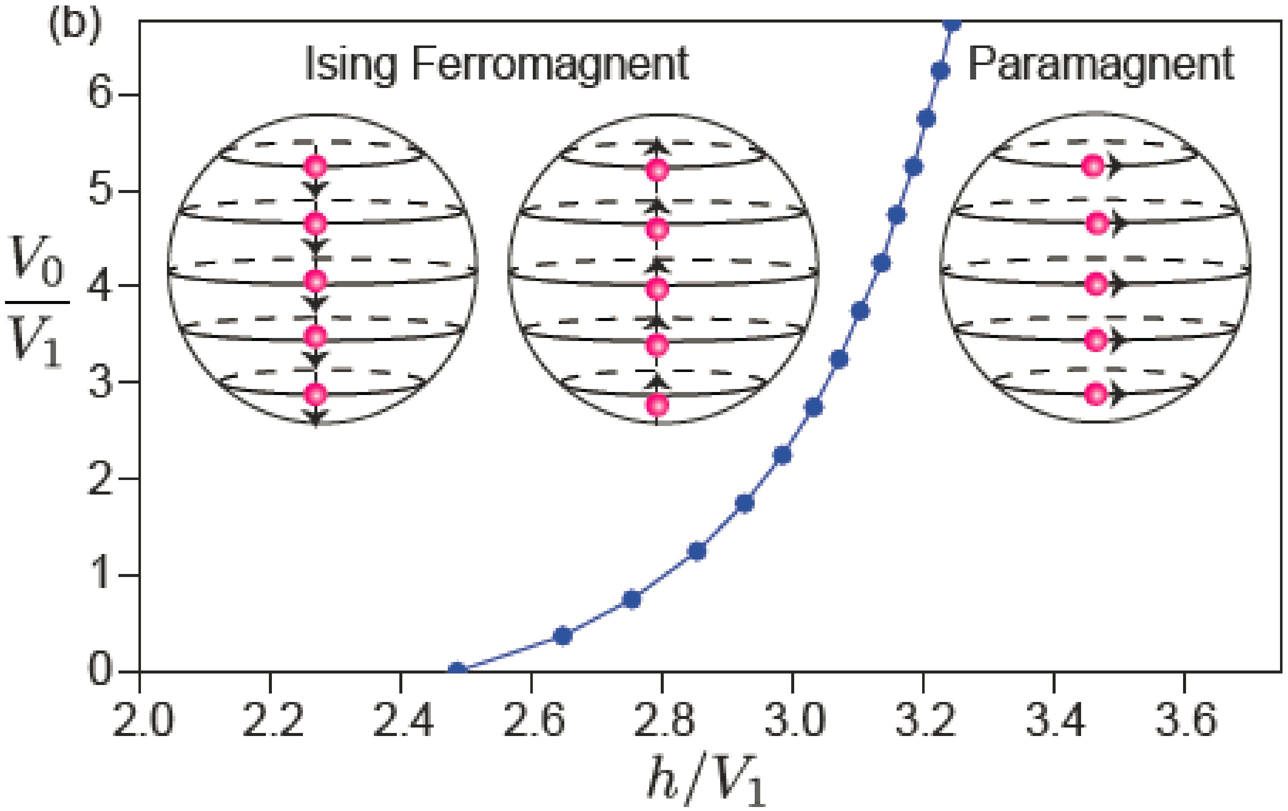
Ising order parameter: $M = \sum_{m=-s}^{s} \mathbf{c}_m^\dagger \frac{\sigma^z}{2} \mathbf{c}_m$

$\mathbf{c}_m^\dagger = (c_{m,\uparrow}^\dagger, c_{m,\downarrow}^\dagger)$

Phase transition: Order parameter scaling



$$M = \sum_{m=-s}^{s} \mathbf{c}_m^\dagger \frac{\sigma^z}{2} \mathbf{c}$$



Ferromagnet $\langle M^2\rangle \sim L_x^4 = N^2$

Paramagnet $\langle M^2\rangle \sim O(N^0)$

Ising Criticality $\langle M^2\rangle \sim L_x^{4-2\Delta} = N^{2-\Delta}$

$\Delta \approx 0.518148$

$$\langle M_z^2\rangle = \langle(\sum_i s_i^z)^2\rangle = \langle \sum_{ij} s_i^z s_j^z\rangle = \sum_{ij}\langle s_i^z s_j^z\rangle \sim (L^2)^2 \frac{1}{L^{2\Delta}} = L^{4-2\Delta_\sigma} \overset{\eta=2\Delta-1}{=} L^{3-\eta}$$

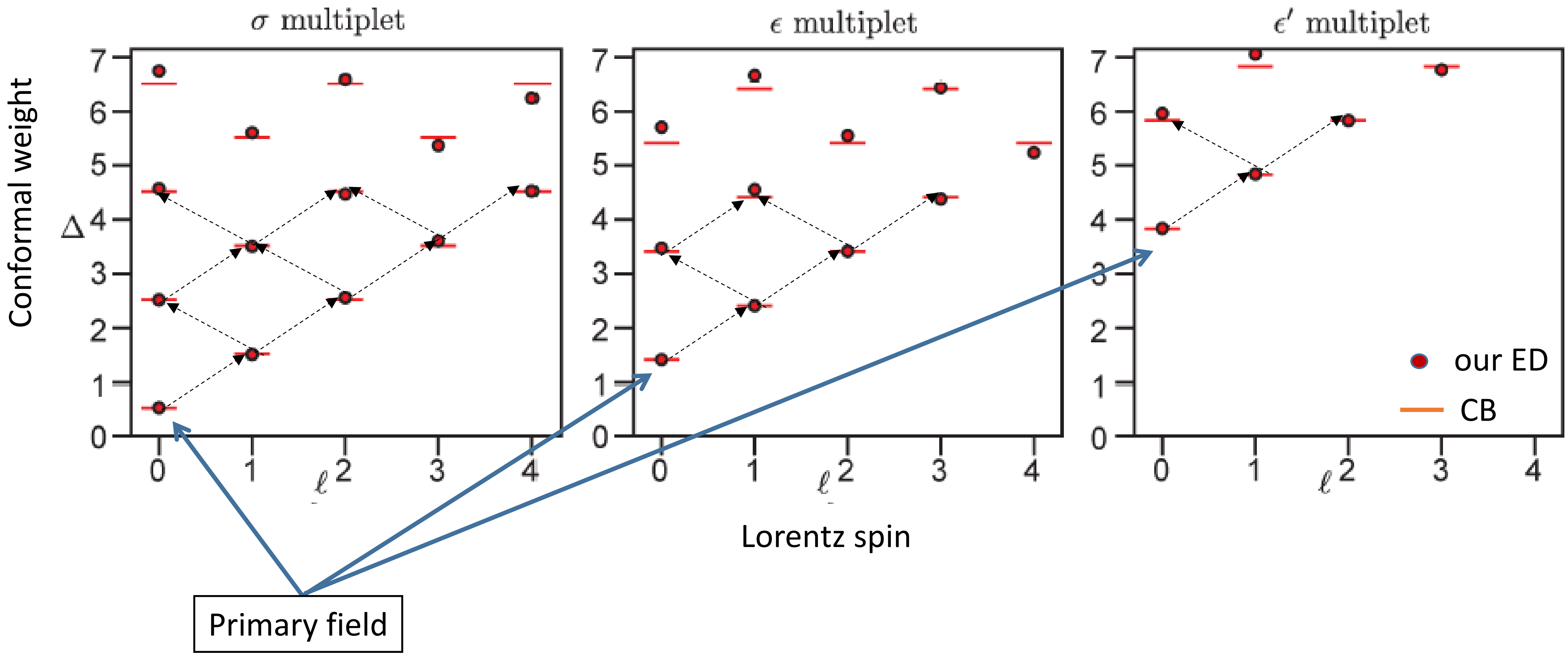
conformal symmetry: $< s_i^z s_j^z > \sim \frac{1}{|\vec{r}_i - \vec{r}_j|^{2\Delta}}$

State-operator correspondence

descendents: $\partial_{\mu_1} \cdots \partial_{\mu_j} \Box^n O, \quad n, j \geq 0$

$D(P_\mu|\phi>) = (\Delta_\phi + 1)|\phi + 1>$

$\frac{E_n - E_0}{E_1 - E_0}$



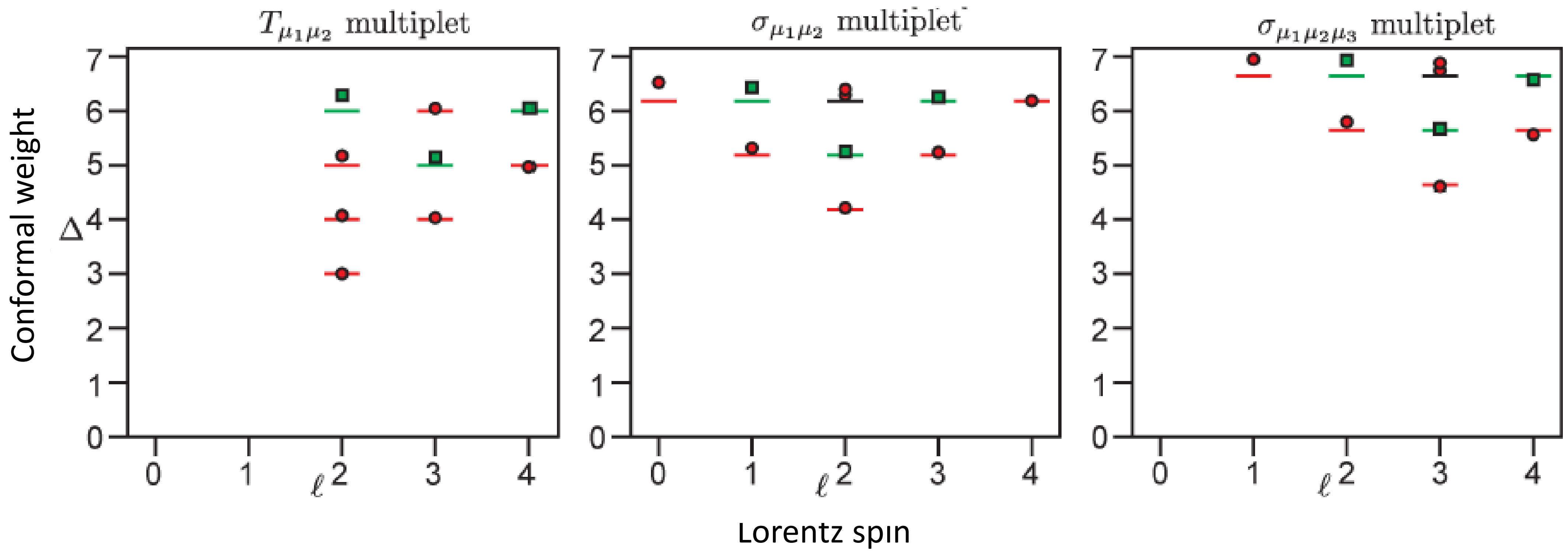
Spectra forms an almost perfect conformal tower structure ⇒ Conformal symmetry in the 3D Ising transition!

State-operator correspondence

Conformal multiplet of spinning operator:

$$\partial_{\nu_1}\cdots\partial_{\nu_j}\partial_{\mu_1}\cdots\partial_{\mu_s}\Box^n O_{\mu_1\cdots\mu_l} \qquad \varepsilon_{\mu_l\rho\tau}\partial_\rho\partial_{\nu_1}\cdots\partial_{\nu_j}\partial_{\mu_1}\cdots\partial_{\mu_s}\Box^n O_{\mu_1\cdots\mu_l}$$

1. Energy momentum tensor is conserved.
2. Parity odd descendant.



Conformal tower in 3D Ising transition!

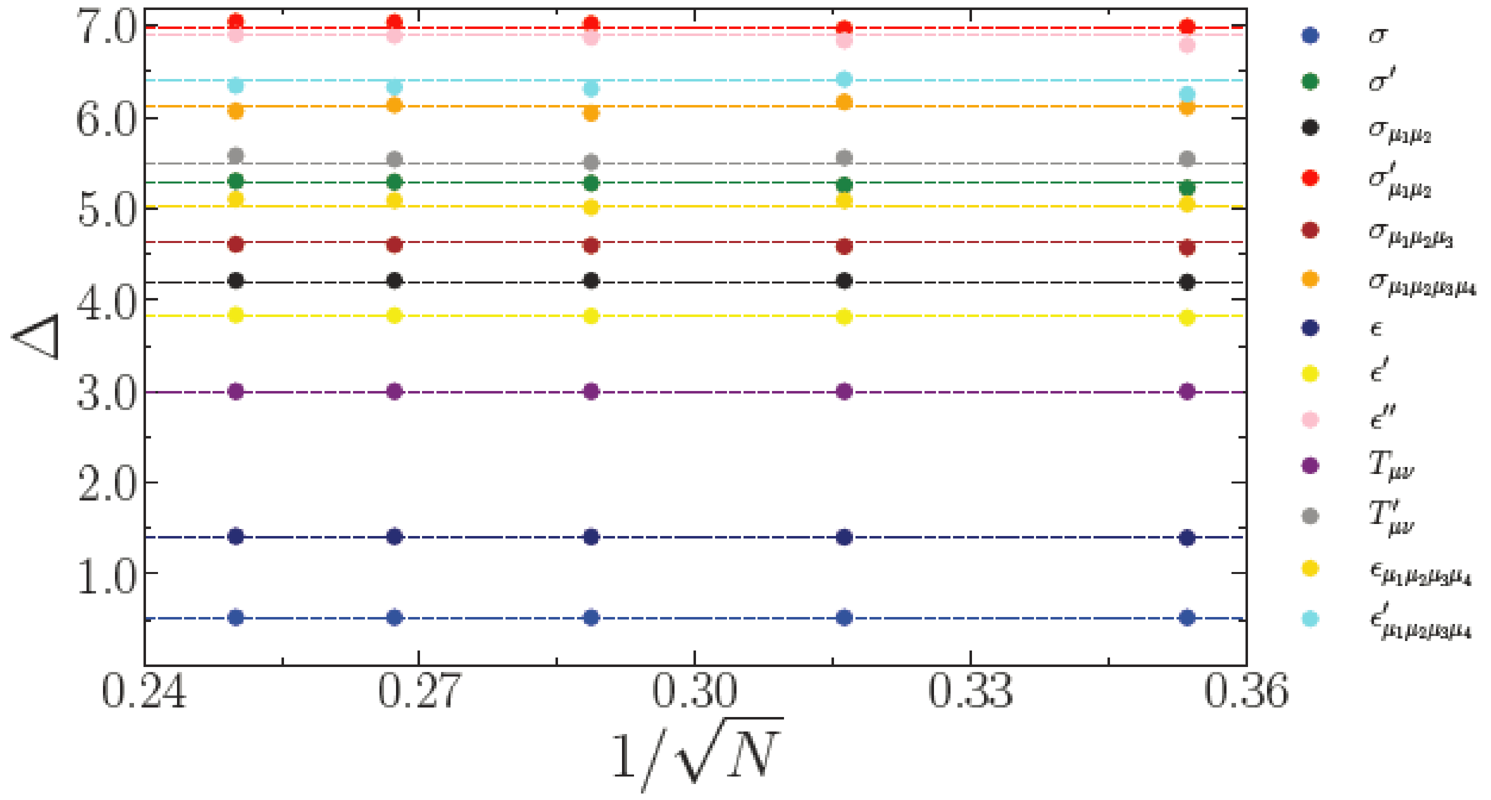
State-operator correspondence

We identified 13 parity even primary fields and 2 parity odd primary fields.

TABLE I. Low-lying primary operators identified via state-operator correspondence on a fuzzy sphere with $N = 16$ electrons. The operators in the first and second row are $\mathbb{Z}_2$ odd and even operators, respectively. We highlight that two new parity-odd primary operators σ^{P-} and ϵ^{P-} are found. The conformal bootstrap data is from Ref. [24].

	σ	σ'	$\sigma_{\mu_1\mu_2}$	$\sigma'_{\mu_1\mu_2}$	$\sigma_{\mu_1\mu_2\mu_3}$	$\sigma_{\mu_1\mu_2\mu_3\mu_4}$		σ^{P-}
Bootstrap	0.518	5.291	4.180	6.987	4.638	6.113		NA
Fuzzy sphere	0.524	5.303	4.214	7.048	4.609	6.069		11.191
	ϵ	ϵ'	ϵ''	$T_{\mu\nu}$	$T'_{\mu\nu}$	$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	$\epsilon'_{\mu_1\mu_2\mu_3\mu_4}$	ϵ^{P-}
Bootstrap	1.413	3.830	6.896	3	5.509	5.023	6.421	NA
Fuzzy sphere	1.414	3.838	6.908	3	5.583	5.103	6.347	10.014

State-operator correspondence

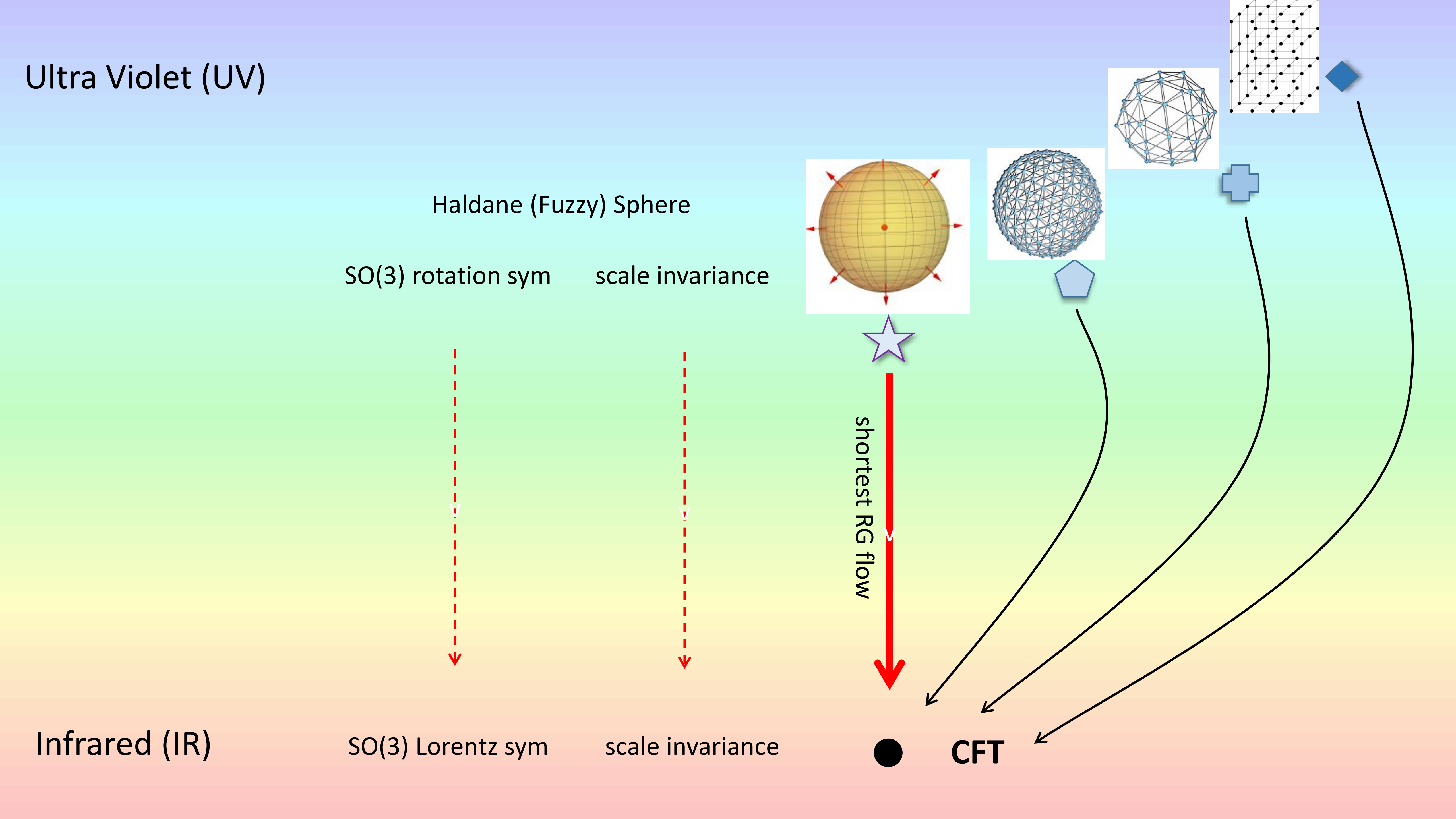


Finite-size effect is negligibly small

incredibly small system sizes, up to 18 spins (ED), 36 spins (DMRG).
(MC 10^9 spins were simulated.)

	CB	16 spins	Error
σ	0.518	0.524	1.2%
σ'	5.291	5.303	0.2%
$\sigma_{\mu_1\mu_2}$	4.180	4.214	0.8%
$\sigma'_{\mu_1\mu_2}$	6.987	7.048	0.9%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.609	0.6%
$\sigma_{\mu_1\mu_2\mu_3\mu_4}$	6.113	6.069	0.7%
σ^{P-}	NA	11.19	–

	CB	16 spins	Error
ϵ	1.413	1.414	0.07%
ϵ'	3.830	3.838	0.2%
ϵ''	6.896	6.908	0.2%
$T_{\mu\nu}$	3	3	–
$T'_{\mu\nu}$	5.509	5.583	1.3%
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	5.023	5.103	1.6%
$\epsilon'_{\mu_1\mu_2\mu_3\mu_4}$	6.421	6.347	1.2%
ϵ^{P-}	NA	10.01	–


Ultra Violet (UV)
Haldane (Fuzzy) Sphere
SO(3) rotation sym
scale invariance
shortest RG flow
Infrared (IR)
SO(3) Lorentz sym
scale invariance
CFT

Operator Product Expansion

Conformal data consists of a list of scaling dimensions and operator product expansion (OPE) coefficients

Conformal data (Δ_i, f_{ijk})
- **scaling dimension** Δ_i
- **operator product expansion** f_{ijk}

Operator product expansion (OPE) [Wilson 69', Kadanoff 69']:

$$\phi_i(r_1)\phi_j(r_2) = \sum_k f_{ijk}\,\phi_k((r_1+r_2)/2)+\dots$$

$$< \phi_i\,(r_1)\phi_j(r_2)\phi_k(r_3) >= \frac{f_{ijk}}{|r_{12}|^{\Delta_i+\Delta_j-\Delta_k}\,|r_{13}|^{\Delta_i+\Delta_k-\Delta_j}\,|r_{23}|^{\Delta_j+\Delta_k-\Delta_i}}$$

OPEs are important in many ways:

$$Z = \mathrm{Tr}\, e^{-\mathcal{H}^* - \sum_i g_i \sum_r a^{x_i} \phi_i(r)}$$

1. Fusion rules $\phi_i \times \phi_j = \sum_k N_{ijk}\,\phi_k$

2. A fixed-point Hamiltonian under perturbative scaling operators

   Renormalization group equations *J. Cardy, Scaling and renormalization in statistical physics*

$$dg_k/dl = (d - \Delta_k)g_k - \sum_{ij} f_{ijk}\, g_i g_j + \dots$$

Operator Product Expansion

Utilizing the state-operator correspondence, OPE calculation can be greatly simplified

$$\lim_{z\to-\infty}\phi_{cyl}(z)|0>=|\phi>$$

$$<0\big|\phi_i(\infty)\phi_j(0)\phi_k(-\infty)\big|0>=<\phi_i\big|\phi_j(0)\big|\phi_k>$$

Three-point correlator reduces to one-point correlator

Example:

$$\langle\sigma|n^z(\vec{\Omega})|0\rangle = \frac{1}{R^{\Delta_\sigma}}\left(c_\sigma + \sum_{n=1}^{\infty}\frac{a_n}{R^{2n}}\right) \qquad \langle\sigma|n^z(\vec{\Omega})|\epsilon\rangle = \frac{f_{\sigma\sigma\epsilon}}{R^{\Delta_\sigma}}\left(c_\sigma + \sum_{n=1}^{\infty}\frac{\tilde{a}_n}{R^{2n}}\right)$$

The OPE coefficient can be computed by:

$$\frac{\langle\sigma|n^z(\vec{\Omega})|\epsilon\rangle}{\langle\sigma|n^z(\vec{\Omega})|0\rangle} = f_{\sigma\sigma\epsilon} + \frac{a_1-\tilde{a}_1}{c_\sigma R^2} + O(R^{-4})$$

$$n^a(\vec{\Omega}) = \psi^\dagger(\vec{\Omega})\sigma^a\psi(\vec{\Omega}) = \sum_{l=0}^{2s}\sum_{m=-l}^{l} n^a_{l,m}Y_{l,m}(\vec{\Omega})$$

From fuzzy sphere operators to CFT operators

$$\mathcal{O}(\tau=0,\vec{\Omega}) = \sum_\alpha c_\alpha\hat{\phi}_\alpha(\tau=0,\vec{\Omega}).$$

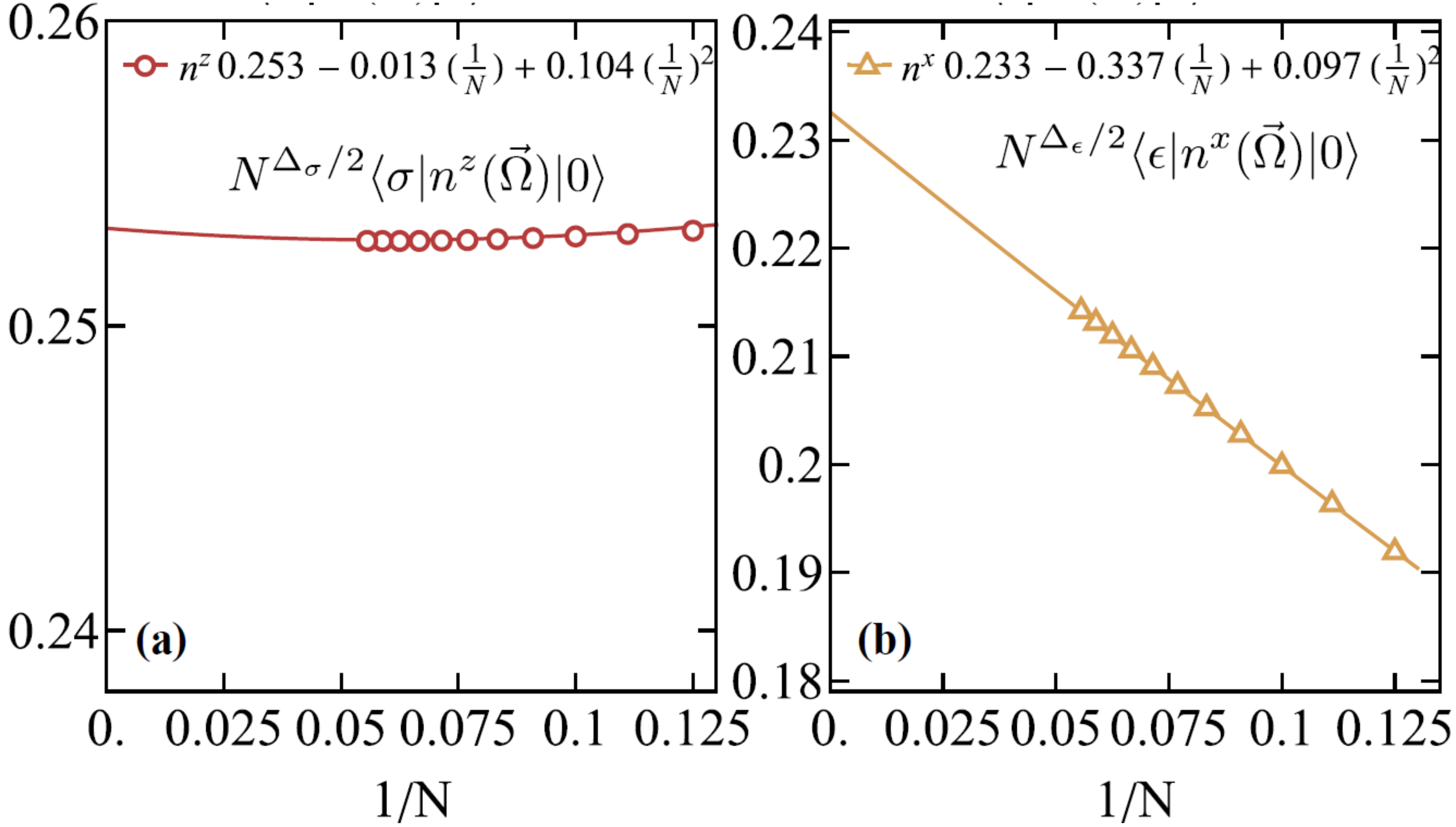
$$\langle\phi_\alpha|\mathcal{O}(\tau=0,\vec{\Omega})|0\rangle = \sum_{n=0}^{\infty}\frac{c_n h_n(\vec{\Omega})}{R^{\Delta_\alpha+n}}$$

$$\langle\phi_\alpha|\mathcal{O}(\tau=0,\vec{\Omega})|\phi_\gamma\rangle = \sum_\beta f_{\alpha\beta\gamma}\frac{c_\beta\tilde{h}_{\alpha\beta\gamma}(\vec{\Omega})}{R^{\Delta_\beta}}$$

Operator Product Expansion

$$\langle\sigma|n^z(\vec{\Omega})|0\rangle = \frac{1}{R^{\Delta_\sigma}}\left(c_\sigma + \sum_{n=1}^{\infty}\frac{a_n}{R^{2n}}\right) \quad \langle\epsilon|n^x(\vec{\Omega})|0\rangle = \frac{1}{R^{\Delta_\epsilon}}\left(c_\epsilon + \sum_{n=1}^{\infty}\frac{\tilde{a}_n}{R^{2n}}\right)$$

$$R \sim \sqrt{N}$$



Operator Product Expansion

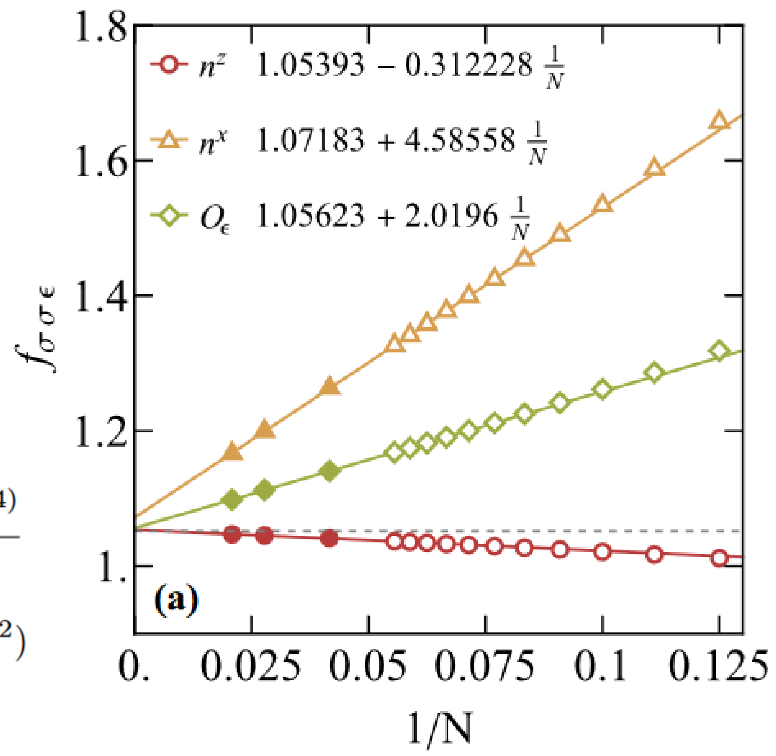
Bootstrap data: $f_{\sigma\sigma\epsilon} = 1.0518537(41)$

$$\frac{\langle\sigma|\hat{n}^z_{0,0}|\epsilon\rangle}{\langle\sigma|\hat{n}^z_{0,0}|0\rangle} \approx \frac{c_\sigma f_{\sigma\sigma\epsilon} R^{-\Delta_\sigma} + c_{\Box\sigma} f_{\sigma,\Box\sigma,\epsilon} R^{-(\Delta_\sigma+2)} + c_{\Box^2\sigma} f_{\sigma,\Box^2\sigma,\epsilon} R^{-(\Delta_\sigma+4)} + c_{\sigma'} f_{\sigma,\sigma',\epsilon} R^{-\Delta_{\sigma'}}}{c_\sigma R^{-\Delta_\sigma} + c_{\Box\sigma} R^{-(\Delta_\sigma+2)} + c_{\Box^2\sigma} R^{-(\Delta_\sigma+4)} + c_{\sigma'} R^{-\Delta_{\sigma'}}}$$

$$\approx f_{\sigma\sigma\epsilon} + \frac{c_1}{R^2} + \frac{c_2}{R^4} + O(R^{-4.77}) \approx f_{\sigma\sigma\epsilon} + \frac{c_1'}{N} + \frac{c_2'}{N^2} + O(N^{-2.38}).$$

$$\frac{\langle\sigma|\hat{n}^x_{0,0}|\sigma\rangle - \langle 0|\hat{n}^x_{0,0}|0\rangle}{\langle\epsilon|\hat{n}^x_{0,0}|0\rangle} \approx \frac{c_\epsilon f_{\sigma\epsilon\sigma} R^{-\Delta_\epsilon} + c_{\Box\epsilon} f_{\sigma,\Box\epsilon,\sigma} R^{-(\Delta_\epsilon+2)} + c_{\epsilon'} f_{\sigma,\epsilon',\sigma} R^{-\Delta_{\epsilon'}} + c_{\Box^2\epsilon} f_{\sigma,\Box^2\epsilon,\sigma} R^{-(\Delta_\epsilon+4)}}{c_\epsilon R^{-\Delta_\epsilon} + c_{\Box\epsilon} R^{-(\Delta_\epsilon+2)} + c_{\epsilon'} R^{-\Delta_{\epsilon'}} + c_{\Box^2\epsilon} R^{-(\Delta_\epsilon+4)}}$$

$$\approx f_{\sigma\sigma\epsilon} + \frac{c_1}{R^2} + \frac{c_2}{R^{2.4173}} + \frac{c_3}{R^4} + O(R^{-4.4173}) \approx f_{\sigma\sigma\epsilon} + \frac{c_1'}{N} + \frac{c_2'}{N^{1.2087}} + O(N^{-2})$$



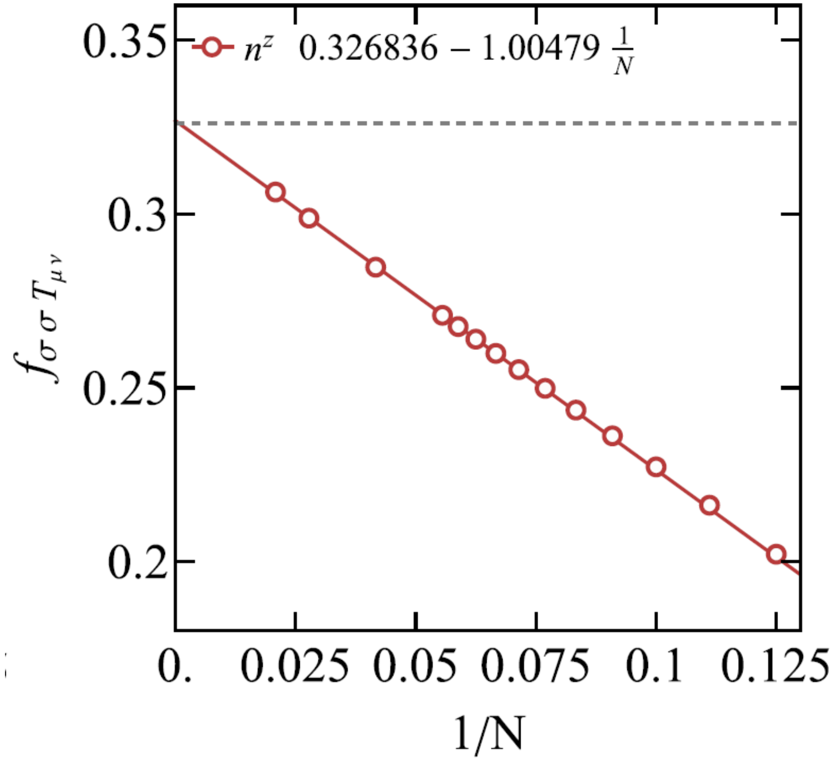
Operator Product Expansion

We can approach the OPE involving energy-momentum tensor.

$$\sqrt{\frac{15}{8}}\frac{\langle\sigma|\hat{n}^z_{2,0}|T_{\mu\nu}\rangle}{\langle\sigma|\hat{n}^z_{0,0}|0\rangle} \approx \frac{c_\sigma f_{\sigma\sigma T_{\mu\nu}} R^{-\Delta_\sigma} + c_{\Box\sigma} f_{\sigma,\Box\sigma,T_{\mu\nu}} R^{-(\Delta_\sigma+2)} + c_{\sigma_{\mu\nu}} f_{\sigma,\sigma_{\mu\nu},T_{\mu\nu}} R^{-\Delta_{\sigma'}} + c_{\Box^2\sigma} f_{\sigma,\Box^2\sigma,T_{\mu\nu}} R^{-(\Delta_\sigma+4)}}{c_\sigma R^{-\Delta_\sigma} + c_{\Box\sigma} R^{-(\Delta_\sigma+2)} + c_{\Box^2\sigma} R^{-(\Delta_\sigma+4)} + c_{\sigma'} R^{-\Delta_{\sigma'}}}$$

$$\approx f_{\sigma\sigma T_{\mu\nu}} + \frac{c_1}{R^2} + \frac{c_2}{R^{3.662}} + O(R^{-4}) \approx f_{\sigma\sigma T_{\mu\nu}} + \frac{c_1'}{N} + \frac{c_2'}{N^{1.831}} + O(N^{-2}).$$

Bootstrap: $f_{\sigma\sigma T_{\mu\nu}} = 0.32613776(45)$

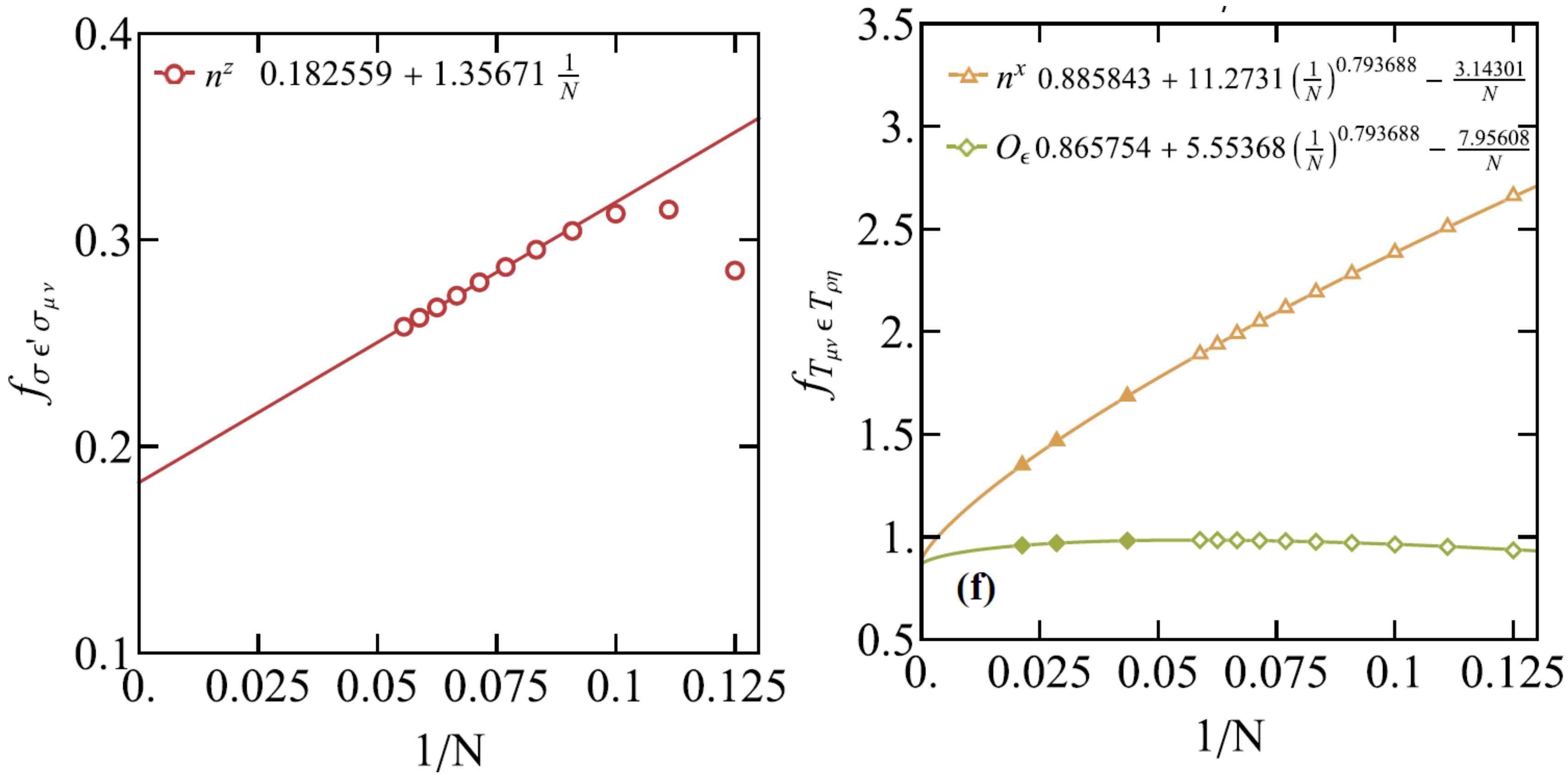


Operator Product Expansion

We can also go beyond conformal bootstrap.

$$\sqrt{\frac{15}{8}}\frac{\langle\epsilon'|\hat{n}^z_{2,0}|\sigma_{\mu\nu}\rangle}{\langle\sigma|\hat{n}^z_{0,0}|0\rangle} \approx f_{\epsilon'\sigma\sigma_{\mu\nu}} + \frac{c_1}{R^2} + \frac{c_2}{R^{3.662}} + \frac{c_3}{R^4} + O(R^{-4.772}) \approx f_{\epsilon'\sigma\sigma_{\mu\nu}} + \frac{c_1'}{N} + \frac{c_2'}{N^{1.831}} + O(N^{-2})$$

$$\frac{\langle T_{\mu\nu}, m=0|\mathcal{O}|T_{\rho\eta}, m=0\rangle - \langle 0|\mathcal{O}|0\rangle}{\langle\epsilon|\mathcal{O}|0\rangle} \approx f_{T_{\mu\nu}\epsilon T_{\rho\eta}} + \frac{c_1}{R^{1.5874}} + \frac{c_2}{R^2} + \frac{c_3}{R^{2.4173}} + O(R^{-3})$$



OPEs for 3D Ising CFT

$$< \phi_i\,(r_1)\phi_j(r_2)\phi_k(r_3) >= \frac{f_{ijk}}{|r_{12}|^{\Delta_i+\Delta_j-\Delta_k}\,|r_{13}|^{\Delta_i+\Delta_k-\Delta_j}\,|r_{23}|^{\Delta_j+\Delta_k-\Delta_i}}$$

Operators	Spin	Z_2	$f_{\alpha\beta\gamma}$ (Fuzzy Sphere)	$f_{\alpha\beta\gamma}$ (Bootstrap)
σ	0	$-$	$f_{\sigma\sigma\epsilon} \approx 1.0539(18)$	$f_{\sigma\sigma\epsilon} \approx 1.0519$
ϵ	0	$+$	$f_{\epsilon\epsilon\epsilon} \approx 1.5441(23)$	$f_{\epsilon\epsilon\epsilon} \approx 1.5324$
ϵ'	0	$+$	$f_{\sigma\sigma\epsilon'} \approx 0.0529(16)$	$f_{\sigma\sigma\epsilon'} \approx 0.0530$
			$f_{\epsilon\epsilon\epsilon'} \approx 1.566(68)$	$f_{\epsilon\epsilon\epsilon'} \approx 1.5360$
σ'	0	$-$	$f_{\sigma'\sigma\epsilon} \approx 0.0515(42)$	$f_{\sigma'\sigma\epsilon} \approx 0.0572$
			$f_{\sigma'\sigma\epsilon'} \approx 1.294(51)$	NA
			$f_{\sigma'\epsilon\sigma'} \approx 2.98(13)$	NA
$T_{\mu\nu}$	2	$+$	$f_{\sigma\sigma T} \approx 0.3248(35)$	$f_{\sigma\sigma T} \approx 0.3261$
			$f_{\sigma'\sigma T} \approx -0.00007(96)$	$f_{\sigma'\sigma T} = 0$
			$f_{\epsilon\epsilon T} \approx 0.8951(35)$	$f_{\epsilon\epsilon T} \approx 0.8892$
			$f_{T\epsilon T} \approx 0.8658(69)$	NA
$\sigma_{\mu\nu}$	2	$-$	$f_{\sigma\epsilon\sigma_{\mu\nu}} \approx 0.400(33)$	$f_{\sigma\epsilon\sigma_{\mu\nu}} \approx 0.3892$
			$f_{\sigma\epsilon'\sigma_{\mu\nu}} \approx 0.18256(69)$	NA

High Energy Physics - Theory

[Submitted on 15 May 2023]

The five-point bootstrap

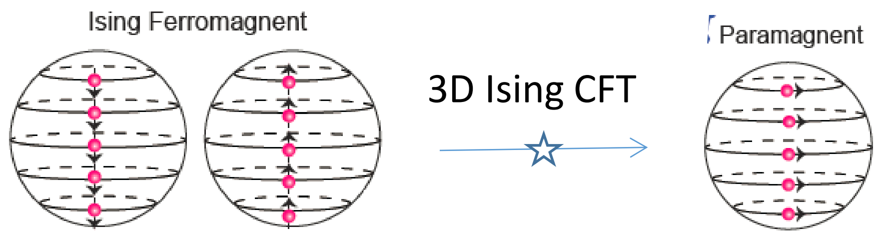
David Poland, Valentina Prilepina, Petar Tadić

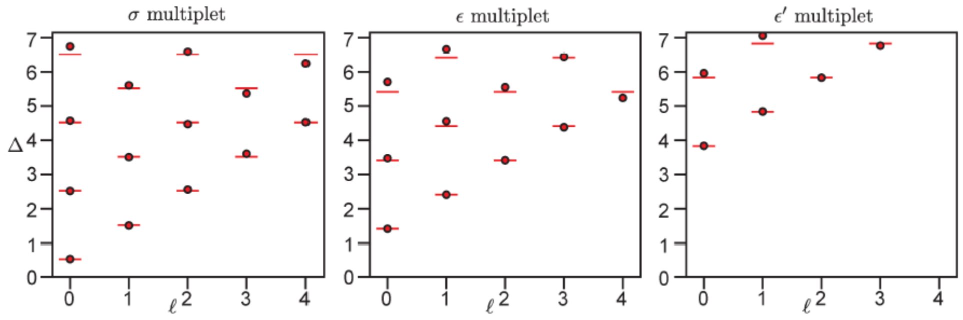
Recent data from bootstrap verifies out prediction:

$$f_{T\epsilon T} = 0.81(5)$$

3D CFT from Fuzzy sphere regularization

- Motivation: Spotting 3D CFT on geometry $S^2 \times R$
- Solution: Fuzzy-sphere scheme to simulate 3D transition
- Results: Conformal data of 3D Ising transition on the sphere $S^2 \times R$
  Conformal data set:
  State perspective: Almost perfect conformal tower structure
  Operator perspective: OPE coefficients: 4 unknown even in boostrap





Scientific metric: state-operator correspondence

The 3D Ising criticality indeed has conformal symmetry
(Conjectured by Polyakov since 1970s)

W.Zhu*, C. Han, E. Huffman, J. Hofmann, Y.-C. He*, **Phys Rev. X** 13, 021009 (2023)
Liangdong Hu, Y.-C. He***, W. Zhu*, Phys. Rev. Lett.** 131, 031601 (2023) [Editor's suggestion]

Outline

- Fuzzy sphere regularization
  a. Motivation from the CFT and State-operator correspondence
  b. Spherical Landau level regularization as a solution of the space-time geometry $S^2 \times R$

- Example of the 3D Ising transition
  a. Emergent conformal symmetry
  b. Scaling dimensions, operator product expansion coefficients, etc.

- Deconfined Quantum Critical Point
  a. Emergent (approximate) conformal symmetry
  b. Pseudo-criticality

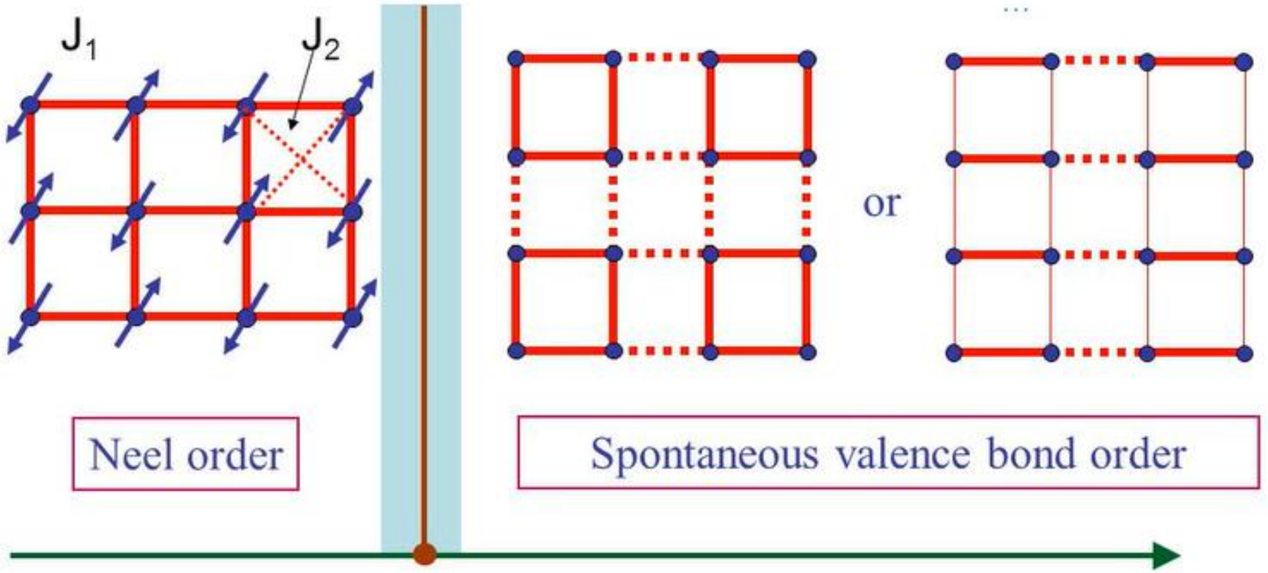
- Outlook and discussion

Deconfined Quantum Critical Point (DQCP)

DQCP

A distinct class of phase transitions that cannot be described with Landau-Ginzberg theory, distinguished by an emergent conserved topological quantity at the critical point.
A possible quantum critical point between two conventional phases [Sachdev & Read 1989; Senthil 2003]

- Non-Landau phase transition
- Natural variables are emergent fractionalized degrees of freedom instead of order parameter
- Enlarged symmetry



Senthil-Sachdev-Balents-Vishwanath-Fisher picture

Science 303, 1490 (2004); PRB 70, 144407 (2003)

Critical field theory for the Neel-VBS transition (NCCP1)

$$L = |\nabla \times a|^2 + |(\nabla - ia)z|^2 + s|z|^2 + u|z|^4$$

CP1 field

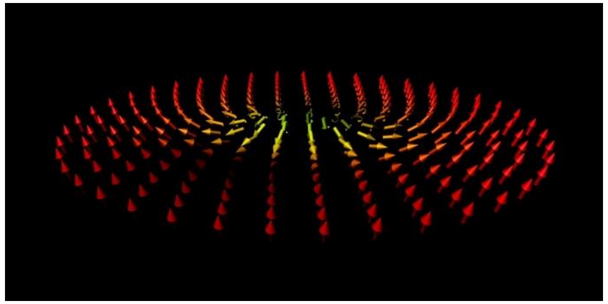
A state of staggered magnetization using CP1 fields

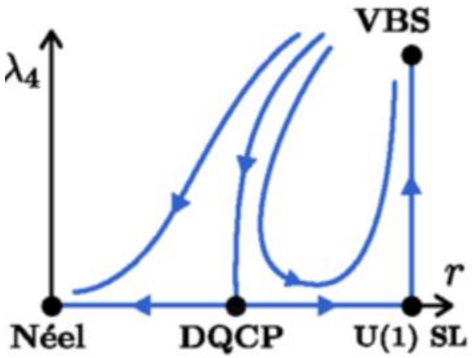
$\vec{S}_r = \epsilon_r \vec{N}_r \qquad \epsilon_r \equiv (-1)^{x+y} \qquad \vec{N} \sim z^\dagger \vec{\sigma} z$

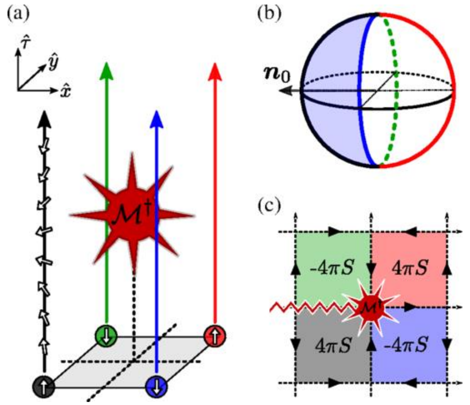
The spinon fields have a U(1) "gauge" redundancy

Monopole operator

- Monopole operator $\mathcal{M}(r)$ creates a source of the "magnetic field" $b = \nabla \times a$.
- In the Neel phase, it is topological excitation (skyrmion) of order parameter N.
- ψ_{VBS} ~$\mathcal{M}(r)$ is the VBS order parameter [PRB 70, 144407 (2003)] $\qquad \mathcal{M}\sim\varphi_x(r)$ +i $\varphi_y(r) \sim \psi_{VBS}$
- This proliferation leads to a "condensation" of the monopole operator, $\mathcal{M} \neq 0$, hence VBS order.







Tanaka-Hu picture

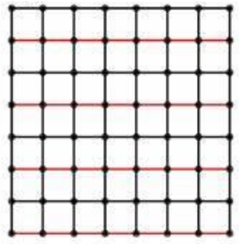
PRL 95, 036402(2005)

The Neel-VBS competition for spin-1/2 magnets on an isotropic two dimensional square lattice is described by this SO(5) superspin non-linear sigma model with the extra topological WZW term.

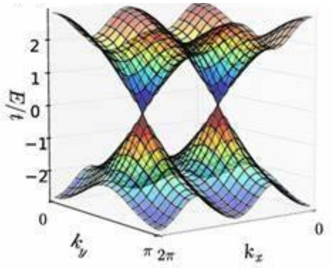
Dirac spinon on the square lattice

$$\mathcal{L} = i\bar{\Psi}\,\partial\!\!\!/\,\Psi$$

Pi-flux square lattice



Spinon band structure



the system can have AFM order.

$$\mathcal{L}_{\mathrm{AF}} = i m_{\mathrm{AF}} \bar{\Psi}(\boldsymbol{n}\cdot\boldsymbol{\sigma})\Psi$$

the translational symmetry breaking leads to chiral mass terms $t \to t + (-1)^{i_\mu}\delta t$

$$\mathcal{L}_{\mathrm{VBS}} = \bar{\Psi}\gamma_3\Psi + \bar{\Psi}\gamma_5\Psi$$

The SDW and the two VBS ordering potentials all belong to the family of chirally rotated mass terms

$$S = \int d^3x \bar{\psi}\left(-i\tau_i\partial_i + im\hat{\phi}.\vec{\Gamma}\right)\psi$$

$$S[\hat{\phi}] = \int d^3x \frac{1}{2g}\left(\partial_i\hat{\phi}\right)^2 - 2\pi i\Gamma[\hat{\phi}]$$

$$\Gamma = \frac{3}{8\pi^2}\int_0^1 du \int d^3x \epsilon_{\alpha\beta\gamma\delta\kappa}\phi_\alpha\partial_x\phi_\beta\partial_y\phi_\gamma\partial_\tau\phi_\delta\partial_u\phi_\kappa$$

Abanov and Wiegmann, Nucl. Phys. B 570, 685 (2000)

Equivalence of NLσM with NCCP1

NLσM $S[\hat{\phi}] = \int d^3x \frac{1}{2g}\left(\partial_i \hat{\phi}\right)^2 - 2\pi i \Gamma[\hat{\phi}]$

$$\Gamma = \frac{3}{8\pi^2}\int_0^1 du \int d^3x \epsilon_{\alpha\beta\gamma\delta\kappa}\phi_\alpha \partial_x \phi_\beta \partial_y \phi_\gamma \partial_\tau \phi_\delta \partial_u \phi_\kappa$$

NCCP1 $L = |\nabla \times a|^2 + |(\nabla - ia)z|^2 + s|z|^2 + u|z|^4$

The derivation of the geometric spin action can be obtained based on the CP1 representation

$$a_t \equiv -i\mathbf{z}^\dagger \partial_t \mathbf{z} = \partial_t \phi[1 - \cos\theta]/2$$

$$\exp[i\mathcal{A}[\mathbf{n}]] = \exp[iS \int dt(\cos\theta - 1)\partial_t \phi] = \exp[-2iS \int dt a_t]$$

WZW term describes the non-trivial Berry phase

$$\mathbf{n}(t,0) = \mathbf{n}_0 \rightarrow \mathbf{n}(t,u) = \mathbf{n}(t)$$
$$\theta(t,0) = 0 \rightarrow \theta(t,u) = \theta(t)$$

$$\frac{1}{2}\mathbf{n} \cdot (\partial_t \mathbf{n} \times \partial_u \mathbf{n}) = \partial_t a_u - \partial_u a_t = -\partial_u a_t$$

$$\exp[-2iS \int dt a_t(t)] = \exp[iS \int dt \int_0^1 du \mathbf{n} \cdot (\partial_t \mathbf{n} \times \partial_u \mathbf{n})] \equiv \exp[iS_{WZW}/\hbar]$$

Monopole creation is related to skyrmion excitation

Deconfined Quantum Critical Points: Symmetries and Dualities
Chong Wang, Adam Nahum, Max A. Metlitski, Cenke Xu, and T. Senthil, Phys. Rev. X 7, 031051 (2017)

NLσM with WZW simulated by the LLL

$$\mathcal{H}_{\mathrm{ZLL}} = \frac{U}{2}\left[\sum_{a=1}^{4}\psi_a^\dagger(x)\psi_a(x)\right]^2 - \sum_{i=1}^{5}\frac{u_i}{2}\left[\sum_{a,b=1}^{4}\psi_a^\dagger(x)\Gamma^i_{ab}\psi_b(x)\right]^2$$

Clifford algebra for SO(5) $\Gamma^i = \{\tau^z\sigma^x, \tau^z\sigma^y, \tau^z\sigma^z, \tau^x, \tau^y\}$

$u_1 = u_2 = u_3 = u_N \quad u_4 = u_5 = u_K$

SO(5) superspin 'vector' $\boldsymbol{n} = [N_x, \; N_y, \; N_z, \; \boldsymbol{\varphi}_x, \; \boldsymbol{\varphi}_y\,]$

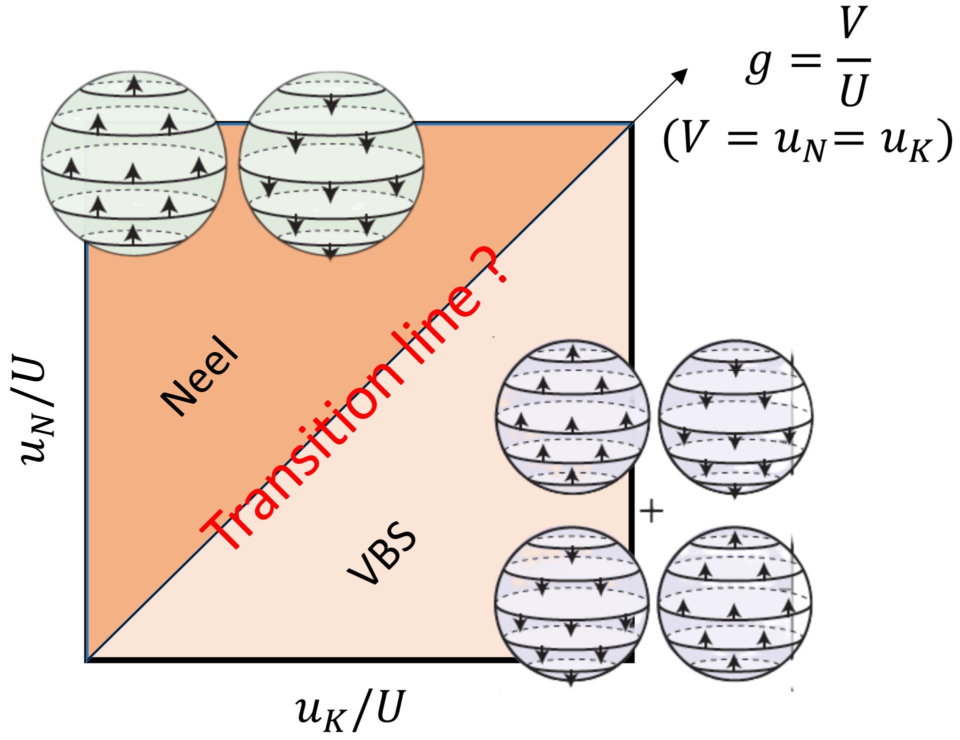
target space: $\dfrac{U(4)}{U(2)\times U(2)} \to \dfrac{Sp(2)}{Sp(1)\times Sp(1)} = S^4$

$\pi_2[\dfrac{U(4)}{U(2)\times U(2)}] = Z$ Skymion quantum number

$$S = \frac{1}{2\gamma}\int d^3r(\partial\mathbf{n})^2 + S_{\mathrm{WZW}}[\mathbf{n}] + \ldots,$$

$$S_{\mathrm{WZW}}[\mathbf{n}] = \frac{2\pi i}{\mathrm{vol}(S^4)}\int dt\, d^3r\, \epsilon^{abcde} n^a \partial_s n^b \partial_x n^c \partial_y n^d \partial_t n^e.$$

The SO(5)-NLσM with topological term has been argued to flow to the DQCP J. Lee and S. Sachdev, PRL **114**, 226801 (2015)



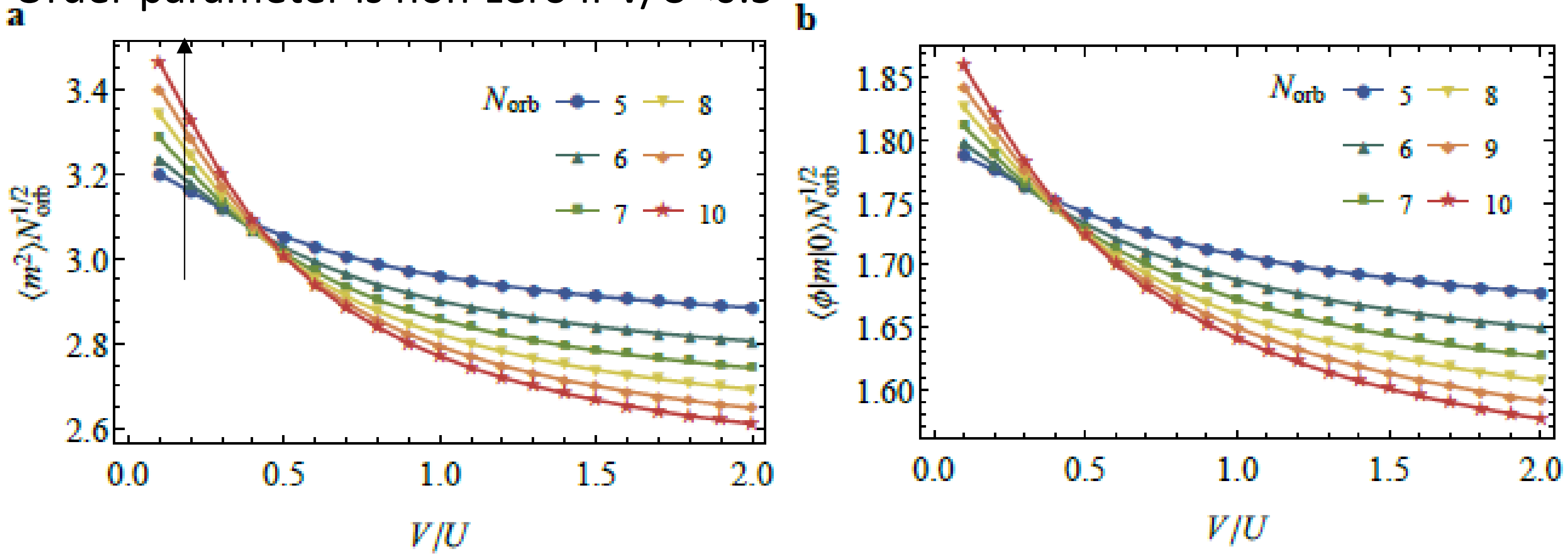
Scaling of order parameter

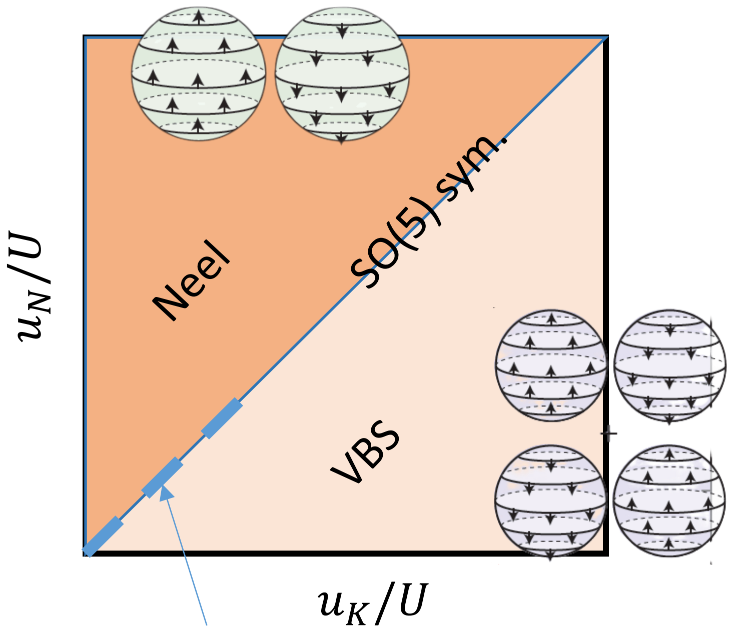
SO(5) symmetry breaking order parameter (i.e. SO(5) vector),

$$m^i = \sum_m \hat{\mathbf{c}}_m^\dagger \gamma^i \hat{\mathbf{c}}_m \qquad \{\mathbb{I}\otimes\tau^x, \mathbb{I}\otimes\tau^z, \sigma^x\otimes\tau^y, \sigma^y\otimes\tau^y, \sigma^z\otimes\tau^y\}$$

Order parameter tends to decrease as system size for V/U >0.5 -> critical phase

Order parameter is non-zero if V/U<0.5





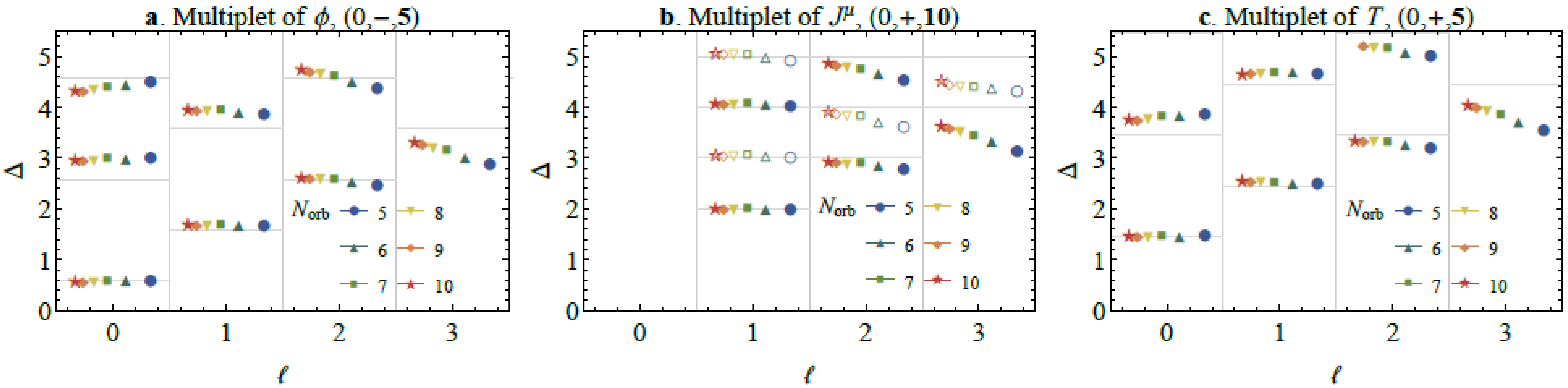
SO5 symmetry is spontaneously broken when V/U<0.5

Approximate conformal symmetry

$$\delta E_k = E_k - E_0 = \text{constant} \times \Delta_k$$

the (approximate) integer-spaced levels

$$(\ell = 0, \mathscr{P}, \text{rep.}; \Delta) \xrightarrow{\partial^{\nu_1} \dots \partial^{\nu_j} (\partial^2)^n \Phi} (j, \mathscr{P}, \text{rep.}; \Delta + 2n + j)$$



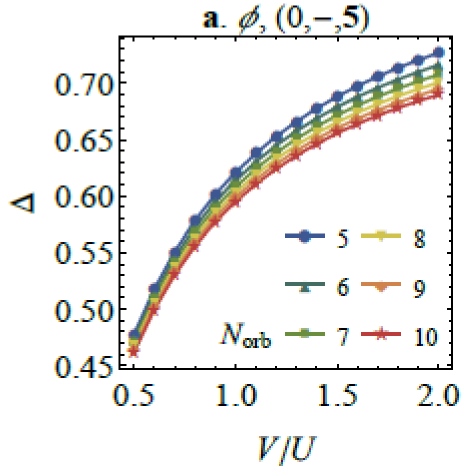
The lack of exact conformal symmetry could be due to finite-size or the pseudo-criticality (discussed later).

The energy spectrum has an emergent (approximate) conformal symmetry

Operator information

Scaling dimensions of Primaries of SO(5) NLσM with WZW (V/U=0.915, N=9)

l	P	Rep.	Δ	Operator	
0	$-$	**5**	0.584	$\phi \sim \mathcal{M}_{2\pi}$	SO(5) order parameter
0	$+$	**14**	1.454	$T \sim \phi^2 \sim \mathcal{M}_{4\pi}$	control Neel-VBS transition
1	$+$	**10**	2.000	J^μ	flavor current
0	$-$	**30**	2.565	$\mathcal{M}_3$	6π monopole
0	$+$	**1**	2.845	S	Parity even singlet $\to$ Pseudo-criticality
0	$+$	**55**	3.885	$\mathcal{M}_4$	8π monopole $\to$ VBS on C_4 lattice
0	$-$	**1**	5.354	S^-	Parity odd singlet $\to$ chiral spin liquid



- The lowest ℓ = 0 parity-odd SO(5) vector ϕ corresponds to the order parameter.
- Its scaling dimension is related to the anomalous dimension $\eta = (\Delta_\phi - 1/2)/2 \sim 0.168$.

	Fuzzy sphere	loop model	J-Q
η	0.168	0.0395	0.35
ν	0.647	0.677	0.455 (< CB bound)

G. J. Sreejith and S. Powell, Phys.Rev.B92, 184413 (2015).
G. J. Sreejith and S. Powell, Phys. Rev. B 89, 014404 (2014).
A. W. Sandvik, Phys. Rev. Lett. 104, 177201 (2010).
A.W. Sandvik and B. Zhao, Chin. Phys. Lett. 37, 057502 (2020).

Operator information

Scaling dimensions of Primaries of SO(5) NLσM with WZW (V/U=0.915, N=9)

l	P	Rep.	Δ	Operator	
0	−	**5**	0.584	$\phi \sim \mathcal{M}_{2\pi}$	SO(5) order parameter
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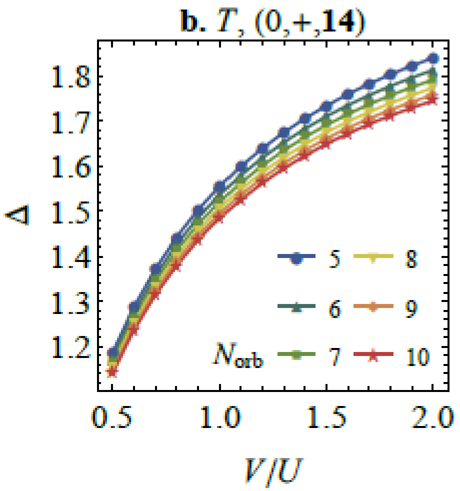
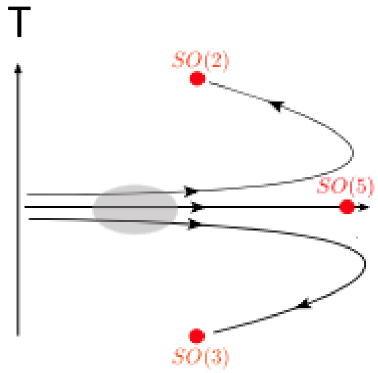


- The lowest parity-even symmetric rank-2 tensor T corresponds to the relevant perturbation that controls the original Neel-VBS transition.
- Its scaling dimension is related to the exponent $\nu = 1/(3 - \Delta_T) \sim 0.647$.



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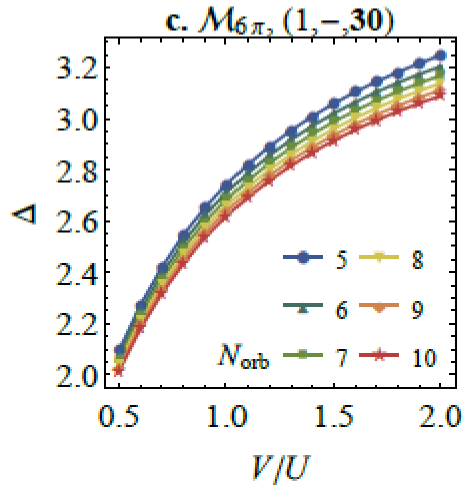
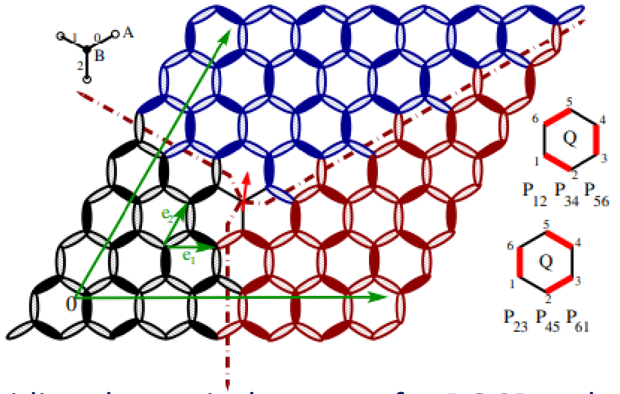


- The lowest ℓ = 0 parity-odd operator in representation corresponds to the 6π-monopole in the CP1 description.
- This operator is forbidden in lattices with C_4 rotation symmetry but allowed for C_3 symmetry.
- Our calculation finds it relevant in all cases, which is likely to imply that the DQCP is not possible on honeycomb lattice as Monopole-6π will drive it away



Providing theoretical support for DQCP on honeycomb: S. Pujari, K. Damle, and F. Alet, PRL 111, 087203 (2013)

6π-monopole is relevant → DQCP on C_3 lattice (e.g. honeycomb lattice) is impossible

Operator information

Scaling dimensions of Primaries of SO(5) NLσM with WZW (V/U=0.915, N=9)

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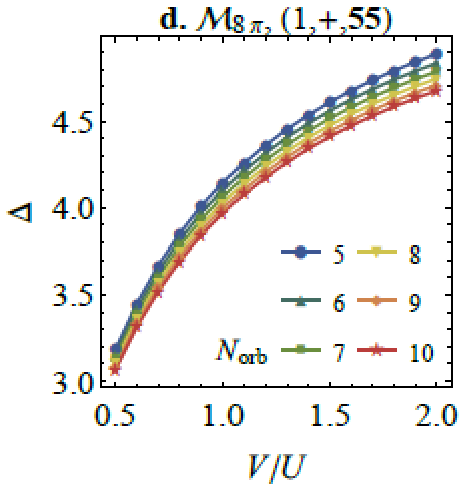
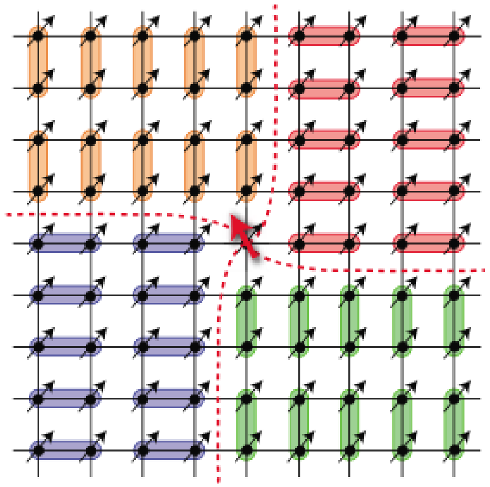


- The lowest parity-even operator corresponds to the 8π-monopole in the CP1 description.
- This operator is related to the irrelevant perturbation in the Neel-VBS DQCP.
- Our calculation confirms its irrelevance.



Providing theoretical support for the argument before:
O. I. Motrunich and A. Vishwanath, PRB 70, 075104 (2004)

8π-monopole is irrelevant → DQCP on C_4 lattice (e.g. square lattice) is possible

Operator information

Scaling dimensions of Primaries of SO(5) NL$\boldsymbol{\sigma}$M with WZW (V/U=0.915, N=9)

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0	$-$	**1**	5.354	S^-	Parity odd singlet $\to$ chiral spin liquid

- The lowest ℓ = 0 parity-odd operator in representation, is related to the instability towards a CSL. Our calculation confirms its irrelevance.

Operator information

Scaling dimensions of Primaries of SO(5) NLσM with WZW (V/U=0.915, N=9)

l	P	Rep.	Δ	Operator	
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- We identify a ℓ = 0 parity-even operator, dubbed as S, which is unknown before.
- It is dangerously relevant. -> It drives the DQCP unstable.

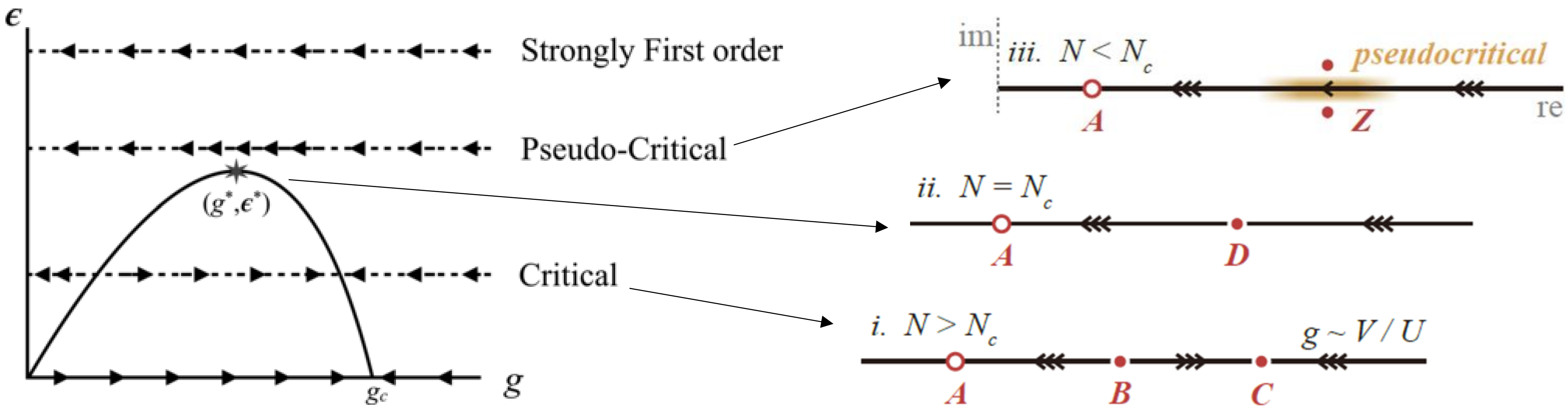
It appears a relevant singlet operator S → driving instability of pseudo-criticality

RG diagram of NLσM

$$S = \frac{1}{2\gamma}\int d^3r(\partial\mathbf{n})^2 + S_{\mathrm{WZW}}[\mathbf{n}] + \ldots,$$

$$S_{\mathrm{WZW}}[\mathbf{n}] = \frac{2\pi i}{\mathrm{vol}(S^4)}\int dt\, d^3r\, \epsilon^{abcde} n^a \partial_s n^b \partial_x n^c \partial_y n^d \partial_t n^e.$$

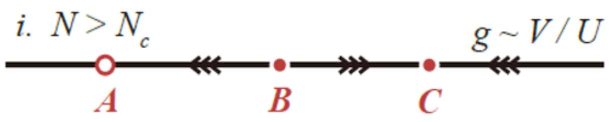
In real parameter space, no fixed point exists.
The RG flow becomes extremely slow on the real axis.



A Nahum, Phys. Rev. B 106, L081109 (2022)
Ruochen Ma and Chong Wang, Phys. Rev. B 102, 020407(R) (2020)

Conformal perturbation for pseudo-criticality

critical scenario



$$\tilde{\beta}(\lambda) = -\alpha(\lambda^2 - x^2) + \mathcal{O}(x^4)$$

fixed points $\lambda = +x$ $\lambda = -x$

$$\tilde{\lambda}(R, \lambda_0) = x \tanh\left[\tanh^{-1}\frac{\lambda_0}{x} + \alpha x \log\frac{R}{R_0}\right] + \mathcal{O}(x^2)$$

Pseudo-critical scenario

$$\beta(\lambda) = R\frac{\mathrm{d}\lambda(R, \lambda_0)}{\mathrm{d}R} = -\alpha(\lambda^2 + y^2) + \mathcal{O}(y^4)$$

fixed points $\lambda^*_{Z_\pm} = \pm iy$

$$\lambda(R, \lambda_0) = y \tan\left[\tan^{-1}\frac{\lambda_0}{y} - \alpha y \log\frac{R}{R_0}\right] + \mathcal{O}(y^2)$$

Using conformal perturbation, we can write the Hamiltonian as

$$H(\lambda) = H_0 + \lambda \int \frac{\mathrm{d}\vec{r}}{4\pi} S(\vec{r})$$

$\lambda = \lambda(R, \lambda0)$ is the factor of the singlet operator S that depend on the linear system size R and a tuning parameter λ in the Hamilonian

The rescaled energy of an arbitrary operator Φ could be interpreted as the scaling dimension

$$\Delta_\Phi(\lambda) = \langle\Phi|H(\lambda)|\Phi\rangle + \mathcal{O}(\lambda^2) = \Delta_{\Phi,0} + \lambda f_{\Phi\Phi S} + \mathcal{O}(\lambda^2)$$

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fixed points $\lambda = +x$ $\lambda = -x$

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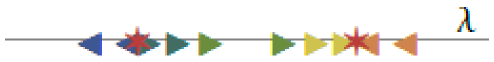
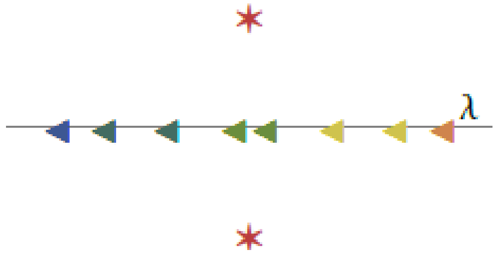
b

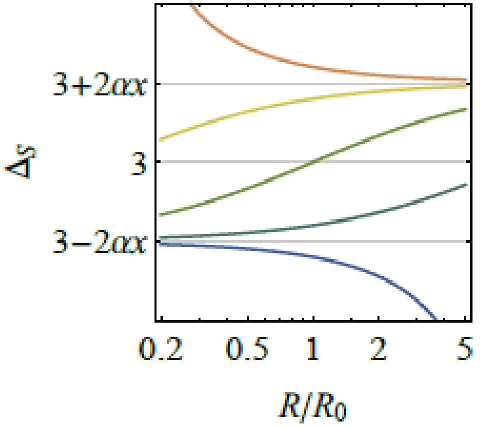


a



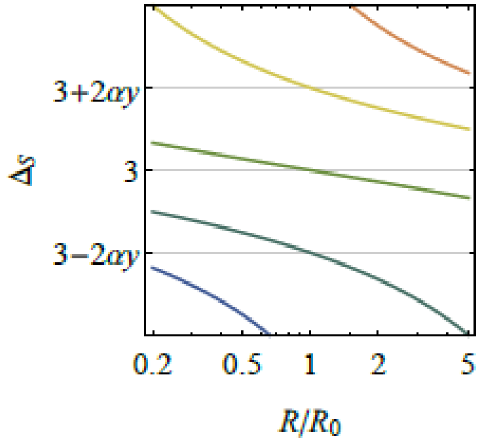
d

$$\tilde{\Delta}_\Phi(\lambda_0, R) = \Delta_\Phi + \tilde{\lambda} f_{\Phi\Phi S} + \mathcal{O}(\tilde{\lambda}^2)$$



c

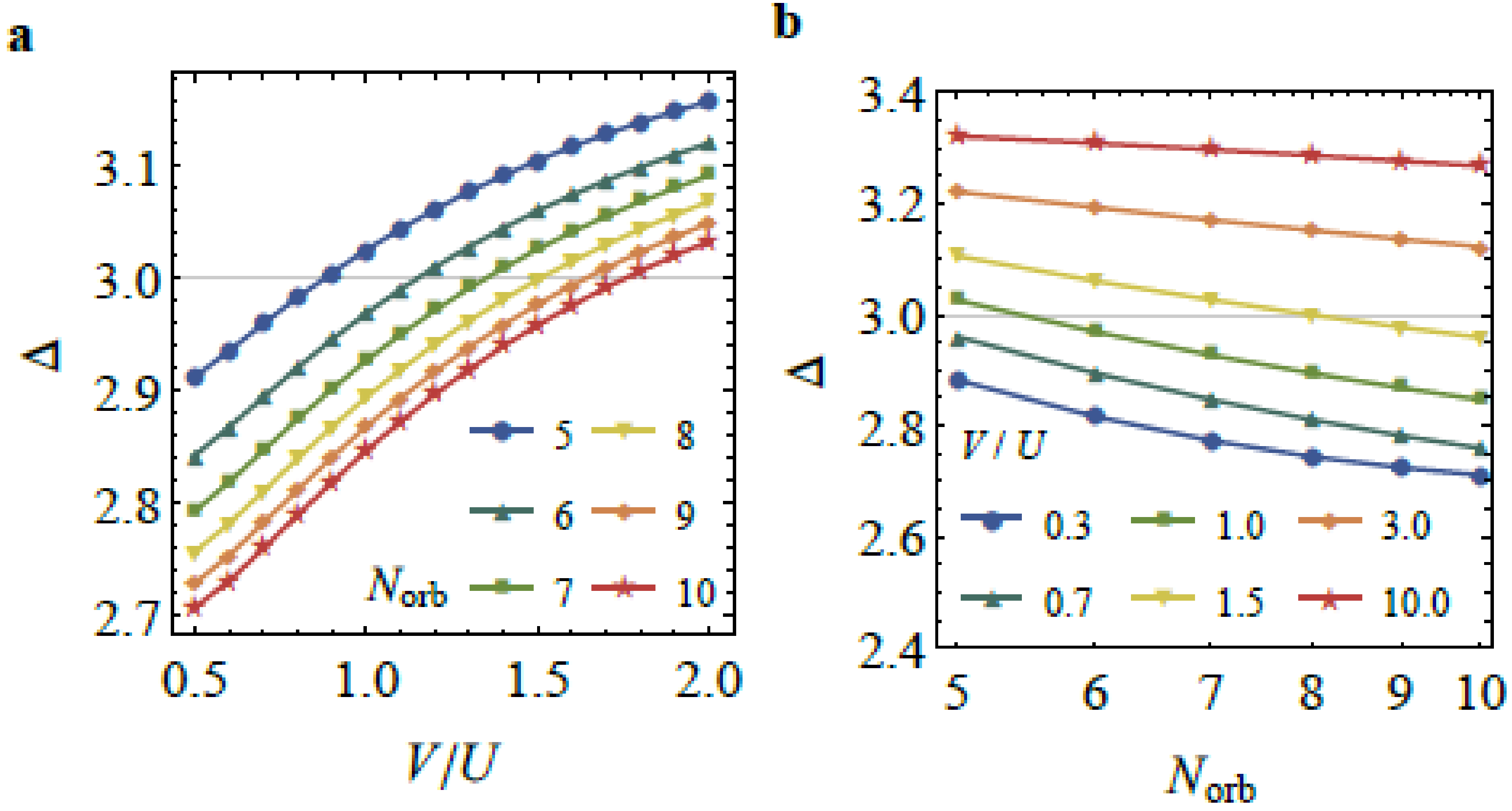
$$\Delta_\Phi(\lambda_0, R) = \Delta_{\Phi,0} + f_{\Phi\Phi S}\lambda(R, \lambda_0) + \mathcal{O}(\lambda^2).$$



the lowest singlet will always flow from irrelevant to relevant as the system size R increases!

Pseudo-criticality

The scaling dimension of the lowest scalar S (a) as a function of V/U for different Norb and (b) as a function of Norb for different V/U.



In the case of real fixed points, Δ will increase towards irrelevance with Δ > 3, while for pseudo-criticality, Δ will decrease from irrelevant (Δ > 3) towards relevant (Δ < 3) along the flow.

DQCP corresponds to not a real CFT, but to a pseudo-critical region that locates near complex CFT fixed points and exhibits approximate conformal symmetry.

Pseudo-criticality

Scaling dimensions of Primaries of SO(5) NL$\boldsymbol{\sigma}$M with WZW (V/U=0.915, N=9)

l	$\mathbf{P}$	Rep.	Δ	Operator	
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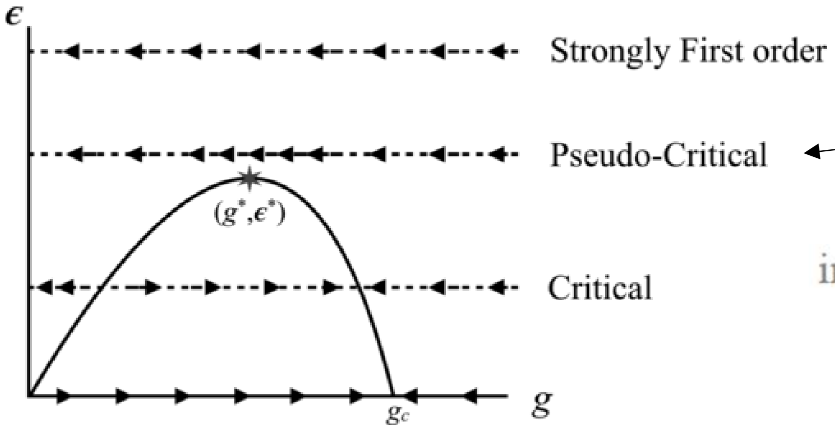
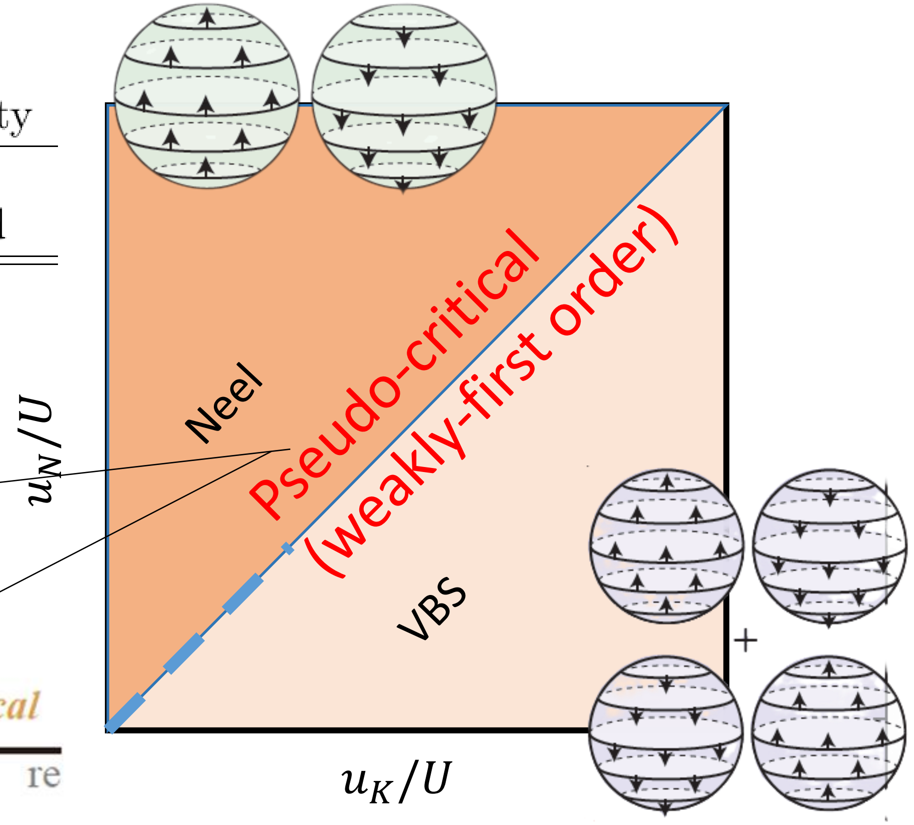




Deconfined quantum criticality --- First order?

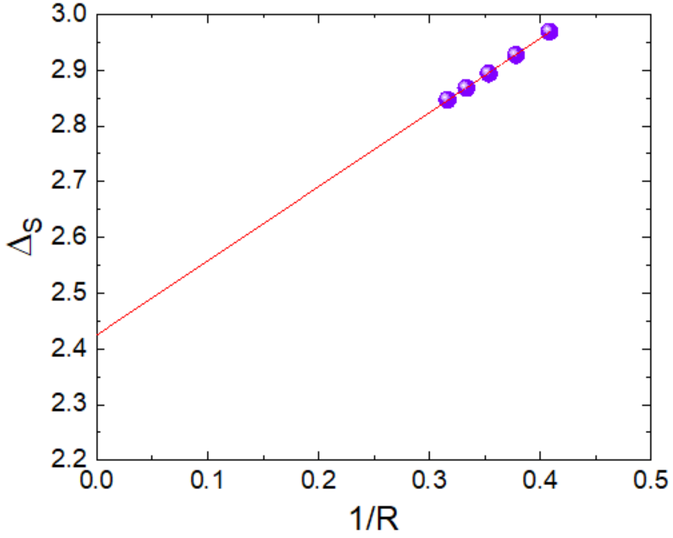
A relevant scalar operator exists, which has been confirmed by different methods.

How to understand this relevant operator is important to the DQCP.

Scaling dims from different methods

		Δ_ϕ	Δ_s	Δ_t	Δ_j	Δ_4
arXiv.2405.06607	J-Q model	0.607(4)	2.273(4)	1.417(7)	2.01(3)	3.723(11)
arXiv.2310.08343	SO(5) CFT	0.630*	2.359	1.519	2*	3.884
arXiv.2306.16435	Fuzzy sphere	0.585	2.831	1.458	2*	3.895

Fuzzy sphere under finite-size scaling



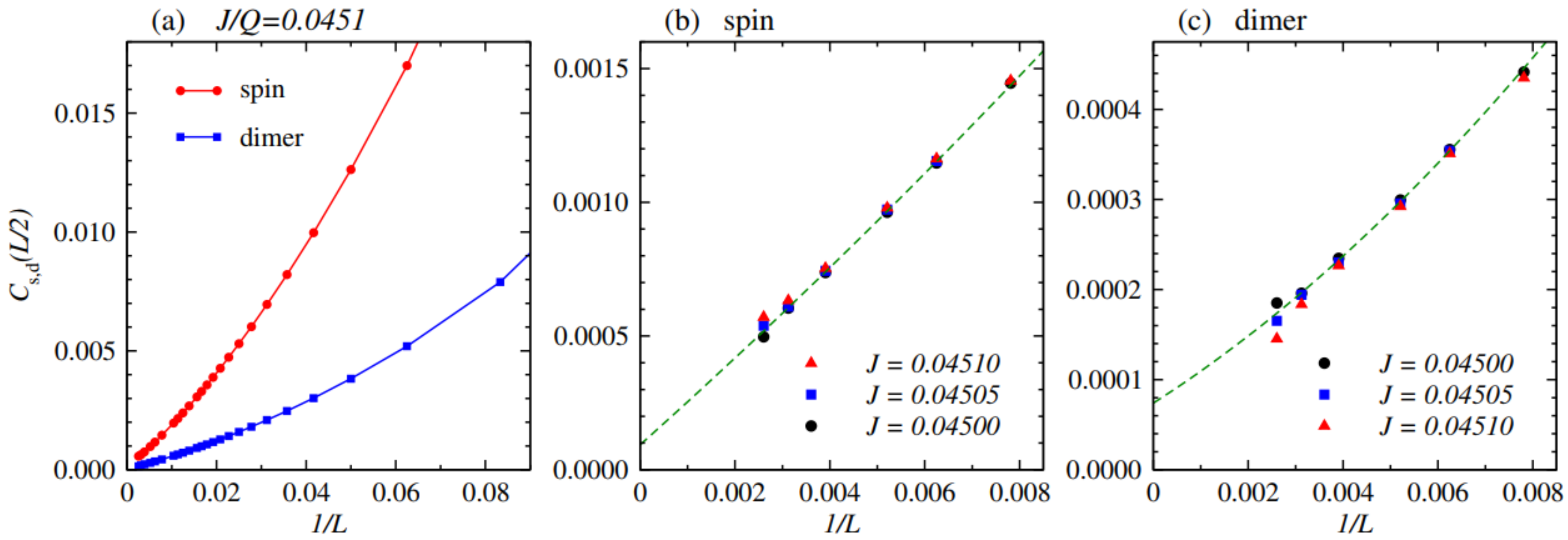
The SO(5) Deconfined Phase Transition under the Fuzzy Sphere Microscope: Approximate Conformal Symmetry, Pseudo-Criticality, and Operator Spectrum,
Zheng Zhou, L. D. Hu, W. Zhu, Y. C. He, arXiv.2306.16435

Bootstrapping Deconfied Quantum Tricriticality,
Shai M. Chester, Ning Su, arXiv.2310.08343

SO(5) multicriticality in two-dimensional quantum magnets,
Jun Takahashi, Hui Shao, Bowen Zhao, Wenan Guo, and Anders W. Sandvik, arXiv.2405.06607

Numerical evidence: Weakly First-order

We resolve the long-standing problem of the nature of the quantum phase transition between a Néel antiferromagnet and a spontaneously dimerized valence-bond solid in two-dimensional spin-1/2 magnets. We study a class of J-Q models, in which the standard Heisenberg exchange J competes with multi-spin interactions Q_n formed by products of n singlet projectors on adjacent parallel links of the lattice. Using large-scale quantum Monte Carlo (QMC) calculations, we provide unambiguous evidence for first-order transitions in these models, with the strength of the discontinuities increasing with n. In the case of the widely studied $n = 2$ and $n = 3$ models, the first-order signatures are very weak, but observable in correlation functions on large lattices. On intermediate length scales (up

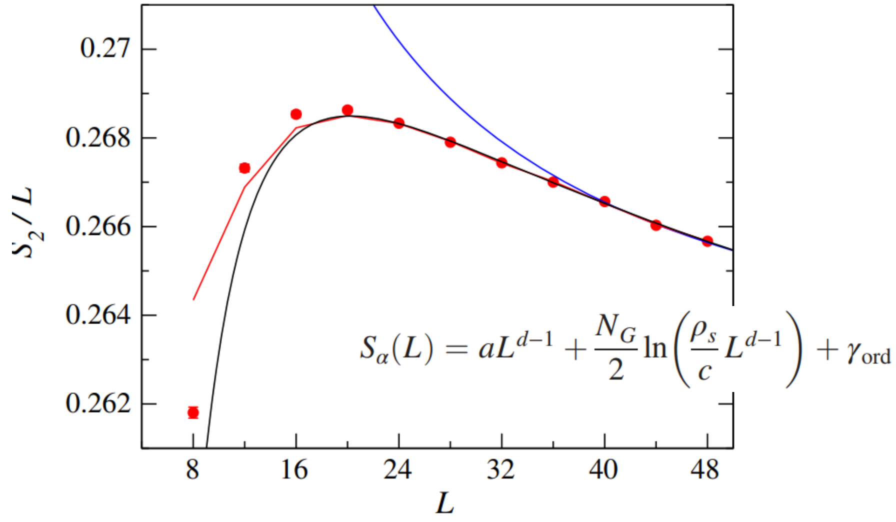


SO(5) multicriticality in two-dimensional quantum magnets,
Jun Takahashi, Hui Shao, Bowen Zhao, Wenan Guo, and Anders W. Sandvik, arXiv.2405.06607

We study the scaling behavior of the Rényi entanglement entropy with smooth boundaries at the putative deconfined critical point separating the Néel antiferromagnetic and valence-bond-solid states of the two-dimensional $J - Q_3$ model. We observe a subleading logarithmic term with a coefficient indicating the presence of four Goldstone modes, signifying the presence of an $SO(5)$ symmetry at the transition point, which spontaneously breaks into an $O(4)$ symmetry in the thermodynamic limit. This result supports the conjecture that an $SO(5)$ symmetry emerges at the transition point, but reveals the transition to be weakly first-order. We demonstrate, through this Letter, a novel approach to detect emergent continuous symmetry and, more importantly, identify weakly first-order phase transitions efficiently, which have been notoriously challenging for conventional methods.

TABLE II. Fitting results of Eq. (4) to $S_2(L)$ at $Q/J = 1.491\,53$ with $L = 8$–48. For results obtained using $I(L, S = 2)$ and $I(L, S = 3)$, see [47].

$L_{\min}$	a	$N_G/2$	$\gamma_{\rm ord}$	χ_r^2/P-value
12	0.2540(2)	1.89(4)	1.10(2)	0.93/0.48
16	0.2543(3)	1.83(6)	1.07(3)	0.82/0.55
20	0.2541(5)	1.88(10)	1.09(5)	0.91/0.47
24	0.2534(7)	2.01(14)	1.16(9)	0.76/0.55
28	0.253(2)	2.1(3)	1.2(2)	0.86/0.46



Z. Deng, L. Liu, Wenan Guo, and Hai-Qing Lin, PRL 133, 100402 (2024)

3D phase transitions and 3D DQCP

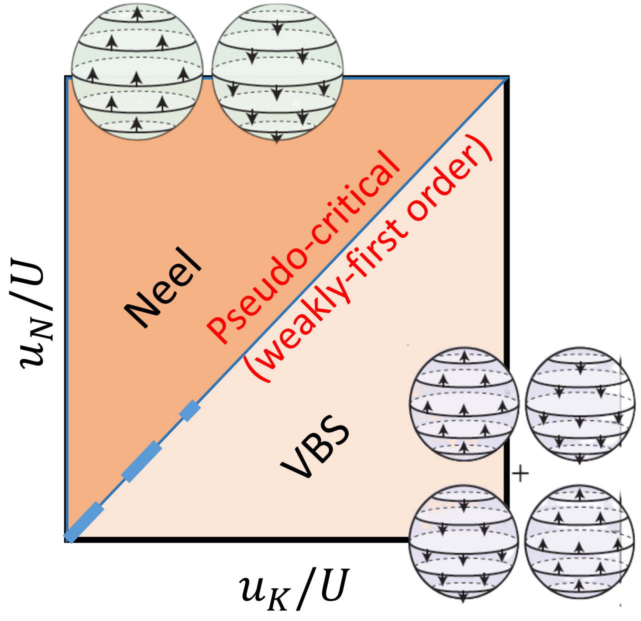
- Solution: Fuzzy-sphere scheme to simulate 3D transition
- Results: **The 3D DQCP has approximately conformal symmetry**

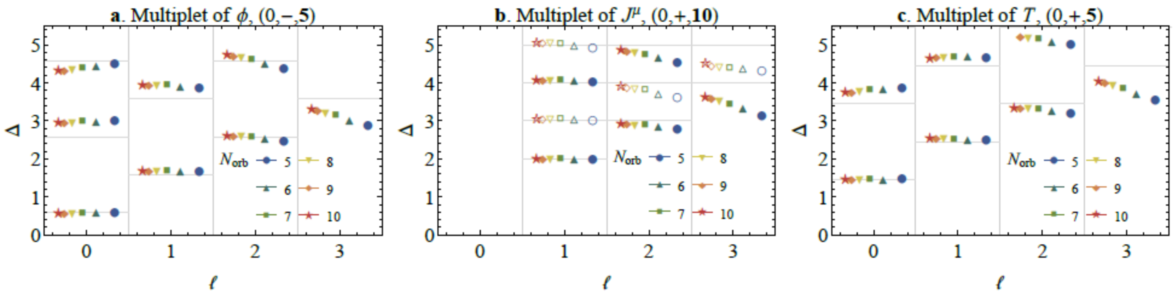
Instability of 8π monopole or CSL is not likely to occur

DQCP cannot be realized on honeycomb lattice or rectangular lattice

A singlet operator is dangerous relevant -> pseudo-criticality

Neel-VBS transition is weakly first-order.





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