Exotic emergences in low-dimensional quantum Ising models

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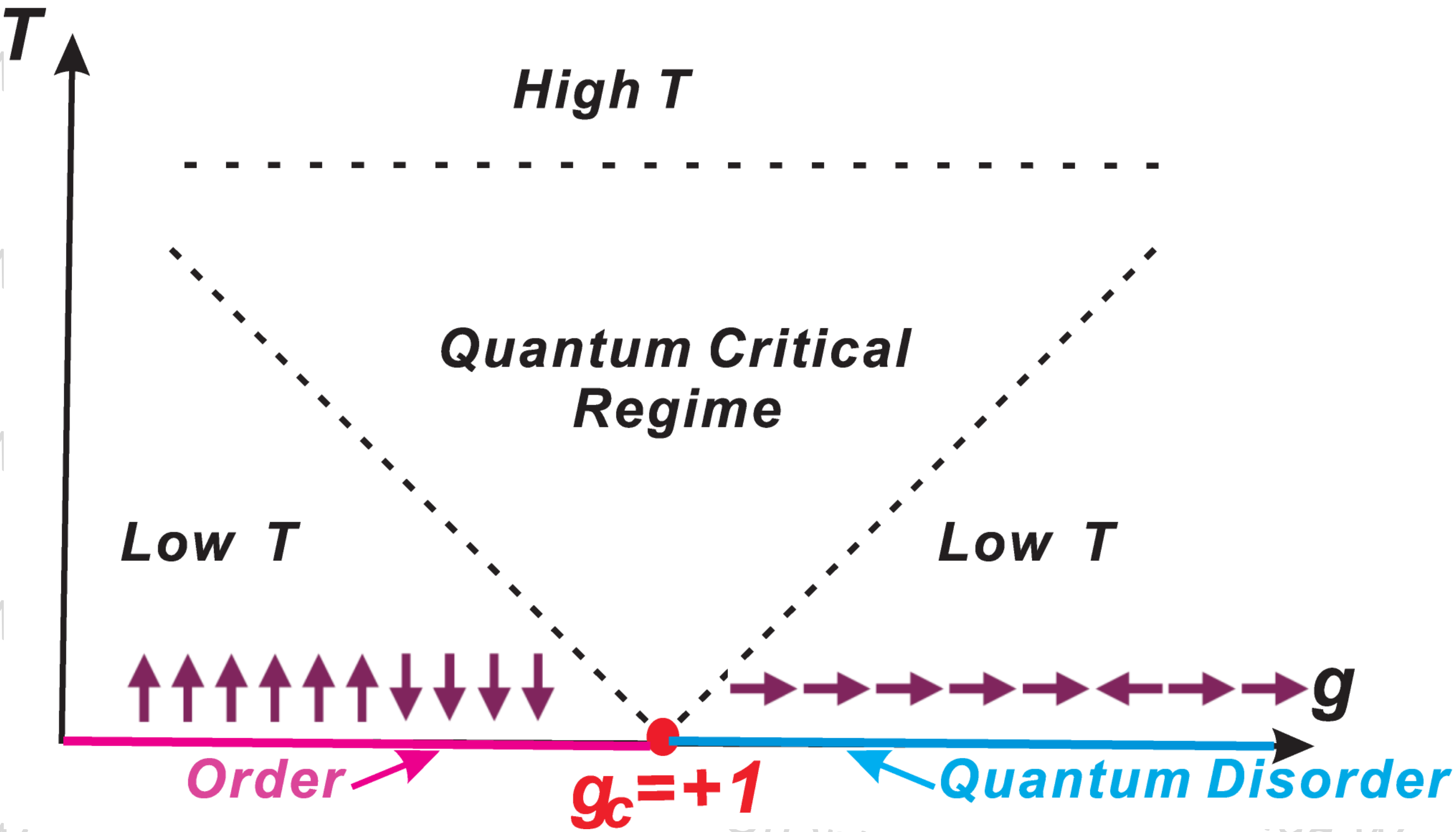
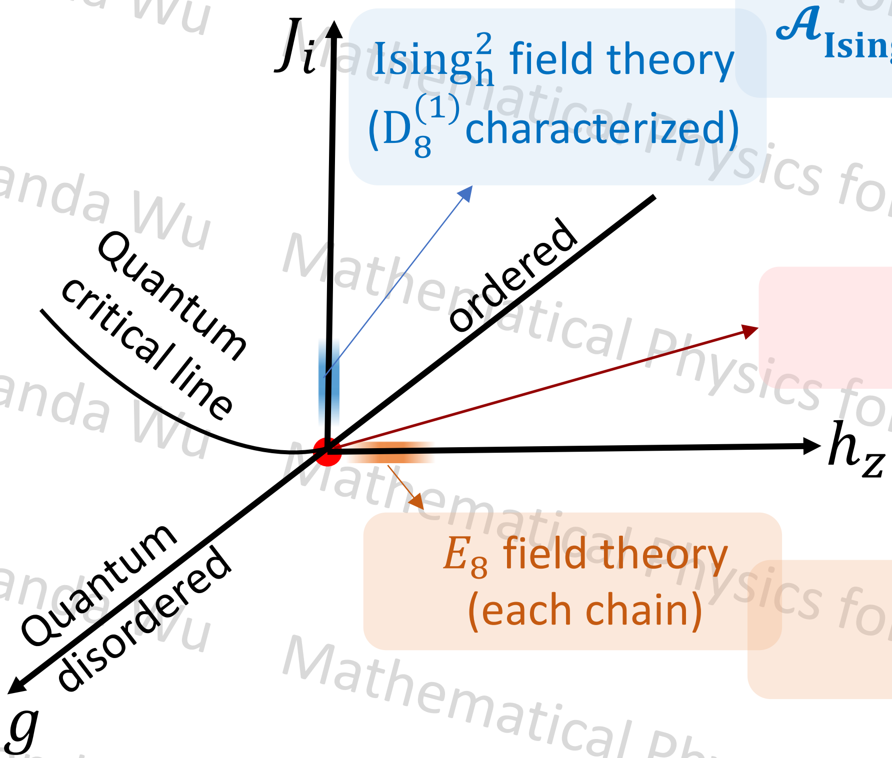
Transverse field Ising chain (TFIC)

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - g \sum_i \sigma_i^x$$



Key: TFIC quantum criticality



$$\mathcal{A}_{\text{Ising}_h^2} = \mathcal{A}^{(1)}_{c=1/2} + \mathcal{A}^{(2)}_{c=1/2} + \lambda \int \sigma^{(1)}\sigma^{(2)} dx d\tau$$

$$m_1 \approx 4.405|h|^{\frac{8}{15}}; \quad m_2 = 2m_1 \cos\frac{\pi}{5};$$

$$m_3 = 2m_1 \cos\frac{\pi}{30}; \quad m_4 = 2m_2 \cos\frac{7\pi}{30};$$

$$m_5 = 2m_2 \cos\frac{2\pi}{15}; \quad m_6 = 2m_2 \cos\frac{\pi}{30};$$

$$m_7 = 2m_4 \cos\frac{\pi}{5}; \quad m_8 = 2m_5 \cos\frac{\pi}{5}.$$

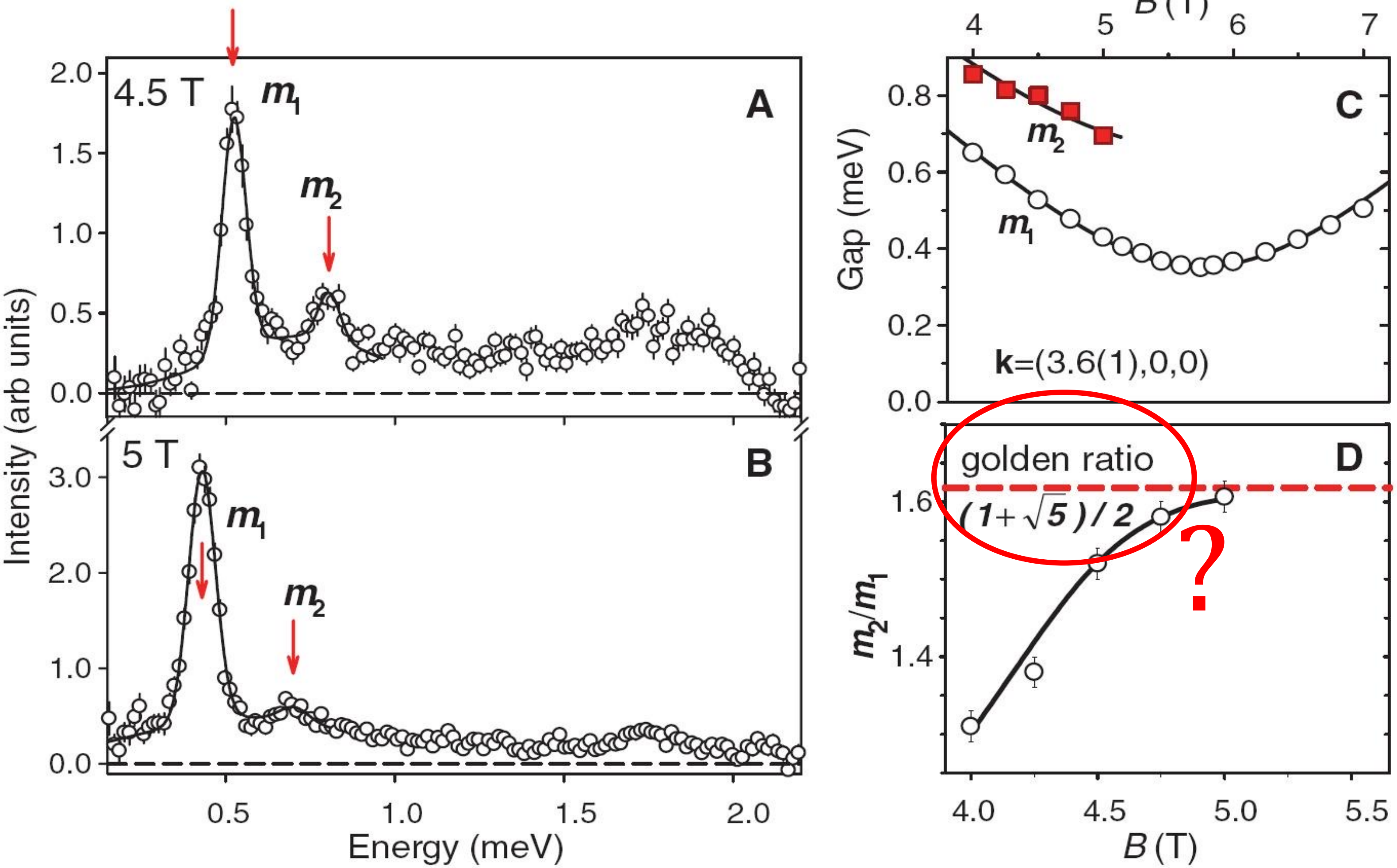
$$\mathcal{A}_{E_8} = \mathcal{A}_{c=1/2} + h \int \sigma(x,\tau) dx d\tau$$

➤ **Key: Accessing quantum criticality of the transverse-field Ising chain**

A. B. Zamolodchikov, Int. J. Mod. Phys. A4, 4235(1989)
A. LeClair, A. Ludwig, and Mussardo, Nucl. Phys. B 512, 523 (1998)

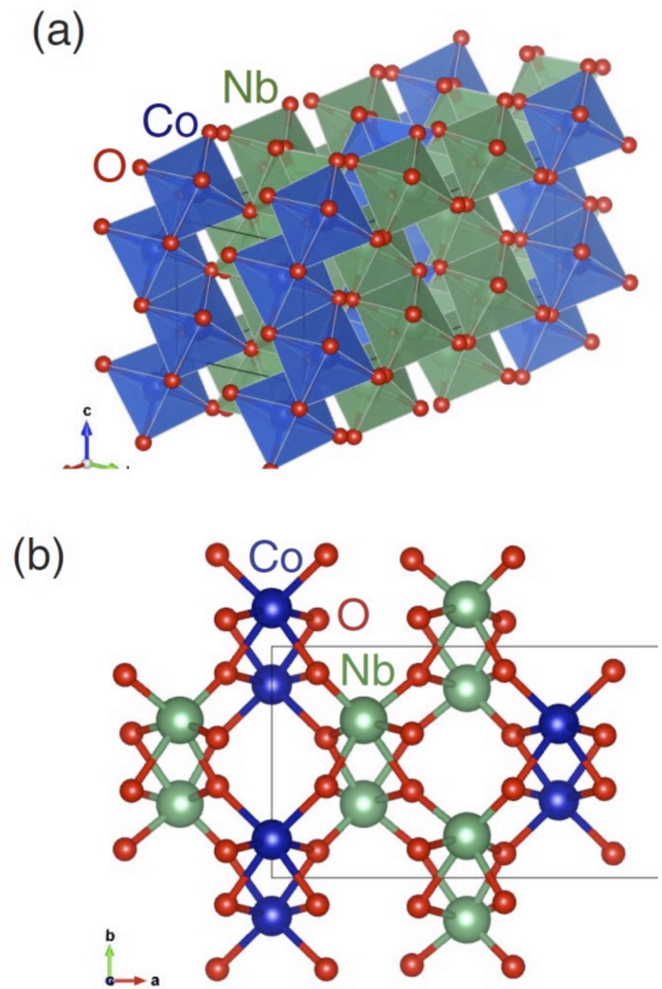
The beginning of all

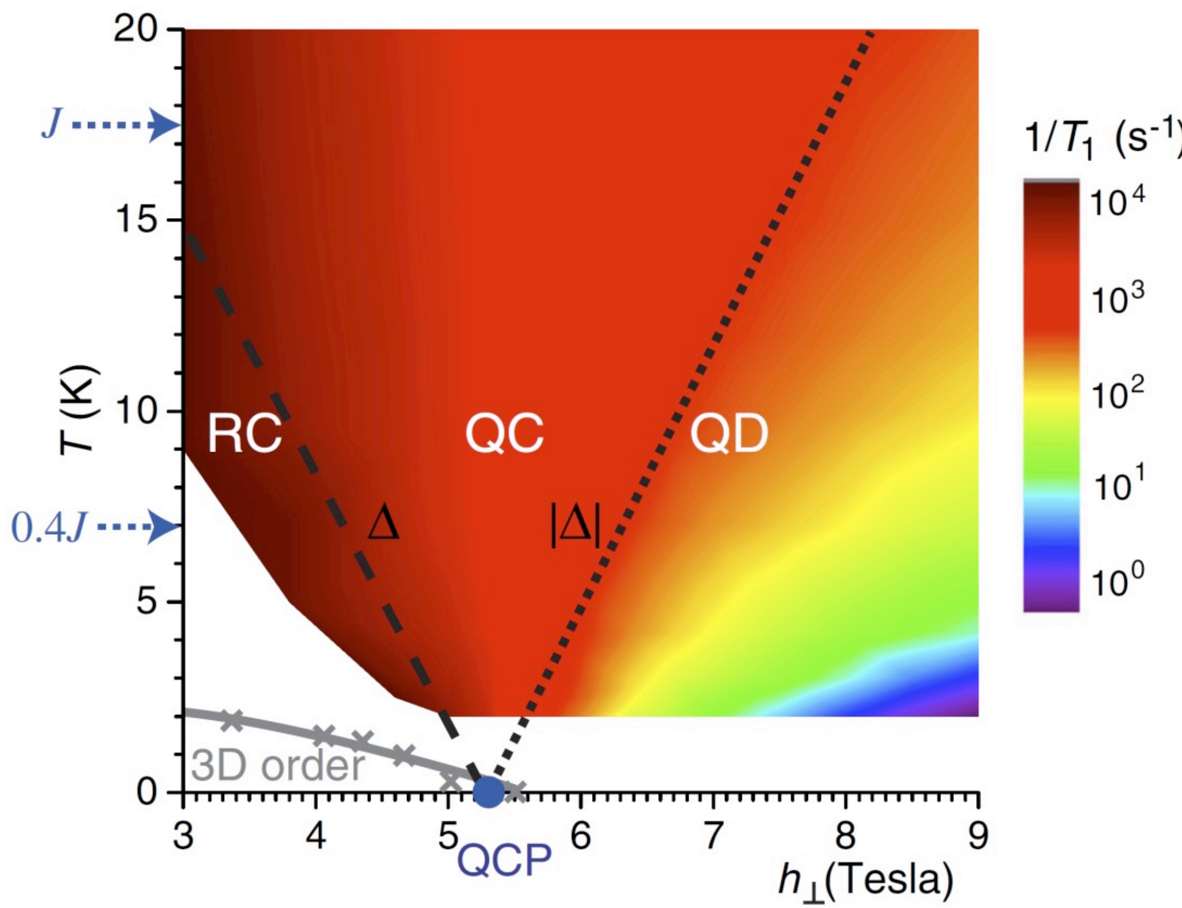
Inelastic Neutron Scattering Experiment on $CoNb_2O_6$



R. Coldea, D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl, K. Kiefer, Science, 327, 177 (2010).

Phase diagram of Quasi-1D Ferromagnet $CoNb_2O_6$

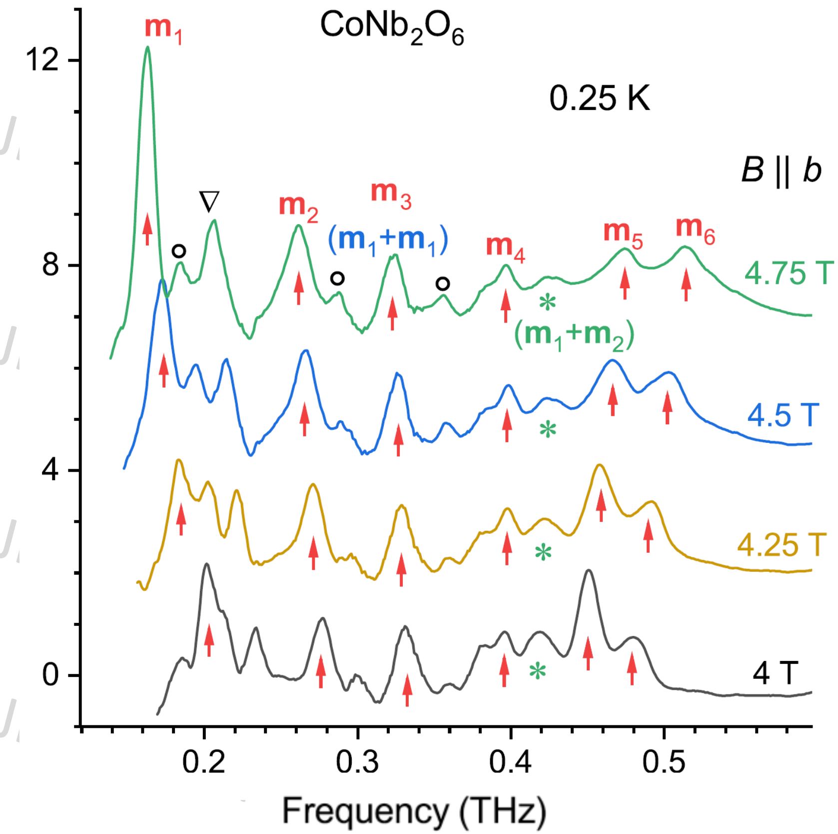


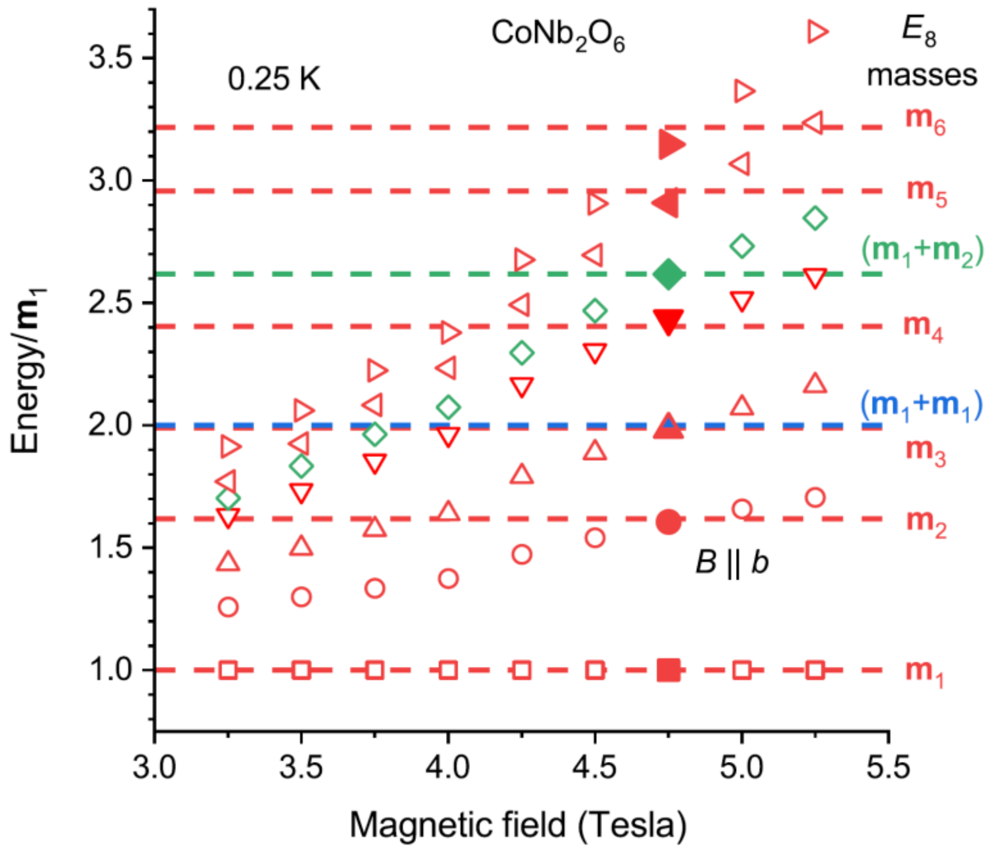


A. W. Kinross, et al., Phys. Rev. X 4, 031008 (2014)

E_8 physics in $CoNb_2O_6$?

- **A more accurate THz spectroscopy spin dynamics measurement**





K. Amelin, *et al.* Phys. Rev. B **102**, 104431 (2020)

A warmup

TFIC critical regions

- Near classical phase transition

$$\xi \sim [(T - T_c)/T_c]^{-\nu}$$

- Near quantum phase transition

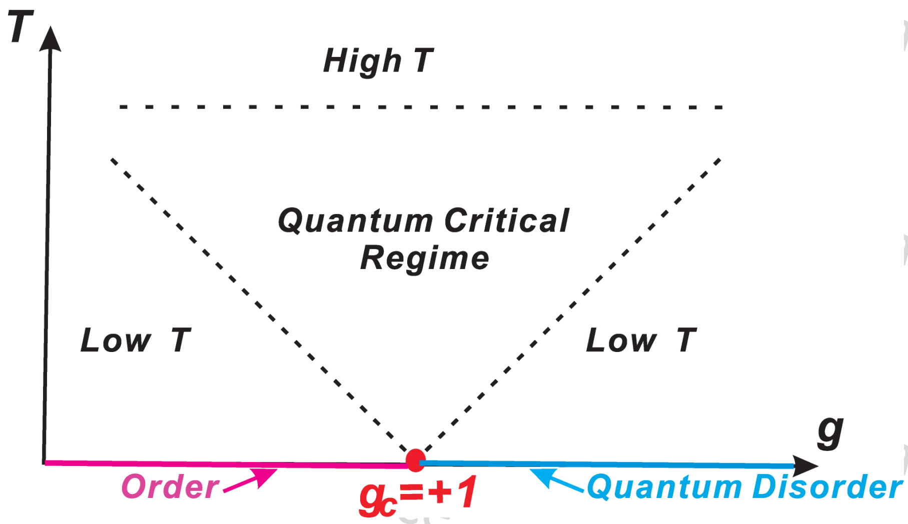
$$\xi \sim \Lambda^{-1}[(g - g_c)/g_c]^{-\nu}$$

$$\Delta \sim \xi^{-z} \sim J[(g - g_c)/g_c]^{\nu z}$$

- Coherent Length scale

$$\xi_\tau \sim \hbar c/\Delta \sim J^{-1}[(g - g_c)/g_c]^{-\nu z}$$

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - g\sum_i \sigma_i^x$$



In low-T disorder regime: $\xi_\tau \ll \frac{\hbar c}{k_B T} \Leftrightarrow \Delta \sim \hbar c/\xi_\tau \gg k_B T$

In quantum critical regime: $\xi_\tau \gg \frac{\hbar c}{k_B T} \Leftrightarrow \Delta \sim \hbar c/\xi_\tau \ll k_B T$

TFIC quantum criticality: thermodynamics

➤ Critical scaling behaviors of Grüneisen ratio

Low-T disordered region $|g-1| \gg t$

$$\Gamma_{cr}(t \to 0, g) = \frac{\alpha}{c_v} \to \frac{\mathrm{sgn}(g-1)}{|g-1|}$$

Quantum critical region $|g-1| \ll t$

$$\Gamma_{cr}(t, g) = \frac{\alpha}{c_v} \sim \mathrm{const}$$

No divergence! Particularly, right at the QCP with finite temperature

$$\boldsymbol{\Gamma_{cr}(g=1, t) = \frac{1}{2}}$$

Jianda Wu, Lijun Zhu, and Qimiao Si, Phys. Rev. B 97, 245127 (2018)

TFIC quantum criticality: dynamics

➢ Critical scaling behaviors: NMR relaxation rate

For the quantum critical transverse-field Ising chain, at finite T

$$\chi(k,\omega)=\frac{Z_c}{T^{7/4}}\frac{G_I(0)\Gamma\left(\frac{7}{8}\right)\Gamma(\frac{1}{16}-\mathrm{i}\frac{\omega+ck}{4\pi T})\Gamma(\frac{1}{16}-\mathrm{i}\frac{\omega-ck}{4\pi T})}{4\pi\Gamma(1/8)\Gamma(\frac{15}{16}-\mathrm{i}\frac{\omega+ck}{4\pi T})\Gamma(\frac{15}{16}-\mathrm{i}\frac{\omega-ck}{4\pi T})}$$

In NMR parameter region ($\omega\approx 0$ compared with other energy scales)

$$\frac{1}{T_1}\sim T\chi(\omega\approx 0)\sim\frac{1}{T^{3/4}}$$

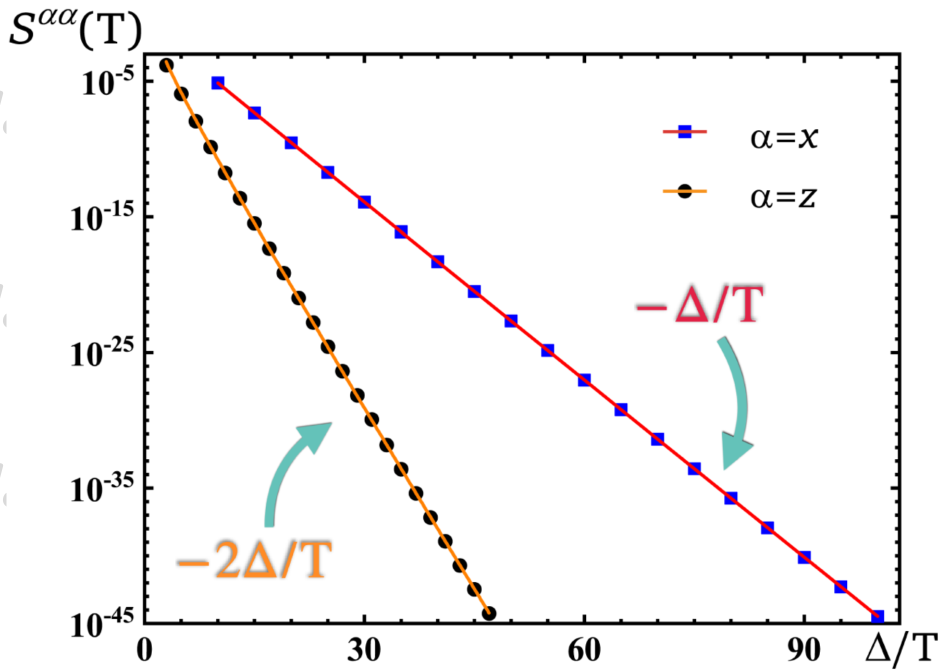
S. Sachdev, Quantum Phase Transitions, Cambridge University Press (2011)

TFIC quantum criticality: Thermal activation gap

- **Local spin dynamics** $\omega \ll T \ll \Delta$

$$S^{zz}(\omega) \approx \frac{\bar{\sigma}^2}{\Delta}\frac{3\sqrt{3}}{2\pi}\left(\frac{k_B T}{\Delta}\right)^2 e^{-\frac{2\Delta}{k_B T}}$$

$$S^{xx}(\omega) \approx \frac{\Delta}{\pi J^2} e^{-\frac{\Delta}{k_B T}}\left[-\ln\left(\frac{\omega}{4 k_B T}\right) + \frac{k_B T}{2\Delta} - \gamma_E\right]$$



J. Yang, W. Yuan, T. Imai, Q. Si*, **JW***, and M. Kormos*, Phys. Rev. B 106, 125149 (2022)

E_8 physics

TFIC with longitudinal field

1D Transverse-field Ising chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - g\sum_i \sigma_i^x + h\sum_i \sigma_i^z$$

Perturbation term

$$m_1 \approx 4.405|h|^{\frac{8}{15}}; \quad m_2 = 2m_1 \cos\frac{\pi}{5};$$
$$m_3 = 2m_1 \cos\frac{\pi}{30}; \quad m_4 = 2m_2 \cos\frac{7\pi}{30};$$
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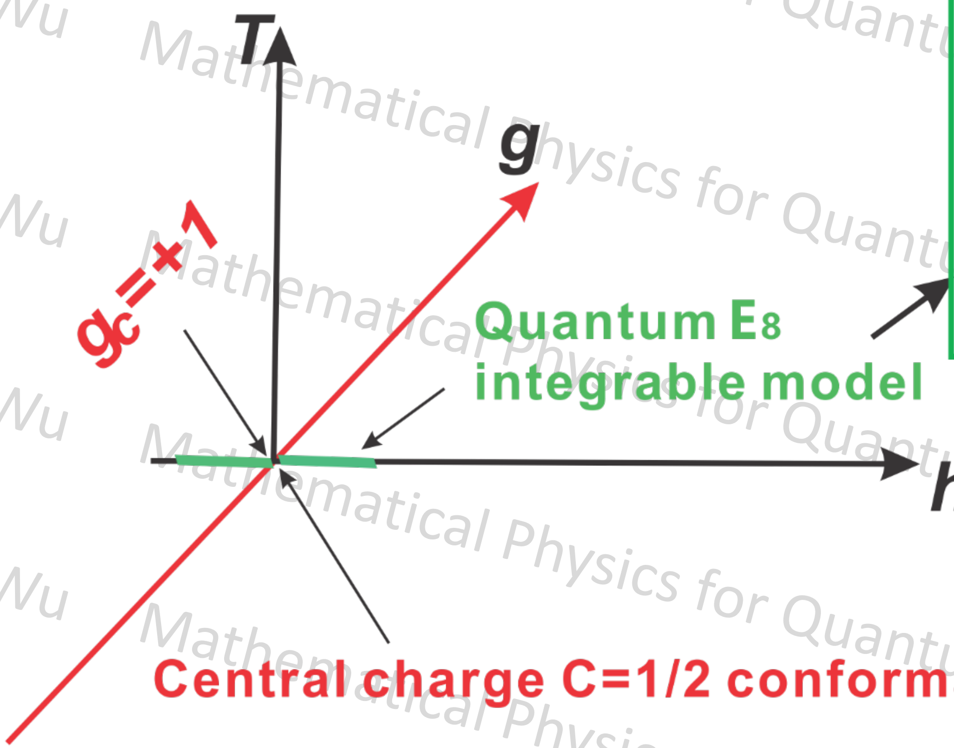


A. B. Zamolodchikov, Int. J. Mod. Phys. A4, 4235(1989).

A "quantum critical point" in the 3D AFM dome of $BaCo_2V_2O_8$

The effective Hamiltonian for $BaCo_2V_2O_8$ on a chain

$$H = J\sum_i [\epsilon(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + \sigma_i^z\sigma_{i+1}^z] - g_x\sum_i \sigma_i^x - g_y\sum_i (-)^i\sigma_i^y$$

⇧

Distortion effect: $\mathcal{H}_{perturbation} = \lambda \mathbf{l}\cdot\mathbf{S} - \delta\left(l_z^2 - \frac{2}{3}\right)$

⇧

Spin-orbit coupling splits the orbital ground state manifold T_1

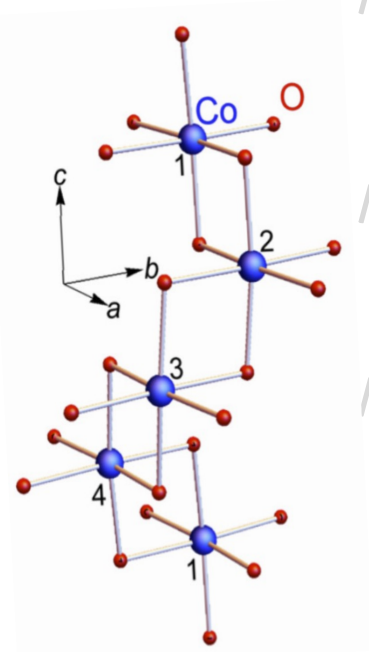
$$T_1 = D_{j=5/2} \oplus D_{j=3/2} \oplus D_{j=1/2}$$

⇧

Crystal field effect for single ion: $\Gamma_{L=3} = A_2 \oplus T_1 \oplus T_2$

⇧

Co^{2+} **ion:** $3d^7$ **has** $L = 3, S = \frac{3}{2}$



J. Yang, X. Wang, and **JW***, J. Phys. A Math.Theor.56.013001 (2023)

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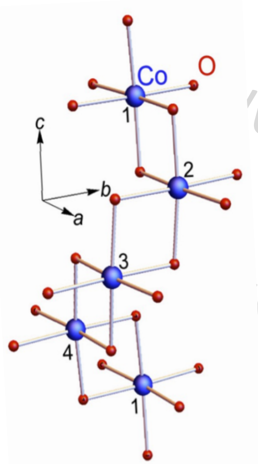


Same universality

$$H = -\sum_i \sigma_i^z\ \sigma_{i+1}^z - g_x\sum_i \sigma_i^x$$

$$U = \exp(-i\alpha\sum_i S_i^z)$$

$$H = -\sum_i \sigma_i^z\ \sigma_{i+1}^z - g_x\sum_i \sigma_i^x - g_y\sum_i \sigma_i^y$$

AFM Ising

$$U = \exp(-i\pi\sum_i S_{2i}^x)$$

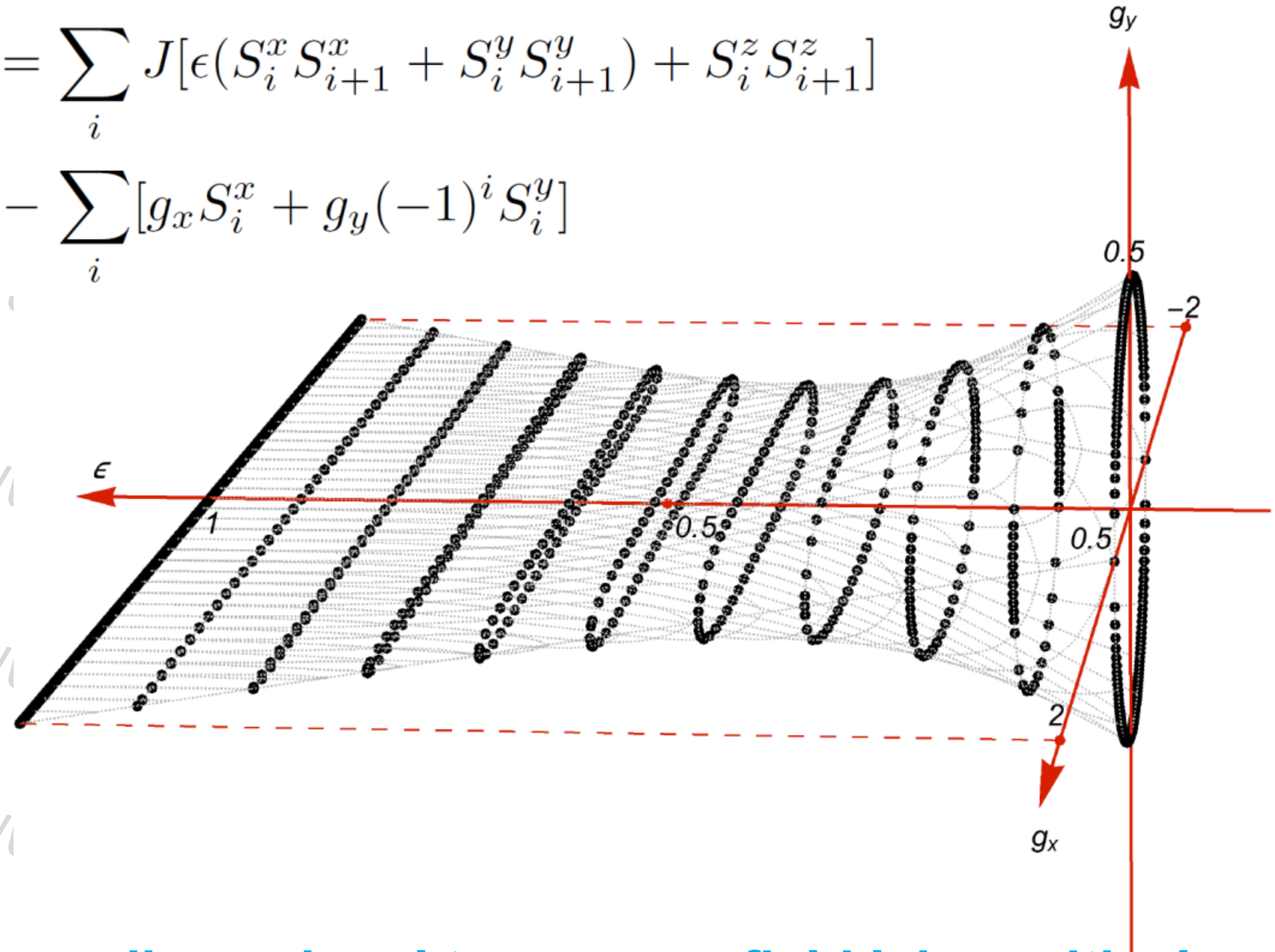
$$H = \sum_i \sigma_i^z\ \sigma_{i+1}^z - g_x\sum_i \sigma_i^x - g_y\sum_i (-)^i\sigma_i^y$$

FM Ising

H. Zou, R. Yu*, and **JW***, J. Phys: Condens. Matter 32, 045602 (2020)

Quantum critical surface

$$H_g = \sum_i J[\epsilon(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + S_i^z S_{i+1}^z] - \sum_i [g_x S_i^x + g_y (-1)^i S_i^y]$$



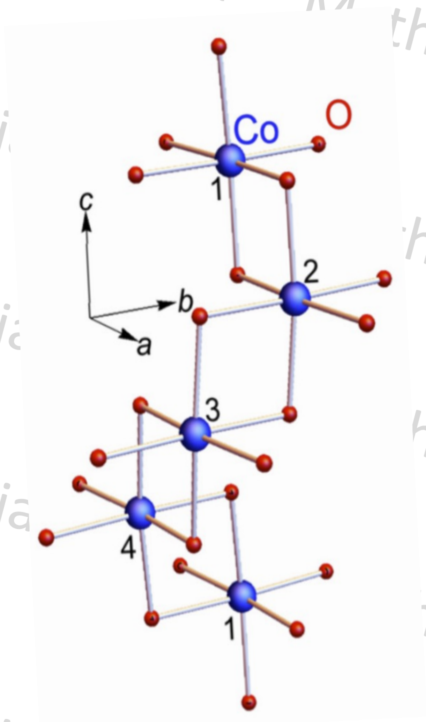
The one-dimensional transverse-field Ising critical surface in the 3D parameter space from iTEBD calculation.

Haiyuan Zou, Rong Yu*, and Jianda Wu*, J.Phys: Condens Matter 32, 045602 (2019)

A “quantum critical point” in the 3D AFM dome of $BaCo_2V_2O_8$

The effective Hamiltonian for $BaCo_2V_2O_8$ on a chain

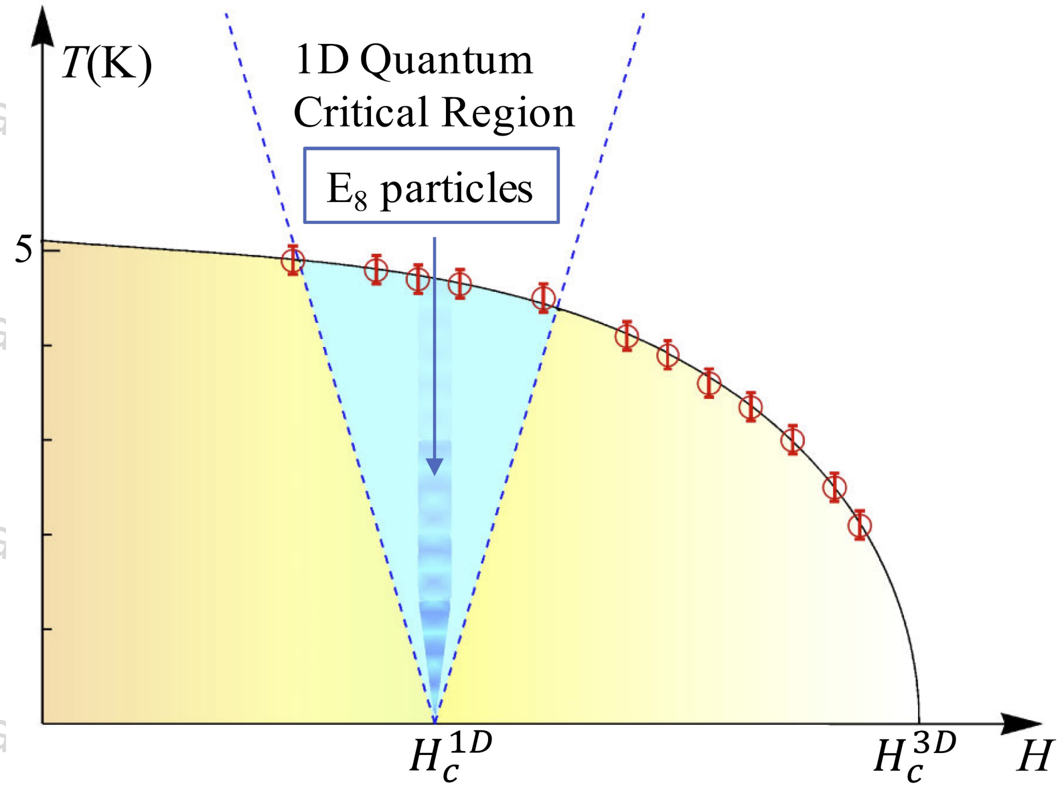
$$H = J\sum_i [\epsilon(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + \sigma_i^z\sigma_{i+1}^z] - g_x\sum_i \sigma_i^x - g_y\sum_i (-)^i\sigma_i^y$$



We need

- **Transverse field Ising chain QCP**
- **A staggered magnetic field at the QCP due to the AFM Ising coupling**

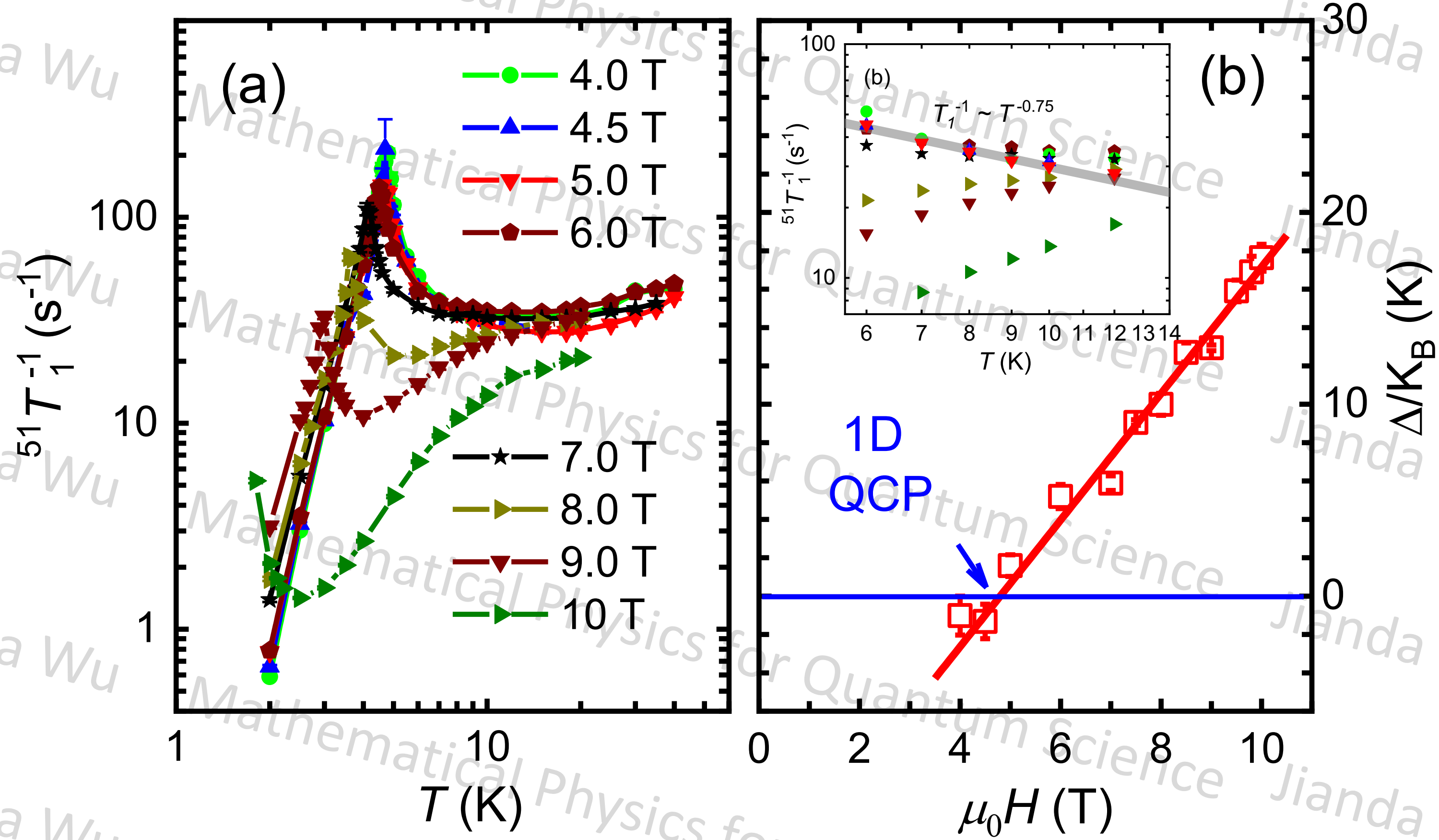
The region for E_8 physics!



- **3D AFM order ⟹ Effective staggered longitudinal field**
- **TFIC quantum criticality confirmed by the NMR measurement**
- **E_8 physics confirmed by the THz ESR and INS measurements**

H. Zou#, Y. Cui#, X. Wang#, ..., Weiqiang Yu*, Jie Ma*, and **JW***, Phys. Rev. Lett. 127, 077201 (2021)

NMR experiment



NMR experiment clearly identifies 1D quantum criticality outside the 3D AFM dome: the power law behavior of the relaxation rate follows transverse-field Ising chain universality.

Phys. Rev. Lett. 127, 077201 (2021)

E_8 spectrum - analytical results

$$\mathcal{A}_{E_8} = \mathcal{A}_{1/2} + h\int \sigma(x,\tau)\,dx\,d\tau$$

$$D^{\alpha\alpha}(\omega, q=0) = \sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{d\theta_1\cdots d\theta_n}{N(2\pi)^{n-2}}\left|\langle 0|\sigma^{\alpha}|A_{a_1}(\theta_1)\cdots A_{a_n}(\theta_n)\rangle\right|^2\delta(\omega - E\{a_i\})\delta(P\{a_i\})$$

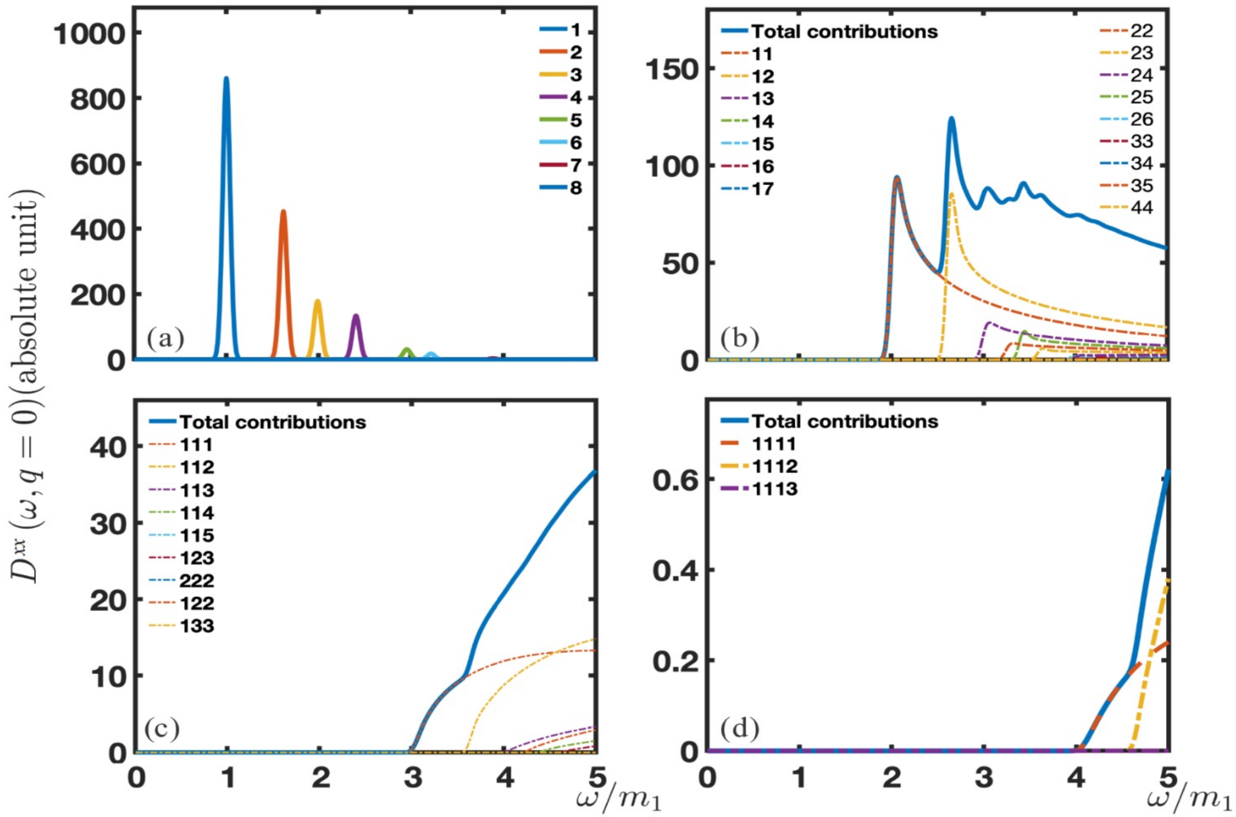
$\lvert n\rangle$	the total energy reduced by m_1	$\lvert n\rangle$	the total energy reduced by m_1
A_1	1	A_2A_5	≥4.5743
A_2	1.6180	A_2A_6	≥4.8364
A_3	1.9890	A_3A_3	≥3.9781
A_4	2.4049	A_3A_4	≥4.3939
A_5	2.9563	A_3A_5	≥4.9453
A_6	3.2183	A_4A_4	≥4.8097
A_7	3.8912	$A_1A_1A_1$	≥3
A_8	4.7834	$A_1A_1A_2$	≥3.6180
A_1A_1	≥2	$A_1A_1A_3$	≥3.9890
A_1A_2	≥2.6180	$A_1A_1A_4$	≥4.4049
A_1A_3	≥2.9890	$A_1A_1A_5$	≥4.9563
A_1A_4	≥3.4049	$A_1A_2A_2$	≥4.2361
A_1A_5	≥3.9563	$A_1A_2A_3$	≥4.6071
A_1A_6	≥4.2183	$A_1A_3A_3$	≥4.9781
A_1A_7	≥4.8912	$A_2A_2A_2$	≥4.8541
A_2A_2	≥3.2361	$A_1A_1A_1A_1$	≥4
A_2A_3	≥3.6071	$A_1A_1A_1A_2$	≥4.6180
A_2A_4	≥4.0229	$A_1A_1A_1A_3$	≥4.9890

Phys. Rev. Lett. 127, 077201 (2021)

E_8 spectrum - analytical results (continue)

$$\mathcal{A}_{E_8} = \mathcal{A}_{1/2} + h\int \sigma(x,\tau)\,dx\,d\tau$$

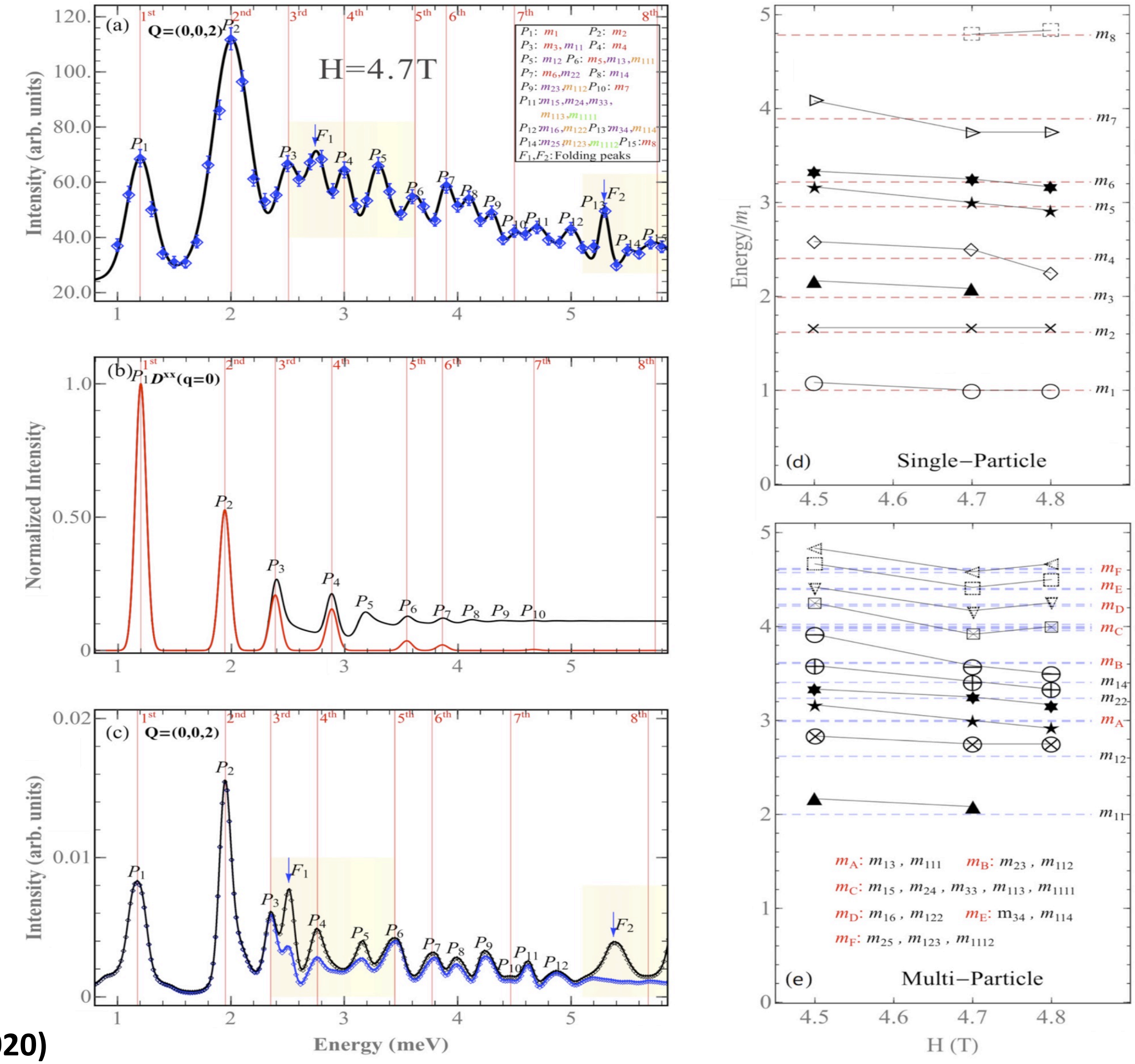
$$D^{\alpha\alpha}(\omega, q=0) = \sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{d\theta_1\cdots d\theta_n}{N(2\pi)^{n-2}}\left|\langle 0|\sigma^{\alpha}|A_{a_1}(\theta_1)\cdots A_{a_n}(\theta_n)\rangle\right|^2\delta(\omega - E\{a_i\})\delta(P\{a_i\})$$



Xiao Wang, …, **JW***, Phys. Rev. B 103, 235117 (2021)
Phys. Rev. Lett. 127, 077201 (2021)

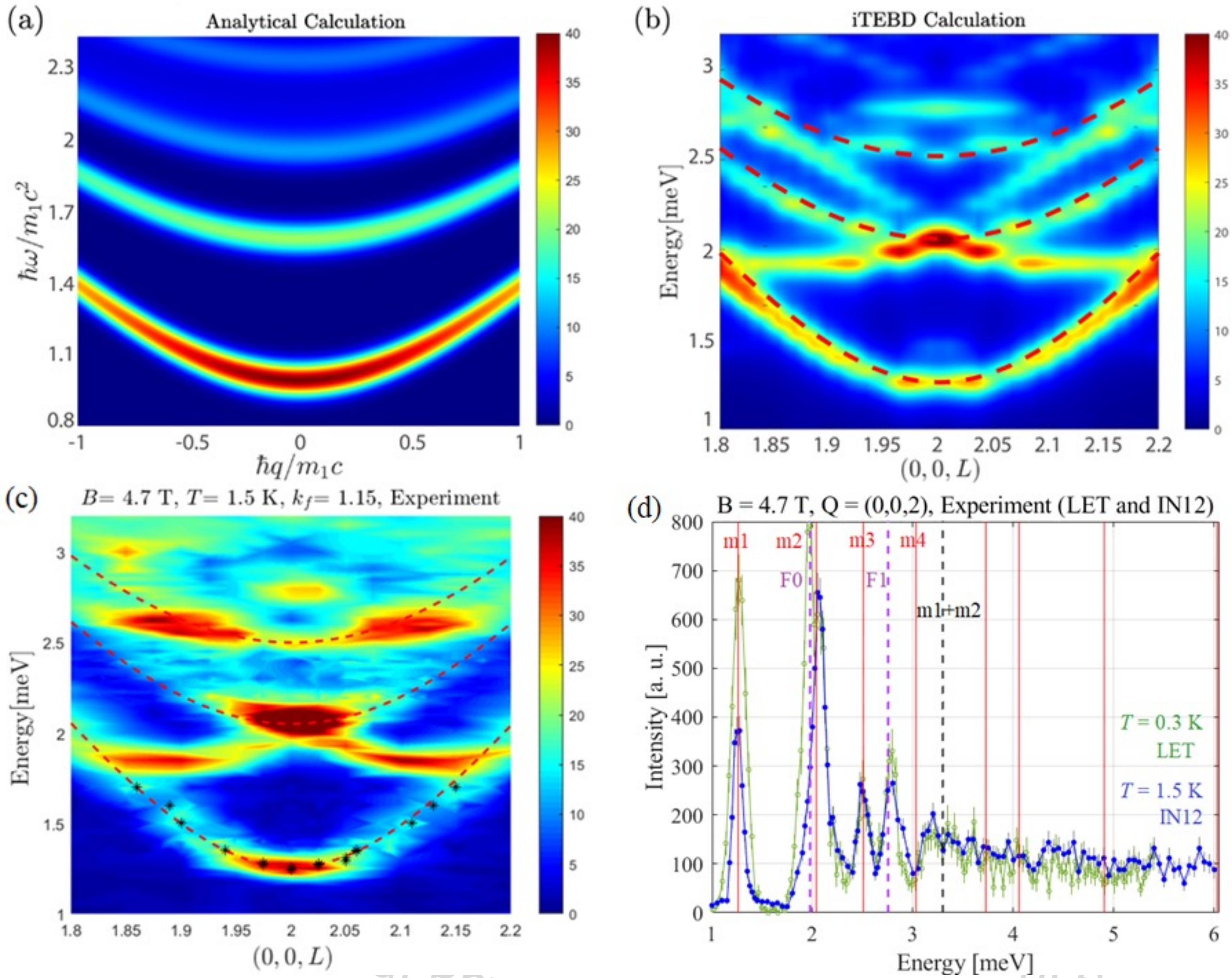
E8 spectrum observed in experiments

Neutron experiment:



Phys. Rev. B 101, 220411(R) (2020)
Phys. Rev. B 103, 235117 (2021)
Phys. Rev. Lett. 127, 077201 (2021)

E_8 particles observed in experiments



X. Wang, K. Puzniak, K. Schmalzl, C. Balz, M. Matsuda, A. Okutani, M. Hagiwara, J. Ma*, **JW***, & B. Lake*, Science Bulletin 69, 2974 (2024)

$D_8^{(1)}$ physics

Weakly-coupled transverse field Ising ladder

$$H = H^{(1)}_{\text{Ising}} + H^{(2)}_{\text{Ising}} + \mathrm{J}_i \sum_{i=1}^{N} \sigma_i^{z(1)} \sigma_i^{z(2)}$$

$$H^{(1,2)}_{\text{Ising}} = -J\left(\sum_{i=1}^{N} \sigma_i^{z(1,2)} \sigma_{i+1}^{z(1,2)} + \sum_{i=1}^{N} \sigma_i^{x(1,2)}\right)$$

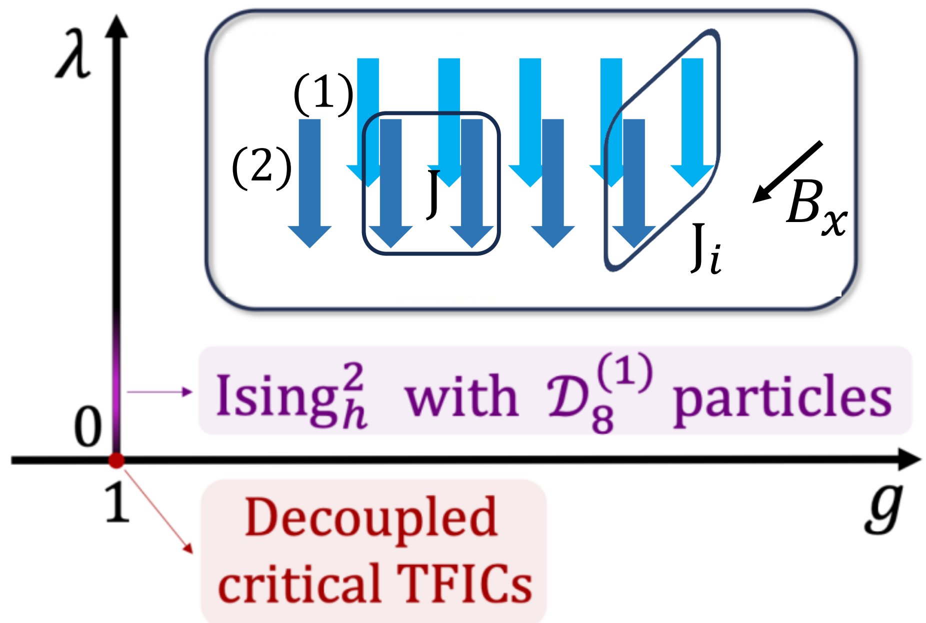
$$m_{A_{+1}} = m_{A_{-1}}$$

$$m_1 = 2m_s \sin\frac{\pi}{14} \qquad m_4 = 2m_s \sin\frac{2\pi}{7}$$

$$m_2 = 2m_s \sin\frac{\pi}{7} \qquad m_5 = 2m_s \sin\frac{5\pi}{14}$$

$$m_3 = 2m_s \sin\frac{3\pi}{14} \qquad m_6 = 2m_s \sin\frac{3\pi}{7}$$

A. LeClair, A. Ludwig, and G. Mussardo, Nucl. Phys. B 512, 523 (1998);



Yunjing Gao
高云静

Y. Gao, X. Wang, N. Xi, Y. Jiang, R. Yu*, **JW*** arXiv: 2402.11229, submitted

Ising_h^2 integrable field theory

- Scaling limit $a,\ J_i \to 0,\ J_i/a$ finite $\quad \mathcal{A}_{\mathrm{Ising}_h^2} = \mathcal{A}_{1/2}^{(1)} + \mathcal{A}_{1/2}^{(2)} + \lambda \int_i \sigma^{(1)}\sigma^{(2)} dx d\tau$

lattice	→	field theory	Scaling dimension
$\sigma_i^{z(1,2)}$	→	$\sigma^{(1,2)}(x)$	$1/8$
$\sigma_i^{x(1,2)}$	→	$\epsilon^{(1,2)}(x)$	1

- Bosonization rules

$$\sigma^{(1)}(x)\sigma^{(2)}(x) \sim :\cos\frac{\phi(x)}{2}:$$
$$\epsilon^{(1)}(x) + \epsilon^{(2)}(x) \sim :\cos\phi(x):$$
$$\epsilon^{(1)}(x) - \epsilon^{(2)}(x) \sim :\cos\Theta(x):$$

$\phi(x)$: bosonic field

$\Theta(x)$: dual field of $\phi(x)$

$$\frac{\partial\Theta}{\partial x} = -\frac{\partial\phi}{\partial t}$$

⇒ $$\mathcal{A} = \int dx dt \left\{ \frac{1}{16\pi}(\partial\phi)^2 + \Lambda\cos\phi/2 \right\}$$ ($\mathbb{Z}_2$ orbifold)

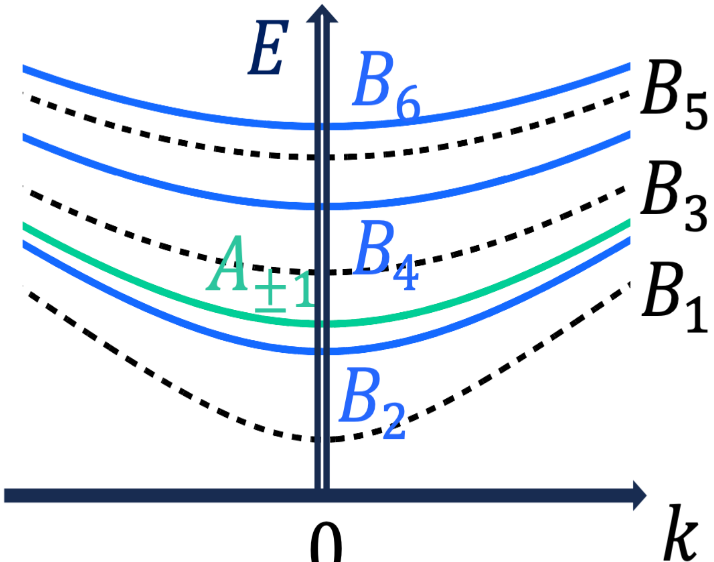
A. LeClair, A. Ludwig, and G. Mussardo, Nucl. Phys. B 512, 523 (1998)
D. Boyanovsky, Phys. Rev. B 39, 6744 (1989)

$\mathrm{Ising}_{\mathrm{h}}^2$ integrable field theory

- Effective $\mathrm{Ising}_{\mathrm{h}}^2$ action

Described by $D_8^{(1)}$ algebra

$$\mathcal{A} = \int dxdt \left\{ \frac{1}{16\pi} (\partial\phi)^2 + \Lambda \cos\phi/2 \right\} \quad (\mathbb{Z}_2 \text{ orbifold})$$



Emerged 8 relativistic particles:

breathers: $m_{B_n} = m_{B_1} \dfrac{\sin\frac{n\pi}{14}}{\sin\frac{\pi}{14}},$

$n = 1,2,\ldots,6$

(anti-)soliton: $m_{A_{\pm 1}} = \dfrac{m_{B_1}}{2\sin\frac{\pi}{14}}$

scaling $m \sim \Lambda^{4/7}$

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW***
arXiv: 2402.11229, submitted

Global properties

- Topological charge $Q = \int_{\infty}^{x} \partial_x \phi$

$\vert B_n\rangle$	$Q = 0$
$\vert A_{+1}\rangle$	$Q = 1$
$\vert A_{-1}\rangle$	$Q = -1$

$\cos\phi(x)$: conserve Q

$e^{i\Theta(x)}$: charge-1 raising

$e^{-i\Theta(x)}$: charge-1 lowering

- Charge parity conjugation $\hat{C}$ B. Schroer and T. Truong. Nucl. Phys. B 144, 80 (1978)

$$\hat{C}|A_{\pm1}(\theta)\rangle = |A_{\mp1}(\theta)\rangle \qquad \hat{C}\phi\hat{C}^{-1} = -\phi$$

$$\hat{C}|B_n(\theta)\rangle = (-1)^n|B_n(\theta)\rangle \qquad \hat{C}\cos\phi(x)\hat{C}^{-1} = \cos\phi(x)$$

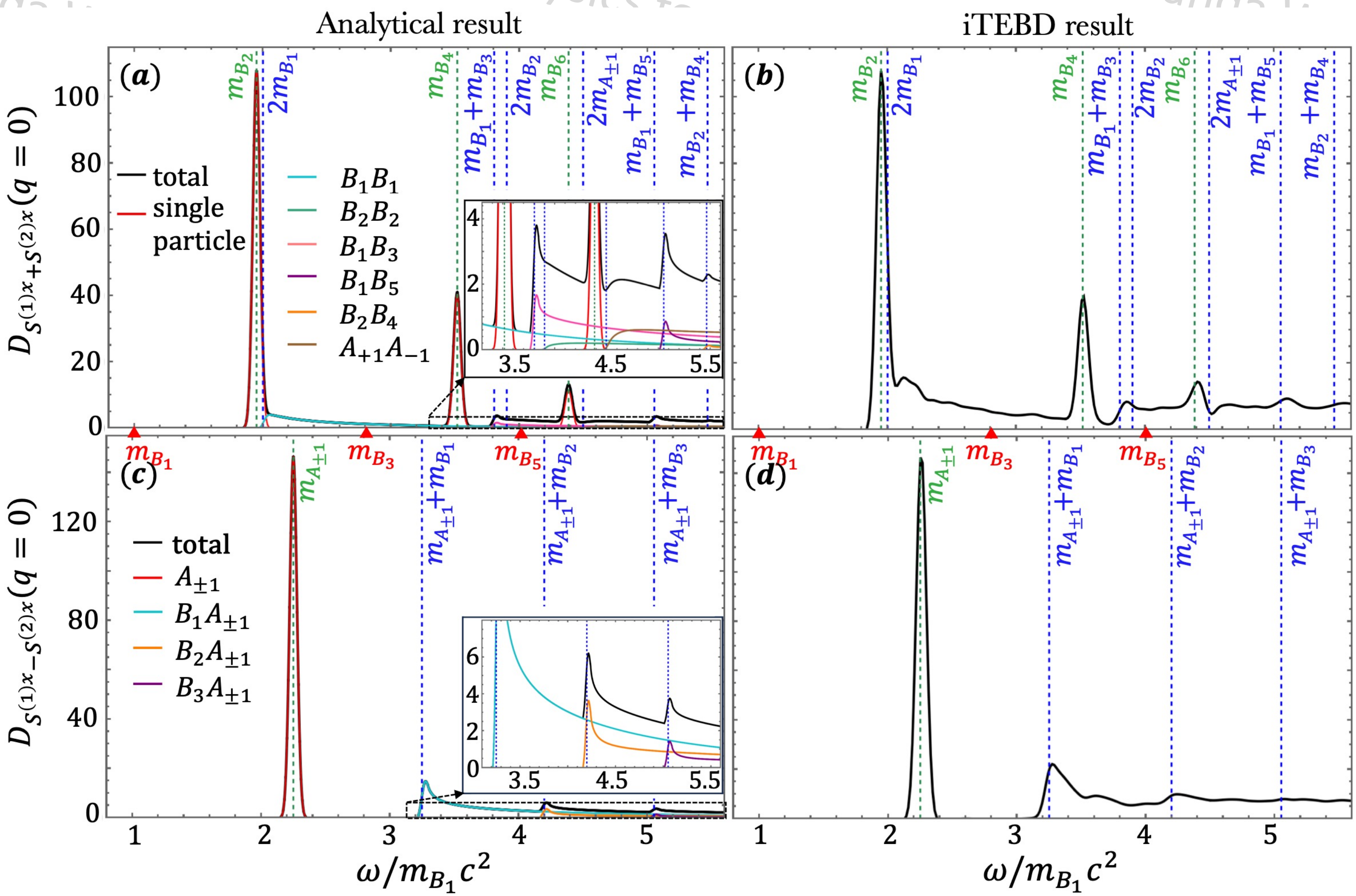
- Dynamic Structure Factor

$$D^{\alpha\alpha}(\omega, q=0) = \sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{d\theta_1\cdots d\theta_n}{N(2\pi)^{n-2}}\left|\langle 0|O^{\alpha}|A_{a_1}(\theta_1)\cdots A_{a_n}(\theta_n)\rangle\right|^2\delta(\omega - E\{a_i\})\delta(P\{a_i\})$$

Dynamic structure factors

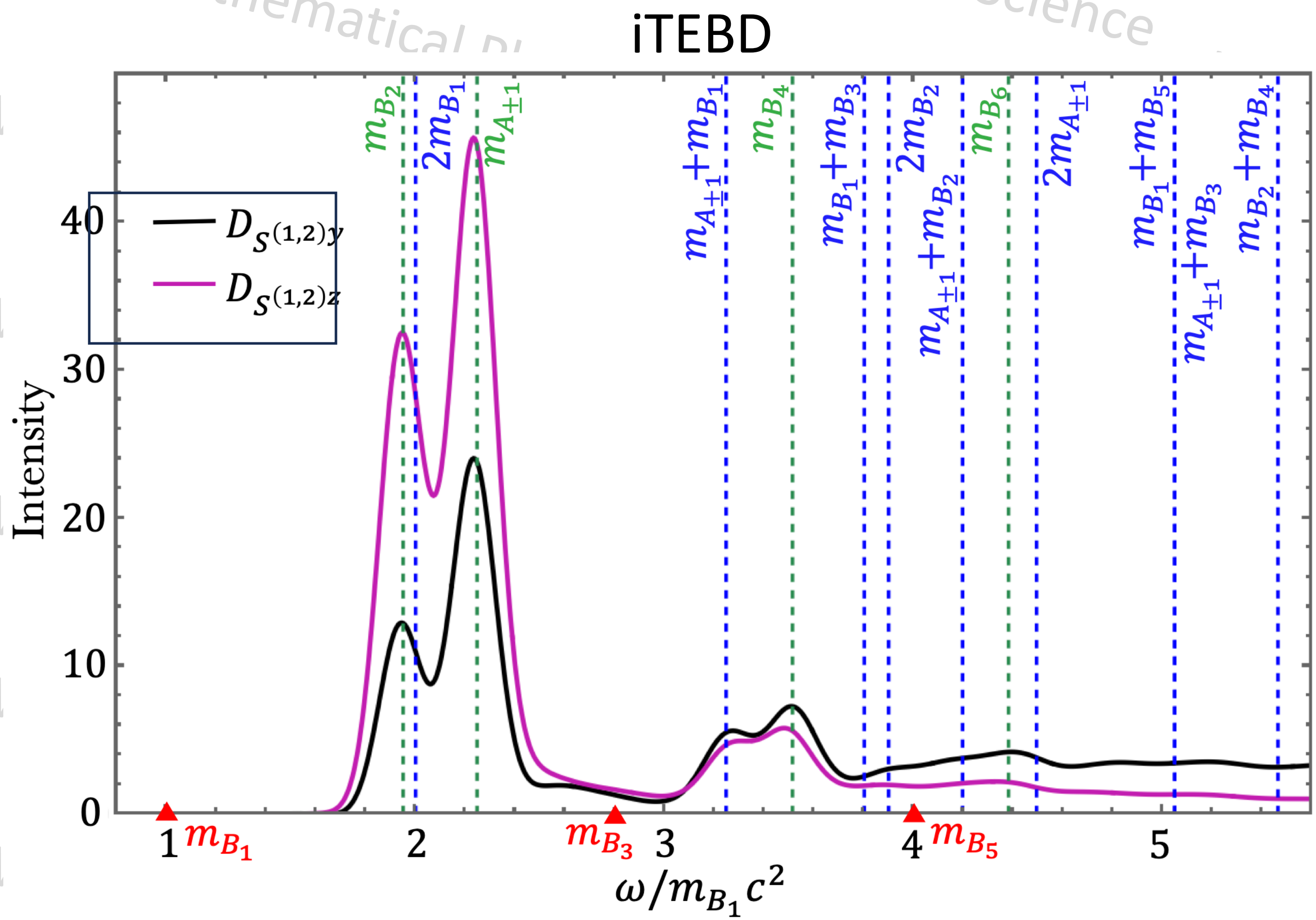
$S_j^{(1)x} + S_j^{(2)x} \sim :\cos\phi(x):$

$S_j^{(1)x} - S_j^{(2)x} \sim :\cos\Theta(x):$



Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

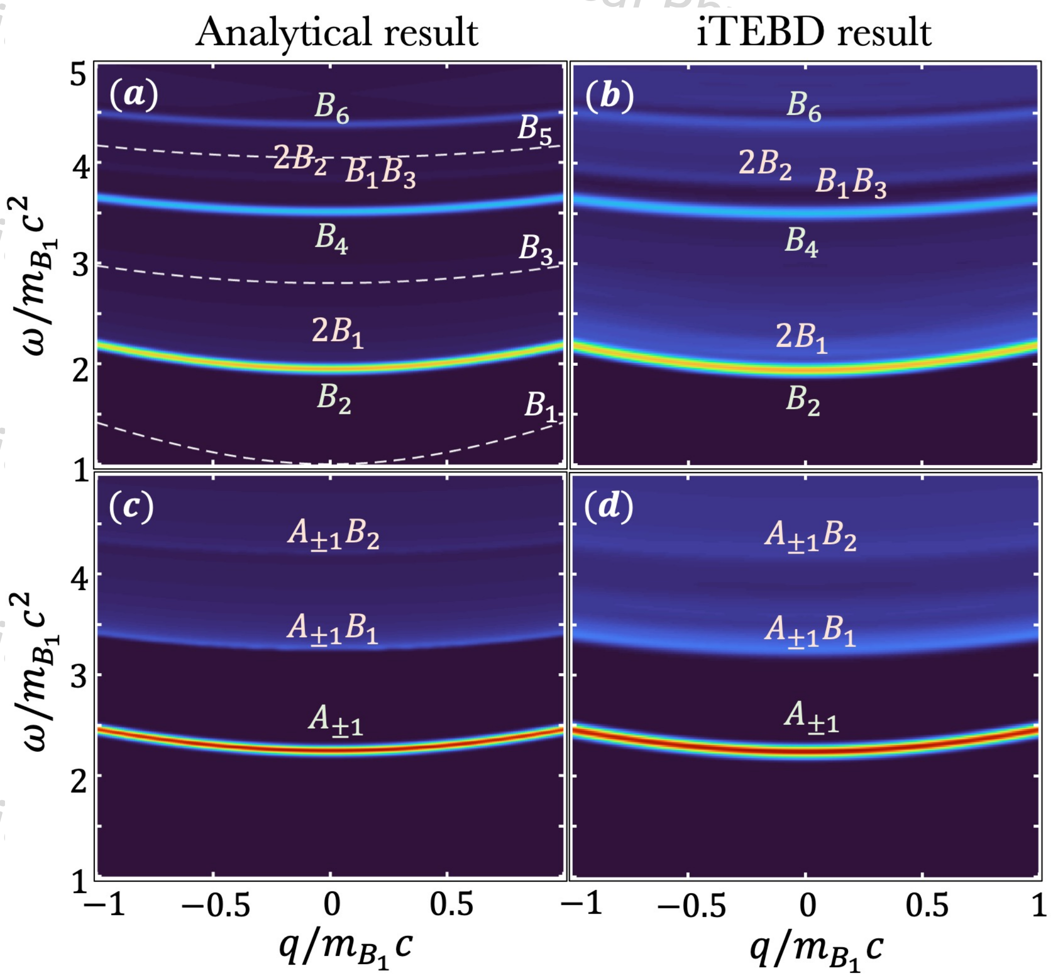
Dynamic structure factors



No B_1, B_3, B_5 excitations from $S^{x,y,z}$ channels !

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

Dynamic structure factors



- $\sim$10% region of the Brillouin zone is plotted
- Single particle channels: strong delta peaks
- Two particle channels: diverge at edges if $m_{P_1} \neq m_{P_2}$

(a,b): $S^{(1)x} + S^{(2)x}$

(c,d): $S^{(1)x} - S^{(2)x}$

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

Dark particle

- The **parity-odd** B_1, B_3, B_5 cannot be excited directly from the ground state through $S^{x,y,z}$ fluctuations (all momentum modes)

$$S^x \to \cos\phi + \cos\Theta$$ +iTEBD

- Parity-odd operator? Permit $|0\rangle \to |B_{1,3,5}\rangle$ view from field theory

$$\mu_j^D = \prod_{l=1}^{j-1} \sigma_l^{x(1)} \sigma_l^{x(2)} \xrightarrow[\text{limit}]{\text{scaling}} \mu^D(x) \xrightarrow{\text{bosonization}} \sin\phi(x)$$

⇩

simultaneous coherent operation on macroscopic number of spins

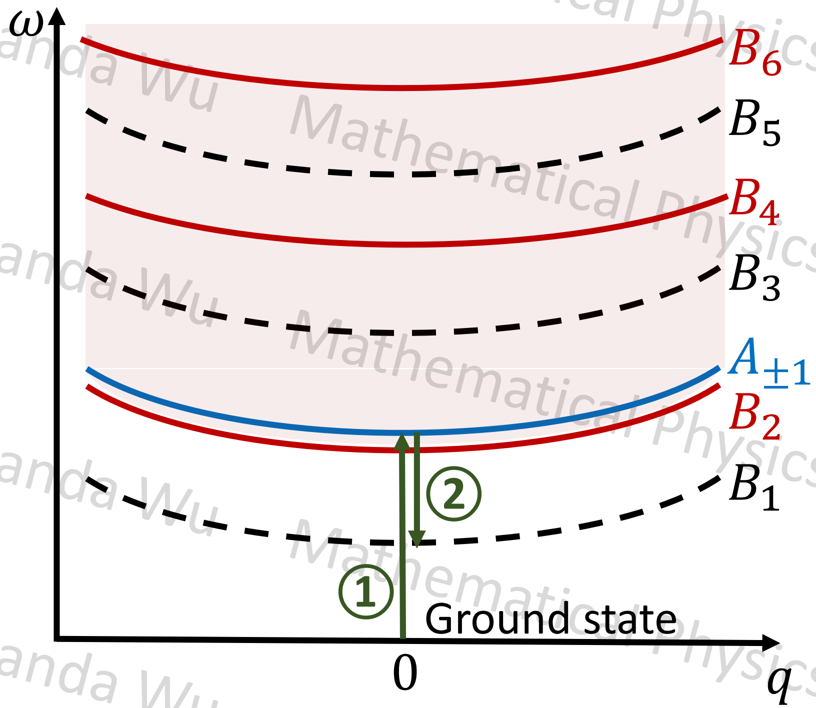
coherent length of vacuum (ground state) fluctuations

$$\xi_0 \sim \frac{\hbar c}{m_1} \approx 5a$$

Vacuum fluctuation of μ_j^D-type is exponentially cutoff as e^{-ja/ξ_0}!

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

Dark particle



Spontaneous decay from B_1 to GS:

- Forbiden from (quasi-) local fluctuation
- Field theory permitted channel is not accessible

$\Longrightarrow$ **Stable and Long life-time B_1**

Magnetic Storage?

Transportation?

Quantum qubit?

...

B_1 preparation:

① excitation

② controlled decay

B_1 cannot decay to ground state through spin fluctuations once being prepared

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

Dark particle detection

Thermal activation detection

Local spin dynamics for $\omega \ll T \ll m_1$

$$C^{\mathcal{O}}(\omega,T) = \frac{1}{\mathcal{Z}}\sum_{i,f} C^{\mathcal{O}}_{i,f}(\omega,T) = \sum_{i,f}^{\infty} D^{\mathcal{O}}_{i,f}(\omega,T)$$

where

$$C^{\mathcal{O}}_{i,f}(\omega,T) = \sum_{\mathbf{i},\mathbf{f}} \int \frac{d\theta'_1 \ldots d\theta'_i}{(2\pi)^i \mathcal{A}_{\mathbf{i}}} \int \frac{d\theta_1 \ldots d\theta_f}{(2\pi)^f \mathcal{A}_{\mathbf{f}}} e^{-E_{\mathbf{i}}/T} \qquad \mathcal{A}_{\mathbf{i}} = \prod_{l\in\mathbf{i}} n_l!$$

$$\cdot |\langle P'_1(\theta'_1)\ldots P'_i(\theta'_i)|\mathcal{O}|P_1(\theta_1)\ldots P_f(\theta_f)\rangle|^2 \delta(\omega + E_{\mathbf{i}} - E_{\mathbf{f}})$$

and

B. Pozsgay and G. Takacs, J. Stat. Mech. 1011, P11012 (2010)

$$D^{\mathcal{O}}_{0,f} = C^{\mathcal{O}}_{0,f},\; D^{\mathcal{O}}_{1,f} = C^{\mathcal{O}}_{1,f} - \mathcal{Z}_1 C^{\mathcal{O}}_{0,f-1}$$

$$D^{\mathcal{O}}_{2,f} = C^{\mathcal{O}}_{2,f} - \mathcal{Z}_1 C^{\mathcal{O}}_{1,f-1} + (\mathcal{Z}_1^2 - \mathcal{Z}_2) C^{\mathcal{O}}_{0,f-2}$$

$$\mathcal{Z}_n = \sum_{\mathbf{n}} \int \frac{d\theta_1 \ldots d\theta_n}{(2\pi)^n \mathcal{A}_{\mathbf{n}}} e^{-\frac{E_{\mathbf{n}}}{T}} \langle P_1(\theta_1)\ldots P_n(\theta_n)|P_1(\theta_1)\ldots P_n(\theta_n)\rangle$$

Thermal activation detection

$$C^x(\omega,T)\approx D^x_{1,1}(\omega,T)\approx\frac{|F^{\mathrm{B_1B_1}}_{\cos\phi}(i\pi)|^2}{\pi m_{B_1}}e^{-m_{B_1}/T}\left[\sqrt{\frac{\pi T}{\omega}}-\frac{\sqrt{\pi}}{4}\left(\frac{T}{\omega}\right)^{\frac{3}{2}}\right]$$

The NMR relaxation rate for a local nucleus

$$\frac{1}{T_1}\sim|A_y|^2C^y(\omega_n)+|A_z|^2C^z(\omega_n)$$

Yunjing Gao
高云静

Jiahao Yang
杨家豪
(Now Postdoc at Peking U)

But

$$C^y=(\omega/J)^2C^z/4\Longrightarrow\frac{1}{T_1}\sim|A_z|^2C^z(\omega_n)$$

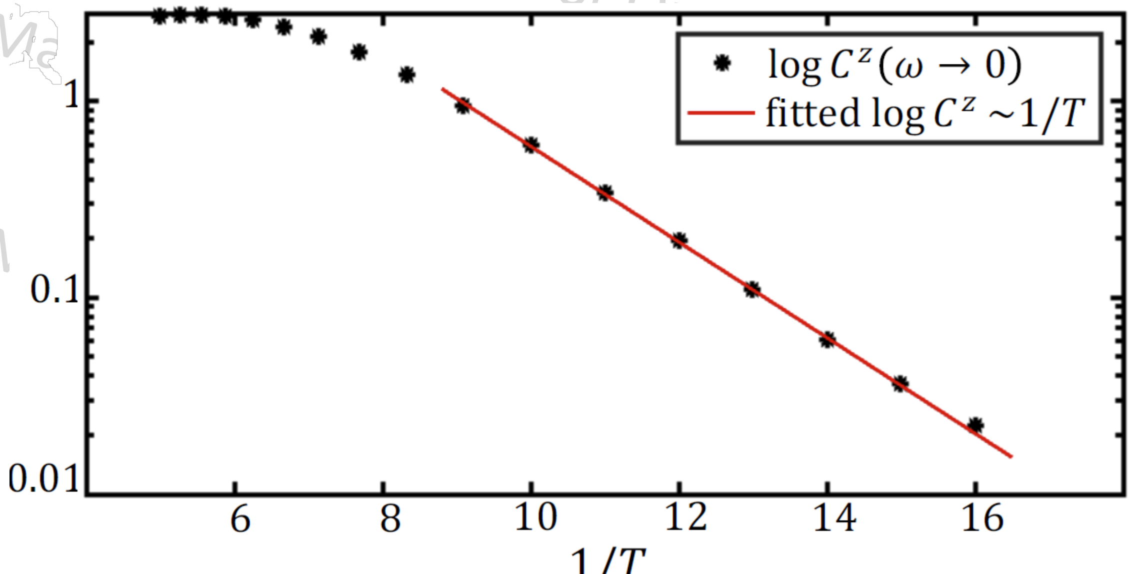
Furthermore,

$$1/T_2=1/T_1'+1/T_2'\Longrightarrow\frac{1}{T_1'}=\frac{A}{T_1},\quad\frac{1}{T_2'}=|A_x|^2C^x(\omega\to0)$$

Y. Gao#, J. Yang#, H. Lin, R. Yu, **JW***, arXiv:2406.15024, submitted

Thermal activation detection

QMC: $J = 1, J_i = 0.1$



$$C^z = 160.4513 e^{-0.56/T}$$

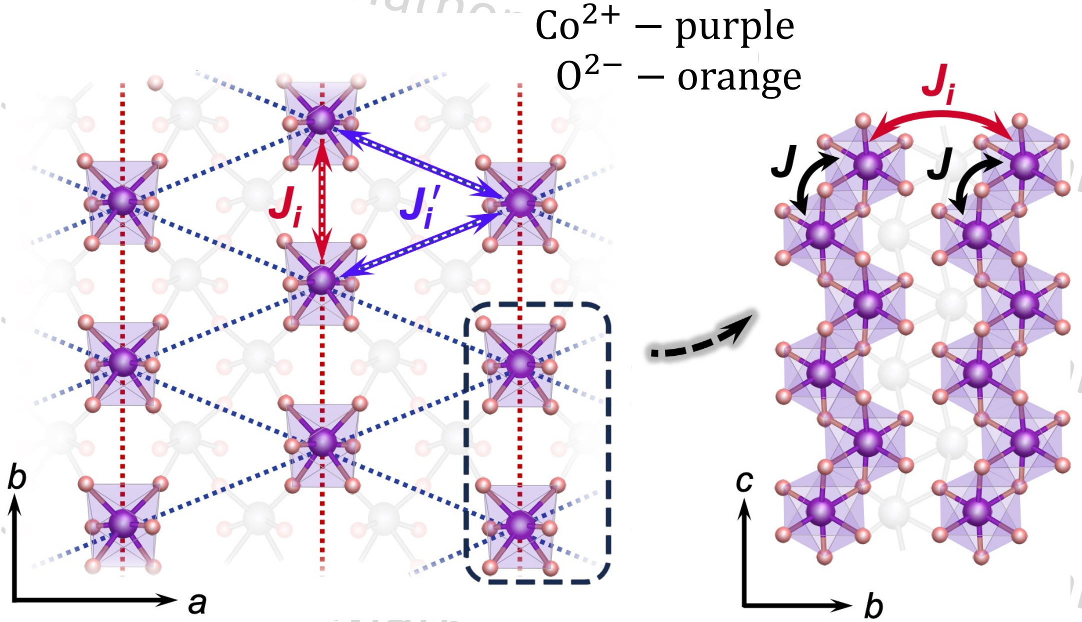
Analytical lattice result

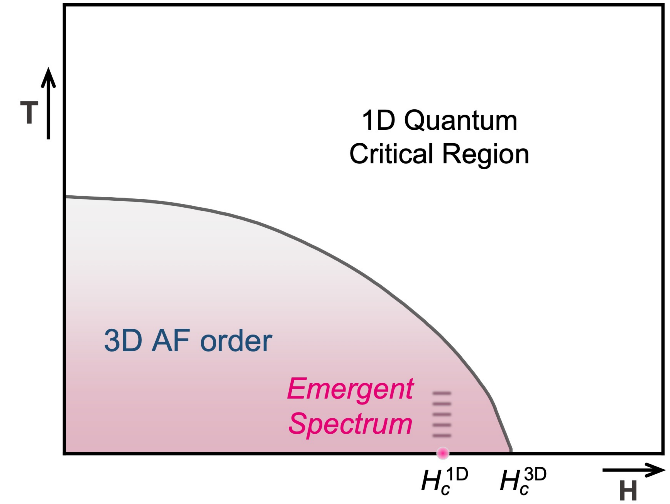
$$m_{B_1} = \left(\frac{2\sqrt{2}}{\mathcal{C}_1 2^{1/6} e^{-1/4} \mathcal{A}^3} \right)^{4/7} J_i^{4/7} J \Longrightarrow m_{B_1} \approx 0.63$$

Y. Gao#, J. Yang#, H. Lin, R. Yu, **JW***, arXiv:2406.15024, submitted

$CoNb_2O_6$

$CoNb_2O_6$





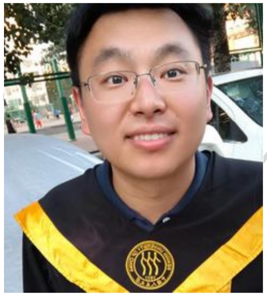
J = 2.7607 meV
ϵ = 0.239 meV
λ_{af} = 0.1507, λ_{dw} = 0.1647
g = 3.1

The model $\quad H = H_{chain} + H_{ic}$

$$H_{chain} = J\sum_j -S_j^z S_{j+1}^z - \epsilon\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right) - \lambda_{af} S_j^z S_{j+2}^z$$
$$+(-1)^j \lambda_{dw}\left(S_j^z S_{j+1}^y + S_j^y S_{j+1}^z\right) - g\mu_0 H \sum_j S_j^x$$

$$H_{ic} = \sum_{j,\langle\alpha,\beta\rangle} J_i S_{j,\alpha}^z S_{j,\beta}^z$$

Xiao Wang
王骁

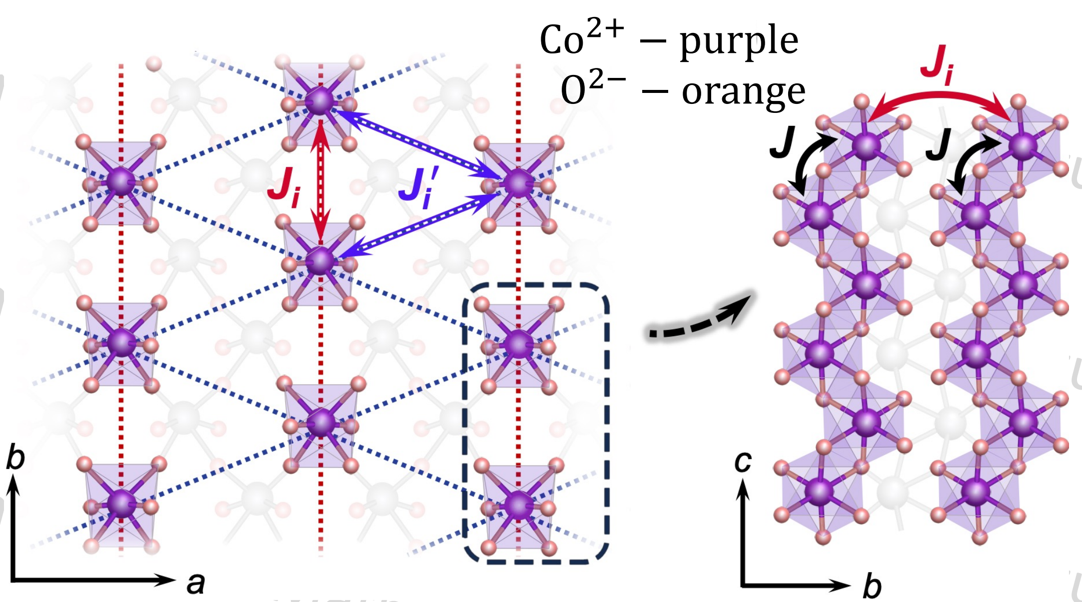


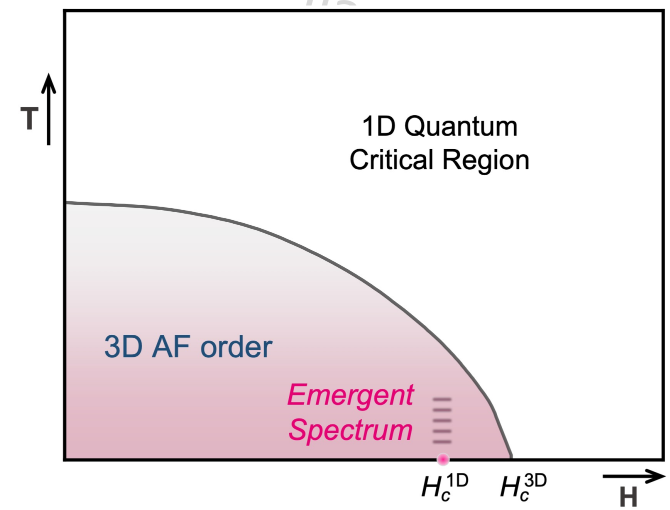
Ning Xi
西宁

M. Fava et. al, Proc. Natl. Acad. Sci. U.S.A. 117, 25219 (2020)

N. Xi#, X. Wang#, Y. Gao, Y. Jiang, Rong Yu*, **JW*** arXiv: 2403.10785, submitted

$CoNb_2O_6$





frustration v.s. proximity to 3D QCP

cluster chain mean-field

$$H_{ic} = \sum_{j,\langle\alpha,\beta\rangle} J_i S^z_{j,\alpha} S^z_{j,\beta} \approx \underbrace{J_i \sum_{j=1}^{N} S^z_{j,1} S^z_{j,2}}_{\text{two N.N. chains as minimal unit}} - \underbrace{\tilde{h} \sum_{j=1}^{N} \sum_{m=1,2} S^z_{j,m}}_{\text{other chains: mean-field}}$$

two N.N. chains as minimal unit

other chains: mean-field

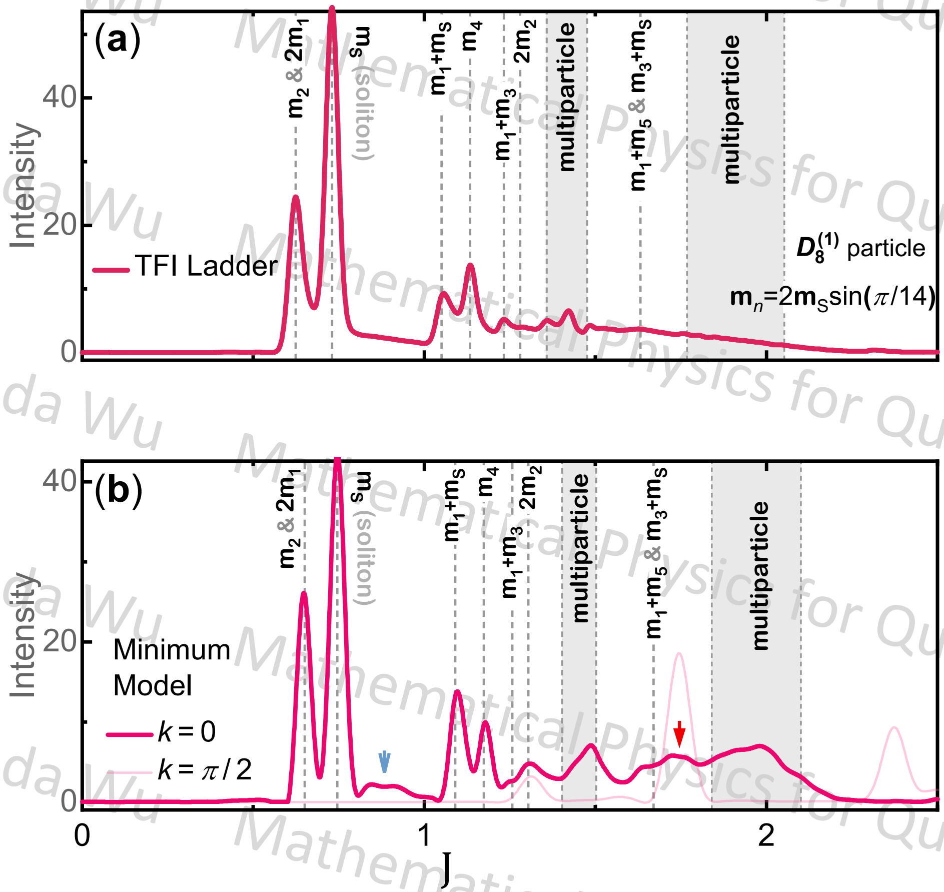
$J_i = 0 \rightarrow E_8$ Hamiltonian

$\frac{J_i}{\tilde{h}} \rightarrow \infty \rightarrow D_8^{(1)}$

N. Xi#, X. Wang#, Y. Gao, Y. Jiang, Rong Yu*, **JW*** arXiv: 2403.10785, submitted

$D_8^{(1)}$ physics in $CoNb_2O_6$

Two chains simulation



Ising ladder
$(\epsilon, \lambda_{af}, \lambda_{dw} = 0)$

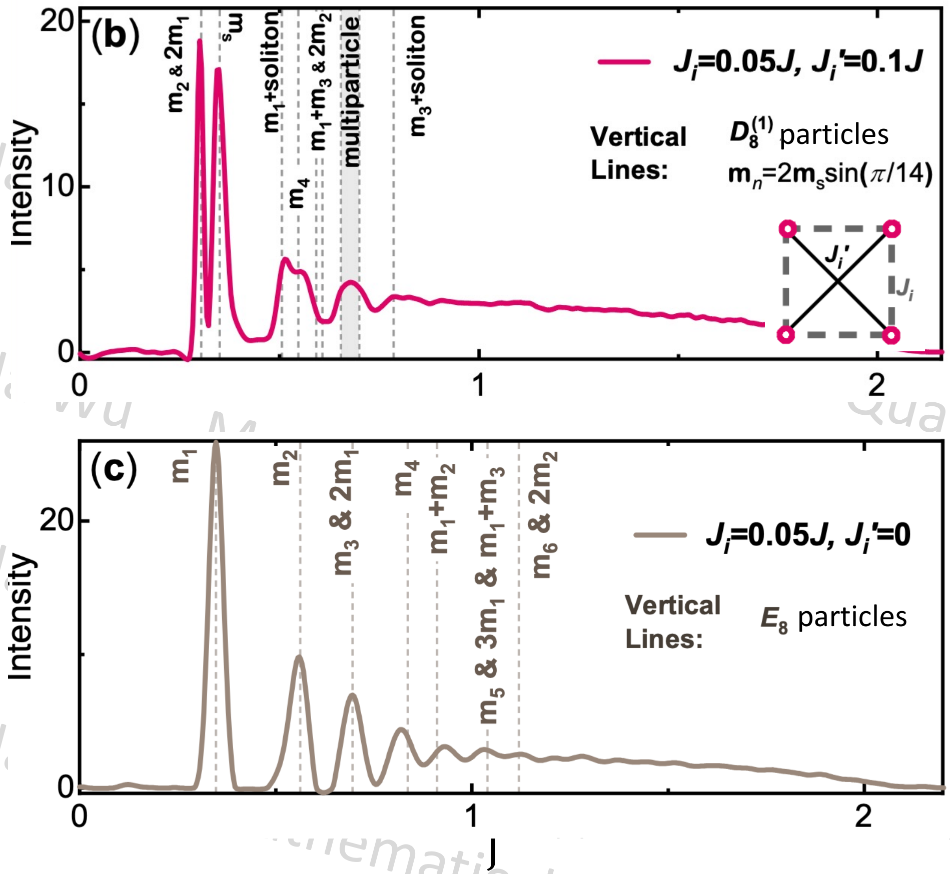
full hamiltonian

$$J = 2.7607\ \mathrm{meV}$$
$$\epsilon = 0.239\ \mathrm{meV}$$
$$\lambda_{af} = 0.1507, \lambda_{dw} = 0.1647$$
$$g = 3.1$$

N. Xi#, X. Wang#, Y. Gao, Y. Jiang, Rong Yu*, **JW*** arXiv: 2403.10785, submitted

$D_8^{(1)}$ physics in $CoNb_2O_6$

Four chains



with frustration

no frustration

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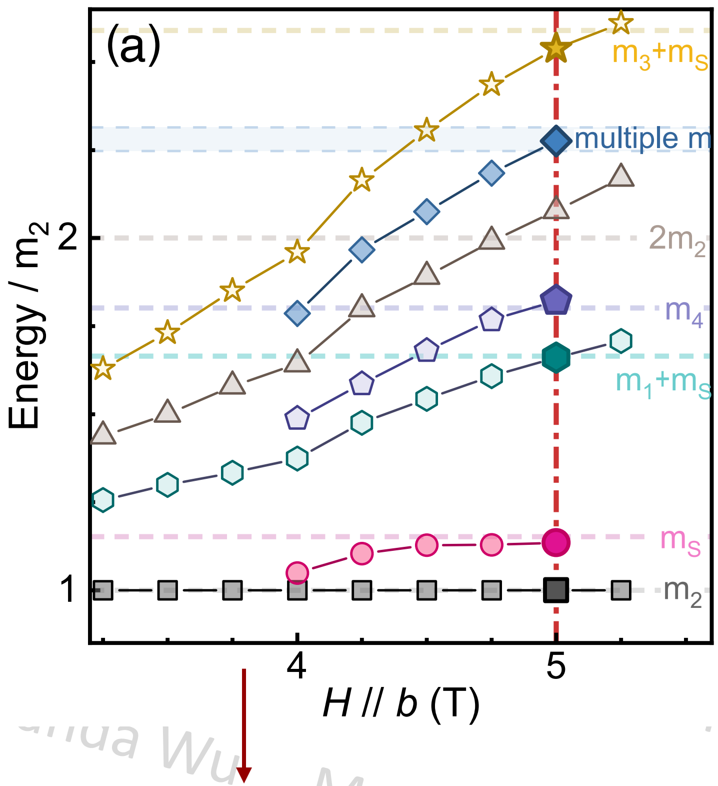
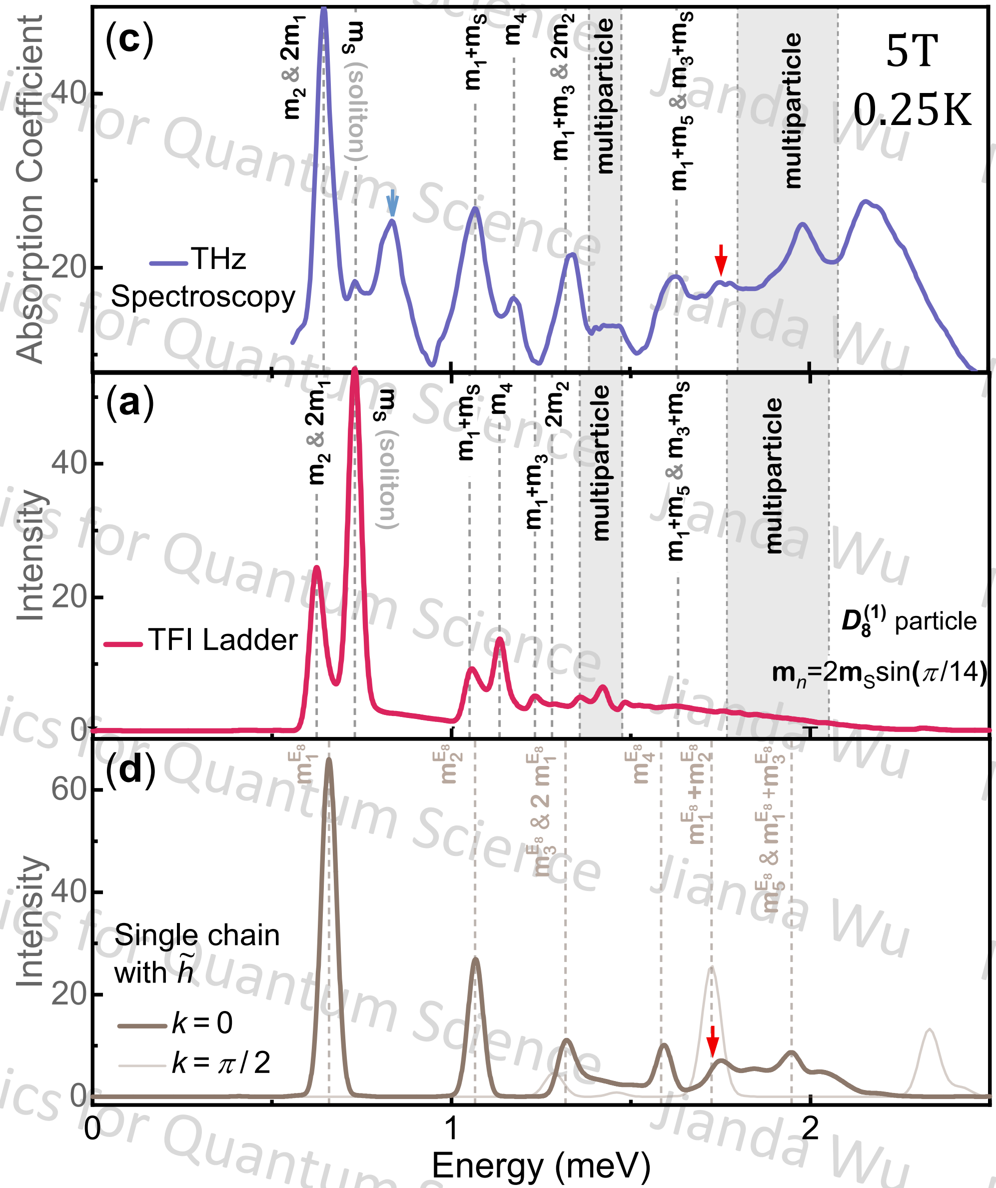
$D_8^{(1)}$ physics in $CoNb_2O_6$



Best fit to $D_8^{(1)}$ mass ratio @ $H_c^{1D} \simeq 5.0T$



N. Xi#, X. Wang#, Y. Gao, Y. Jiang, Rong Yu*, **JW*** arXiv: 2403.10785, submitted

Poster: E8 dynamics in a perturbed quantum critical Ising chain and its experimental realization in Antiferromagnet BaCo2V2O8

Xiao Wang
王骁

Poster: Dark particle in a quantum Ising ladder

Yunjing Gao
高云静

Y. Gao, X. Wang, N. Xi, Y. Jiang, Rong Yu*, **JW*** arXiv: 2402.11229, submitted

Y. Gao#, J. Yang#, H. Lin, R. Yu, **JW*** arXiv:2406.15024, submitted

N. Xi#, X. Wang#, Y. Gao, Y. Jiang, Rong Yu*, **JW*** arXiv: 2403.10785, submitted

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Thanks!