Confinement and localisation in out-of-equilibrium spin chains

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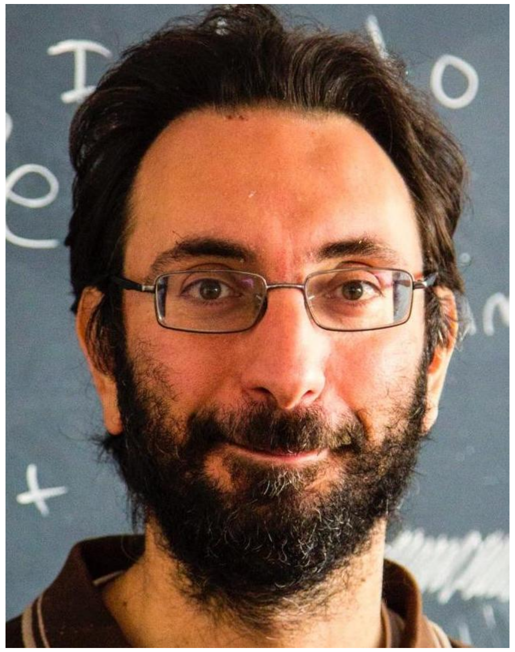
MOMENTUM OF INNOVATION

Collaborators



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Outline

1. Introduction: quantum quenches
2. Confinement and suppression of light-cone dynamics
3. Decay of the false vacuum and Bloch oscillations
4. 3-state Potts model: partial localization and baryons
5. Local quenches and escaping fronts
6. Summary and outlook

Breaking integrability

Quantum quench: a paradigmatic non-equilibrium protocol

Initial state: ground state of some local Hamiltonian

$$H_0|\Psi(0)\rangle = \mathcal{E}_0|\Psi(0)\rangle$$

Quantum quench: a sudden change in the Hamiltonian

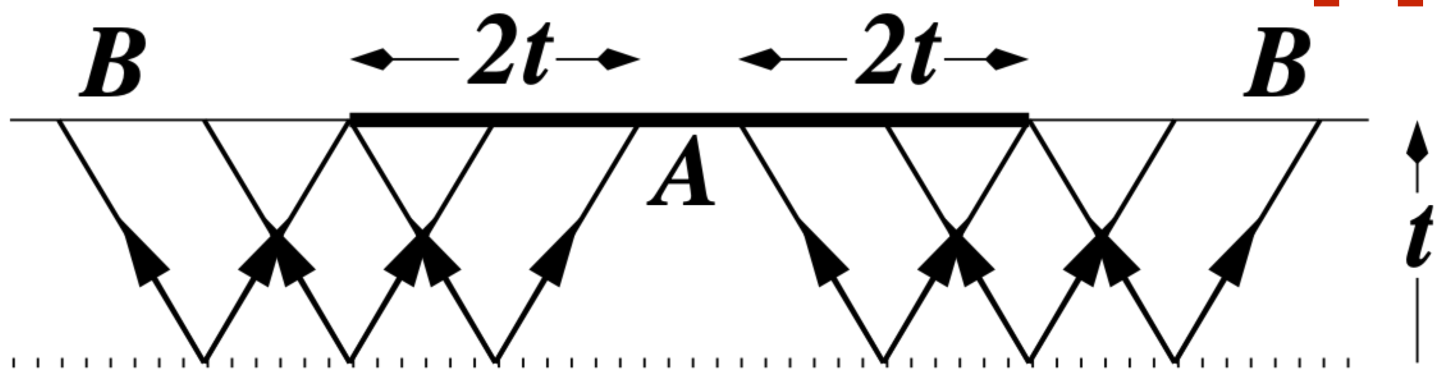
$$H_0 \xrightarrow[t=0]{} H \quad : \quad |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

Global quantum quench:
both H_0 and H are translationally invariant

$$\langle\Psi(0)|H|\Psi(0)\rangle \propto \mathrm{vol}(S)$$

Initial state is "thermodynamical" with finite energy density

How does relaxation happen?



P. Calabrese and J. Cardy, 2005

Initial state: source of quasi-particles

$$v(k) = \frac{dE}{dk} \qquad v_{LR} = \max_k v(k)$$

→ light-cone propagation of correlations and entanglement

Non-integrable dynamics: thermalization is expected

M. Rigol, V. Dunjko, and M. Olshanii, 2007

Integrable dynamics: Generalized Gibbs Ensemble

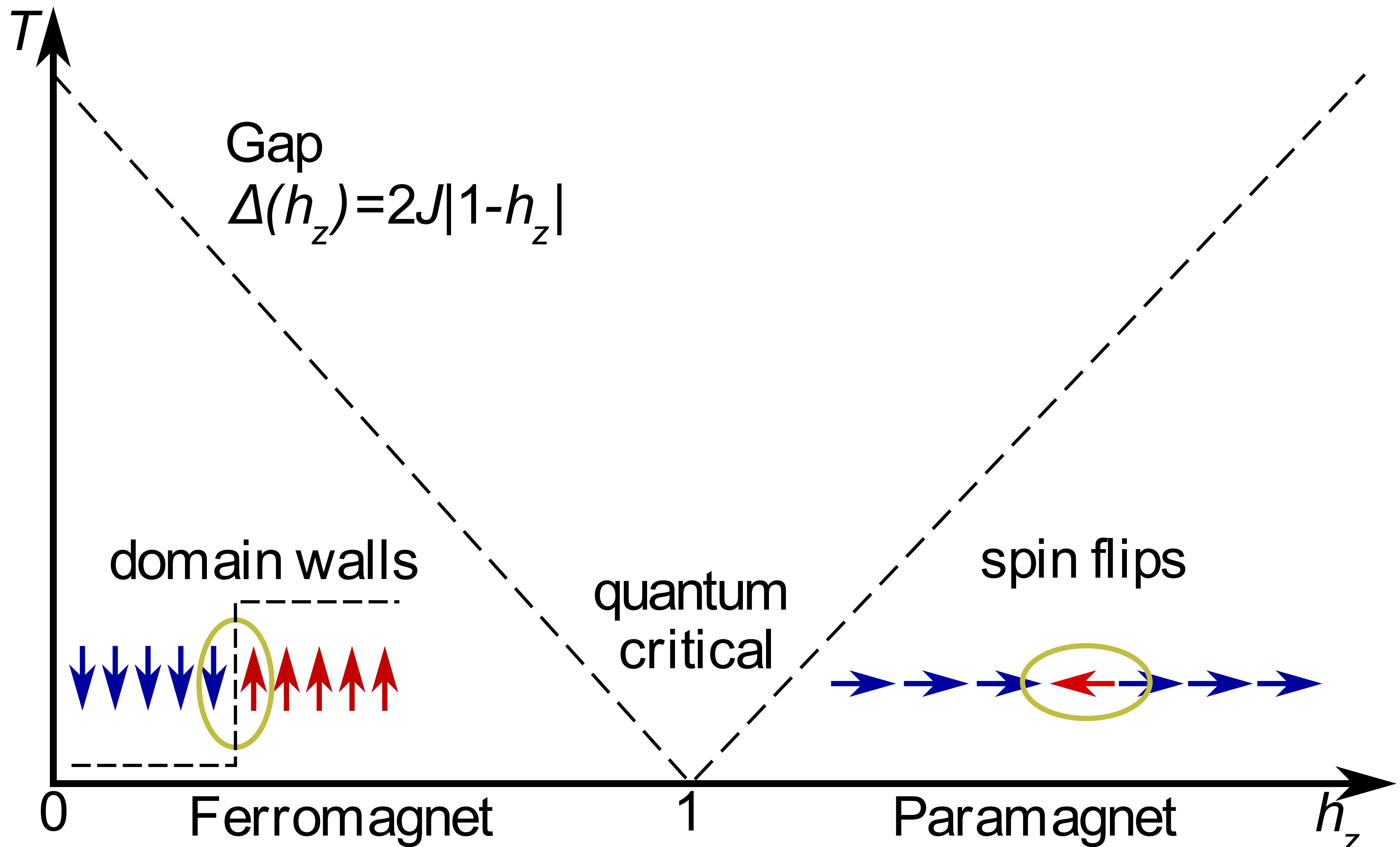
M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, 2007

Transverse field Ising model

$$H_{TFIM} = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z \right)$$

Model exactly solvable in terms of free fermions

$$\epsilon(k) = 2J\sqrt{1 + h_z^2 - 2h_z \cos(k)}$$



$\langle \sigma_i^x \rangle = (1 - h_z^2)^{1/8} \neq 0$ $\langle \sigma_i^x \rangle = 0$

Quantum quench in TFIM

$$H_{TFIM} = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z \right)$$

Model exactly solvable in terms of free fermions

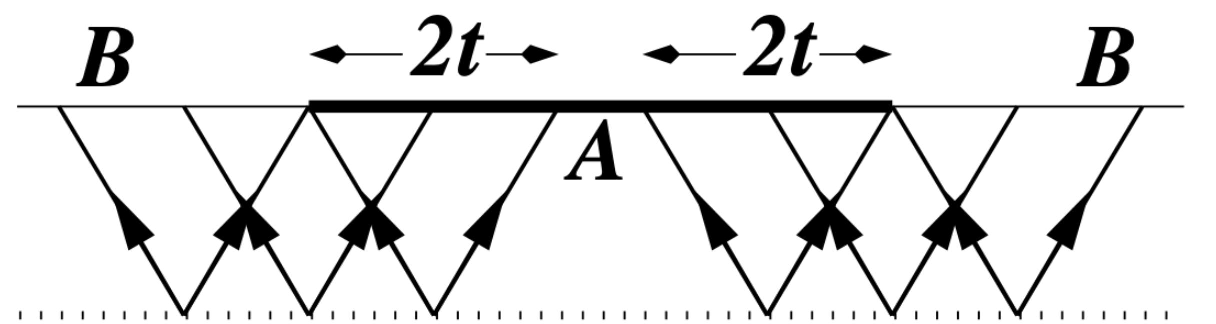
$h_z < 1$: ordered (FM) phase

$$\langle \sigma_i^x \rangle = (1 - h_z^2)^{1/8} \neq 0$$

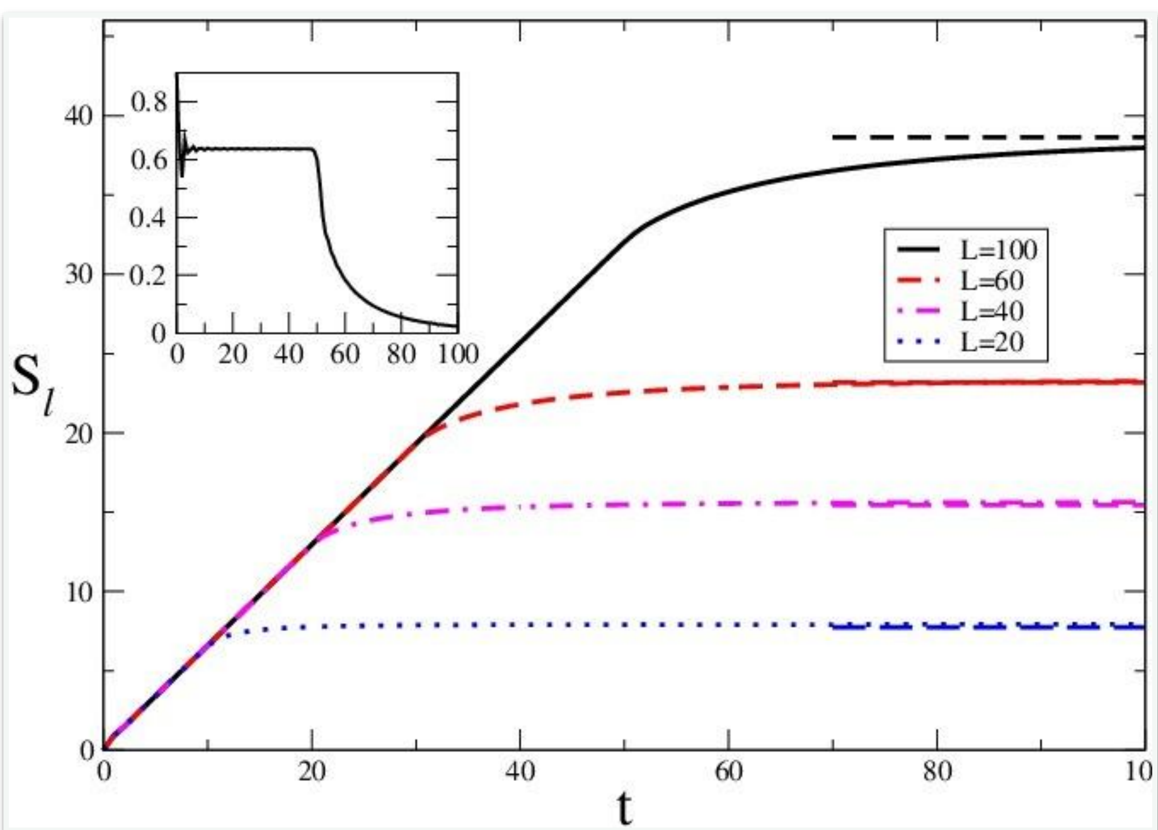
$h_z > 1$: disordered (PM) phase

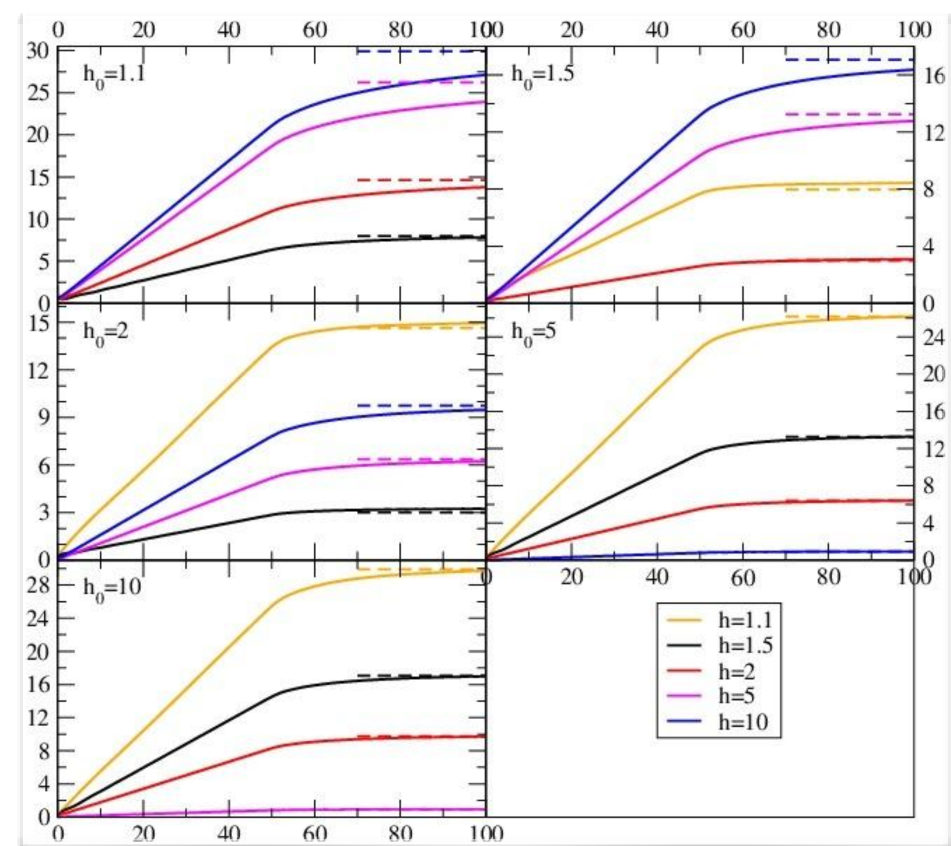
$$\langle \sigma_i^x \rangle = 0$$

Entanglement entropy between interval and rest of the system



P. Calabrese and J. Cardy, 2005





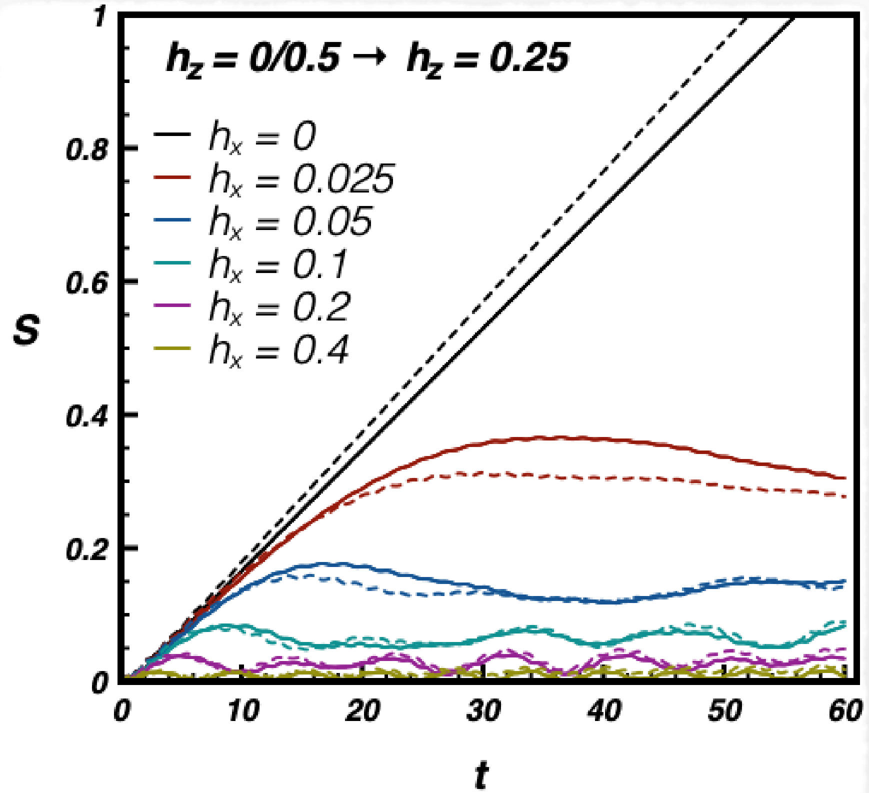
Non-integrable Ising chain in FM phase $h_z < 1$

$$H = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$

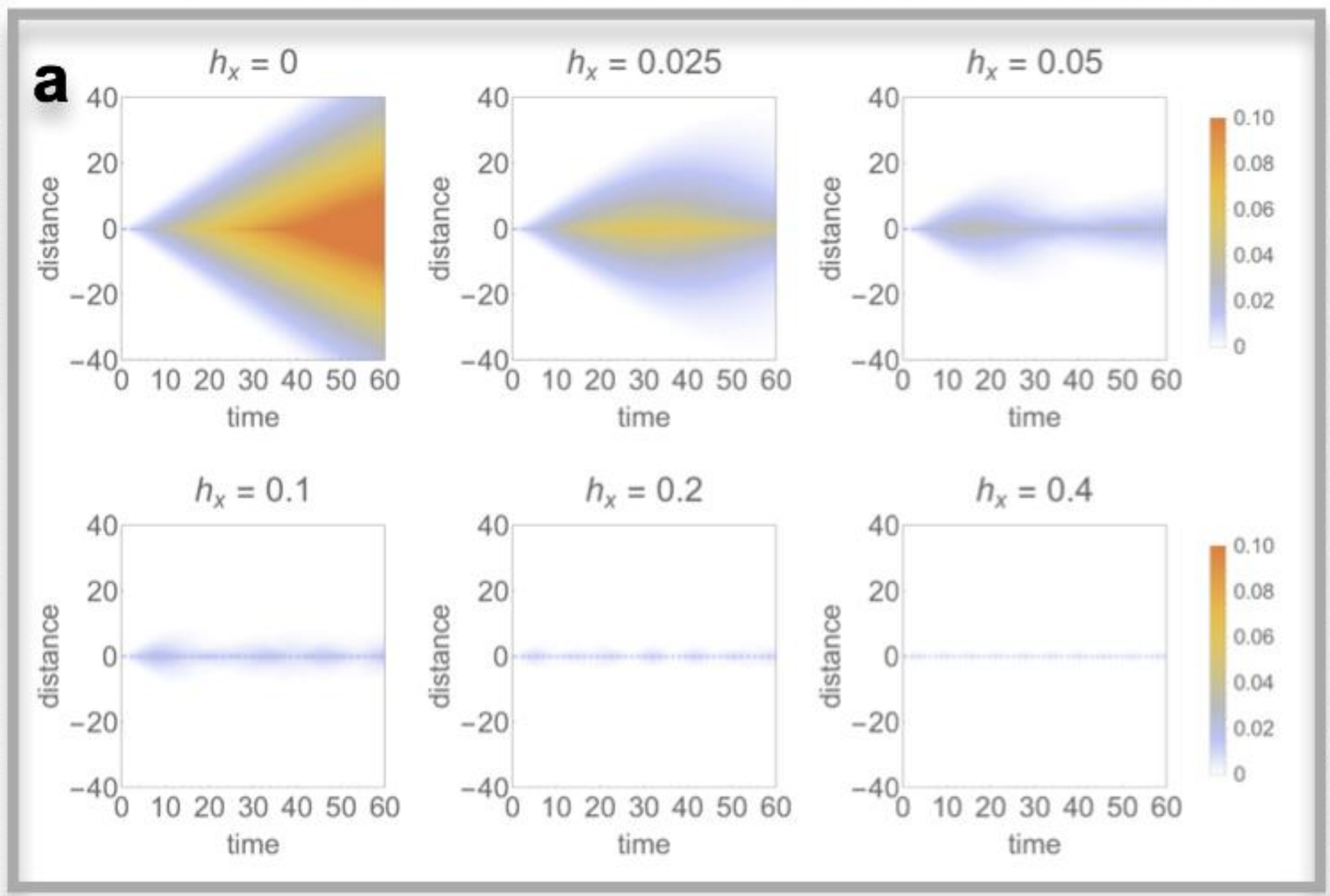
$$|\Psi(0)\rangle = \cdots \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \cdots \qquad h_x > 0$$

$\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$ **: light-cone gets suppressed!**

Entanglement entropy between left and right halves



iTEBD simulation by M. Collura (SISSA)

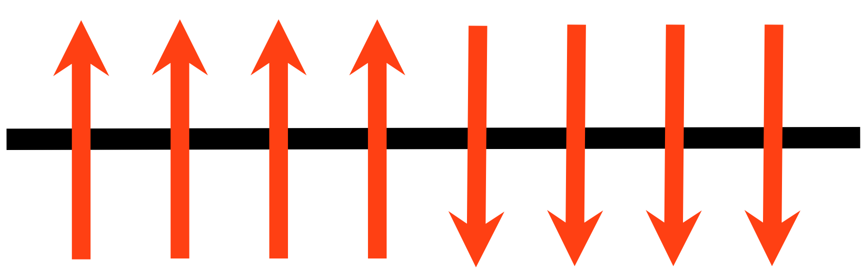


Confinement in the Ising model

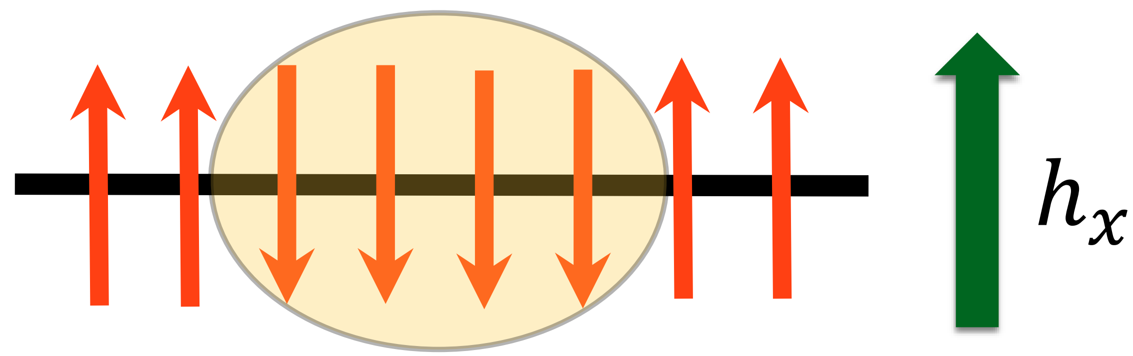
McCoy & Wu '78

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

- For $h_x = 0$ free fermions with dispersion $\epsilon(k) = 2J\sqrt{1 + h_z^2 - 2h_z \cos(k)}$
- For $h_z < 1$ (FM phase), the massive fermions correspond to domain walls separating domains of magnetisation $\sigma = \pm(1 - h_z^2)^{1/8}$



Free DW



Bound state = meson

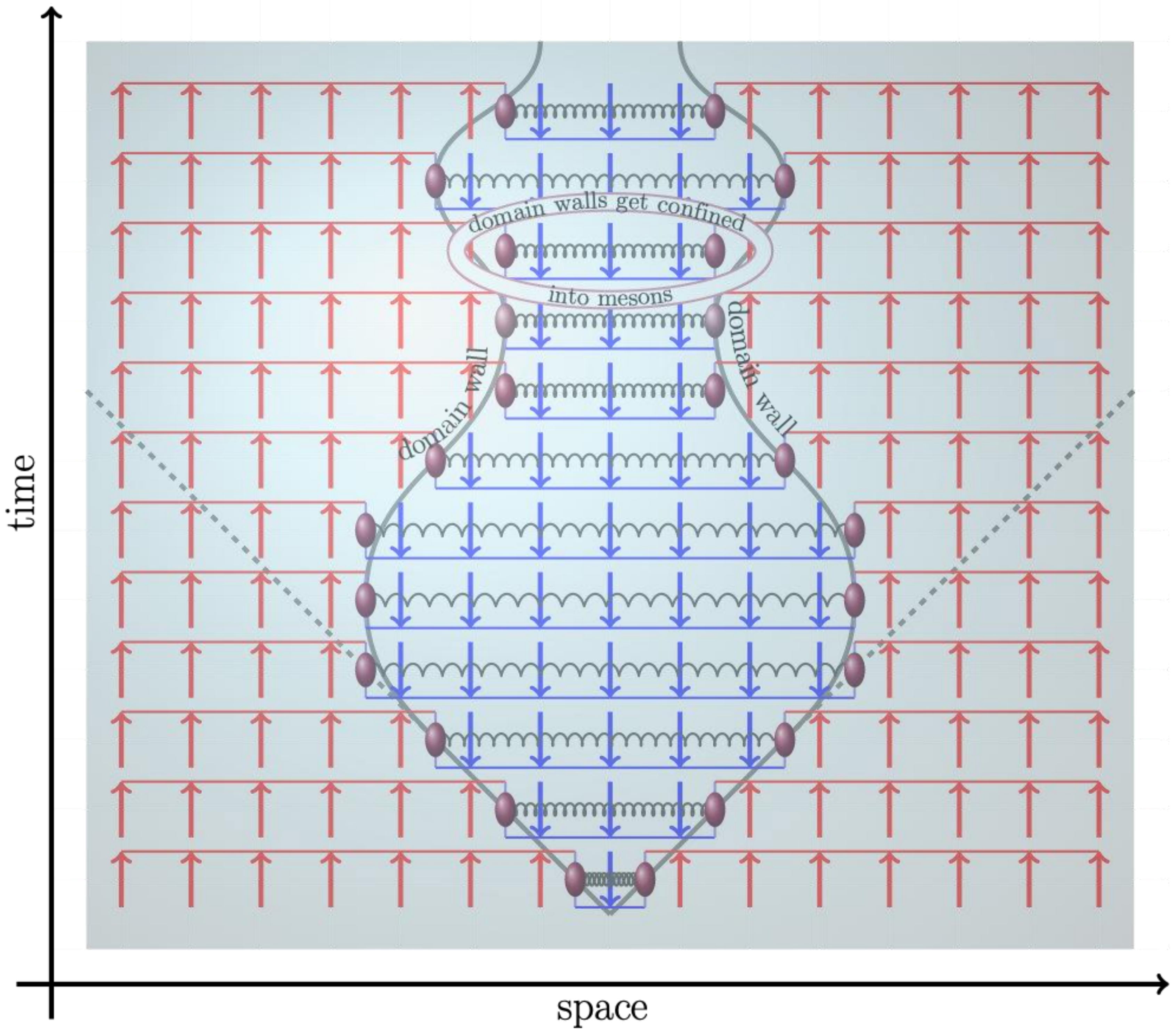
- h_x induces an attractive interaction between DWs that for small enough h_x can be approximated with a linear potential

$$V(x) = 2Jh_x\sigma|x|$$

- DWs do not propagate freely but get confined into **mesons**

Real-time confinement

Localisation of quasi-particles / suppression of entropy growth



M. Kormos, M. Collura, G. Takács, and P. Calabrese,
Nature Physics 13, 246–249 (2017)

The meson spectrum

Consider two fermions in 1D with Hamiltonian

$$H = \epsilon(k_1) + \epsilon(k_2) + \chi|x_2 - x_1| = \omega(k;K) + \chi|x|$$

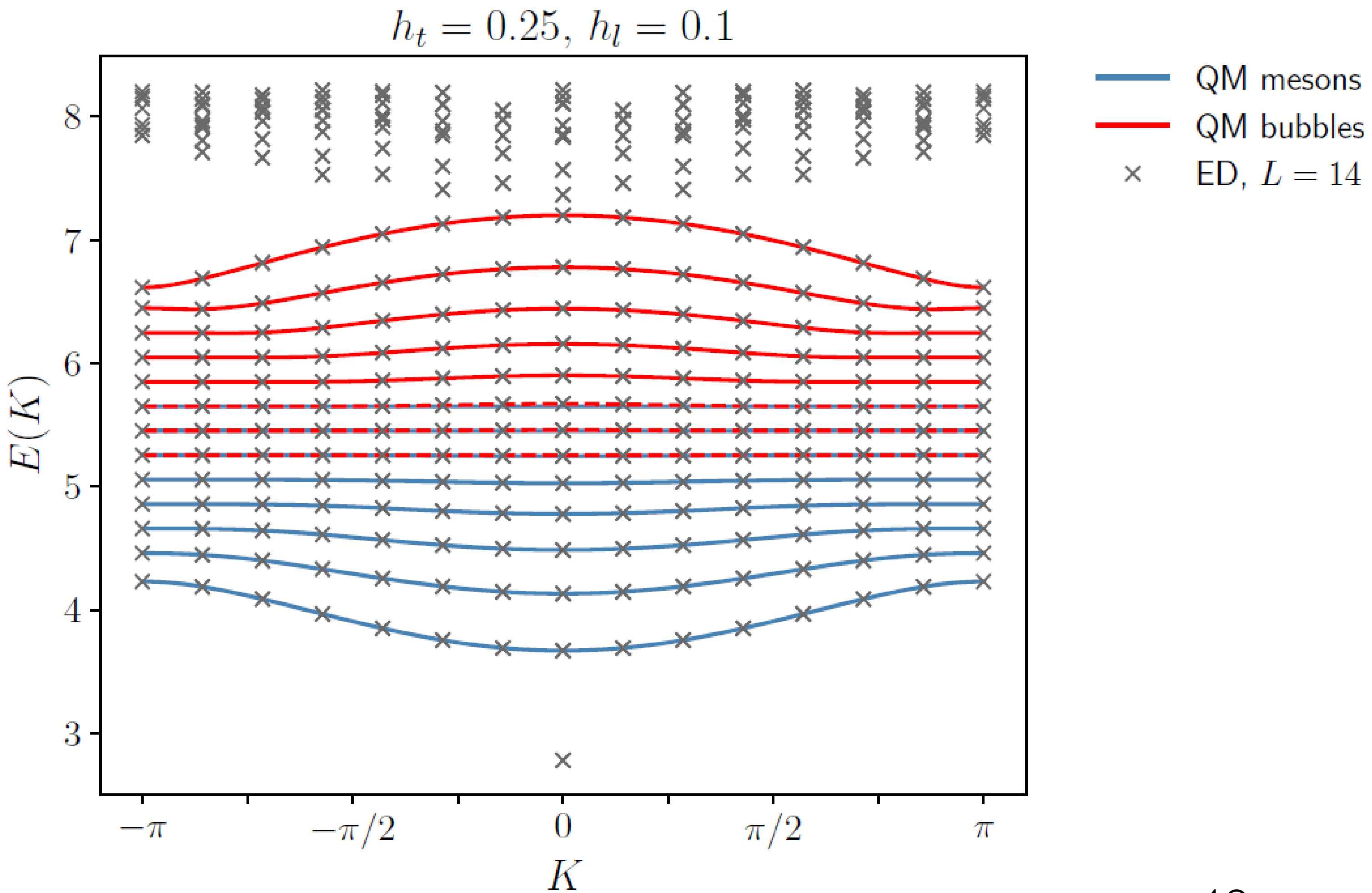
Rutkevich, 2008

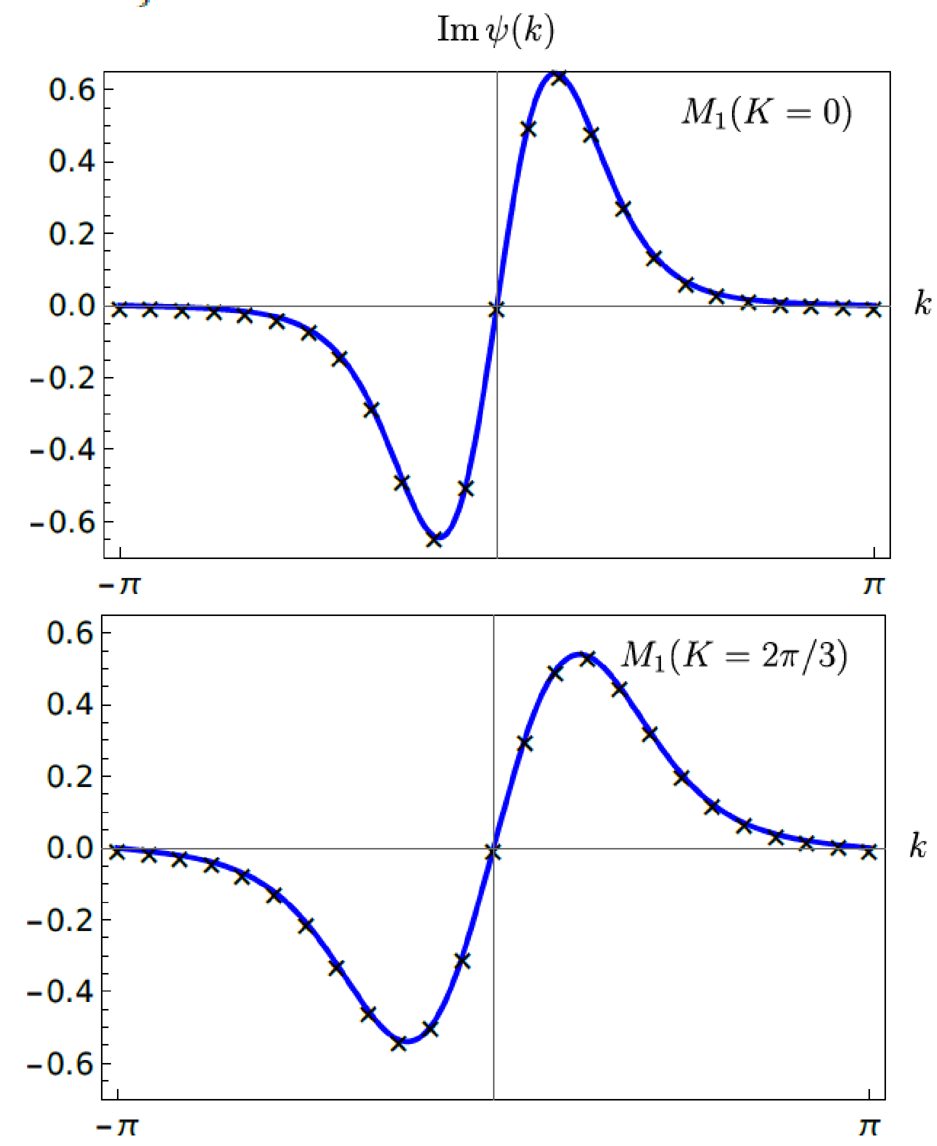
$$k_{1,2} = K/2 \pm k \qquad \chi = 2Jh_l(1-h_t^2)^{1/8}$$

Schrödinger equation → mesons labelled by species number

$$H\psi_{n,K}(x) = \sum_{x'} H(x,x';K)\psi_{n,K}(x') = E_n(K)\psi_{n,K}(x)$$

Krasznai & Takács, 2024

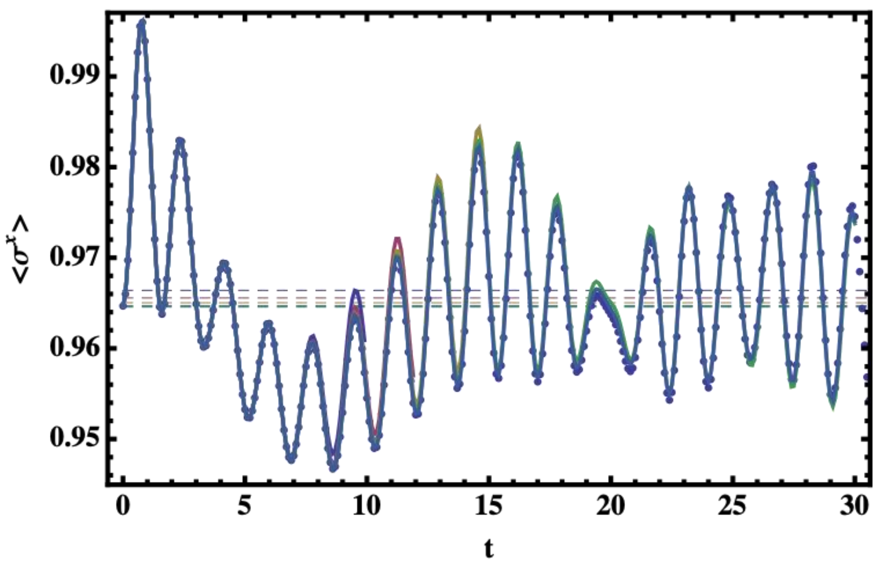




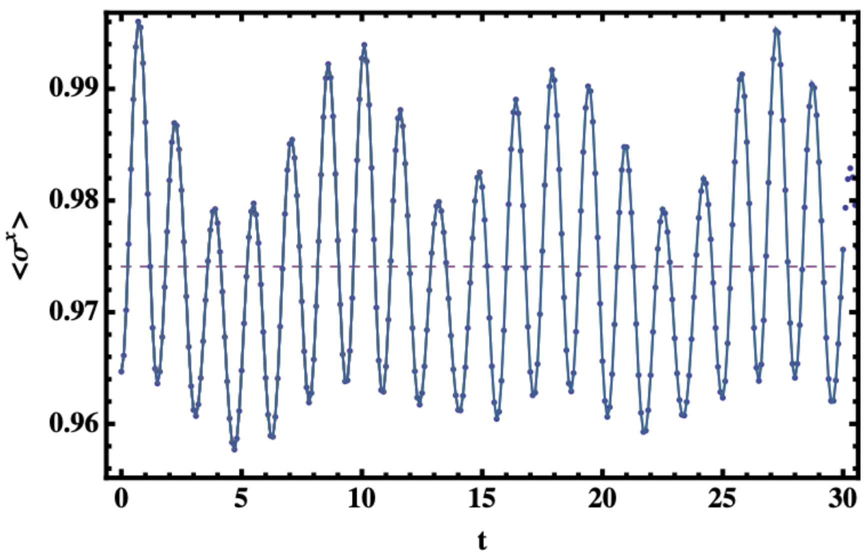
Quench spectroscopy from time evolution of magnetisation

Quenches from FM to FM: no relaxation observed!

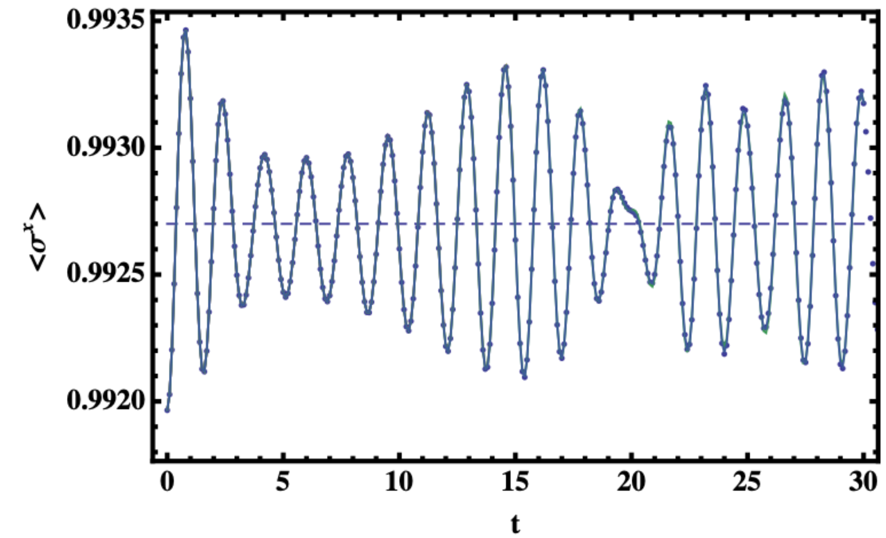
$h_z^0=0.5,\ h_x^0=0,\ h_z=0.25,\ h_x=0.1$



$h_z^0=0.5,\ h_x^0=0,\ h_z=0.25,\ h_x=0.2$

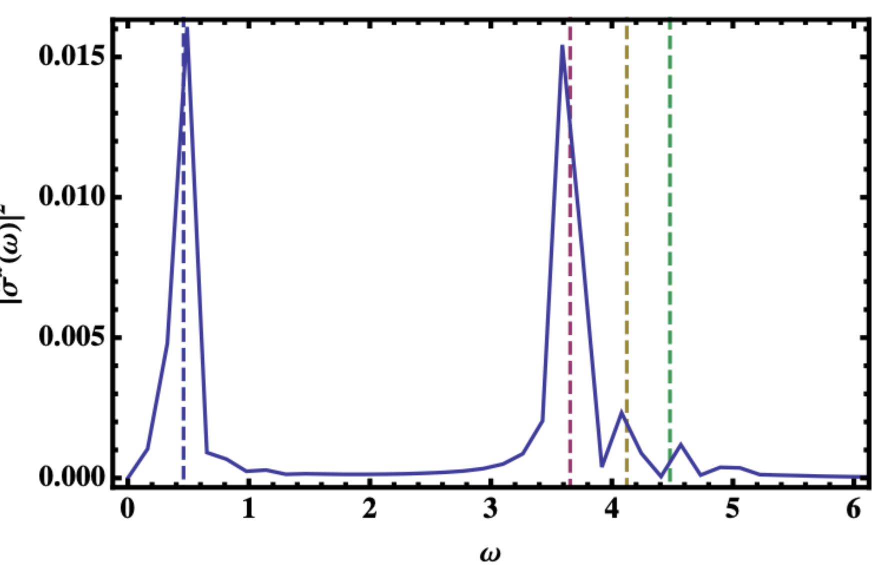


$h_z^0=0.25,\ h_x^0=0,\ h_z=0.25,\ h_x=0.1$



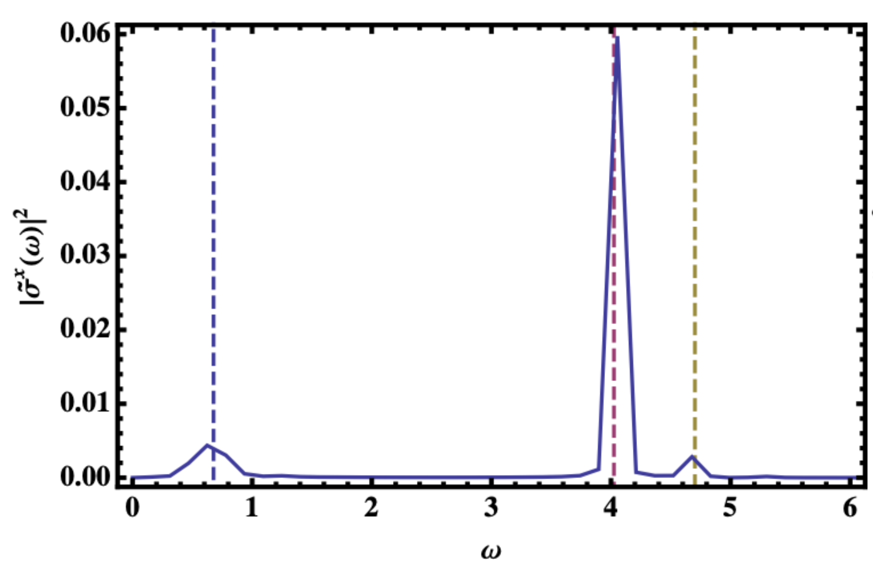
iTEBD vs. ED with L=8,10,12

Power spectrum of $\langle \sigma_x \rangle$ compared to semiclassical meson spectra



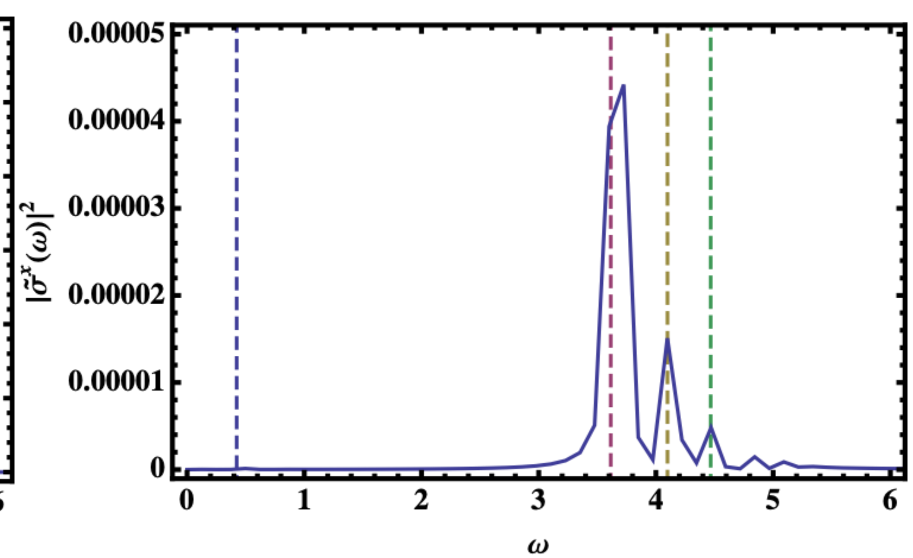
$m_2-m_1=0.46,\ m_1=3.7,$

$m_2=4.1,\ m_3=4.5$



$m_2-m_1=0.68,\ m_1=4.0,$

$m_2=4.7$



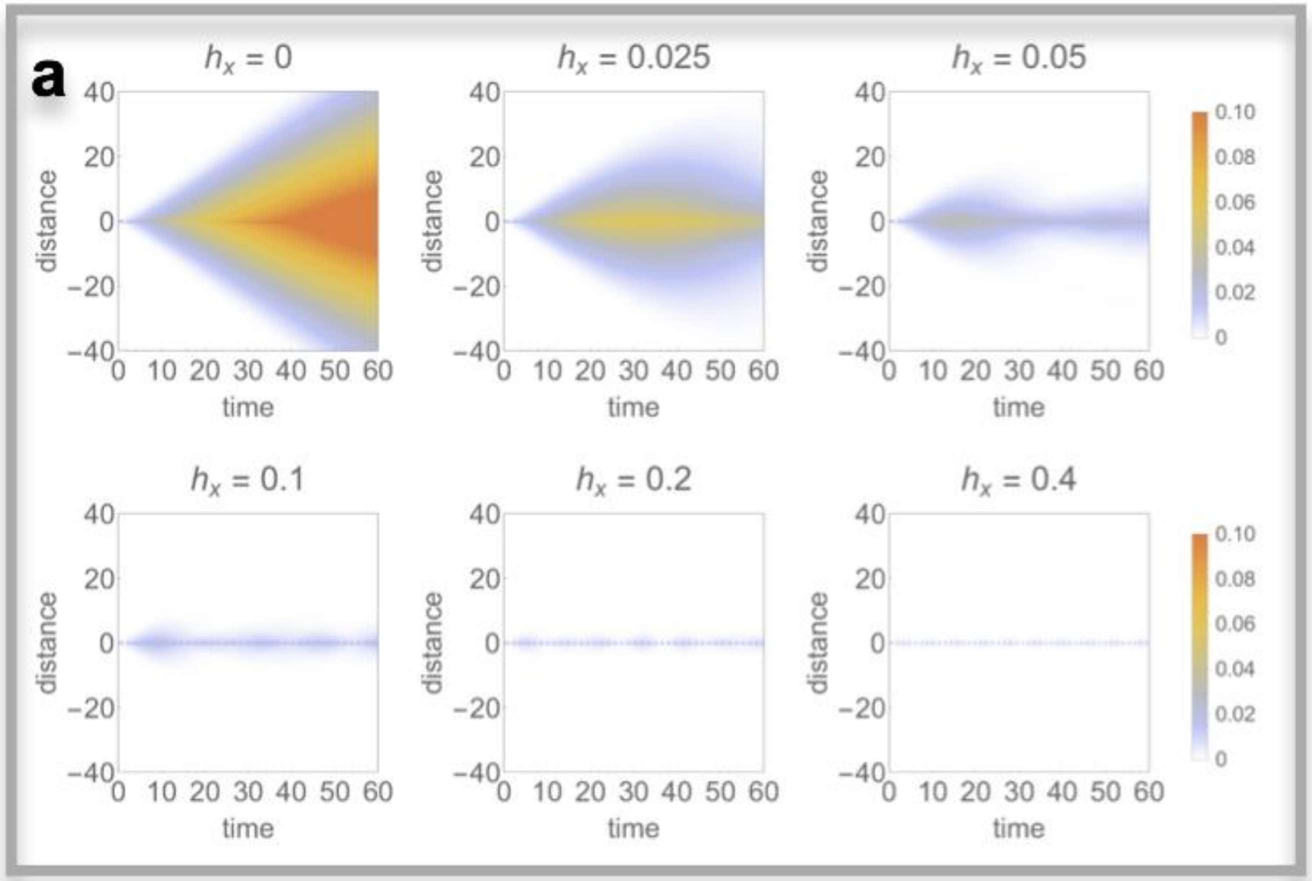
$m_2-m_1=0.46,\ m_1=3.7,$

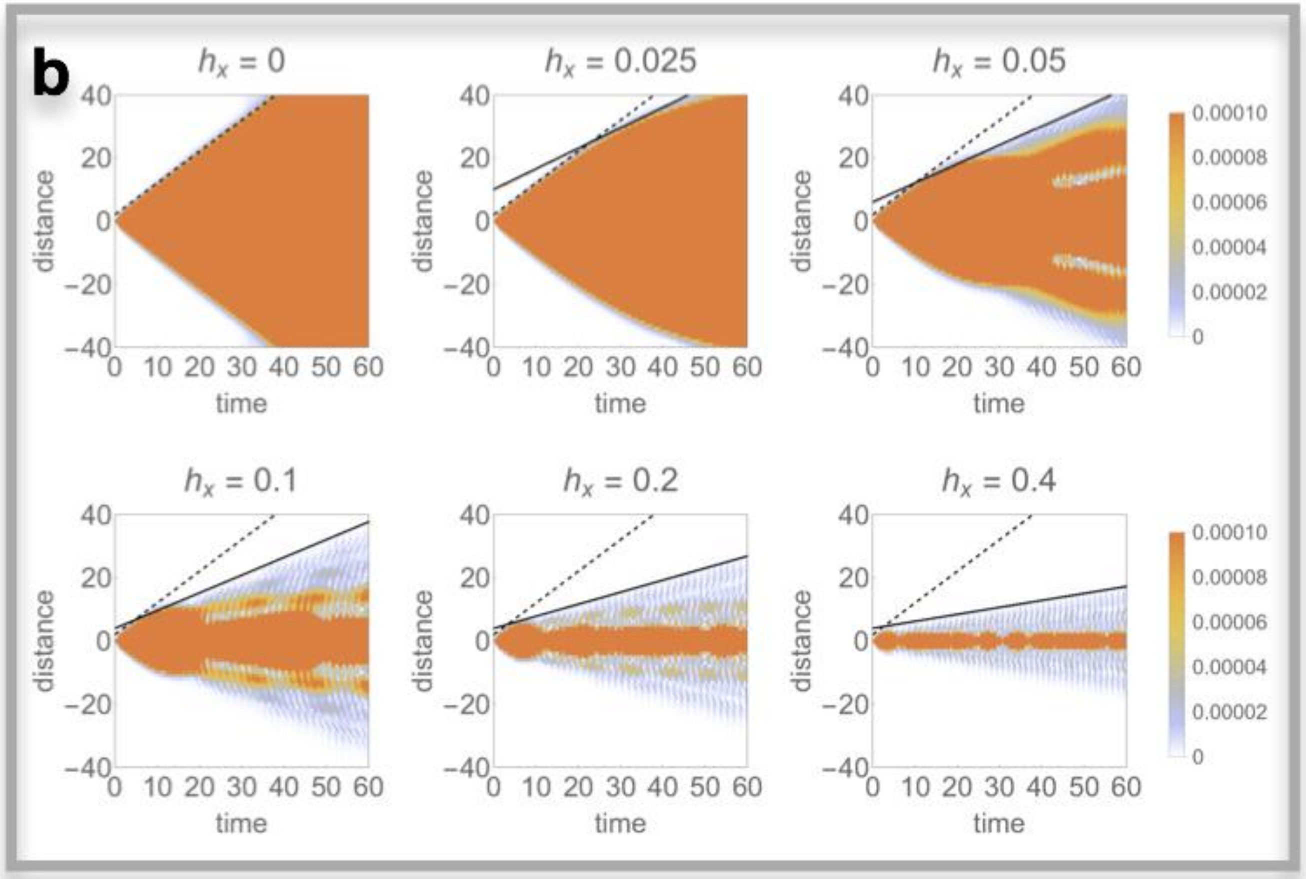
$m_2=4.1,\ m_3=4.5$

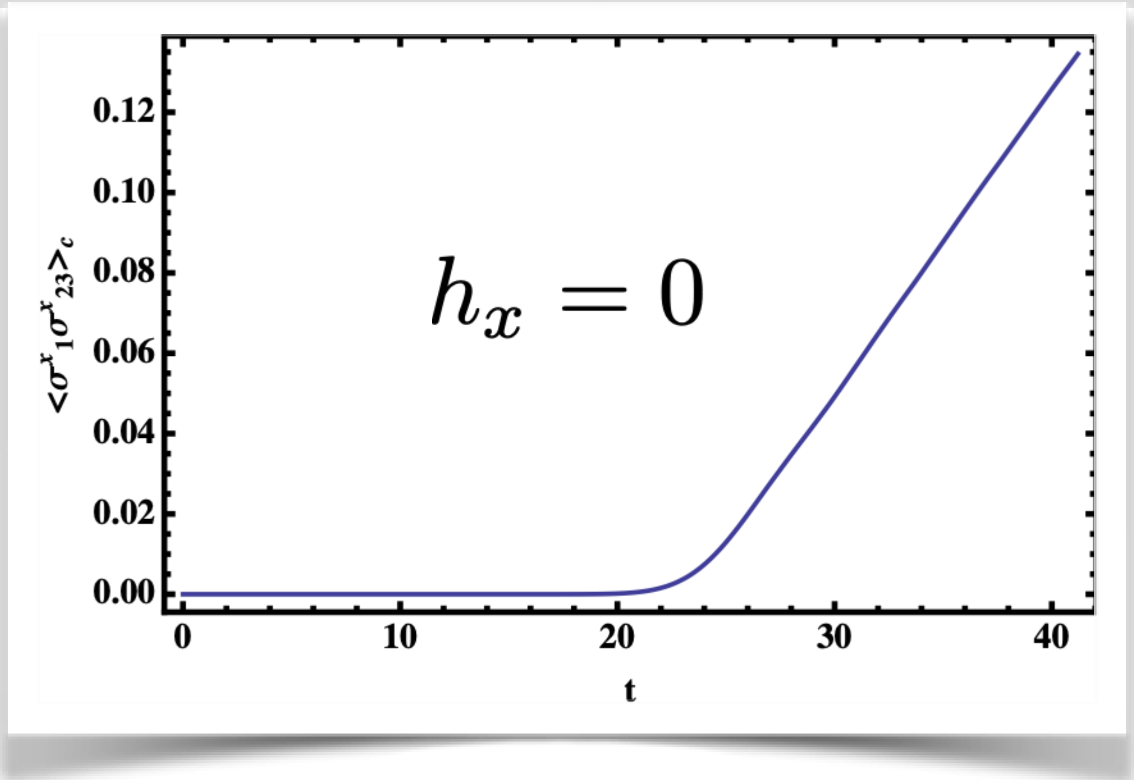
It is the mesons that are the source of the persistent oscillations!

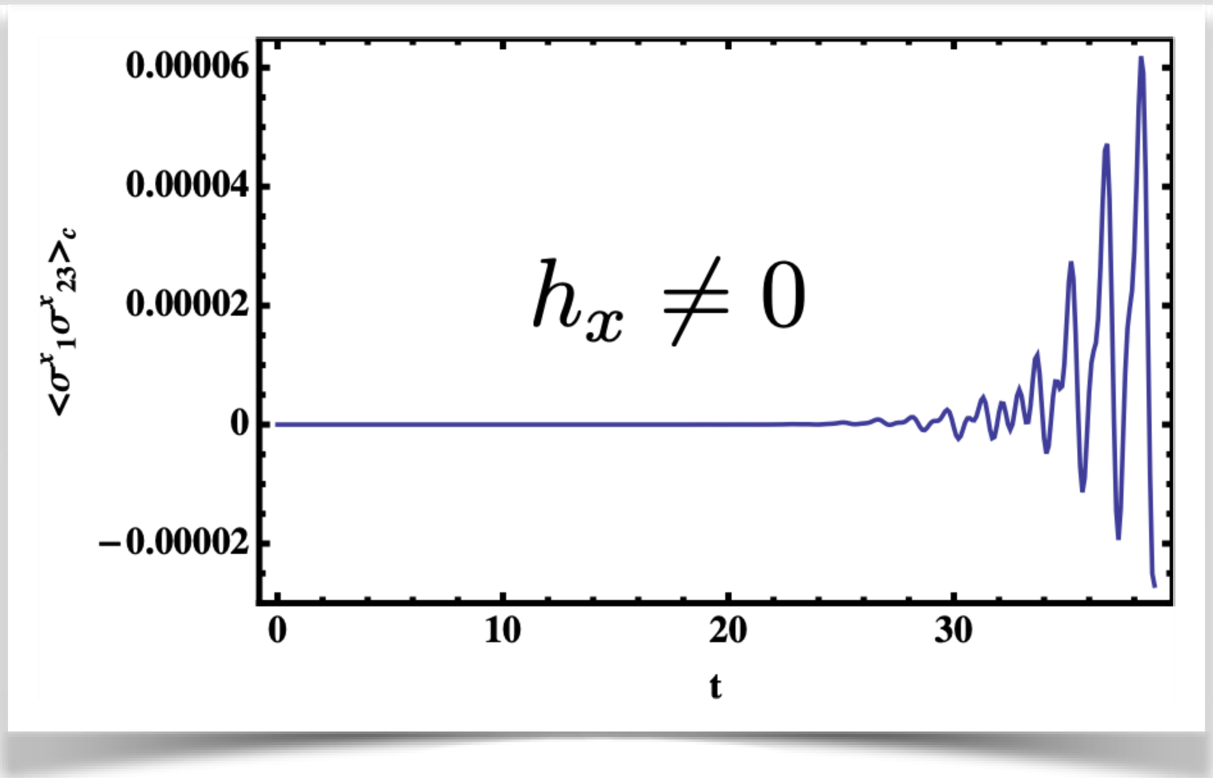
Another effect of mesons: escaping correlations

$$\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$$



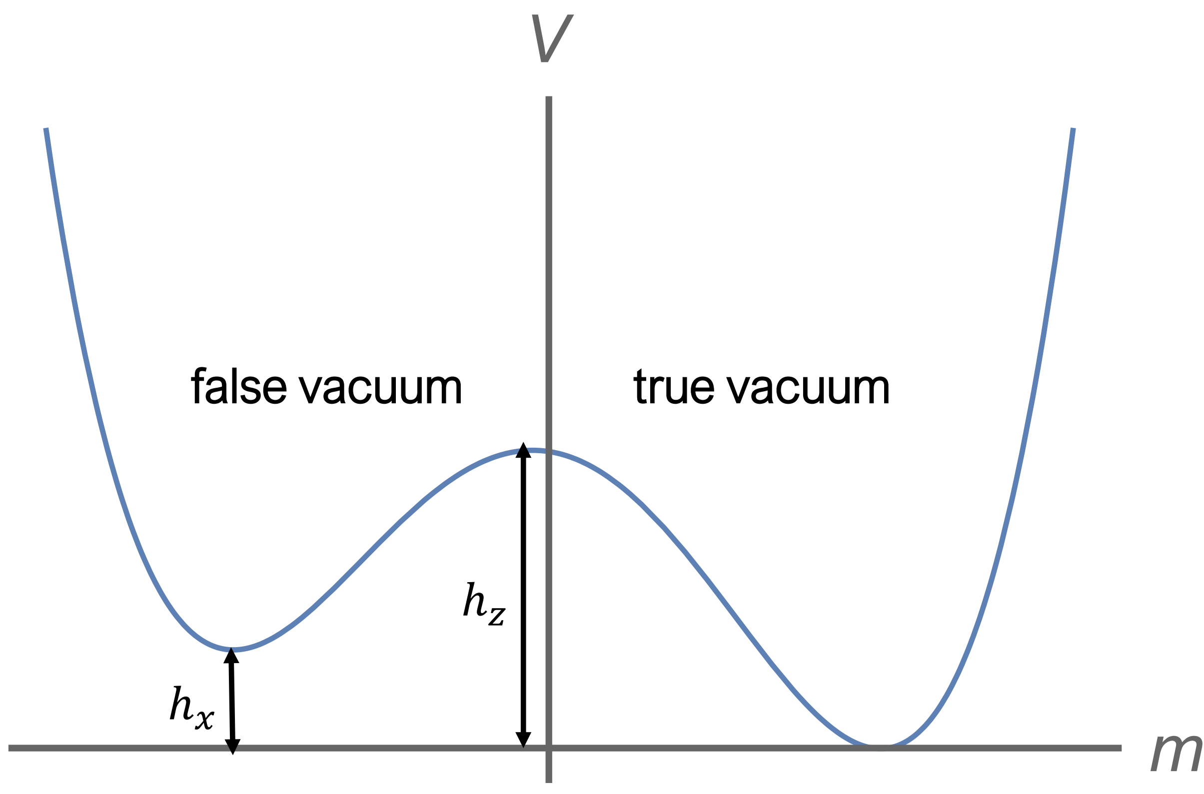






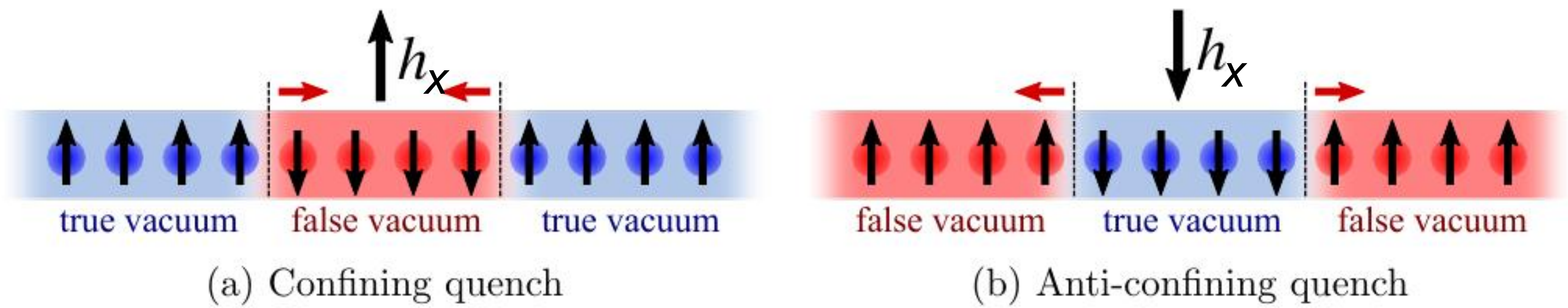
Decay of the false vacuum

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$



So far: confining quench –
h_x parallel to initial magnetisation

Other option: anti-confining quench

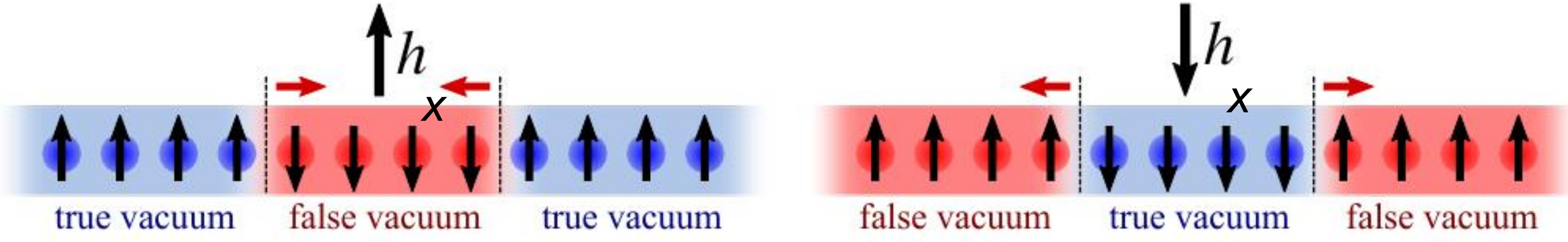


Attractive force **Repulsive force**

Expectation: nucleated bubbles of the true vacuum expand

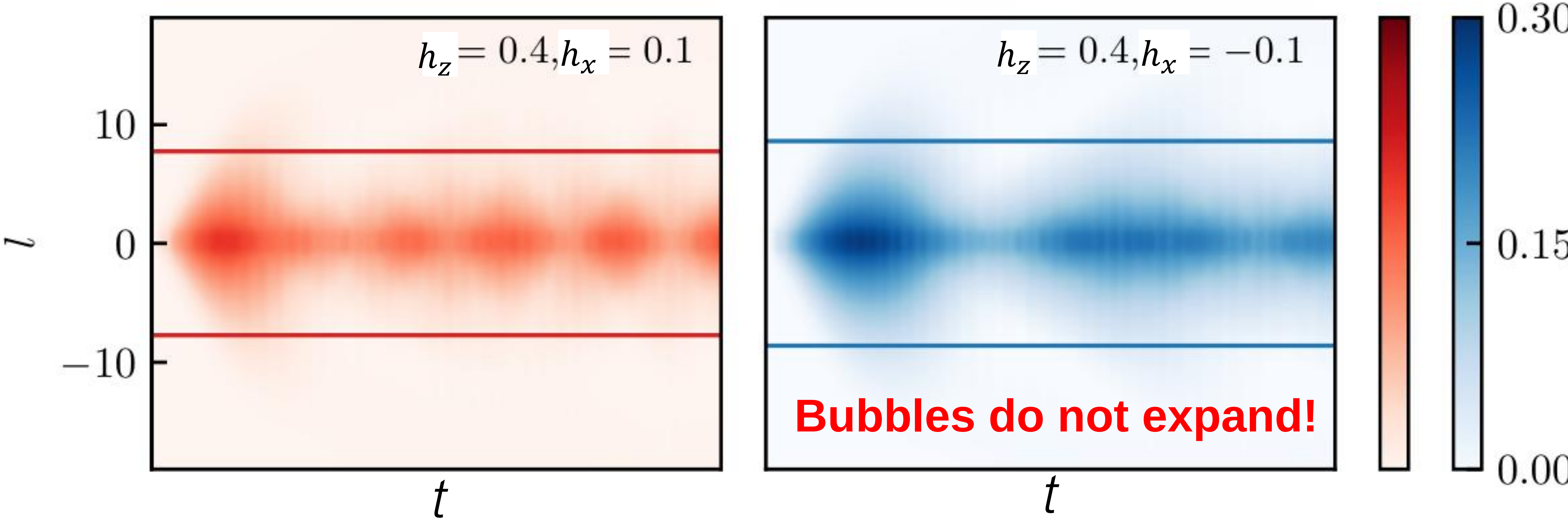
Decay of the false vacuum (QFT: Coleman scenario)

Localisation in anti-confining quenches



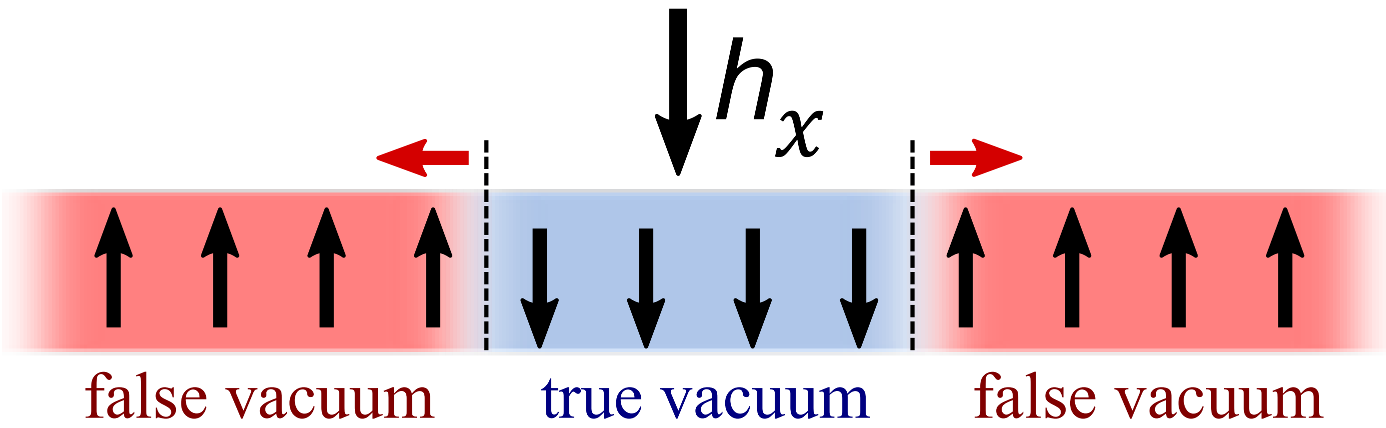
(a) Confining quench (b) Anti-confining quench

$$\langle \sigma_1^x \sigma_{l+1}^x \rangle_c$$



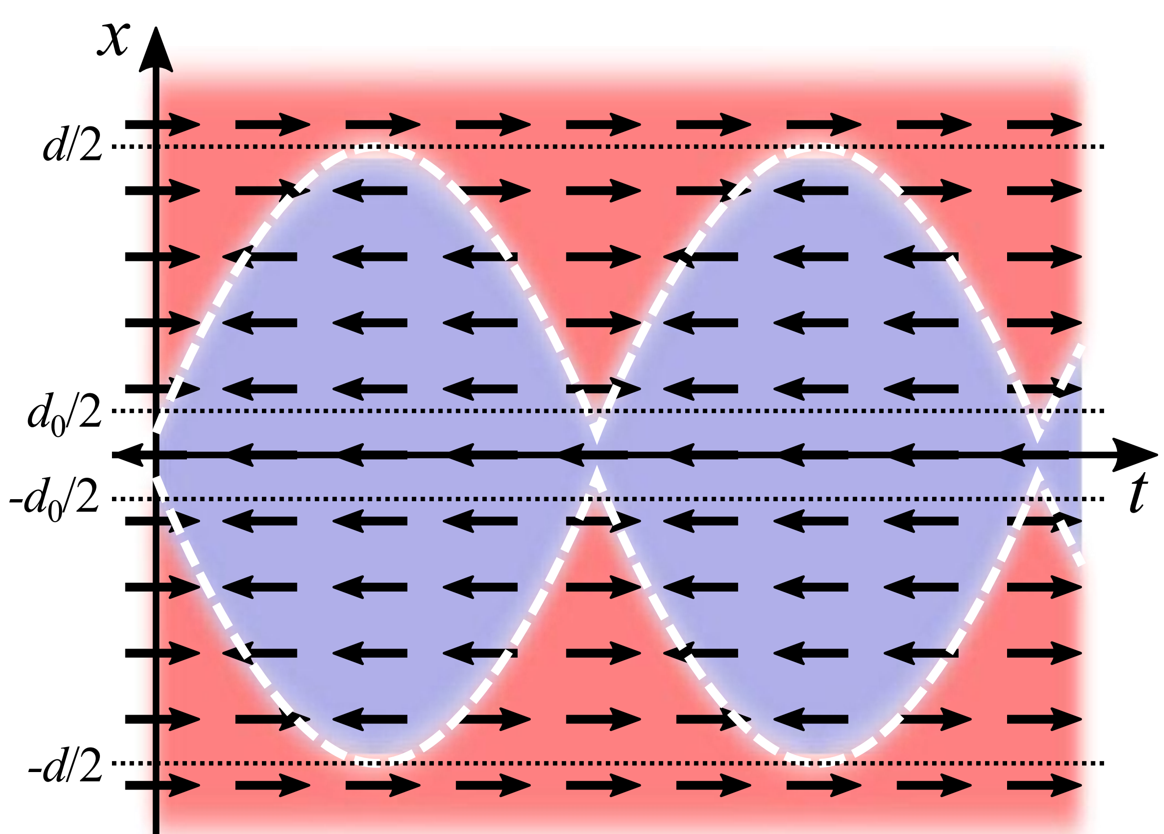
O. Pomponio, M. A. Werner, G. Zaránd and G. Takács,
SciPost Phys. 12, 061 (2022)

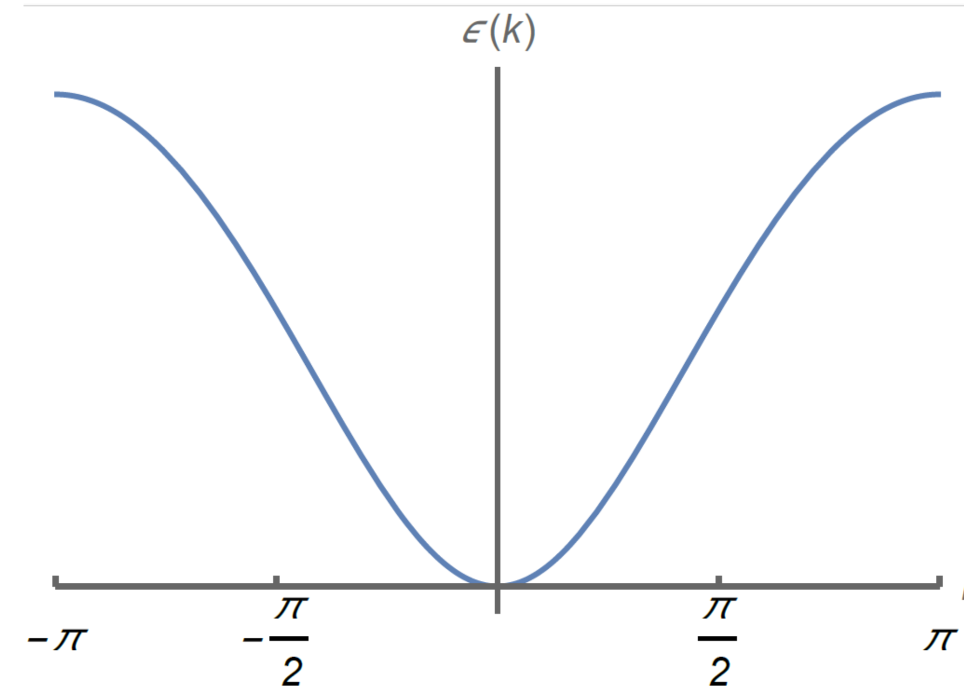
Bloch oscillations



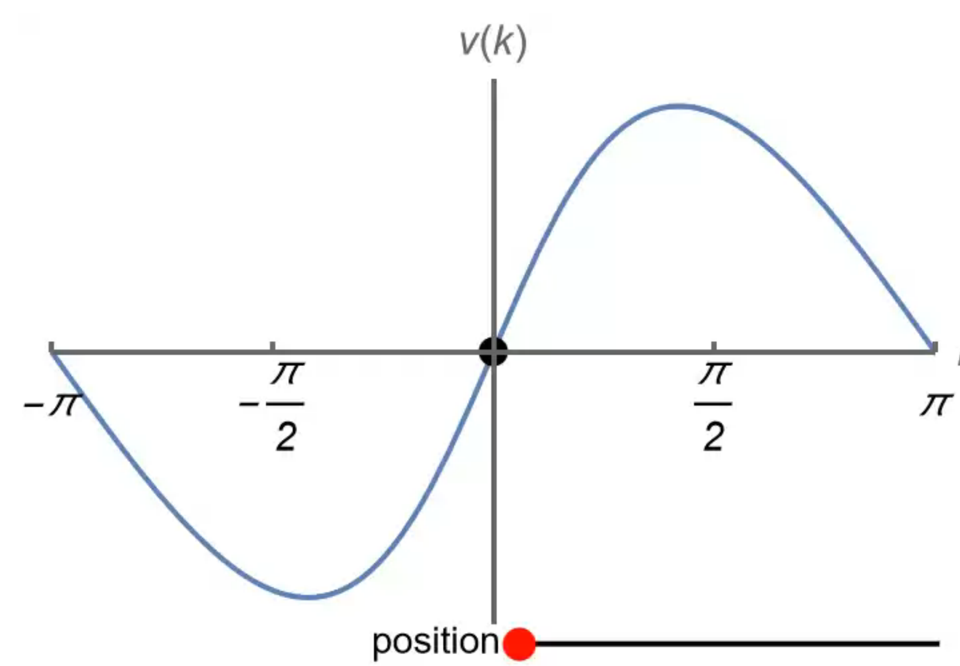
$$V(x) = -\chi|x| \rightarrow \frac{dk}{dt} = \chi$$

$$\omega_{Bloch} = \frac{2\pi}{\chi}$$





$$v(k) = \frac{\partial \epsilon(k)}{\partial k}$$

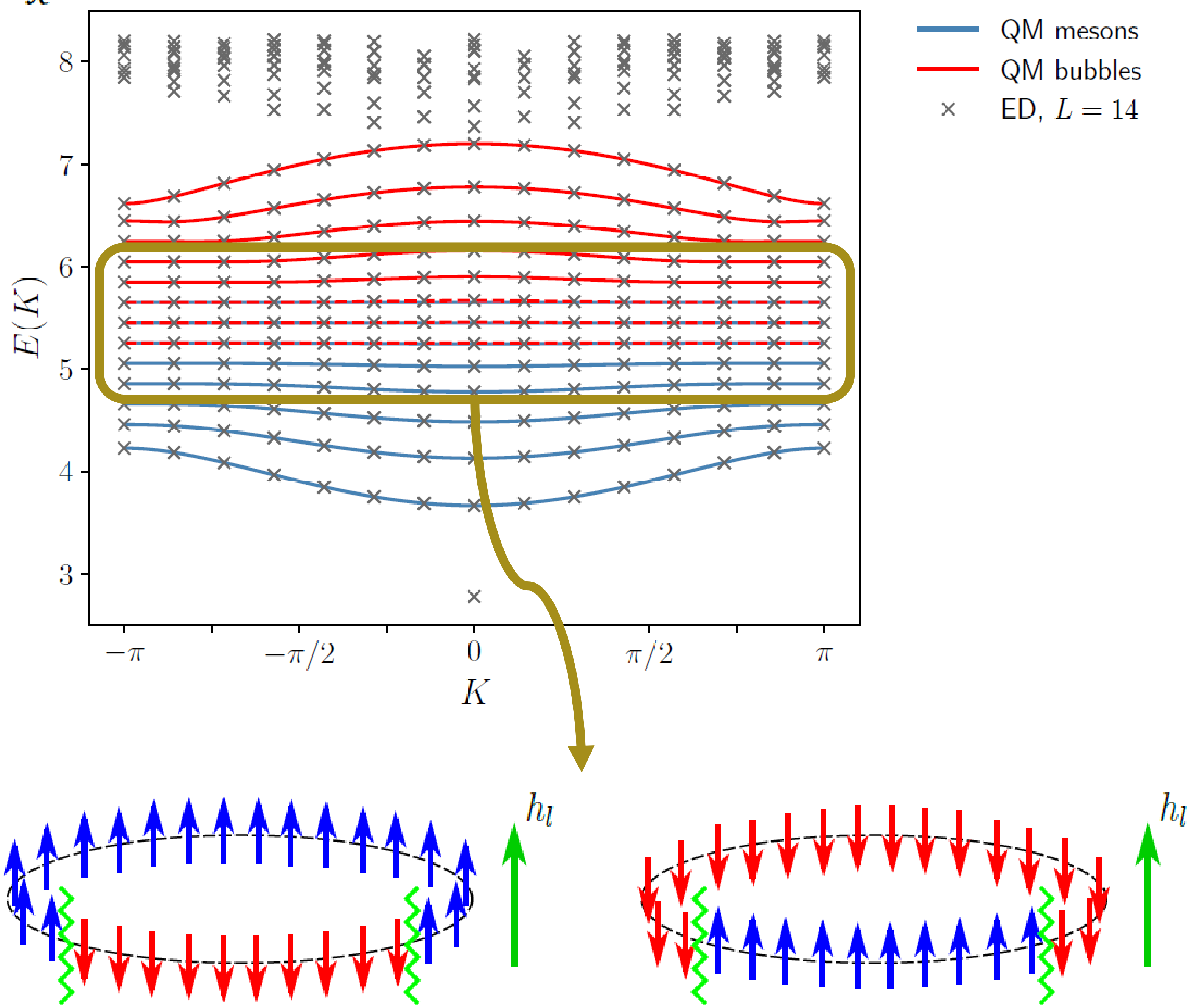


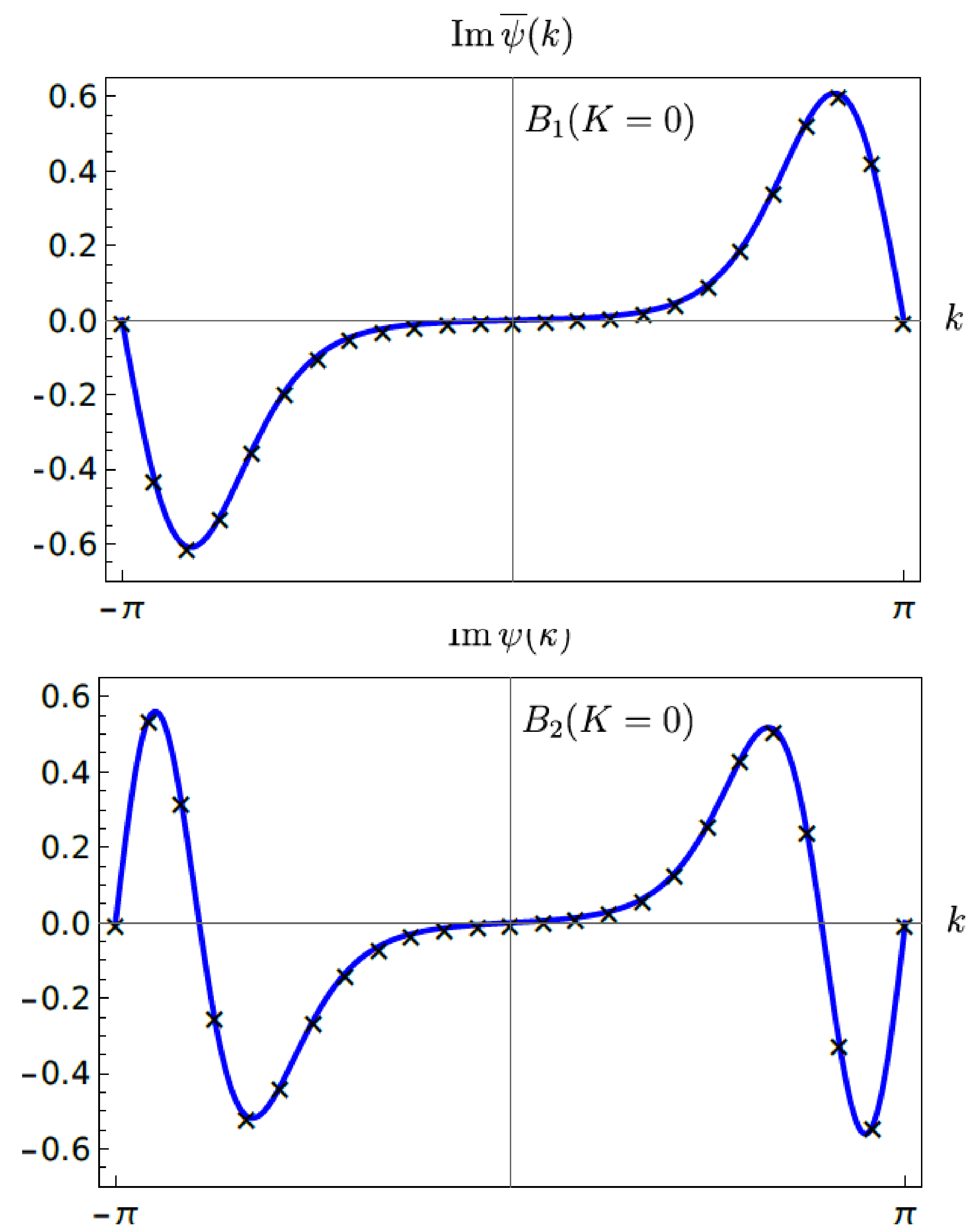
The bubble spectrum

Wannier-Stark localization/Bloch oscillation → localized bubble states

$$\sum_{x'}\left[H_0(x-x',K)-|\chi||x|\right]\overline{\psi}_{n,K}(x')=\overline{E}_n(K)\overline{\psi}_{n,K}(x)$$

Krasznai & Takács, 2024



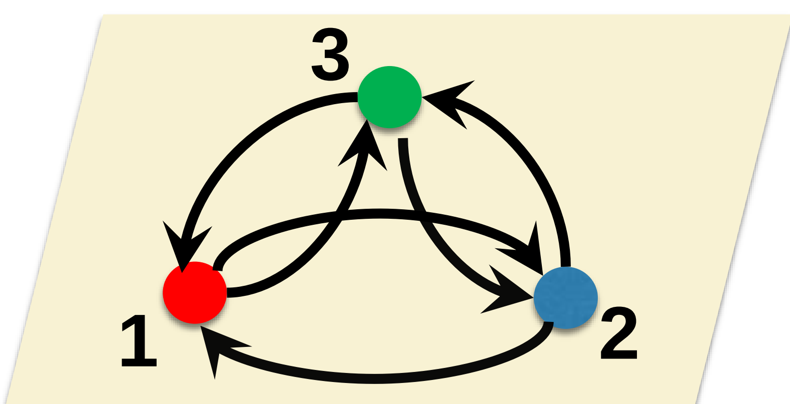


3-state Potts model

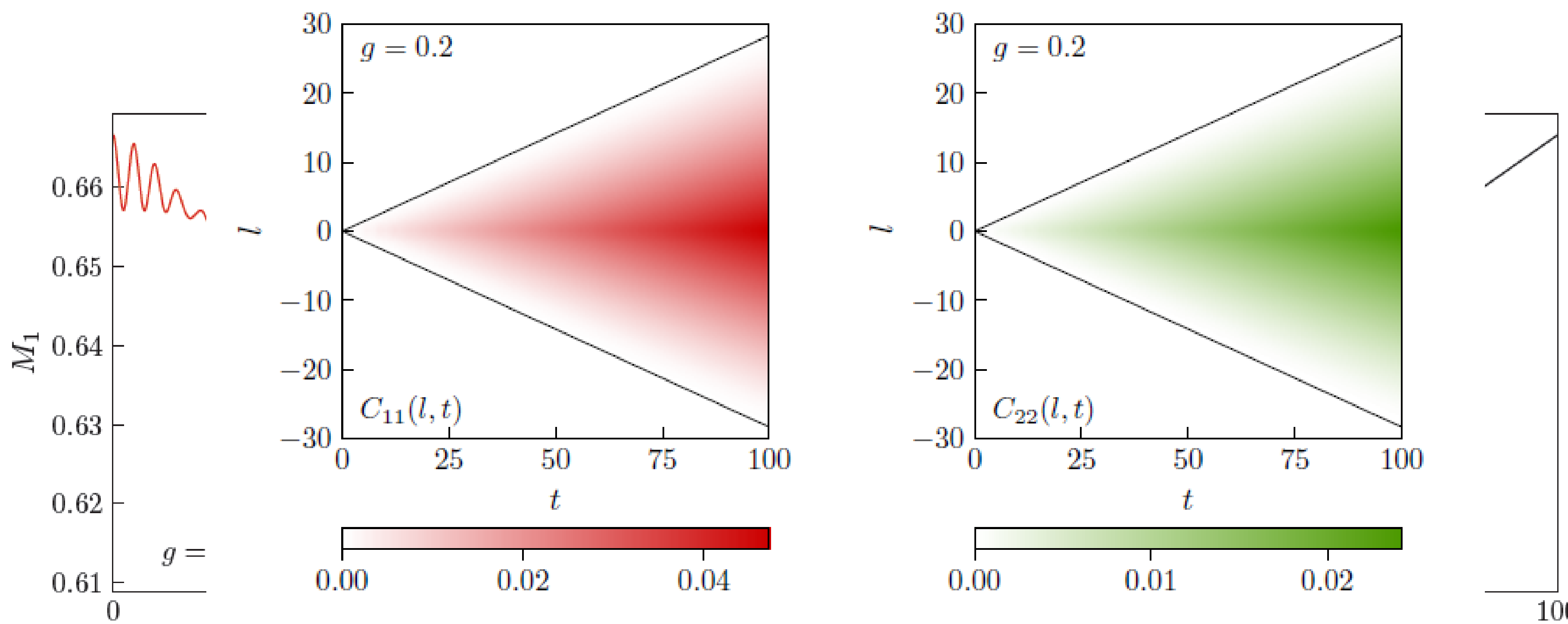
$$H_{trans} = -J \sum_i \left(\sum_{\mu=1}^{3} P_i^{\mu} P_{i+1}^{\mu} + g\tilde{P}_i \right)$$

$$P^0 = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad P^1 = \frac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad P^2 = \frac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad \tilde{P} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Quenches from "red" state



Elementary excitations: interacting kinks



(a) Magne

(a) Connected 11 (*red-red*) correlations.

(b) Connected 22 (*green-green*) correlations.

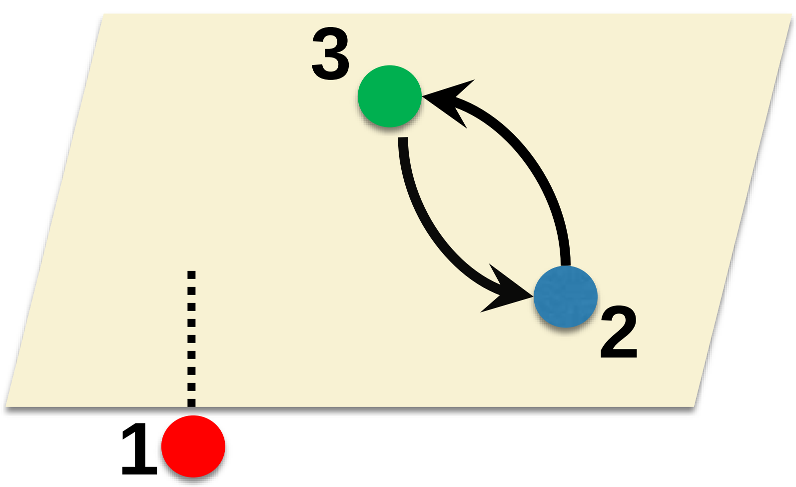
O. Pomponio, A. Krasznai and G. Takács, 2024

Aligned quenches

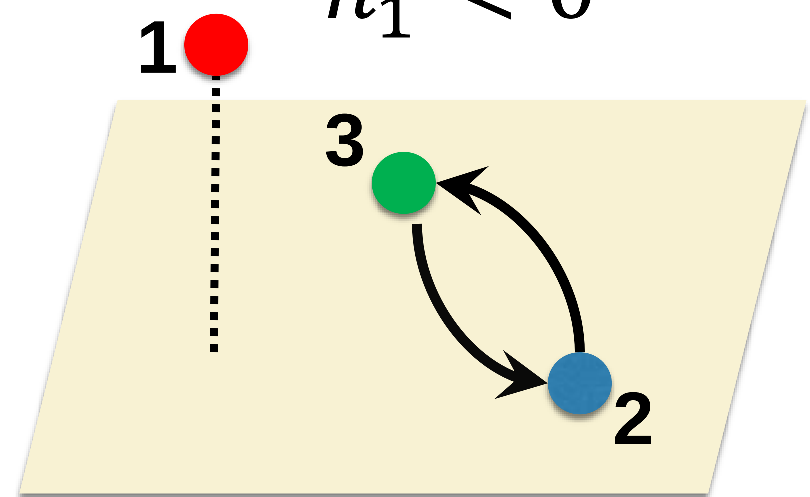
Start from “red” state

$$H_{postquench} = -J \sum_i \left(\sum_{\mu=1}^{3} P_i^\mu P_{i+1}^\mu + g\tilde{P}_i + h_1 P_i^1 \right)$$

$h_1 > 0$



$h_1 < 0$



Elementary excitations:
mesons + baryons
+ metastable Ising kinks

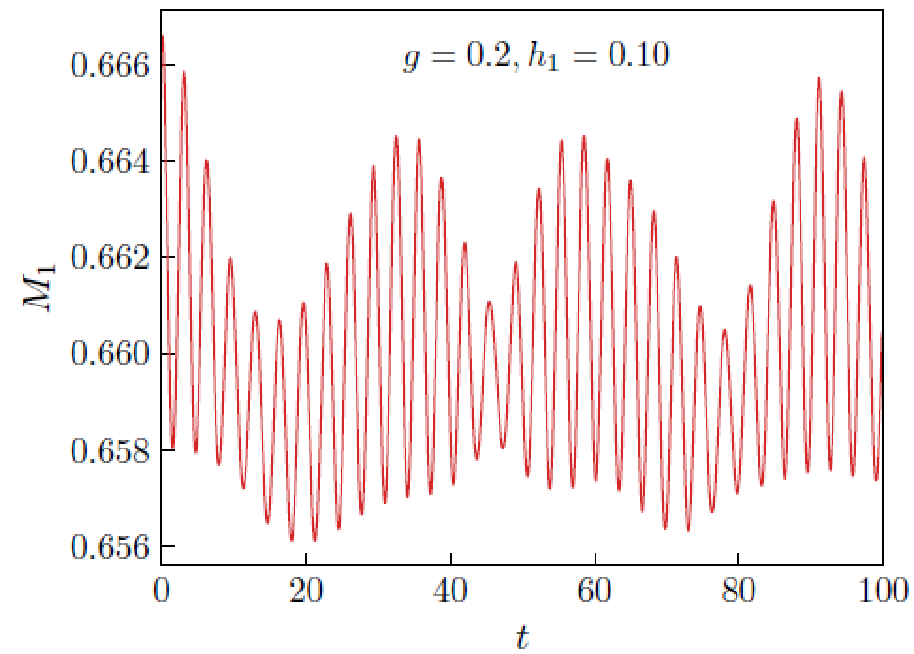
Elementary excitations:
bubbles (mesonic/baryonic)
+ Ising kinks

Even/odd mesons/bubbles **Even/odd baryons/baryonic bubbles**

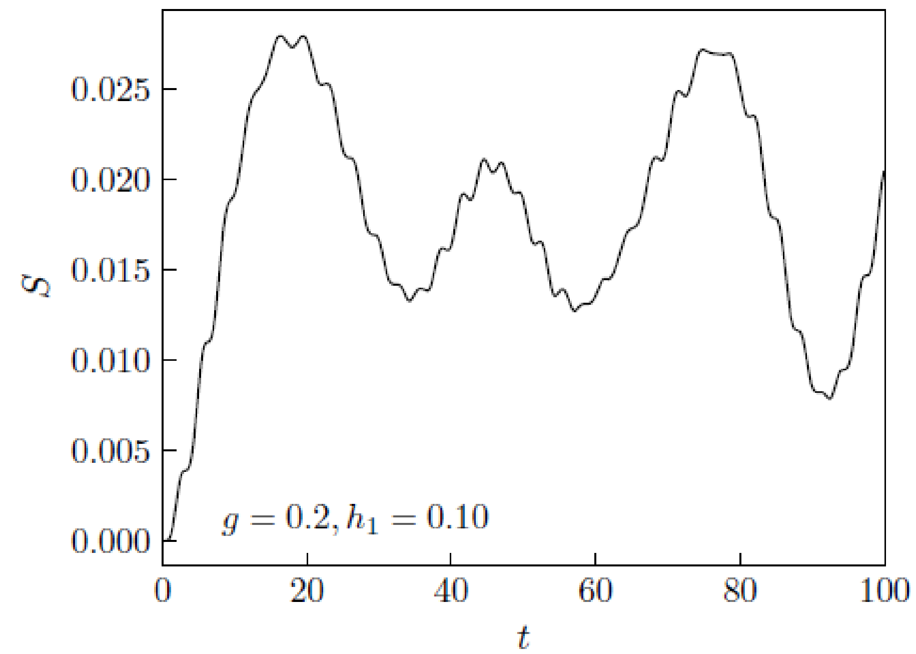
Baryons: better toy model for strong interactions + richer phenomenology

Aligned quenches

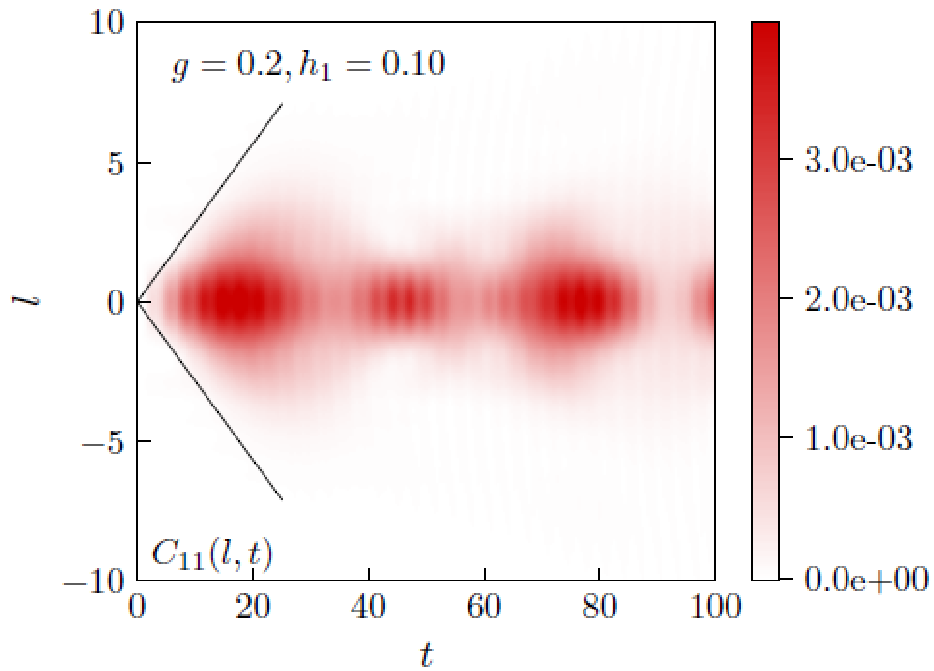
$h_1 > 0$: confinement



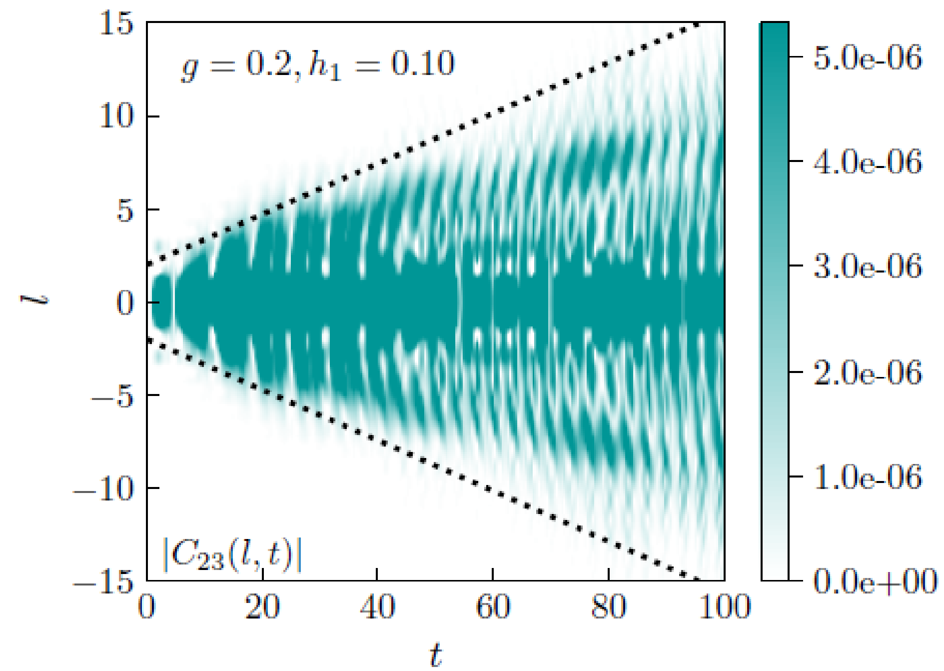
(a) Magnetisation $M_1(t)$.



(b) Entanglement entropy $S(t)$.



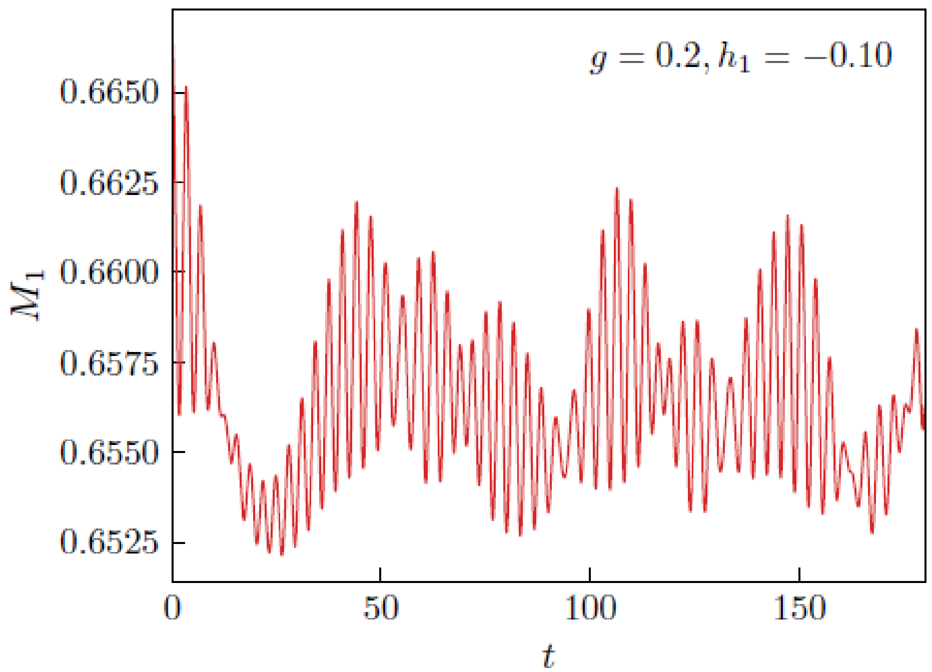
(a) Connected 11 (*red-red*) correlations.

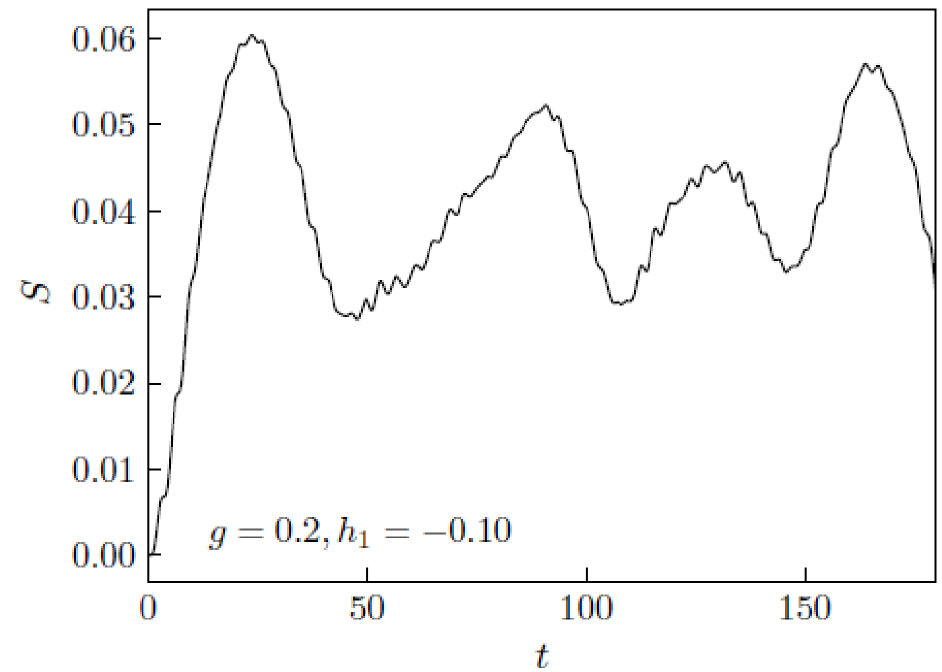


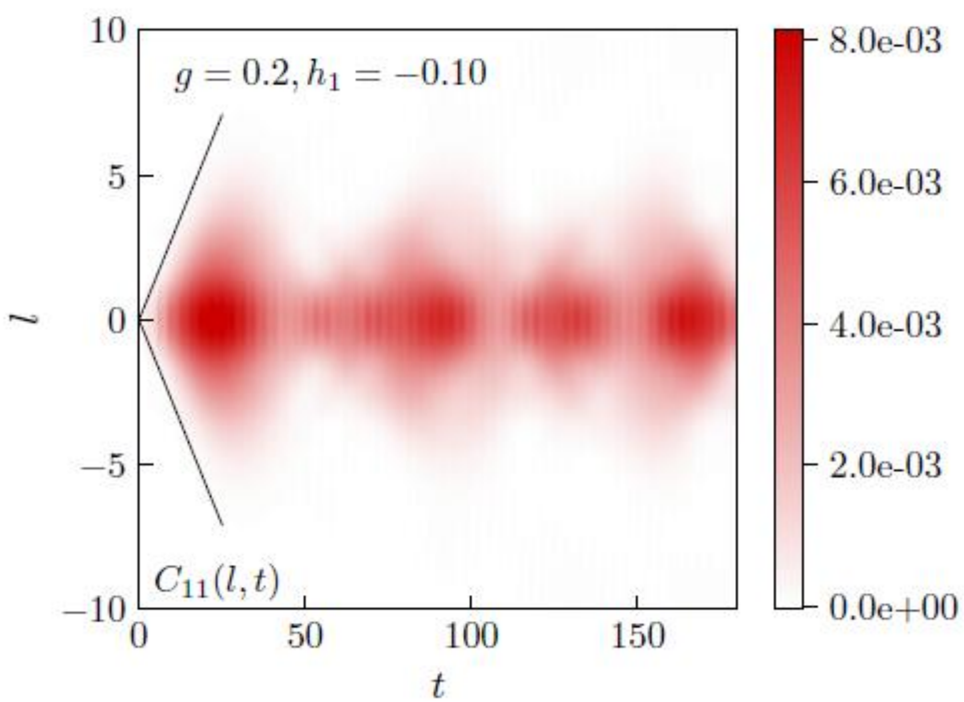
(b) Connected 23 (*cyan*) correlations.

Aligned quenches

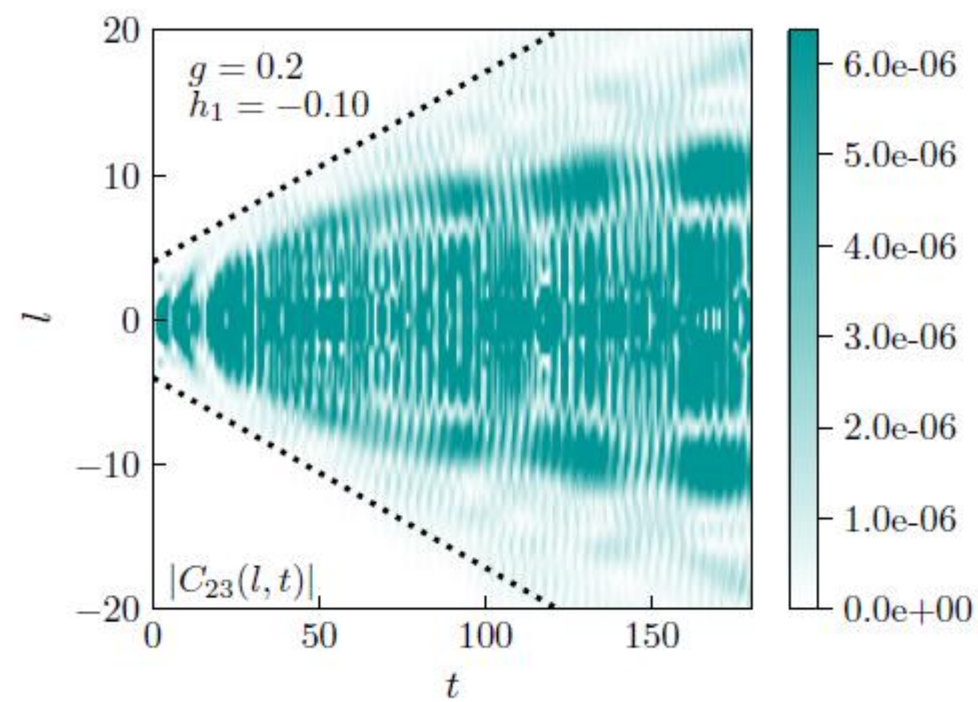
$h_1 < 0$: Wannier-Stark localisation







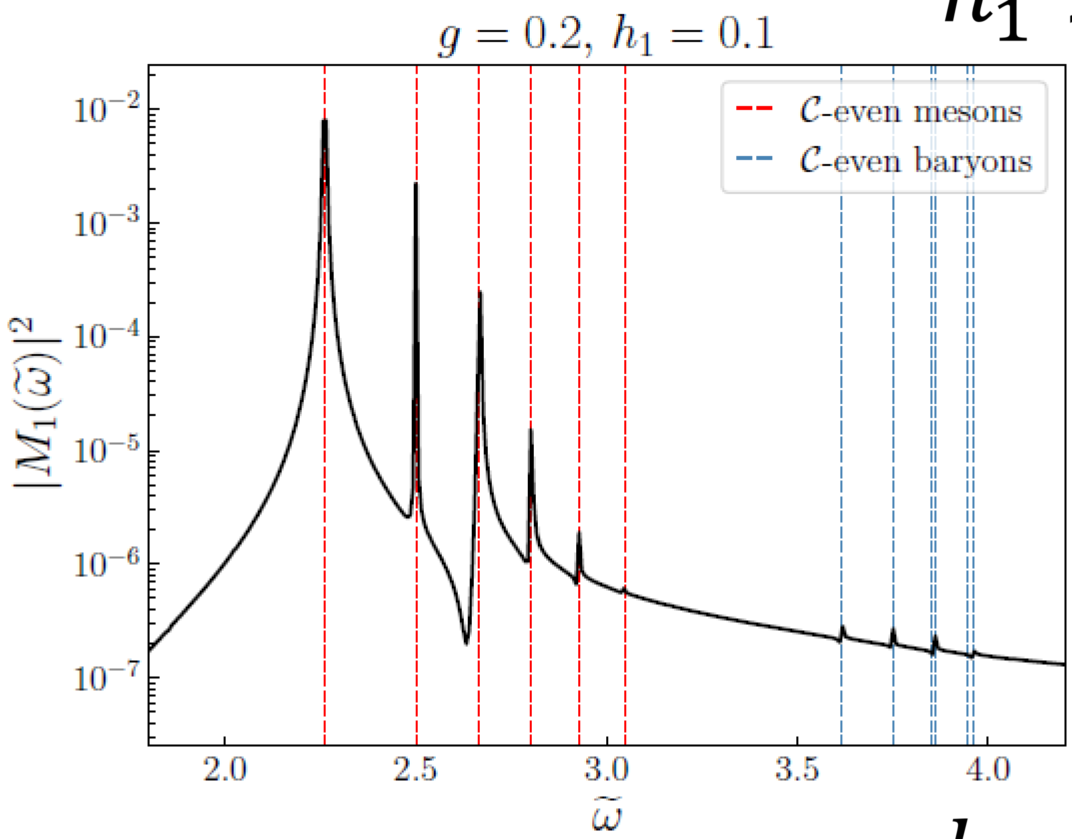
(a) Connected 11 (*red-red*) correlations.

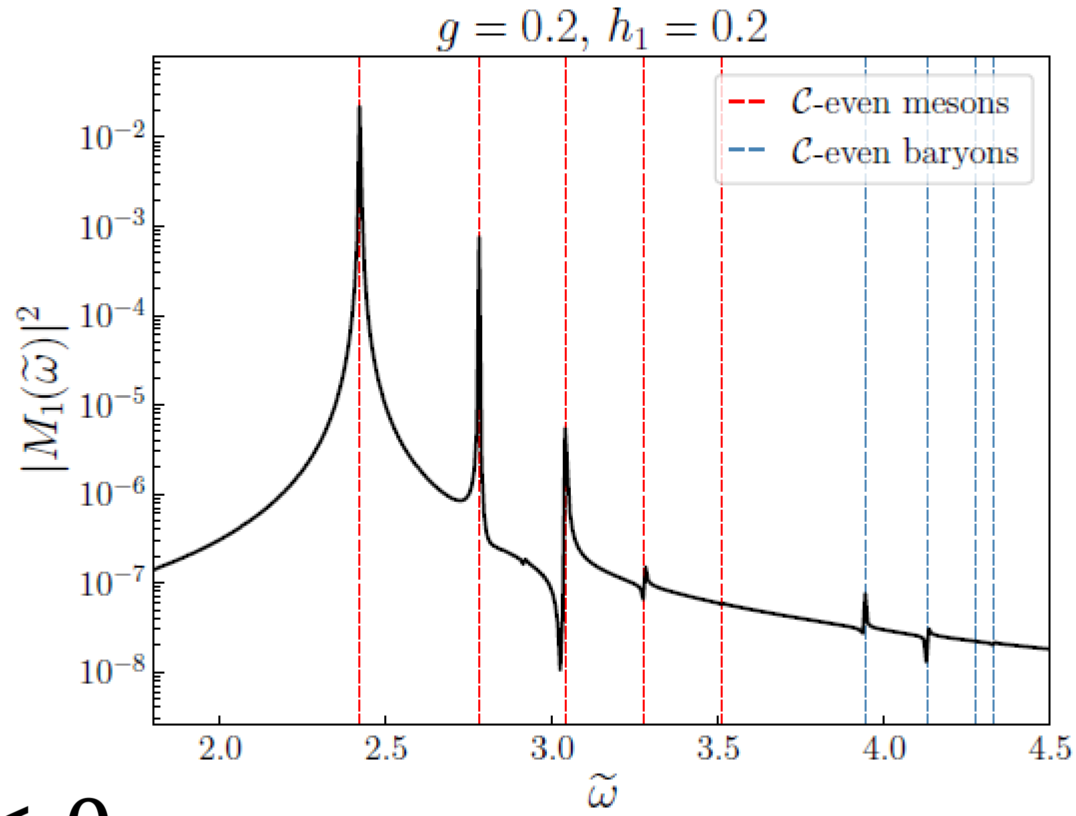


(b) Connected 23 (*cyan*) correlations.

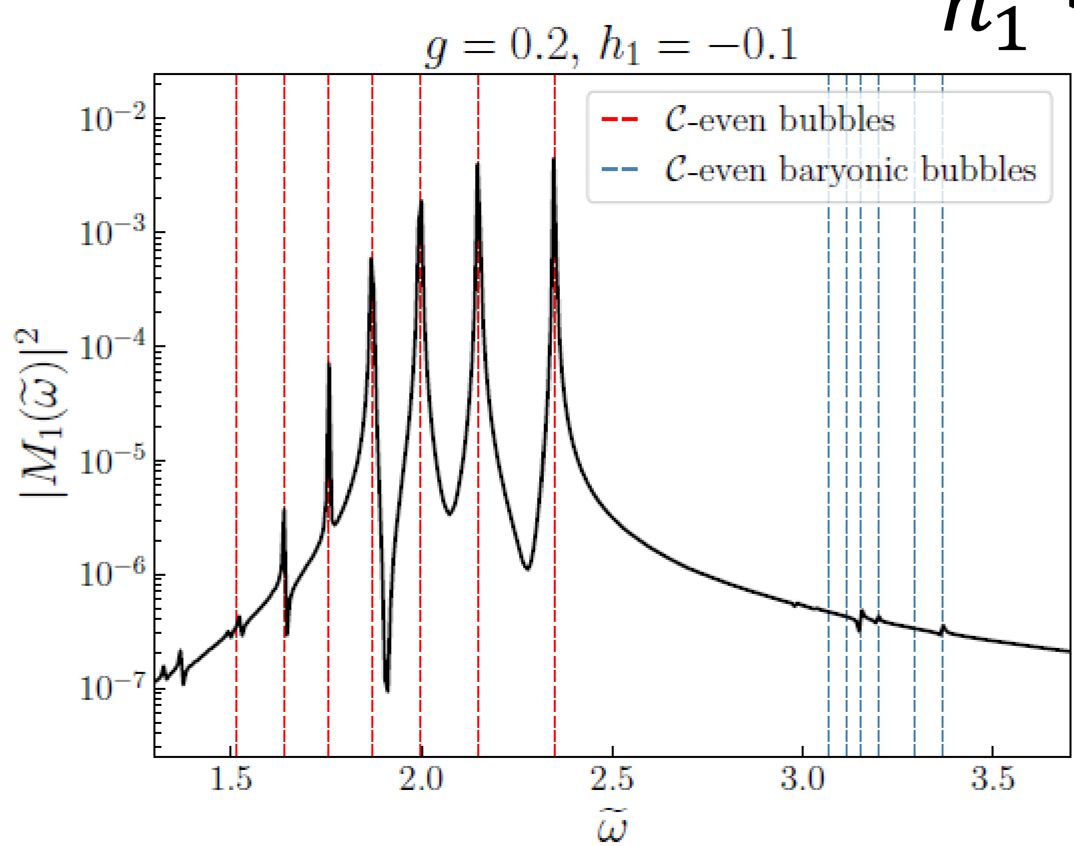
Spectroscopy of aligned quenches

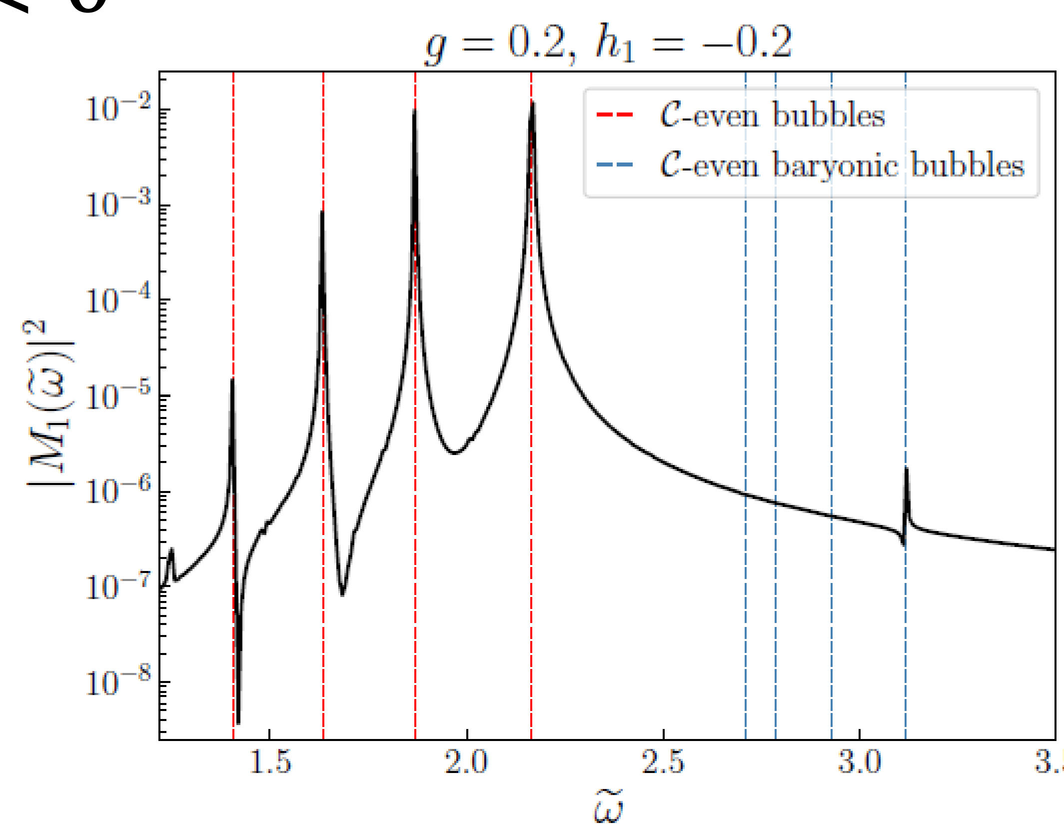
$h_1 > 0$



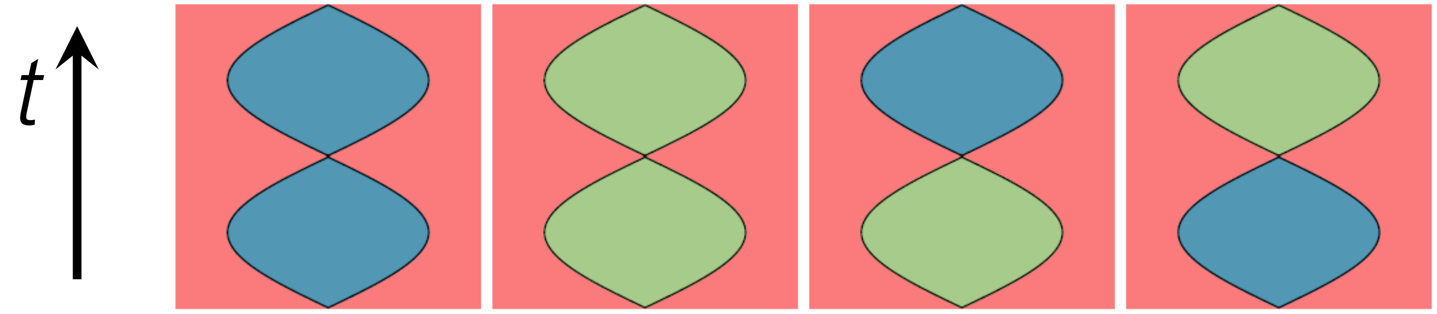


$h_1 < 0$





Semiclassical quantisation



Neutral mesons

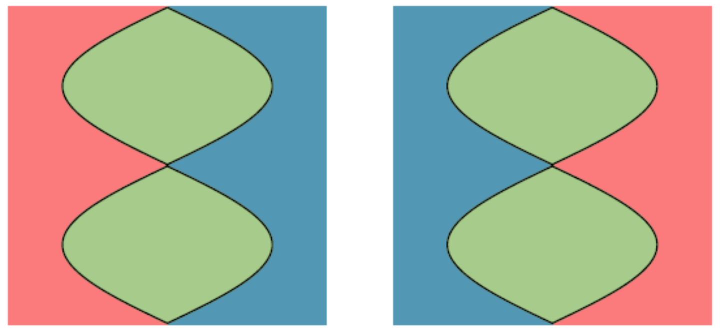
$$2E_n(K)k_a - \int_{-k_a}^{k_a} \omega(k;K) = \chi\left[2\pi(n-1/2+(-1)^\kappa/4) - \delta^{(\kappa)}(K/2-k_a, K/2+k_a)\right]$$

Neutral bubbles

$$2\mathcal{E}_n(K)(\pi-k_a) - 2\int_{k_a}^{\pi} dk\,\omega(k;K) = -\chi\left[2\pi(n-1/2+(-1)^\kappa/4) - \delta^{(\kappa)}(K/2+k_a, K/2-k_a)\right]$$

$\kappa = 0$ or 1: even/odd $\delta^{(\kappa)}$: $h_1 = 0$ kink phase shift (from ED) χ: string tension

Charged mesons



$$2\,E_n(K)\,k_a - \int_{-k_a}^{k_a} dk\,\omega(k;K) = \chi[2\pi(n-1/4) - \hat{\delta}(K/2-k_a, K/2+k_a)]$$

Charged bubbles

$$2\,\mathcal{E}_n(K)\,(\pi-k_a) - 2\int_{k_a}^{\pi} dk\,\omega(k;K) = -\chi[2\pi(n-1/4) + \hat{\delta}(K/2-k_a, K/2+k_a)]$$

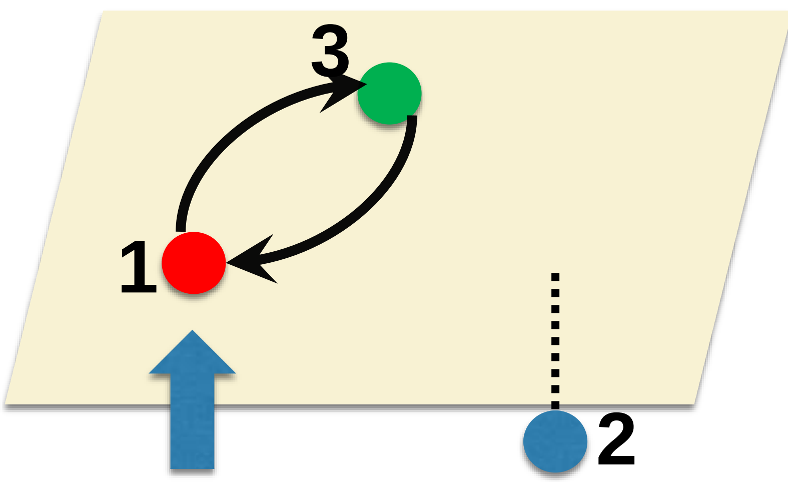
Baryons: no theory yet – from exact diagonalisation

Oblique quenches

Start from "red" state

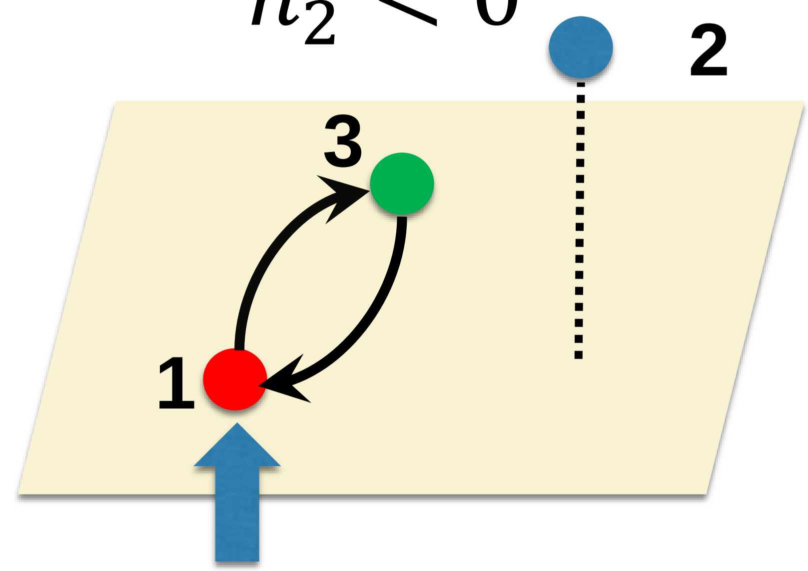
$$H_{postquench} = -J\sum_i \left(\sum_{\mu=1}^{3} P_i^\mu P_{i+1}^\mu + g\tilde{P}_i + h_1 P_i^1 \right)$$

$h_2 > 0$



Relevant excitations:
bubbles + metastable Ising kinks

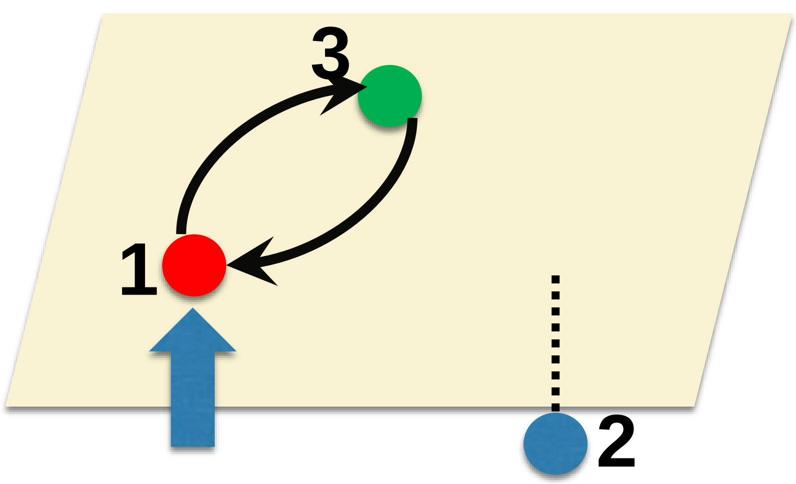
$h_2 < 0$

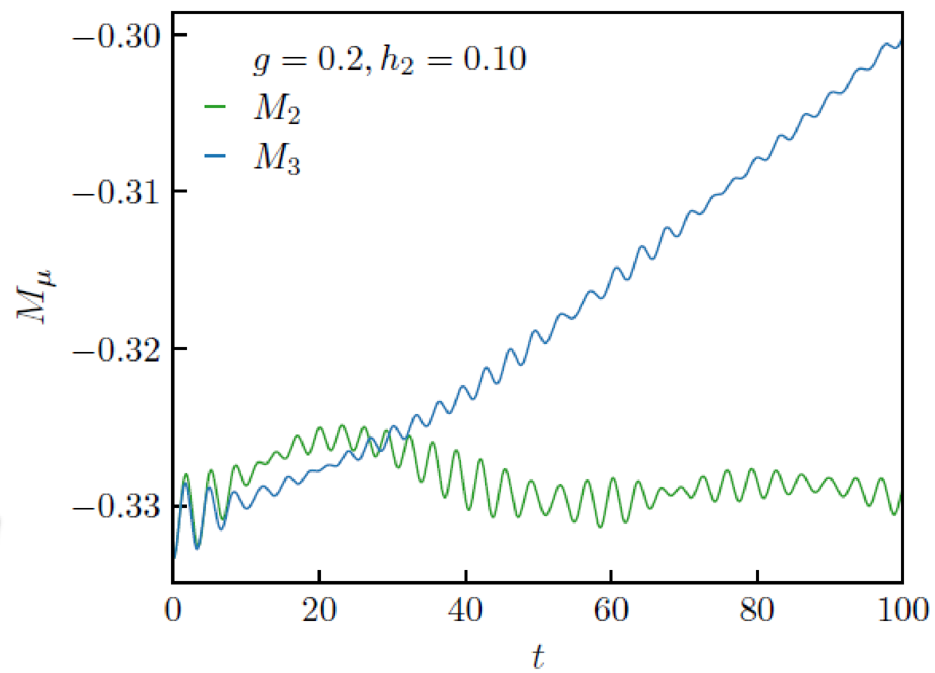


Relevant excitations:
mesons + Ising kinks

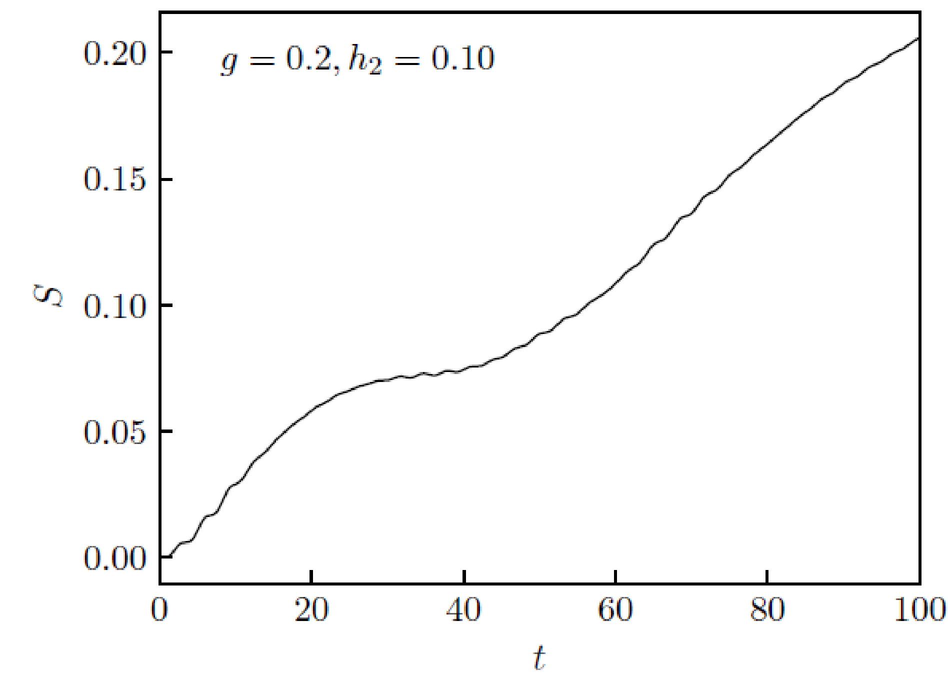
Oblique quenches

$h_2 > 0$: **partial WS localisation**

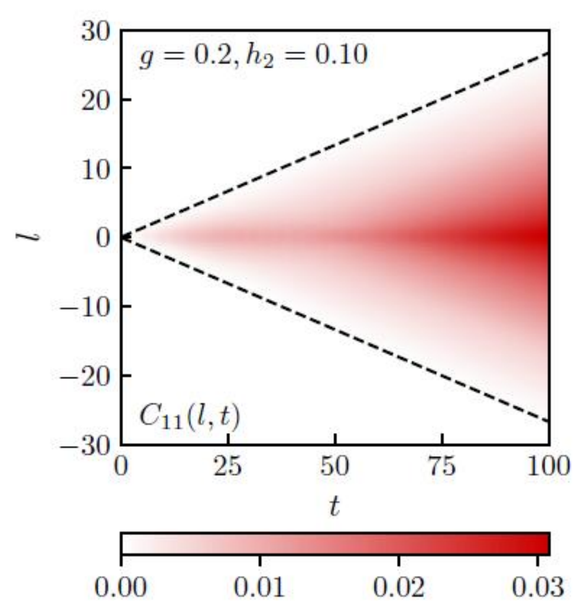




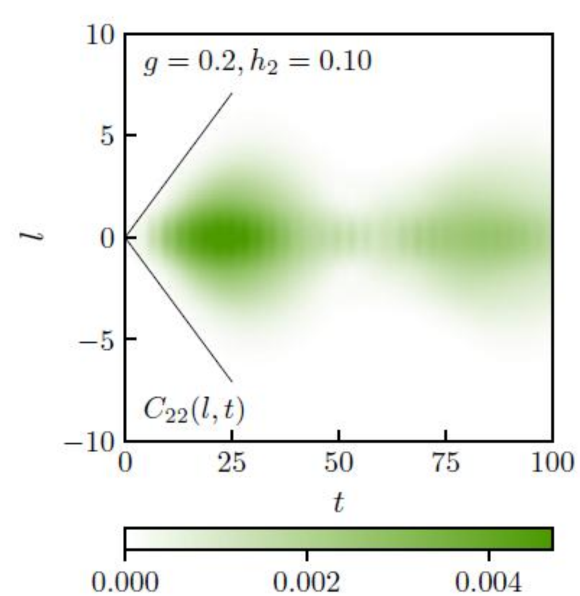
(a) Magnetisations $M_2(t)$ and $M_3(t)$.



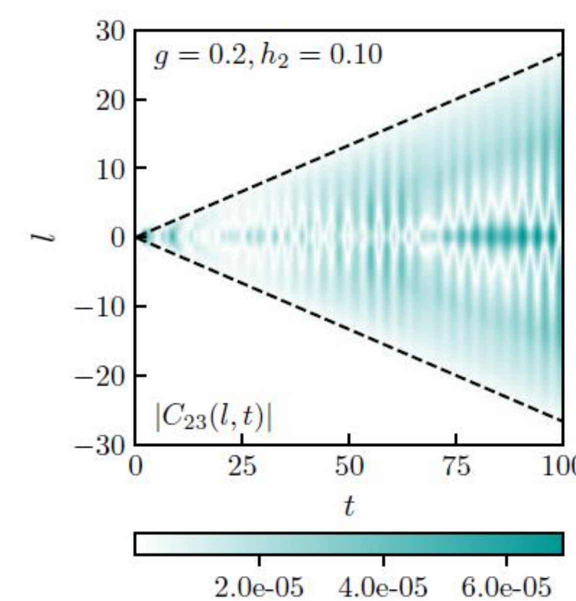
(b) Entanglement entropy $S(t)$.



(a) Connected 11 (*red-red*) correlations.



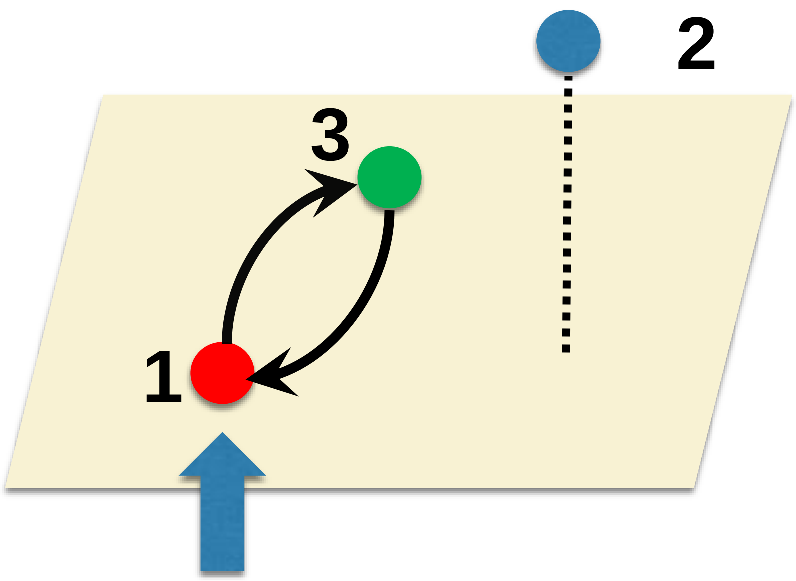
(b) Connected 22 (*green-green*) correlations.

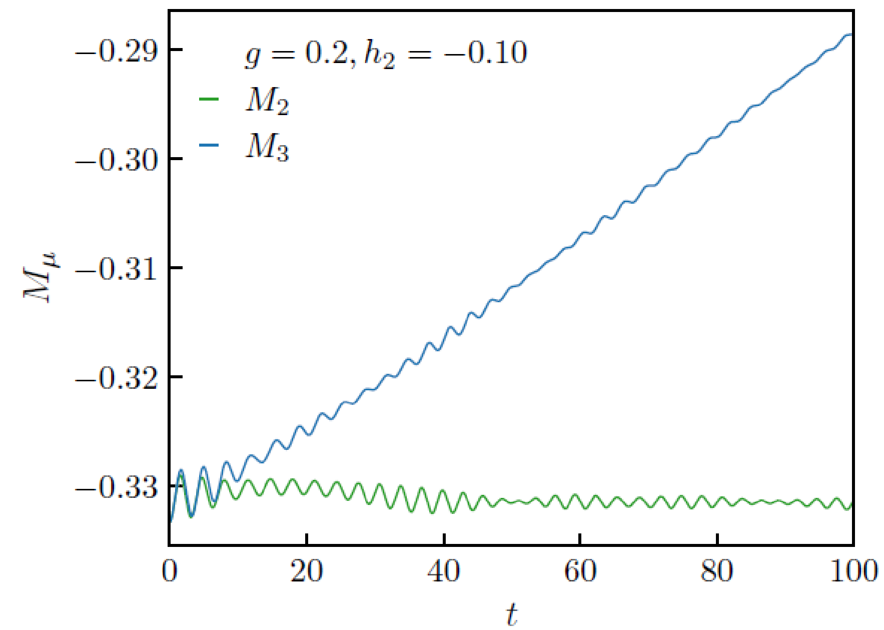


(c) Connected 23 (*cyan*) correlations.

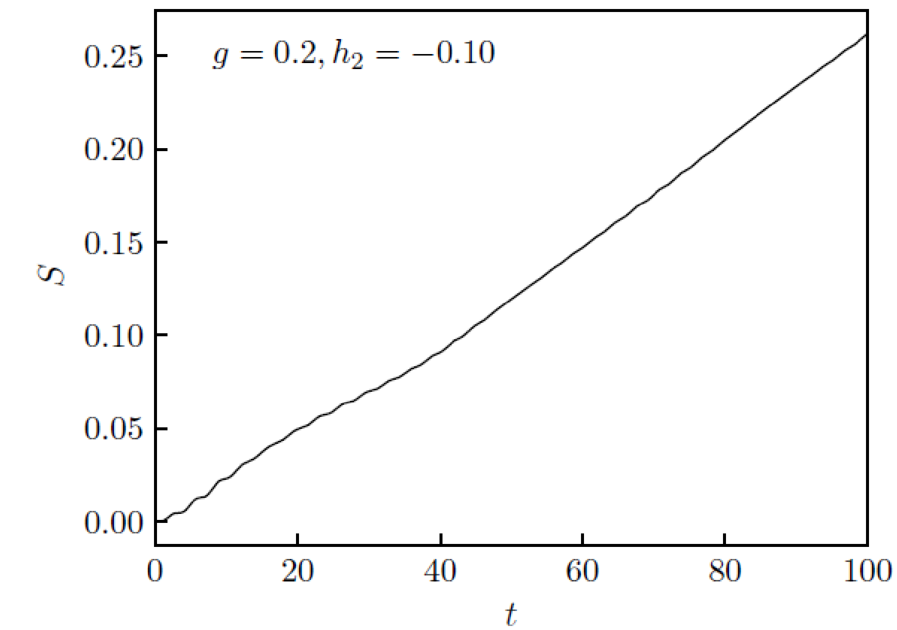
Oblique quenches

$h_2 < 0$: partial confinement

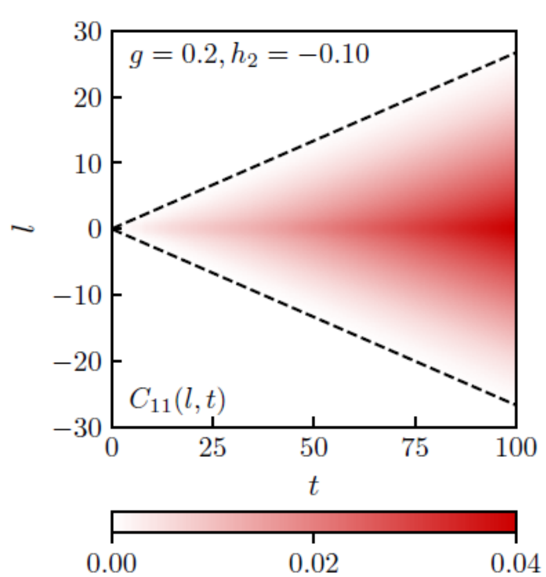




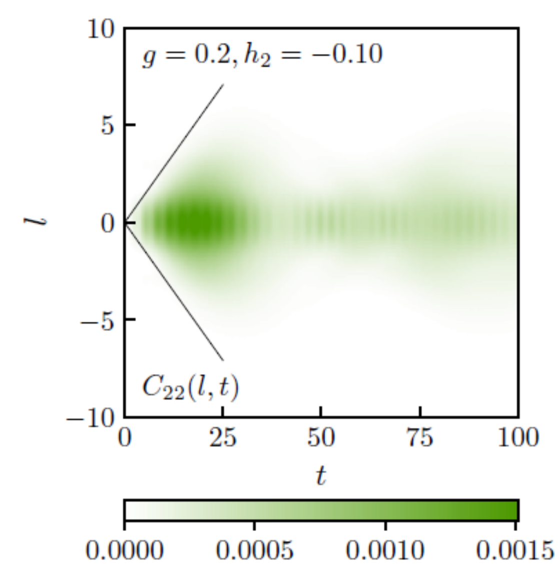
(a) Magnetisations $M_2(t)$ and $M_3(t)$.



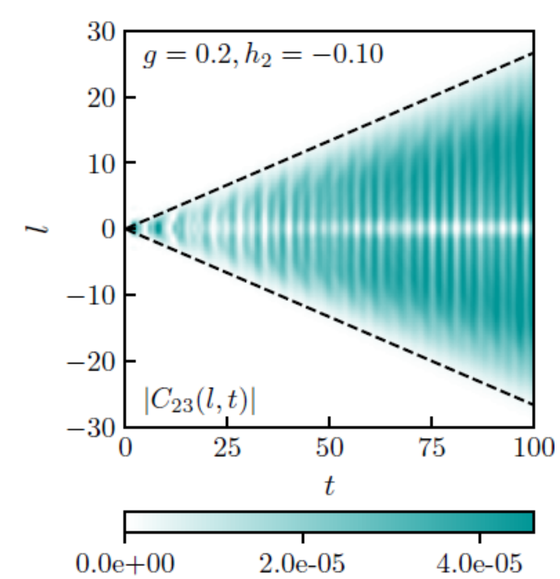
(b) Entanglement entropy $S(t)$.



(a) Connected 11 (*red-red*) correlations.



(b) Connected 22 (*green-green*) correlations.

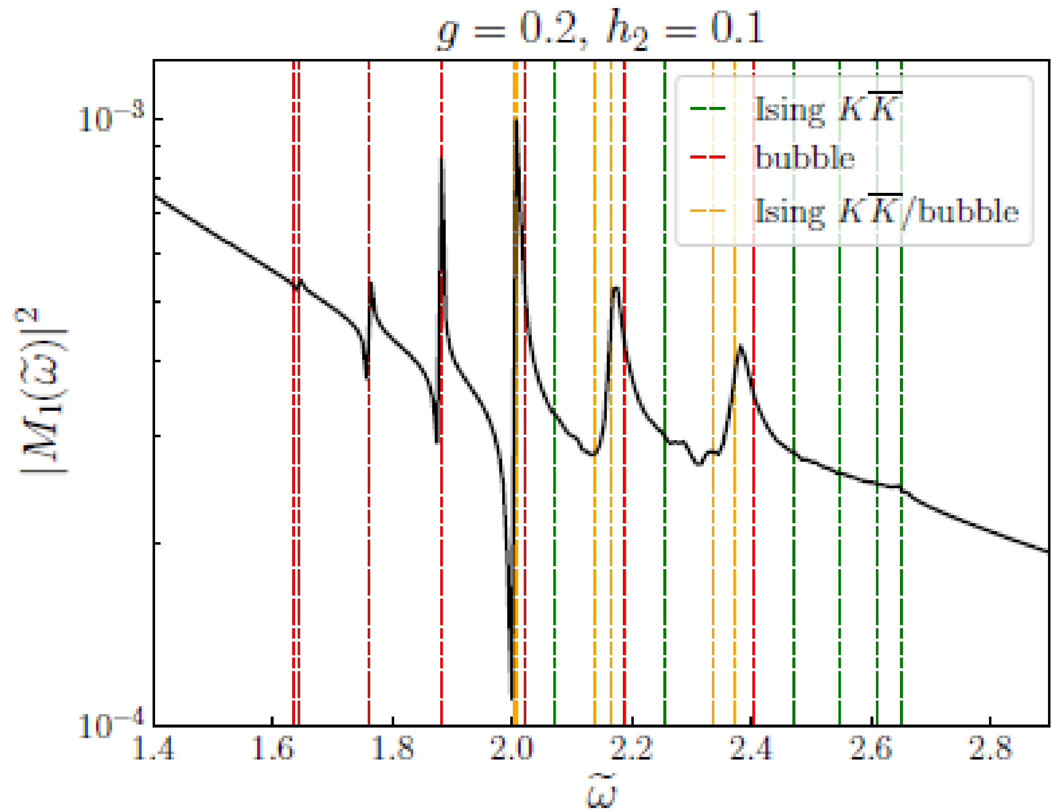


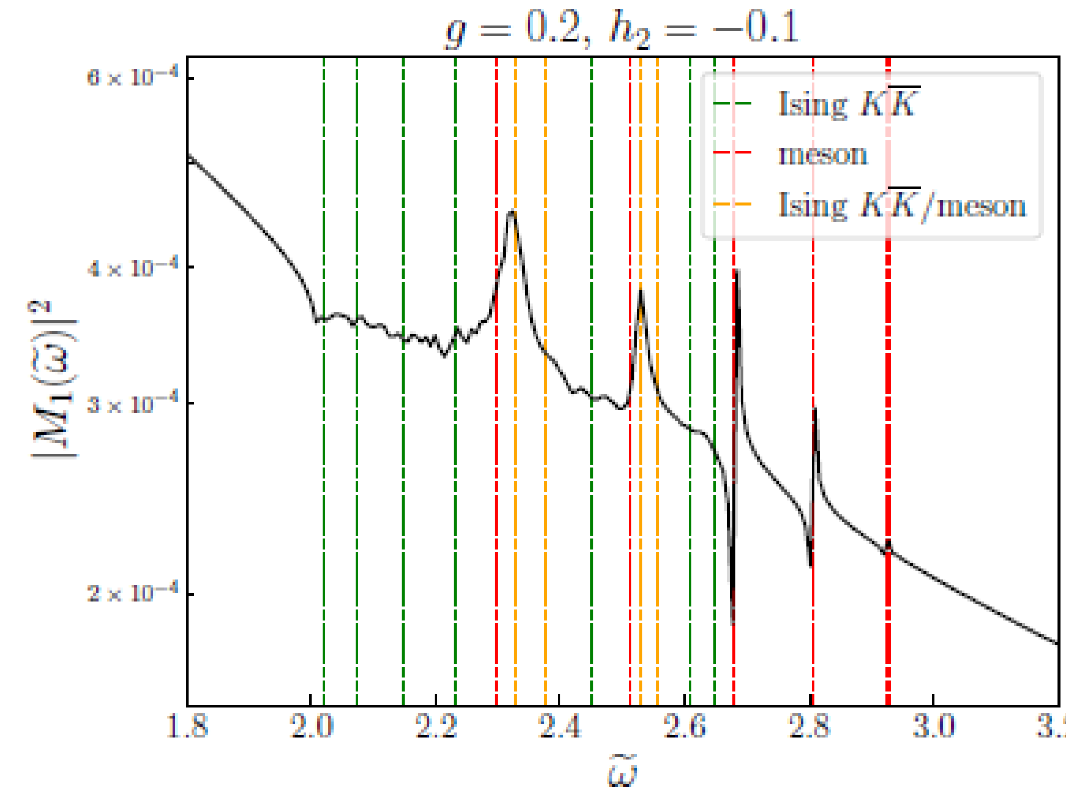
(c) Connected 23 (*cyan*) correlations.

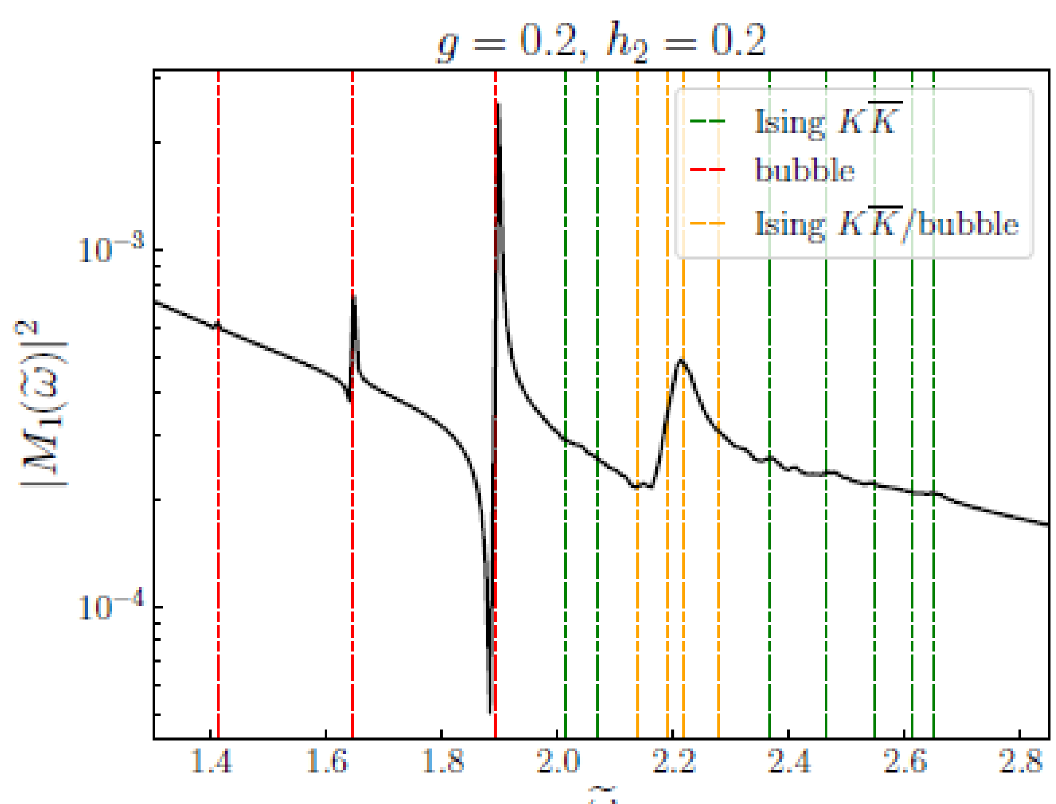
Spectroscopy of oblique quenches

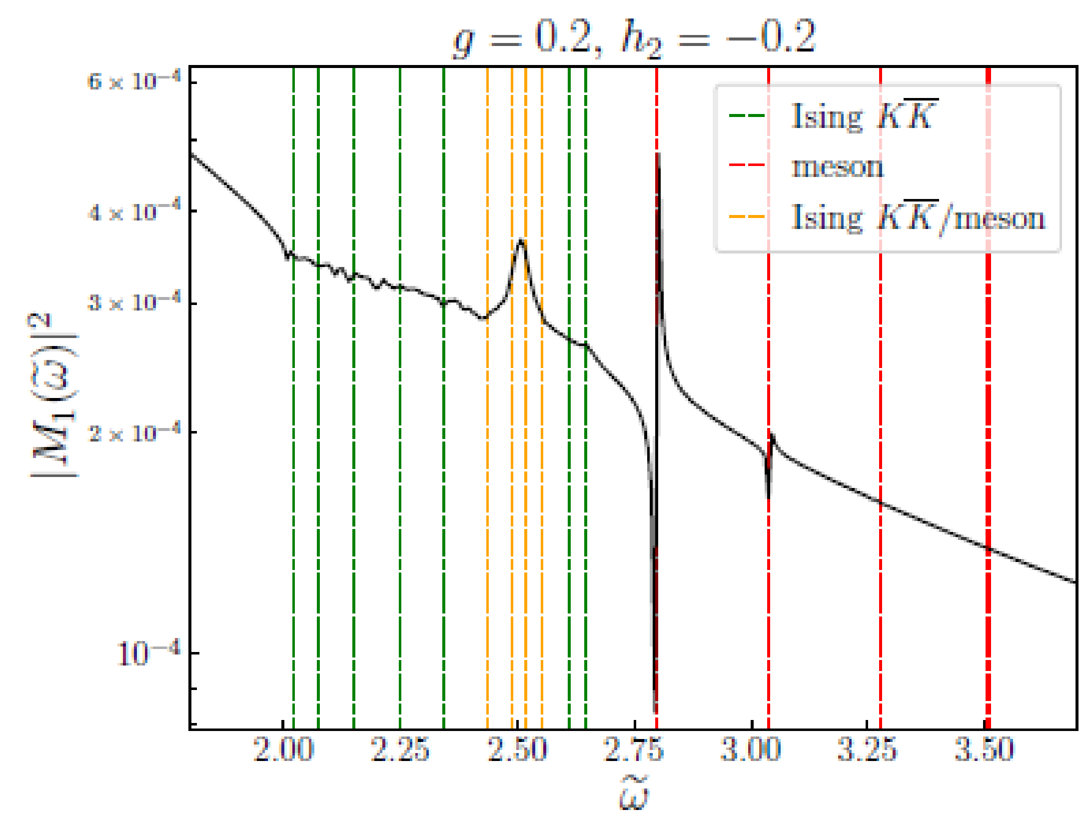
$h_2 > 0$

$h_2 < 0$





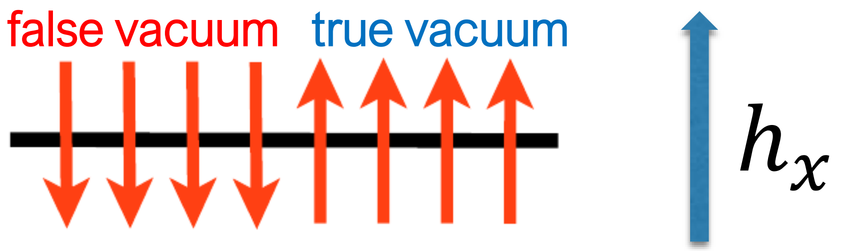




Local quenches: transport

Start system in domain wall initial state

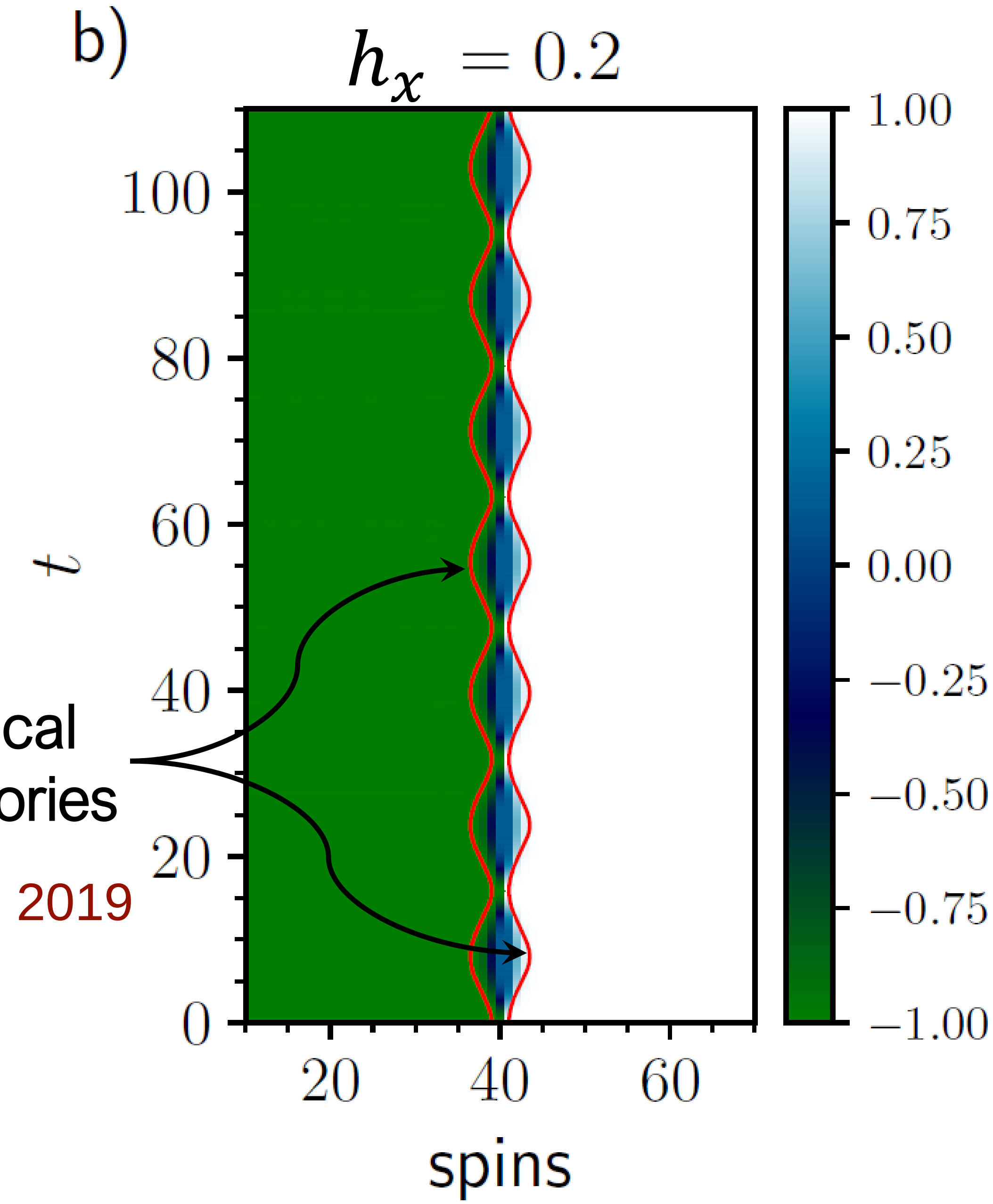
$$H = -J \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$



Right half: confinement

Left half: Bloch oscillations (Wannier-Stark localization)

semiclassical kink trajectories

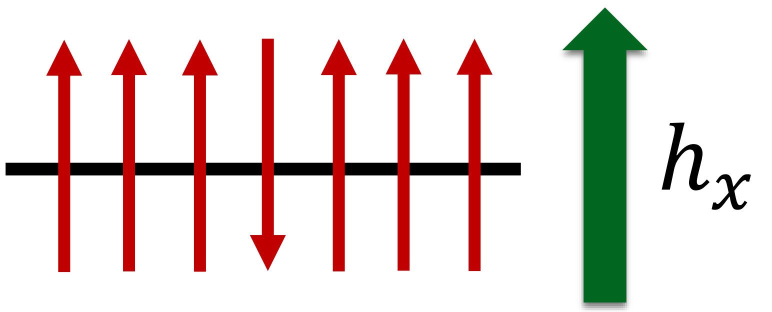


P.P. Mazza, G. Perfetto, A. Lerose, M. Collura, and A. Gambassi, 2019

No surprise here…

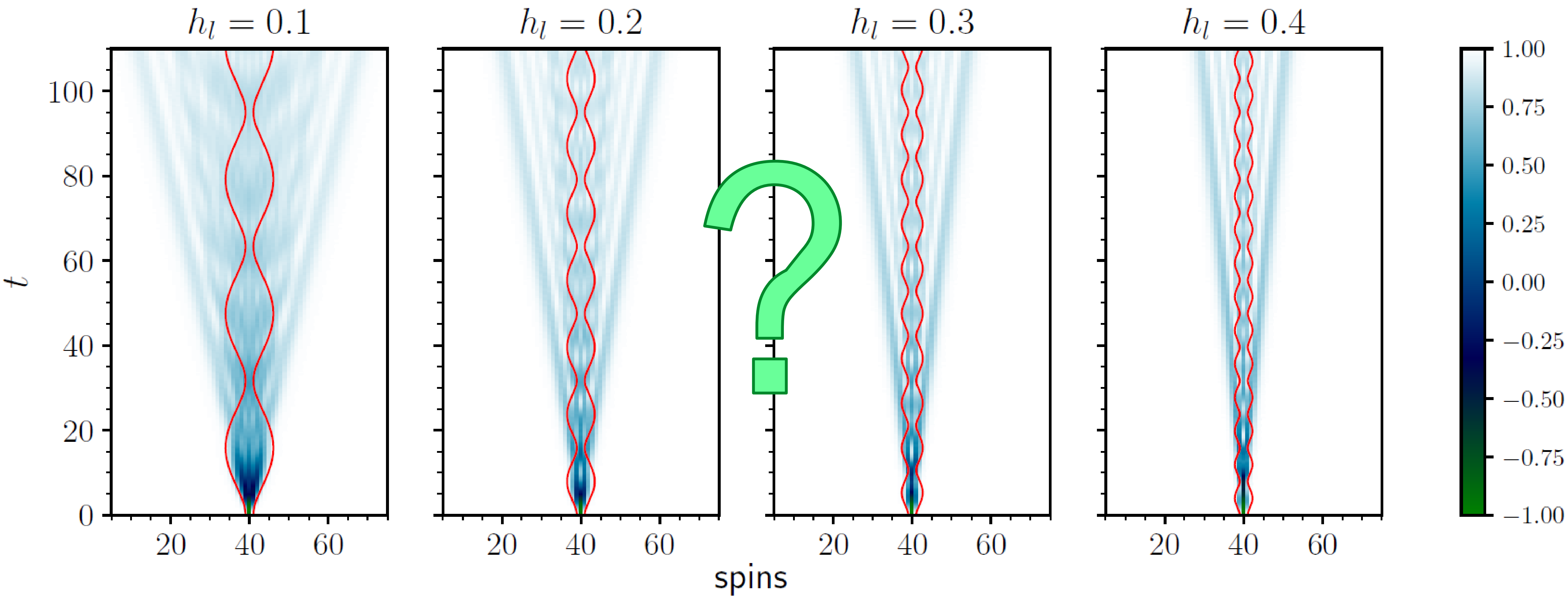
Simulation:
A. Krasznai and G. Takács, 2024

Escaping fronts



Start system in spin-flip initial state

$$H = -J\sum_{i=1}^{L}\left(\sigma_i^x\sigma_{i+1}^x + h_z\sigma_i^z + h_x\sigma_i^x\right)$$

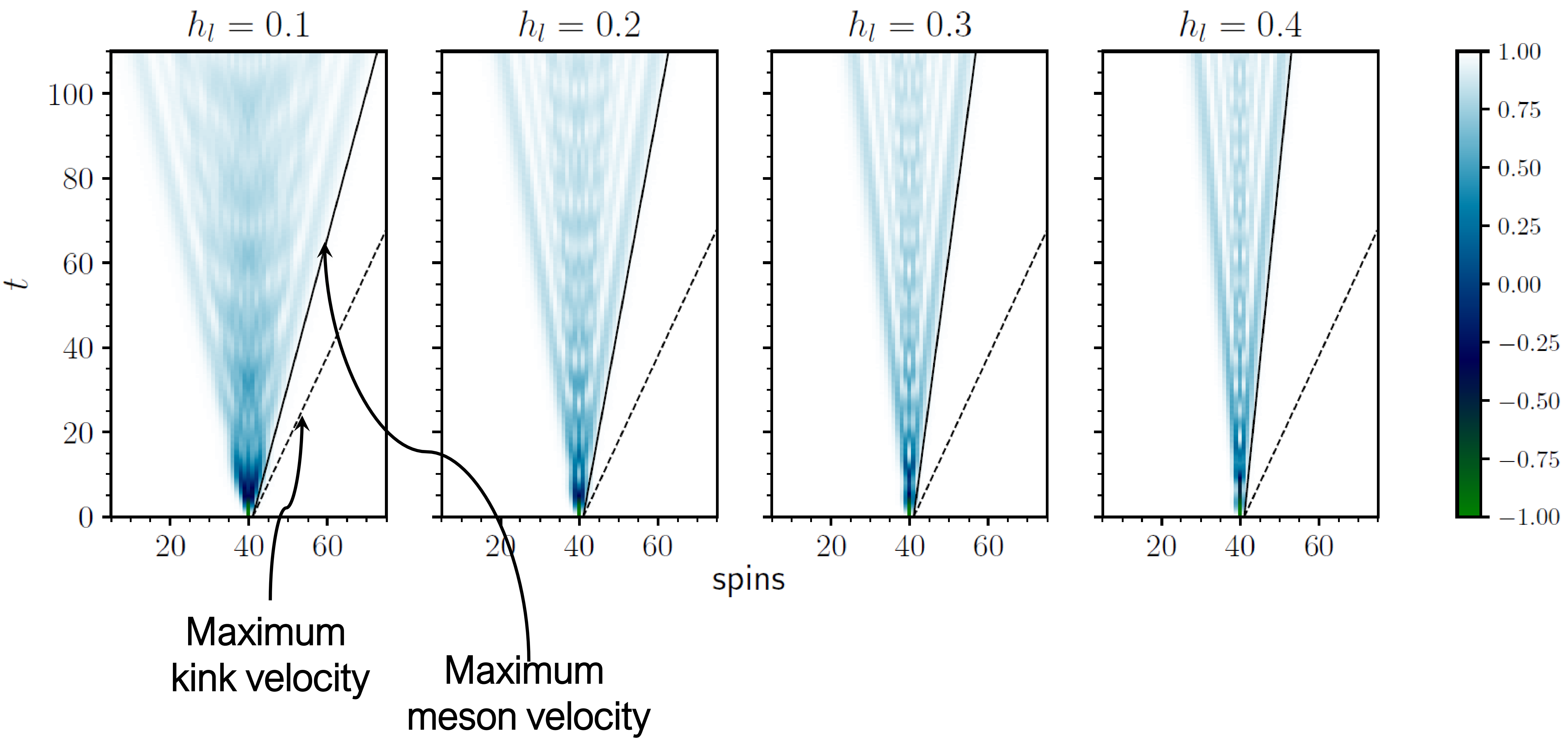


A. Krasznai and G. Takács, 2024

Schrödinger kittens escape confinement

Combining analytic and numerical methods:

Escaping fronts are superpositions of left/right moving single mesons



A. Krasznai and G. Takács, 2024

Overlaps of mesons with initial state

Initial state: using Jordan-Wigner transformation

$$|\Psi(0)\rangle = \hat{\sigma}^z_{L/2}\,|+\rangle_{h_t,h_l=0} = \frac{1}{\sqrt{2}}\left(|\Psi(0)\rangle_{\mathrm{NS}} + |\Psi(0)\rangle_{\mathrm{R}}\right)$$

$$|\Psi(0)\rangle_{\mathrm{NS}} = -\left\{1 - \frac{2}{L}\sum_{k\in\mathrm{NS}} u_k^2 - \frac{2i}{L}\sum_{k_1,k_2\in\mathrm{NS}} e^{-i(k_1-k_2)L/2} v_{k_1} u_{k_2} \eta^\dagger_{-k_1}\eta^\dagger_{k_2}\right\}|0\rangle_{\mathrm{NS}}$$

$$u_q = \frac{h_t - \cos q + \sqrt{1+{h_t}^2 - 2h_t\cos q}}{\left(\sin^2 q + \left(h_t - \cos q + \sqrt{1+{h_t}^2-2h_t\cos q}\right)^2\right)^{1/2}}$$

$$v_q = \frac{-\sin q}{\left(\sin^2 q + \left(h_t - \cos q + \sqrt{1+{h_t}^2-2h_t\cos q}\right)^2\right)^{1/2}}$$

$$|\Psi(0)\rangle_{\mathrm{R}} = -\left\{1 - \frac{2}{L}\sum_{\substack{p\in\mathrm{R}\\ p\neq 0}} u_p^2 - \frac{2i}{L}\sum_{\substack{p_1,p_2\in\mathrm{R}\\ p_1p_2\neq 0}} e^{-i(p_1-p_2)L/2} v_{p_1} u_{p_2}\eta^\dagger_{-p_1}\eta^\dagger_{p_2} - \frac{2}{L}\sum_{\substack{p\in\mathrm{R}\\ p\neq 0}} e^{ipL/2} u_p \eta^\dagger_0 \eta^\dagger_p\right\}|0\rangle_{\mathrm{R}}$$

Meson wave functions: from Schrödinger equation

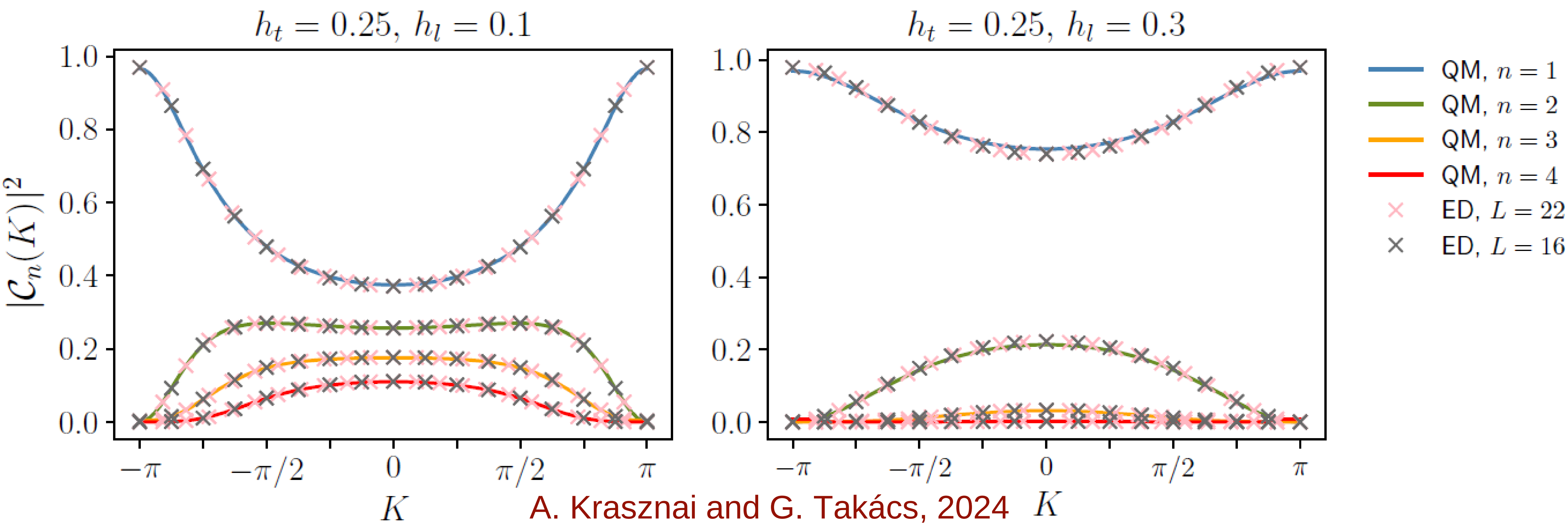
$$|M_n(K)\rangle = \sum_{k\in\mathrm{NS}+K/2}{}' \widetilde{\psi}_{n,K}(k)_L \eta^\dagger_{K/2-k}\eta^\dagger_{K/2+k}|0\rangle_{\mathrm{NS}} + \sum_{k\in\mathrm{R}+K/2}{}' \widetilde{\psi}_{n,K}(k)_L \eta^\dagger_{K/2-k}\eta^\dagger_{K/2+k}|0\rangle_{\mathrm{R}}$$

Note: phase redefinitions!

Overlaps of mesons with initial state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{L}} \sum_{n,K} \mathcal{C}_n(K)\, |M_n(K)\rangle$$

Comparison to exact diagonalisation



A. Krasznai and G. Takács, 2024

Schrödinger kittens escape confinement

Global quench:

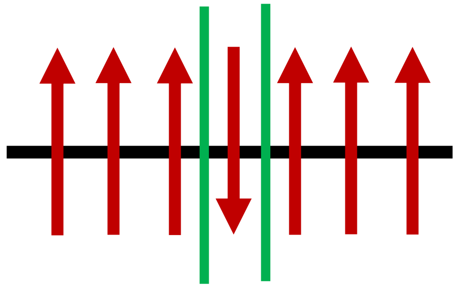
translational invariance only allows to create moving mesons in opposite momentum pairs
- energy threshold!

-> Escaping fronts are strongly suppressed by small probability of tunneling (string breaking/Schwinger effect)

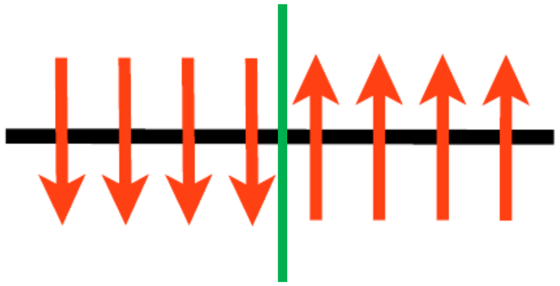
Spin-flip quench:

Single mesons can be created

No suppression: locally available energy from spin-flip

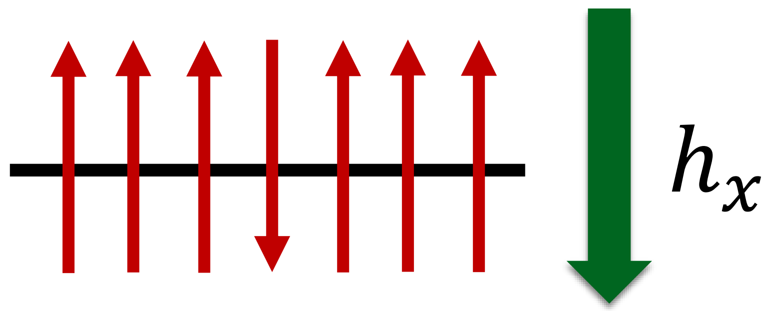


Domain wall quench:
not enough energy to create a meson

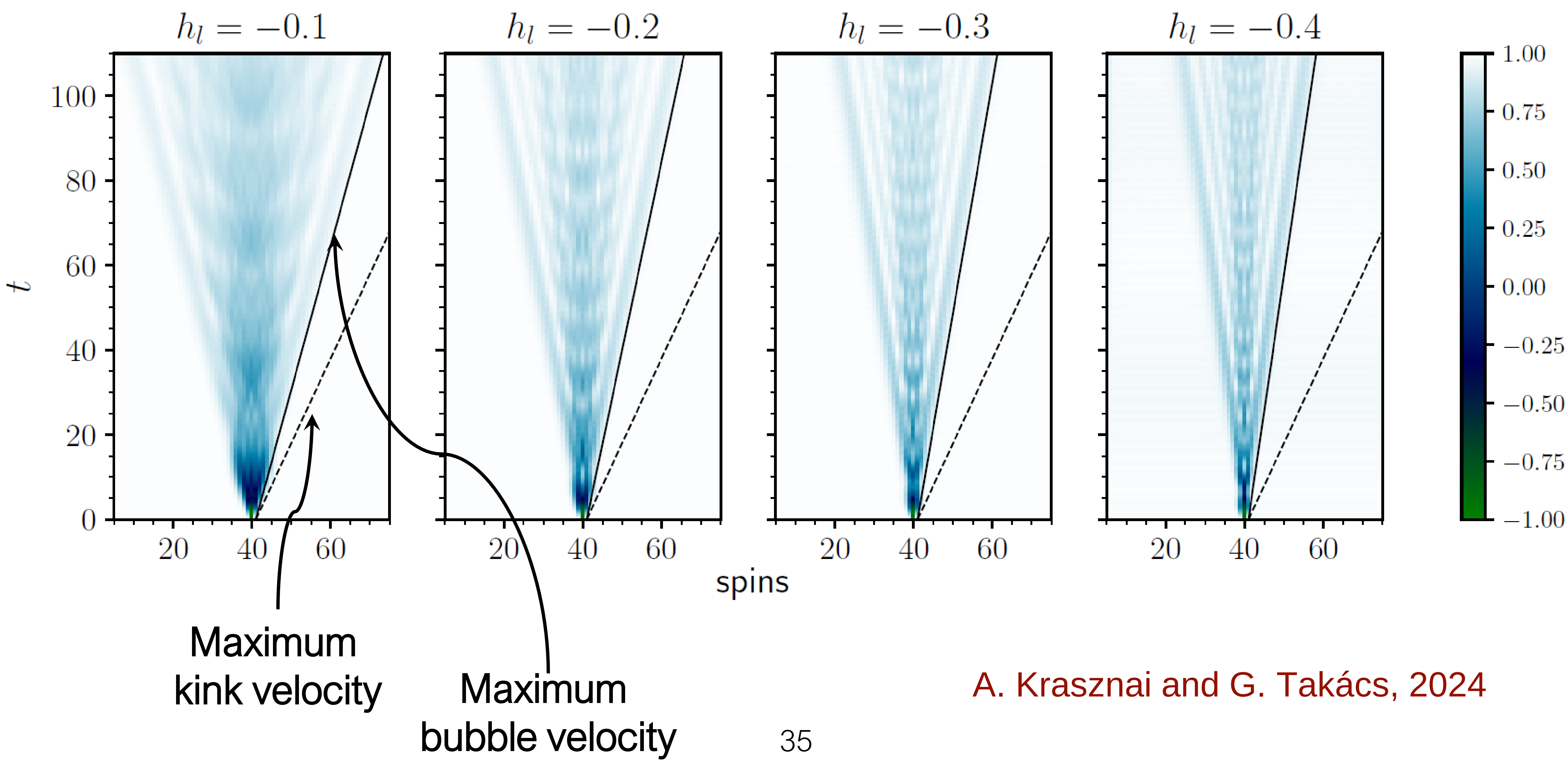


Local quenches induced by spin-flip over the false vacuum

Global quenches: fronts suppressed by Wannier-Stark localization (Bloch oscillations)



Escaping fronts in local quenches: superpositions of left/right moving single, nucleated true vacuum bubbles



A. Krasznai and G. Takács, 2024

Summary

- **Thermalisation of closed quantum systems is nontrivial**
- **Quantum quench is a paradigmatic, experimentally feasible protocol to study non-equilibrium dynamics**
- **Confinement strongly alters dynamics, suppressing light cone**
- **False vacuum decay can be suppressed by Bloch oscillations: Wannier-Stark localisation provides another mechanism to suppress light cone**
- **3-state Potts model: baryonic excitations, partial localization**
- **Local quenches: Schrödinger kittens can escape confinement / Wannier-Stark localisation**

Outlook

1. **Confinement alters dynamics in many other systems (including 1+1D QCD, 2d transverse Ising model etc.)**

2. **Experimental realizations (Ising class!)**

   Confinement: Rydberg atoms
   Quantum simulations

   Vacuum decay: fermionic superfluids

3. **Connection to high energy physics**

4. **Meta-stability of vacuum can be detected by local quenches by difference between meson and bubble spectra!**

F. Wilczek et al., 2023